# Stark Broadening of Isolated Spectral Lines from Heavy Elements in a Plasma* 

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#### Abstract

The frequency distributions of neutral cesium and argon lines broadened by local fields of both electrons and ions in a plasma are calculated in the classical path approximation. The electron collisions are treated by an impact theory originally developed for neutral helium lines which takes into account deviations from adiabaticity. For the ion effects the adiabatic and in most instances also the quasi-static approximation can be used, if necessary corrected for ion-ion correlations and Debye shielding. Atomic matrix elements are obtained with the Bates and Damgaard method and are checked against measured Stark coefficients. Stark broadening parameters are tabulated for the argon lines arising from $2 p$ and $3 p$ levels (Paschen's notation) and for the cesium lines in the series $6 S-n P, 6 P-n D$ with $n$ up to $20,6 P-n S$ with $n$ up to 15 , and $5 D-n F$ with $n$ up to 10. Approximate formulas for total widths and shifts are suggested. For the widths, an accuracy of $20 \%$ is indicated by the good agreement with experiments. Measured shifts are only available for argon. Here the calculated shifts tend to be $20 \%$ larger than the measured values.


## INTRODUCTION

IN a preceding paper ${ }^{1}$ general methods have been described for the calculation of Stark profiles of atomic lines that are isolated, i.e., whose widths are much smaller than the separation between levels that contribute to the perturbation. Numerical results were obtained for 24 neutral helium lines and compared with a pulsed arc experiment. ${ }^{2}$ The consistency ( $\pm 10 \%$ ) in the electron densities resulting from the widths of various lines suggested that the remaining errors in the line broadening calculations were rather small.

This was substantiated by new measurements ${ }^{3}$ on a shock-heated helium plasma, in which electron density and temperature could be determined independently of line broadening. Calculated and measured linewidths coincided within $10 \%$. The agreement in the wavelength shifts of the intensity maxima of the line profiles was poorer (possibly because of the neglect of ion-ion correlations and Debye shielding), but still much better than that obtained by using the adiabatic impact approximation. ${ }^{4,5}$

In the helium calculations ${ }^{1}$ uncertainties introduced by errors in atomic wave functions were of the same order as those stemming from the schematic treatment of close electron collisions and the approximations in the treatment of the broadening by ions. The estimated over-all accuracy of the helium calculations was $20 \%$ for the widths, but is probably $10 \%$ or better as indicated by the good agreement with experiment.

For heavier elements somewhat larger errors must be expected from uncertainties in the atomic matrix elements required to compute the perturbation. How-

[^0]ever, practically all the broadening is caused by interactions between the upper state of the line in question and its neighbors, and lower state interactions can be neglected. Therefore, the relevant wave functions will not deviate too much from Coulomb wave functions (with modified energy parameters) at distances from the nucleus that contribute significantly to the matrix elements. The Bates and Damgaard ${ }^{6}$ procedure should accordingly be applicable without any large deterioration in the accuracy. This is somewhat different from the situation for oscillator strengths, where also lower state wave functions are required, for which the Coulomb approximation is less reliable.
To check the above expectation, extensive calculations were made for a large number of argon and cesium lines. Cesium was chosen as an example because there is little doubt about the validity of the Coulomb approximation for excited state wave functions of alkali atoms, whereas for argon the situation is much less clear. For both argon ${ }^{7}$ and cesium ${ }^{8}$ experimental data are available. If deviations between experiment and theory should turn out to be of the same order in both cases, one would conclude that errors from using the Coulomb approximation of Bates and Damgaard are, indeed, usually not significant in line broadening calculations, and be encouraged to extend such calculations to other elements.
The choice of argon and cesium was further motivated by the recent interest in the spectroscopic properties of these gases. Because it is monatomic and heavy, argon is often used in diaphragm-type shock tubes. Cesium, with its high vapor pressure and low ionization potential, has many applications in plasma sources. It is hoped that the present calculations will serve to make possible more precise measurements of electron densities in these gases, especially at high electron

[^1]densities where microwave techniques are no longer practical, and at fractional ionizations at which optical interferometry is not yet capable of a good precision or too inconvenient.

## ELECTRON BROADENING

The electron contribution can practically always be calculated in the impact approximation. The corresponding profiles are, therefore, of dispersion (LorentzWeisskopf) type. Because of the long-range nature of the (dipole-monopole) interaction between emitting atoms and perturbing electrons, perturbation theory can be used to calculate the dominant terms in widths and shifts of these dispersion profiles. For the same reason, the perturbing electrons can be assumed to move along their classical paths, and the perturbation is accordingly a known function of time.

The following set of equations was used for the computations of electron impact widths (w) and shifts (d) of neutral helium lines (in angular frequencies):

$$
\begin{align*}
& w+i d=N \int d v f(v)\left\{\pi v \rho_{\min }^{2}+\frac{4 \pi}{3 v}\left(\frac{\hbar}{m}\right)^{2}\right. \\
& \left.\left.\quad \times \sum_{\nu \alpha^{\prime}}\left|\langle\alpha| r_{\nu} / a_{0}\right| \alpha^{\prime}\right\rangle\left.\right|^{2}\left[a\left(z_{\alpha \alpha^{\prime}}{ }^{\min }\right)+i b\left(\frac{3}{4} z_{\alpha \alpha^{\prime}}{ }^{\min }\right)\right]\right\}, \tag{1}
\end{align*}
$$

where $\rho_{\min } \equiv z_{\alpha \alpha^{\prime}} \min _{v} / \omega_{\alpha \alpha^{\prime}}$ was defined by

