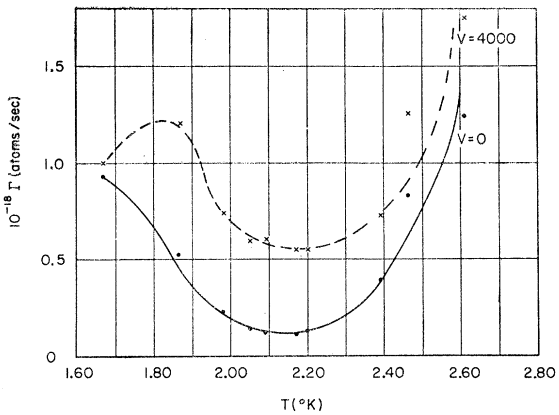

FIG. 12. Flow near the λ point in an A channel.

cooled from 4.2°K to a temperature below the λ point. After the sudden onset of flow at a temperature which ranged from 1.9 to 1.7°K, a normal pattern was observed throughout the entire temperature range. That is, flow occurred above the λ point and increased sharply below it.

Summary

In addition to the pressure gradients, gravitational forces, and thermomechanical forces ordinarily encountered in liquid-helium hydrodynamics, body forces of electrostrictive origin were present and dominant in this work. Superfluid flow at less than critical velocity was brought to flow at critical velocity, as long as part of the flow channel was filled with gas, by electric pressures which affected both the normal and superfluid components of the liquid. The apparatus may be considered to be an electrohydrodynamic flow regulator.

ACKNOWLEDGMENTS

Discussions of many aspects of the research with Professor H. W. Meissner, Professor F. Pollock, and Professor S. I. Rubinow were of substantial value. The authors wish to note, also, the assistance of W. Steinert in the construction of the apparatus.

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Film Flow of Liquid Helium at Low-Pressure Heads*†

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A steady-state method was developed and used to measure the flow rate of saturated liquid-helium films at small pressure heads. In this method a steady heat input into a glass vessel partially immersed in liquid helium generates the film flow inwards. A steady state is established when the rate of film flow is equal to the rate of distillation out of the vessel. The flow rate was derived from the temperature difference and level difference between the liquid in the vessel and the bath. Measurements in the vicinity of 1.2°K indicate a marked pressure-head dependence of the flow rate below a pressure head of 0.01 cm of liquid helium. A maximum flow rate for zero pressure head (±0.001 cm of liquid helium) of about 5×10^{-5} cc/sec cm is observed. This rate is suggested to be the true critical flow rate of the film.

INTRODUCTION

ONE of the characteristic features of the superfluid flow of a saturated liquid-helium film is the relatively constant rate at which it flows regardless of the pressure head. This observation was one of the factors which led to the concept of critical velocity for the superfluid component in the two-fluid theory of liquid helium. The critical velocity is a velocity below which it is believed that the superfluid can flow with essentially no dissipation of its kinetic energy of flow. Such a frictionless flow can occur with zero pressure head.[1]

The observed flow rate of the helium film is usually expressed in cc/sec cm, so the average flow velocity is given by the quotient of the film-flow rate and the thickness of the film. However, the film thickness has been observed to vary with the height of the film above the bath, though the exact dependence of the thickness on the height is still open to debate. The interpretation of the flow rate is further complicated by the fact that the flow rate and thickness can be critically altered by condensation of small amounts of solid impurities and by the roughness of the substrate material. Con-

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† This work was supported in part by the National Science Foundation.

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[1] Bibliography and better coverage may be found in the following review articles: (a) W. H. Keesom, *Helium* (Elsevier Publishing Company, Inc., Amsterdam, 1942). (b) R. G. Dingle, *Advances in Physics*, edited by N. F. Molt (Taylor and Francis, Ltd., London, 1952), Vol. 1, p. 111. (c) F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1954). (d) K. R. Atkins, *Liquid Helium* (Cambridge University Press, New York, 1959).

tamination and roughness of the surface tend to increase the flow rate in a manner which is not completely understood.

Observations indicate that, even with considerable care to eliminate condensation of gas and roughness, the flow rate is not strictly independent of the pressure head and, for a given pressure head, the observed rates may differ from experiment to experiment by as much as 15%. As a result it has been somewhat difficult to specify an exact value for the critical flow rate of the helium film.

Generally, the flow rate of the film is measured by observing the rate at which a partially submerged vessel is either emptied or filled through the film. A typical result showing the flow rate, σ, vs pressure head, p, by Atkins[2] is shown in Fig. 1. It has become customary to accept the relatively flat portion of the curve as the "critical flow rate" of the film. The initial high rate of emptying is attributed to the film being thicker at small heights above the inner level. The decrease in the flow rate at small pressure head has been explained by Eselson and Lazarew[3] as due to thermal effects. Because of the mechanocaloric effect, the superflow of the film causes a temperature difference between the liquid in the vessel and the bath. In accordance with the thermomechanical effect the effective total pressure head, $[\Delta h-(S/g)\Delta T]$, becomes zero at a finite level difference. This level difference then decays to zero as the temperature difference is dissipated. They demonstrated this with experiments using vessels with varying thermal contact with the bath, and the effect became more pronounced with increased thermal

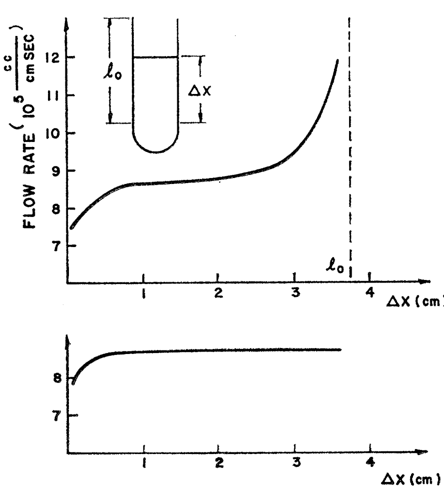

FIG. 1. Film flow rate vs pressure head measured by Atkins (see reference 2). The upper graph gives the emptying rate and the lower graph gives the filling rate.