$$
\begin{align*}
& \frac{2}{3}\left(\hbar / m v \rho_{\min }\right)^{2}\left\{\left.\left[\sum_{\nu \alpha^{\prime}}\left|\langle\alpha| r_{\nu} / a_{0}\right| \alpha^{\prime}\right\rangle\right|^{2} A\left(z_{\alpha \alpha^{\prime}} \min \right)\right]^{2} \\
& \left.\left.+\left.\left[\sum_{\nu \alpha^{\prime}}\left|\langle\alpha| r_{\nu} / a_{0}\right| \alpha^{\prime}\right\rangle\right|^{2} B\left(z_{\alpha \alpha^{\prime}}{ }^{\min }\right)\right]^{2}\right\}^{1 / 2} \\
& =\left[\frac{1}{2} \Gamma\left(\frac{1}{3}\right)\right]^{-3 / 2} . \tag{2}
\end{align*}
$$

These are Eqs. (3.15) and (3.16) of reference 1, in which $N$ is the electron density, $v$ the velocity, $f(v)$ the velocity distribution function, $\langle\alpha| r_{\nu} / a_{0}\left|\alpha^{\prime}\right\rangle$ a component of the matrix element (in atomic units) of the atomic electron coordinate vector taken between upper state $\alpha$ and some interacting state $\alpha^{\prime}$ whose energies differ by $\hbar \omega_{\alpha \alpha^{\prime}}$. Finally, $A, B, a$, and $b$ are functions obtained in reference 1 from the second-order term in the iterated solution of the time dependent Schrödinger equation, i.e., the usual adiabatic approximation ${ }^{4,5}$ was not made.

If the Schrödinger equation for the excited electron wave functions is separable in spherical coordinates, the only nonvanishing matrix elements are in terms of angular momentum quantum numbers $l$ and radial integrals $\sigma$,

$$
\begin{align*}
& \left.\left.\sum_{v}\left|\langle\alpha| r_{v} / a_{0}\right| \alpha^{\prime}\right\rangle\left.\right|^{2}=\sum_{v}\left|\langle l| r_{v} / a_{0}\right| l \pm 1\right\rangle\left.\right|^{2} \\
& \quad=(l+1)(2 l+3)\left[\sigma\left(n_{l}, l ; n_{l+1}, l+1\right)\right]^{2}, \quad l \rightarrow l+1 \\
& \quad=l(2 l-1)\left[\sigma\left(n_{l-1}, l-1 ; n_{l}, l\right)\right]^{2}, \quad l \rightarrow l-1 . \tag{3}
\end{align*}
$$

Following Bates and Damgaard ${ }^{6}$ one writes, e.g.,

$$
\begin{align*}
\sigma\left(n_{l-1}, l-1 ; n_{l}, l\right) & =\left[\frac{-n_{l}}{3}\left(\frac{n_{l}^{2}-l^{2}}{4 l^{2}-1}\right)^{1 / 2}\right] \varphi\left(n_{l-1}, n_{l}, l\right) \\
& \equiv F\left(n_{l}, l\right) \varphi\left(n_{l-1}, n_{l}, l\right) \tag{4}
\end{align*}
$$

with the effective principal quantum numbers to be determined from the excitation energies $E_{n l}$, namely,

$$
\begin{equation*}
n_{l}=\left[E_{\mathrm{H}} /\left(E_{\infty}-E_{n l}\right)\right]^{1 / 2} . \tag{5}
\end{equation*}
$$

Here, $E_{\mathrm{H}}$ is the ionization energy of hydrogen and $E_{\infty}$ the ionization energy of the element in question.
The functions $\varphi\left(n_{l-1}, n_{l}, l\right)$ were tabulated ${ }^{6}$ for $l=1$, 2,3 using Coulomb wave functions. But for $n_{l-1}=n_{l}$ one has $\varphi\left(n_{l-1}, n_{l}, l\right)=1$, and since for larger $l$ the difference between $n_{l}$ and $n_{l-1}$ is always very small, one can then simply use $\varphi\left(n_{l-1}, n_{l}, l\right)=1$. Also, if effective principal quantum numbers are larger than those for which $\varphi\left(n_{l-1}, n_{l}, l\right)$ is tabulated $\left(n_{l}>7\right)$, there is no need for an extension of these tables, because again $n_{l}-n_{l-1}$ will be small. Only for some lines from very low-lying excited states where the validity of the Coulomb approximation is doubtful, an extrapolation to small principal quantum numbers is necessary. This will, however, not cause any significant additional uncertainties, because $\varphi\left(n_{l-1}, n_{l}, l\right)$ is a very slowly varying function of $n_{l}$.
In case of complicated systems like argon there is some ambiguity in the selection of interacting levels due to the breakdown of $l$-s coupling. Therefore, the following additional selection rule was adopted: $\Delta J=\Delta l$, and no change in the inner configuration was allowed. Energy levels and configurations were obtained from the tables in "Atomic Energy Levels." For cesium it was necessary to calculate some additional levels using effective quantum numbers whose deviations from the next integers were determined by extrapolation from the available measured levels in the same series.

## ION BROADENING

The contribution of ion broadening to the widths of isolated lines is usually less than $\sim 20 \%$. It is therefore convenient to express frequencies (or wavelengths) in terms of the electron impact width $w$ and to measure them from the location of the line already shifted by $d$ (due to electron impacts), i.e., to introduce as variable

$$
\begin{equation*}
x=\left(\omega-\omega_{0}-d\right) / w, \tag{6}
\end{equation*}
$$

$\omega_{0}$ being the frequency of the unperturbed line.
If the broadening ions are heavy like cesium or argon, their motion can almost always be neglected. Then one can use both the adiabatic and the quasistatic approximation, which result in reduced line

[^2]profiles
\[

$$
\begin{equation*}
j_{H}(x, \alpha)=(1 / \pi) \int_{0}^{\infty} d \beta W_{H}(\beta) /\left[1+\left(x-\alpha^{4 / 3} \beta^{2}\right)^{2}\right] \tag{7}
\end{equation*}
$$

\]

[Eq. (4.14) of reference 1]. Here $W_{H}(\beta)$ is the Holtsmark ${ }^{10}$ distribution function for the normalized ion field strength $\beta=F / F_{0}$, with $F_{0}=2.61 e N^{2 / 3}$, and $\alpha$ is a parameter that characterizes the quasi-static ion broadening (not to be confused with the quantum numbers $\alpha, \alpha^{\prime}$ ). According to Sec. 4 of reference 1, it is

$$
\begin{equation*}
\left.\alpha=\left.(4 \pi / 3) N\left[\frac{1}{3}(\hbar / m)^{2} \sum_{\alpha^{\prime}}\left|\langle\alpha| r_{\nu} / a_{0}\right| \alpha^{\prime}\right\rangle\right|^{2} / \omega_{\alpha \alpha^{\prime}} w^{3}\right]^{3 / 4}, \tag{8}
\end{equation*}
$$

if one ignores the small dependence of the static quadratic Stark splitting on the magnetic quantum numbers. Since $w$ is proportional to $N$, the parameter $\alpha$ is only proportional to $N^{\frac{1}{4}}$ and hardly ever becomes larger than $\sim 0.5$ for densities at which isolated lines exist.

At small $\alpha$ values, the cores of the reduced line profiles (7) are quite close to dispersion profiles whose half-widths and shifts are about $1+1.75 \alpha$ and $2.0 \alpha$, respectively. The actual profiles have accordingly approximate widths $(1+1.75 \alpha) w$ and shifts $d+2.0 \alpha w$. This follows by comparison with the numerically calculated profiles listed in Table III of reference 1.
The quasi-static approximation will be valid provided the frequencies characterizing the ion field, namely, $v / \rho_{m}$ [ $v$ being a typical ion velocity, $\rho_{m}=(4 \pi N / 3)^{-1 / 3}$ the mean ion-ion separation], are considerably smaller than $w$, which essentially determines the width of the profile. If the parameter

$$
\begin{equation*}
\sigma=w \rho_{m} / v=w / v(4 \pi N / 3)^{1 / 3} \tag{9}
\end{equation*}
$$

becomes of order 1 or smaller, the time dependence of the ion field can no longer be neglected. Then the reduced profiles $j(x, \alpha, \sigma)$ should be employed, which can also be found in Table III of reference 1. From (9) it follows that $\sigma$ is proportional to $N^{2 / 3}$. It therefore increases faster than $\alpha$ with increasing ion densities, implying that the quasi-static approximation becomes more applicable as the relative contribution of ions to the Stark broadening increases, which is mainly determined by the magnitude of $\alpha$.

In the calculation of $j_{H}(x, \alpha)$ and $j(x, \alpha, \sigma)$, it has been implicitly assumed that the perturbing ions were statistically independent. At high densities this is no longer the case and $W_{H}(\beta)$ in (7), e.g., should be replaced by a distribution function in which ion-ion correlations and Debye shielding by electrons are taken into account. Such distribution functions have been calculated by Mozer and Baranger ${ }^{11}$ using a cluster integral expansion. They depend on the ratio of mean

[^3]distance and Debye radius
\[

$$
\begin{align*}
& R=\rho_{m} / \rho_{D}=6^{1 / 3} \pi^{1 / 6} e N^{1 / 6}(k T)^{-1 / 2} \\
& \approx 2^{-1 / 6} 15^{1 / 3} e N^{1 / 6}(k T)^{-1 / 2} \tag{10}
\end{align*}
$$
\]

and (7) should be replaced by

$$
\begin{equation*}
j_{R}(x, \alpha)=(1 / \pi) \int_{0}^{\infty} d \beta W_{R}(\beta) /\left[1+\left(x-\alpha^{4 / 3} \beta^{2}\right)^{2}\right] . \tag{11}
\end{equation*}
$$

This integral was evaluated numerically and the results are listed in Table I. The asymptotic wing formulas (Appendix $Z$ of reference 1) remain unchanged, because the probability of high field strengths at a neutral atom is not affected.

The influence of the above corrections on the linewidths is rather small, because the latter are mostly determined by electron broadening. But shifts can be reduced considerably since here electron and ion contributions are often comparable. Again approximate formulas for total widths and shifts of the complete profiles can be obtained by inspection, namely,

$$
\begin{equation*}
w_{\text {total }} \approx[1+1.75 \alpha(1-0.75 R)] w \tag{12}
\end{equation*}
$$

and

$$
\begin{equation*}
d_{\mathrm{tota}} \approx[d / w+2.0 \alpha(1-0.75 R)] w \tag{13}
\end{equation*}
$$

These expressions are sufficiently accurate in view of the uncertainties introduced by the various other approximations, as long as $\alpha$ is below 0.5 and $R$ below 0.8 .

## NUMERICAL RESULTS

Electron impact widths $w$ and relative shifts $d / w$ were calculated from Eqs. (1)-(5) using up to five interacting states for electron densities $N^{\prime}=10^{16} \mathrm{~cm}^{-3}$. The results are summarized in Tables II (argon) and III (cesium). They can be applied to other electron densities $N$ after multiplication of $w$ with $N / N^{\prime}$ if the lines are isolated, i.e., have no forbidden components, and as long as Debye shielding does not reduce the electron broadening, i.e., as long as $N<N_{\max } \approx m \omega_{\alpha \alpha^{\prime}}{ }^{2} /$ $2 \pi^{2} e^{2}$, where $\omega_{\alpha \alpha^{\prime}}$ is the separation of the nearest interacting level in angular frequency units [Eq. (3.19) of reference 1]. Also tabulated are the ion broadening parameter $\alpha$ defined by Eq. (8) and (for cesium) the static quadratic Stark coefficient C. Again $\alpha$ pertains to the electron density $N^{\prime}=10^{16} \mathrm{~cm}^{-3}$ and should be multiplied with $\left(N / N^{\prime}\right)^{\frac{1}{2}}$, whereas the ratio $d / w$ is obviously independent of $N$. All line broadening parameters depend only weakly on the electron temperature. Interpolation for intermediate temperatures should therefore not cause any difficulty.

Total (half) half-widths and shifts of the line intensity maxima are most readily obtained from the tabulated Stark broadening parameters and Eqs. (12) and (13) with (10), observing the appropriate scaling with the electron density. This procedure is, however,

Table I. Reduced line profiles $j_{R}(x, \alpha)$ for various values of the ion broadening parameter $\alpha$ and the Debye-shielding parameter $R$.

only applicable if the quasi-static approximation holds for the ion broadening, which is sufficiently justified for $\sigma>1$. If more detailed profiles are desired, Table I should be used for $\sigma>1$, and profiles in the usual wavelength scale follow then from the transformation given by Eq. (6). In the rare cases in which $\sigma$ is too small for the quasi-static approximation but $\alpha$ still too large as to make the ion broadening negligible, the reduced profiles $j(x, \alpha, \sigma)$ (Table III of reference 1) would be applicable. [Then $\sigma$ must be determined from Eq. (9) with $w$ in angular frequencies, using the mean velocity of the perturbing ions for $v$.

## COMPARISON WITH EXPERIMENT

Static quadratic Stark shifts have been measured for a large number of argon lines ${ }^{12}$ and for two cesium lines. ${ }^{13}$ In Table IV these measured shifts are compared with the present calculations to provide a check on the radial matrix elements and the coupling scheme used in the line broadening calculations and, in case of argon, also with earlier calculations ${ }^{12}$ assuming pair ( $j-j$ )

[^4]coupling. The agreement is almost always within $20 \%$. Such errors cause slightly smaller uncertainties in the linewidths and shifts.
Stark broadening parameters of 38 argon lines have been measured ${ }^{7}$ as emitted by a plasma produced in a high-current arc burning through an array of watercooled copper diaphragms. Photographic and photoelectric observations of the spectrum were made along the axis to ensure approximate homogeneity of the plasma along the line of sight. The temperature of the arc plasma was determined from the absolute intensity of the ionized argon line at 4348 Å, using a measured value of its transition probability ( $A=6.77 \times 10^{7} \mathrm{sec}^{-1}$ ). The electron density could then be calculated from temperature and pressure (which was atmospheric), using the Saha equation.

Before comparison can be made, the originally quoted plasma conditions must be revised, because the transition probability was erroneous. The newly measured value ${ }^{14}$ is $A=11.7 \times 10^{7} \mathrm{sec}^{-1}$ and compares very favorably with calculations accounting for intermediate coupling, ${ }^{15}$ which yielded $A=12.8 \times 10^{7} \mathrm{sec}^{-1}$. If one adopts the calculated value, the changes in temperature and density are, e.g., $T=11,750^{\circ} \mathrm{K}$ to $T=11,400^{\circ} \mathrm{K}$ and $N=6.4 \times 10^{16} \mathrm{~cm}^{-3}$ to $N=5.0 \times 10^{16} \mathrm{~cm}^{-3}$. Also, the original plasma conditions were apparently calculated using the Unsöld correction ${ }^{16}$ for the ionization potential, which involves the mean distance between ions. Actually, one should have used instead a relation with the Debye radius ${ }^{17,18}$ because the latter is larger than the mean distance between ions. Correction for this finally leads ${ }^{19}$ to $N=4.6 \times 10^{16} \mathrm{~cm}^{-3}$, i.e., an electron density that is smaller by a factor 1.4 than calculated before. (This implies that the measured values of the oscillator strengths of Arr lines ${ }^{7}$ should be revised upwards by a similar factor.)

In Table V full widths $\left(2 w_{\text {total }}\right)$ and shifts ( $\left.d_{\text {total }}\right)$ as measured at this revised electron density are compared with calculated values obtained from Table II using Eqs. (12), (13), and (10) for 13 lines for which both measured and calculated widths and shifts were available. The quoted experimental errors are $10 \%$ or $20 \%$. Except for one line, the agreement of the widths is within experimental error. On the other hand, the measured widths are larger than those calculated with the adiabatic impact theory ${ }^{4,5}$ by a factor 1.5 to 2 . Also, the adiabatic theory consistently yielded shift to width ratios that were too large by a factor of about

[^5]Table II. ${ }^{\text {a }}$ Calculated Stark broadening parameters for argon lines at $N^{\prime}=10^{16} \mathrm{~cm}^{-3}$ : electron impact (half) half-width $w$ (in $\AA$ units), relative electron impact shift $d / w$, and ion broadening parameter $\alpha$.

${ }^{\text {a }}$ The (upper) levels are in Paschen's notation. The values of $d / w$ and $\alpha$ apply to all lines arising from the same level, those of $w$ only to the first line listed whose wavelength is in italics. For the others, it should be multiplied by the ratio of the wavelengths squared.

2 or more, whereas now the calculated shifts are only too large by about $20 \%$. However, it should be noted that all the lines listed in Table V come from $3 p$ levels (Paschen's notation) and that the calculated line

Table III. ${ }^{\text {a }}$ Calculated Stark broadening parameters for cesium lines at $N^{\prime}=10^{16} \mathrm{~cm}^{-3}$ : electron impact (half) half-width $w$ (in $\AA$ units), relative electron impact shift $d / w$, ion broadening parameter $\alpha$, and static quadratic Stark coefficient $C$ in units of $\mathrm{cm}^{-1} /(100 \mathrm{kV} / \mathrm{cm})^{2}$.

|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Table III (continued)

| Line | $T\left({ }^{\circ} \mathrm{K}\right)$ | 2500 |  | 5000 |  | 10000 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $6 P_{1 / 2}-n D_{3 / 2} ; 6 P_{3 / 2}-n D_{5 / 2}$ |  |  |  |  |  |  |  |
| $n=10$ | $w$ | 9.43 | 11.9 | 11.6 | 13.9 | 13.4 | 15.5 |
| 5466 A ; $5635 \AA$ | $d / w$ | -0.43 | -0.44 | -0.17 | -0.19 | -0.03 | -0.04 |
| $C=1.36 \times 10^{2} \quad ; 1.97 \times 10^{3}$ | ${ }_{\alpha}$ | 0.22 | 0.25 | 0.19 | 0.23 | 0.17 | 0.21 |
| $n=11$ | $w$ | 16.9 | 20.7 | 20.3 | 24.0 | 22.9 | 26.4 |
| $5340 \AA$; $5503 \AA$ | $d / w$ | -0.36 | $-0.39$ | -0.13 | -0.15 | 0.00 | -0.01 |
| $C=3.36 \times 10^{2} \quad ; 4.87 \times 10^{2}$ | ${ }_{\alpha}$ | 0.27 | 0.32 | 0.23 | 0.28 | 0.21 | 0.27 |
| $n=12$ | $w$ | 28.1 | 34.4 | 33.5 | 39.5 | 37.4 | 42.9 |
| $5257 \AA$ ¢ 5414 £ | $d / w$ | -0.34 | -0.35 | -0.12 | -0.13 | 0.00 | 0.00 |
| $C=7.71 \times 10^{2} \quad ; 1.11 \times 10^{3}$ | $\alpha$ | 0.33 | 0.39 | 0.29 | 0.35 | 0.27 | 0.33 |
| $n=13$ | $w$ | 44.4 | 54.2 | 52.6 | 61.7 | 58.1 | 66.3 |
| $5197 \AA$; 5349 A | $d / w$ | $-0.31$ | -0.32 | -0.10 | $-0.12$ | 0.01 | 0.01 |
| $C=1.60 \times 10^{3} \quad ; 2.31 \times 10^{3}$ | $\alpha$ | 0.40 | 0.48 | 0.35 | 0.43 | 0.33 | 0.41 |
| $n=14$$5153 \AA$$\AA$ | $w$ | 68.2 | 82.3 | 79.6 | 92.7 | 87.0 | 98.8 |
|  | $d / w$ | -0.29 | $-0.30$ | $-0.10$ | -0.11 | 0.01 | 0.01 |
| $C=3.18 \times 10^{3} \quad ; 4.49 \times 10^{3}$ | $\boldsymbol{\alpha}$ | 0.48 | 0.56 | 0.43 | 0.52 | 0.40 | 0.49 |
| $\begin{gathered} n=15 \\ 5119 \AA: 5268 \AA \end{gathered}$ |  | $100 .$ | $123 .$ | $116 .$ | $136 .$ |  |  |
|  | $d / w$ | $-0.27$ | $-0.29$ | $-0.09$ | $-0.10$ | $0.01$ | $0.02$ |
| $C=5.83 \times 10^{3} \quad ; 8.55 \times 10^{3}$ | $\alpha$ | 0.56 | 0.67 | 0.50 | 0.62 | 0.47 | 0.60 |
| $n=16$ | $w$ | 145. | 178. | 166. | 195. | 178. | 203. |
| 5093 A ; 5241 A | $d / w$ | -0.26 | -0.28 | -0.09 | -0.10 | 0.02 | 0.02 |
| $C=1.05 \times 10^{4} \quad ; 1.55 \times 10^{4}$ | $\alpha$ | 0.66 | 0.79 | 0.59 | 0.74 | 0.56 | 0.71 |
| $\frac{n=17}{\circ} \mathrm{C}$ | ${ }^{w}$ | 199. | 252. | 228. | 271. | 244. | 279. |
| 5073 A ; 5219 A | $d / w$ | -0.23 | -0.26 | -0.07 | -0.09 | 0.02 | 0.02 |
| $C=1.65 \times 10^{4} \quad ; 2.70 \times 10^{4}$ | $\alpha$ | 0.72 | 0.92 | 0.65 | 0.87 | 0.62 | 0.85 |
| $n=18$ | w | 273. | 343. | 309. | 368. | 328. | 376. |
| 5056 A ; 5201 ${ }^{\circ}$ | $d / w$ | -0.22 | -0.25 | $-0.06$ | -0.08 | 0.03 | 0.03 |
| $C=2.69 \times 10^{4} \quad ; 4.44 \times 10^{4}$ | $\alpha$ | 0.82 | 1.05 | 0.74 | 0.99 | 0.71 | 0.98 |
| $n=19$5042$\AA$ | $w$ | 365. | 453. | 411. | 485. | 432. | 494. |
|  | $d / w$ | -0.21 | -0.24 | $-0.06$ | -0.08 | 0.03 | 0.03 |
| $C=4.15 \times 10^{4} \quad ; 6.78 \times 10^{4}$ | $\alpha$ | 0.91 | 1.16 | 0.83 | 1.11 | 0.80 | 1.09 |
|  | ${ }^{w}$ | 483. | 596. | 538. | 634. | 562. | 642. |
|  | $d / w$ | -0.20 | -0.23 | -0.06 | -0.07 | 0.03 | 0.04 |
| $C=6.43 \times 10^{4} \quad ; 1.06 \times 10^{5}$ | $\alpha$ | 1.02 | 1.32 | 0.94 | 1.26 | 0.91 | 1.25 |
| $6 P_{1 / 2}-n S_{1 / 2} ; 6 P_{3 / 2}-n S_{1 / 2}$ |  |  |  |  |  |  |  |
| $n=7$13589$\AA$ | $w$ | 0.39 | 0.46 | 0.49 | 0.57 | 0.64 | 0.75 |
|  | $d / w$ | 1.76 | 1.76 | 1.65 | 1.65 | 1.38 | 1.38 |
| $C=-2.43 \times 10^{-1} ;-2.43 \times 10^{-1}$ | $\boldsymbol{\alpha}$ | 0.08 | 0.08 | 0.07 | 0.07 | 0.06 | 0.06 |
| $n=8$$7609 \AA$ | ${ }^{2}$ | 0.44 | 0.48 | 0.57 | 0.62 | 0.77 | 0.84 |
|  | $d / w$ | 1.70 | 1.70 | 1.47 | 1.47 | 1.11 | 1.11 |
| $C=-1.48 \quad ;-1.48$ | $\alpha$ | 0.12 | 0.12 | 0.10 | 0.10 | 0.08 | 0.08 |
| $n=9$6355$\AA$ |  | 0.82 | 0.89 | 1.13 | 1.21 | 1.55 | 1.66 |
|  | $d / w$ | 1.60 | 1.60 | 1.25 | 1.25 | 0.88 | 0.89 |
| $C=-5.84 \quad ;-5.84$ | $\boldsymbol{\alpha}$ | 0.16 | 0.16 | 0.13 | 0.13 | 0.10 | 0.10 |
| $n=10$ | $w$ | 1.61 | 1.72 | 2.27 | 2.42 | 3.11 | 3.32 |
| $5839 \AA$ ¢ $6034 \AA$ | $d / w$ | 1.44 | 1.44 | 1.05 | 1.05 | 0.71 | 0.71 |
| $C=-1.78 \times 10^{1} ;-1.78 \times 10^{1}$ | $\alpha$ | 0.20 | 0.20 | 0.15 | 0.15 | 0.12 | 0.12 |
| $\begin{gathered} n=11 \\ 5568 \AA ; 5746 \AA \end{gathered}$ | ${ }^{v}$ | 3.27 | 3.48 | 4.65 | 4.95 | 6.31 | 6.72 |
|  | $d / w$ | 1.37 | 1.36 | 0.98 | 0.98 | 0.67 | 0.67 |
|  | $\alpha$ | 0.26 | 0.26 | 0.20 | 0.20 | 0.16 | 0.16 |
| $n=12$ | $w$ | 6.05 | 6.43 | 8.48 | 9.01 | 11.2 | 11.9 |
| $\begin{gathered} 5407 \AA 7^{\mathrm{A}} ; 5573 \AA \mathrm{~A} \\ C=-1.55 \times 10^{2} ;-1.55 \times 10^{2} \end{gathered}$ | $d / w$ | 1.36 | 1.36 | 0.99 | 0.99 | 0.70 | 0.70 |
|  | $\alpha$ | 0.33 | 0.33 | 0.26 | 0.26 | 0.21 | 0.21 |
| $n=13$ |  | 10.5 | 11.1 | 14.6 | 15.5 | 19.1 |  |
| $5301 \AA$ ¢ $5463 \AA$ | $d / w$ | 1.30 | 1.30 | 0.93 | 0.93 | 0.66 | 0.66 |
| $C=-3.53 \times 10^{2} ;-3.53 \times 10^{2}$ | $\alpha$ | 0.39 | 0.39 | 0.31 | 0.31 | 0.25 | 0.25 |
| n=14 | $w$ | 16.5 | 17.5 | 23.2 | 24.6 | 30.0 | 31.8 |
| $C=-6.45 \times 10^{2} ;{ }^{5229} ; 5385 \AA$ | $d / w$ | 1.18 | 1.18 | 0.82 | 0.82 | 0.58 | 0.58 |
|  | $\alpha$ | 0.43 | 0.43 | 0.33 | 0.33 | 0.28 | 0.28 |
| $n=15$ | $w$ | 25.8 | 27.4 | 36.2 | 38.3 | 46.1 | 48.9 |
| 5177 A ; 5329 A | $d / w$ | 1.09 | 1.09 | 0.76 | 0.76 | 0.53 | 0.53 |
| $C=-1.20 \times 10^{3} ;-1.20 \times 10^{3}$ | $\alpha$ | 0.48 | 0.48 | 0.37 | 0.37 | 0.31 | 0.31 |

Table III (continued)

${ }^{\text {a }}$ For the diffuse series lines $6 P_{3 / 2}-n D_{3 / 2}$ the data for $6 P_{1 / 2}-n D_{3 / 2}$ can be used, except that the width $w$ should be increased by a factor of the wavelength ratio squared $(10-6 \%)$
broadening parameters for the near infrared lines arising from $2 p$ levels may be somewhat less reliable.

By varying the arc current from 45 to 120 A the temperature was changed slightly and with it the electron density by a factor of 2 in both directions. Within the experimental error both widths and shifts were found to be proportional to the electron density. This is as expected because the ion broadening parameter $\alpha$ changes only by a factor 1.4 over this range of electron densities and temperatures and the Debye shielding parameter $R$ by a factor 1.15 . Furthermore, the effects of these changes on the (small) relative contribution of the ion broadening partially compensate each other. Therefore, the total widths and shifts scale essentially as the electron contribution, which is

Table IV. ${ }^{a}$ Measured and calculated static Stark shifts in units of $\mathrm{cm}^{-1}$ at field strengths of $100 \mathrm{kV} / \mathrm{cm}$.

| Level | Measured shift | Calculated shift |  |
| :---: | :---: | :---: | :---: |
| Cesium |  |  |  |
| $7 P_{1 / 2}$ | 1.17 | 1.20 | $\ldots$ |
| $7 P_{3 / 2}$ | 1.49 | 1.50 | $\cdots$ |
| Argon (Paschen notation) | $(l-s)$ | $(j-j)$ |  |
| $3 p_{1}$ | 0.41 | 0.62 | 0.42 |
| $3 p_{2}$ | 0.59 | 0.50 | 0.54 |
| $3 p_{3}$ | 0.51 | 0.46 | 0.48 |
| $3 p_{4}$ | 0.46 | 0.50 | 0.50 |
| $3 p_{5}$ | 0.60 | 0.51 | 0.66 |
| $3 p_{6}$ | 0.52 | 0.39 | 0.44 |
| $3 p_{7}$ | 0.43 | 0.41 | 0.40 |
| $3 p_{8}$ | 0.42 | 0.41 | 0.40 |
| $3 p_{9}$ | 0.43 | 0.46 | 0.42 |
| $3 p_{10}$ | 0.35 | 0.40 | 0.42 |

a All shifts are to the red.
proportional to the electron density and practically independent of the temperature in the range of interest.
For cesium profiles of 26 plasma-broadened lines have been measured. ${ }^{8}$ The lines were emitted from the interelectrode region of a parallel-geometry hot-cathode cesium diode. In a typical run, the cathode, made of $\mathrm{UC}: \operatorname{ZrC}(\varphi \approx 3.35 \mathrm{eV})$, was heated to $2150^{\circ} \mathrm{K}$. The voltage applied to the electrodes was 4 V dc and the current 60 A . The cesium vapor pressure, about 2.2 mm Hg , was controlled by maintaining equilibrium with cesium liquid at a condensation temperature of $305^{\circ} \mathrm{C}$.

Observations were made by scanning a monochromator with photomultiplier attachment. They showed homogeneous emission from the interior region of the discharge with insignificant emission from edge

Table V. Measured and calculated widths and shifts of some argon lines at $N=4.6 \times 10^{16} \mathrm{~cm}^{-3}$ and $T=11400^{\circ} \mathrm{K}$.

| Wavelength (Å) | Full width ( $\AA$ ) |  | Shift ( ${ }_{\text {( }}$ ) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Measured | Calculated | Measured | Calculated |
| 4345 | $1.1 \pm 0.1$ | 1.2 | $0.50 \pm 0.05$ | 0.69 |
| 4335 | $1.5 \pm 0.3$ | 1.3 | $0.55 \pm 0.11$ | 0.66 |
| 4333 | $1.1 \pm 0.1$ | 1.2 | $0.54 \pm 0.05$ | 0.64 |
| 4300 | $1.0 \pm 0.1$ | 1.1 | $0.49 \pm 0.05$ | 0.57 |
| 4272 | $1.0 \pm 0.1$ | 1.0 | $0.44 \pm 0.04$ | 0.58 |
| 4266 | $1.1 \pm 0.1$ | 1.1 | $0.50 \pm 0.05$ | 0.54 |
| 4259 | $1.2 \pm 0.1$ | 1.3 | $0.56 \pm 0.06$ | 0.76 |
| 4251 | $0.7 \pm 0.2$ | 1.0 | $0.40 \pm 0.08$ | 0.55 |
| 4201 | $1.0 \pm 0.1$ | 1.1 | $0.44 \pm 0.04$ | 0.58 |
| 4198 | $1.0 \pm 0.2$ | 1.2 | $0.54 \pm 0.11$ | 0.64 |
| 4182 | $1.2 \pm 0.1$ | 1.2 | $0.50 \pm 0.05$ | 0.61 |
| 4164 | $1.0 \pm 0.1$ | 1.0 | $0.42 \pm 0.04$ | 0.55 |
| 4159 | $1.1 \pm 0.1$ | 1.0 | $0.50 \pm 0.05$ | 0.52 |

regions, except for resonance lines. The temperature and density of the plasma were determined by absolute intensity measurements of both lines and continua. Theoretical oscillator strengths ${ }^{20,21}$ were used for the lines and Mohler's ${ }^{22}$ experimental results for freebound transitions for the continuum data. The latter gave an electron temperature of $5500^{\circ} \mathrm{K}$ and an electron density of $1.5 \times 10^{15} \mathrm{~cm}^{-3}$. Line-intensity measurements gave a temperature of $6400^{\circ} \mathrm{K}$ and, from the Saha equation, an electron density of $1.66 \times 10^{15} \mathrm{~cm}^{-3}$. The measured plasma conditions were therefore with a conservative error estimate stated as $N=(1.6 \pm 0.3)$ $\times 10^{15} \mathrm{~cm}^{-3}$ and $T=6000 \pm 500^{\circ} \mathrm{K}$, corresponding to over $99 \%$ ionization.

For these particular conditions, full half-widths of 21 lines were measured. They are compared in Table VI with calculated widths obtained from Table III using Eqs. (12), (13), and (10). The indicated estimated experimental errors are especially serious for low members of the sharp and diffuse series because of the necessity to correct for reabsorption and instrumental width. Except for two cases, the agreement is within the experimental error. However, all calculated widths would fall within the error brackets if a $5 \%$ error were added to account for uncertainties in the electron density.

More extensive measurements on fundamental series lines ${ }^{8}$ showed that linewidths were proportional to the electron density within the limits of applicability of the electron impact theory for isolated lines. These measurements also indicated that linewidths were insensitive

[^6]Table VI. Measured and calculated widths of some cesium lines at $N=1.6 \times 10^{15} \mathrm{~cm}^{-3}$ and $T=6000^{\circ} \mathrm{K}$.

| Transition | Wavelength (̊) | Full width ( $\AA$ ) |  |
| :---: | :---: | :---: | :---: |
|  |  | Measured | Calculated |
| $6 P_{1 / 2}-9 S_{1 / 2}$ | 6355 | $0.5 \pm 0.2$ | 0.4 |
| $6 P_{3 / 2}-9 S_{1 / 2}$ | 6587 | $0.6 \pm 0.2$ | 0.5 |
| $6 P_{1 / 2}-10 S_{1 / 2}$ | 5839 | $1.0 \pm 0.2$ | 0.9 |
| $6 P_{3 / 2}-10 S_{1 / 2}$ | 6034 | $1.1 \pm 0.2$ | 0.9 |
| $6 P_{1 / 2}-11 S_{1 / 2}$ | 5568 | $1.8 \pm 0.2$ | 1.9 |
| $6 P_{3 / 2}-11 S_{1 / 2}$ | 5746 | $1.7 \pm 0.2$ | 2.0 |
| $6 P_{1 / 2}-7 D_{3 / 2}$ | 6723 | $0.5{ }_{-0.4}^{+0.2}$ | 0.5 |
| $6 P_{3 / 2}-7 D_{3 / 2}$ | 6984 | $0.6{ }_{-0.4}^{+0.2}$ | 0.5 |
| $6 P_{3 / 2}-7 D_{5 / 2}$ | 6973 | ${ }_{0.8}^{+0.3}$ | 0.6 |
| $6 P_{1 / 2}-8 D_{3 / 2}$ | 6010 | $1.3 \pm 0.3$ | 1.1 |
| $6 P_{3 / 2}-8 D_{3 / 2}$ | 6217 | $1.3 \pm 0.2$ | 1.1 |
| $6 P_{3 / 2}-8 D_{5 / 2}$ | 6213 | $1.5{ }_{-0.3}^{+0.2}$ | 1.3 |
| $6_{6} P_{1 / 2}-9 D_{3 / 2}$ | 5664 | $2.1 \pm 0.2$ | 2.2 |
| $6 P_{3 / 2}-9 D_{5 / 2}$ | 5845 | $2.8 \pm 0.3$ | 2.7 |
| $5 D_{3 / 2}-5 F_{5 / 2}$ | 8018 | $2.3 \pm 0.3$ | 2.1 |
| $5 D_{5 / 2}-5 F_{7 / 2}$ | 8081 | $2.3 \pm 0.3$ | 2.1 |
| $5 D_{3 / 2}-6 F_{5 / 2}$ | 7229 | $5.4 \pm 0.7$ | 6.2 |
| $5 D_{5 / 2}-6 F_{7 / 2}$ | 7280 | $5.4 \pm 0.7$ | 6.3 |
| $5 D_{3 / 2}-7 F_{5 / 2}$ | 6825 | $12.2 \pm 2.0$ | 12.5 |
| $5 D_{5 / 2}-7 F_{7 / 2}$ | 6871 | $12.2 \pm 2.0$ | 12.7 |
| $5 D_{5 / 2}-8 F_{7 / 2}$ | 6629 | $20.5 \pm 4.0$ | 24.4 |

to temperature in the range from 2500 to $7000^{\circ} \mathrm{K}$. Line shifts have been observed for several transitions in the diffuse and fundamental series, but only the direction of the shifts could be determined with some certainty. It always agreed with the calculated sign of $d / w$.

In conclusion, the good agreement with experiments suggests an accuracy of about $20 \%$ in line broadening calculations using the Coulomb approximation for the atomic matrix elements. This has encouraged similar calculations for both neutral and ionized carbon, nitrogen, and oxygen lines, the results of which will be available shortly. ${ }^{23}$

[^7]
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    ${ }_{21}$ In addition, the first two parts of reference 20 contain calculated Stark widths for the fundamental series lines of cesium, which appear to be too small by about $20 \%$. Also, the temperature dependence is stated to be smaller than that obtained here. These discrepancies are due to the use of calculated energy levels in reference 20 instead of the more accurate extrapolated observed levels that were employed in the present calculations. (See also reference 8.)
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