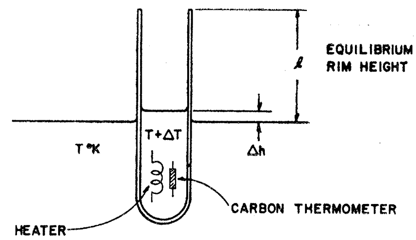

FIG. 2. Schematic diagram of experimental apparatus.

insulation. However, quantitative consideration shows that this explanation is difficult to maintain.[2,4]

The exact behavior of the flow rate in this low pressure-head region has long been of considerable interest. There have been two notable experiments which have demonstrated that there is practically no pressure head associated with flow below the critical rate. One is the double beaker experiment of Mendelssohn *et al.*[5] which showed that the pressure head was less than 2×10^{-2} cm of liquid helium for subcritical rates determined by the dimension of the vessels they used. The other is the experiment by Picus[6] using a plunger where it was shown that the pressure head was of the order of, or less than, 5×10^{-3} cm of liquid helium.

The work reported here is on the results of measurements in this low pressure head region by a truly steady-state method. A vessel with a thermometer and heater inside is partially submerged into the liquid helium bath as seen in the schematic illustration of Fig. 2. After equilibrium is established, heat is fed into the heater at a constant rate. As a result of this, the temperature of the liquid inside the vessel rises and distillation of the helium out into the bath takes place, while superfluid flows into the tube through the film due to the thermomechanical effect. The rate of evaporation can be expected to increase with the temperature difference between the bath and the liquid in the tube. If the flow rate of the film is a monotonically increasing function of the effective pressure head, which is the sum of the thermomechanical and gravitational pressure head, the temperature and the level inside the tube should adjust themselves until a steady state is established in which the film flow rate equals the rate of evaporation. This was actually found to occur.

When a steady state has been established,

$$dV/dt=2\pi r\sigma \text{ cc/sec}, \tag{1}$$

where r is the radius of the tube, and σ is the flow rate of the film in cc/sec cm.

If it is assumed that the walls of the tube are good insulators and all the heat exchange is due to the evaporation and the film flow, the conservation of heat

[2] K. R. Atkins, Proc. Roy. Soc. (London) **A203**, 240 (1950).
[3] B. N. Eselson and D. G. Lazarew, Doklady Akad. Nauk S.S.S.R. **81**, 537 (1951); J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 552 (1952).
[4] See discussion with Eq. (6).
[5] J. G. Daunt and K. Mendelssohn, Nature **157**, 829 (1946); B. S. Chandrasekhar and L. Mendelssohn, Proc. Phys. Soc. (London) **A68**, 857 (1955).
[6] G. S. Picus, Phys. Rev. **94**, 1459 (1954).

gives the relation

$$\dot{Q}=2\pi r\sigma\rho ST+(dV/dt)\rho L, \quad (2)$$

where $\dot{Q}$ is the rate of heat input by the heater. Combining this with Eq. (1)

$$\sigma=\frac{\dot{Q}}{2\pi r\rho(L+ST)}, \quad (3)$$

where L is the latent heat and S is the entropy. The effective pressure head is given by the relation

$$P_{eff}=(S/g)\Delta T-\Delta h \quad \text{cm of liquid helium,} \quad (4)$$

where ΔT is the temperature difference and Δh is the evel difference. Thus, the functional relationship between the flow rate and the effective pressure head can be obtained by measuring the temperature difference, level difference, and heat input for various heat inputs.

It can be shown that for a small temperature difference, the distillation rate is given by[7]

$$\frac{dV}{dt}=\frac{\alpha A}{\rho}\left(\frac{M}{2\pi RT}\right)^{1/2}\left(\frac{\partial P}{\partial T}\right)\Delta T \text{ cc of liquid helium/sec,} \quad (5)$$

where α is the vaporization coefficient, A is the effective area of vaporization, ρ is the density of liquid helium, M is the molecular weight of the helium, and $\partial P/\partial T$ is the slope of the vapor pressure curve. Combining Eqs. (1), (3), and (5), it is also possible to determine the product, αA, of the vaporization coefficient and the effective surface area by

$$\alpha A=\frac{\dot{Q}\qquad\qquad\rho}{\rho(L+ST)(M/2\pi RT)^{1/2}(\partial P/\partial T)\Delta T}. \quad (6)$$

Using Eq. (2), it can also be shown that for film flow at large pressure head, when the flow rate is constant, with an open test tube of radius r, and $\dot{Q}=0$, the temperature difference that develops from the mechanocaloric effect is

$$\Delta T=\frac{2\pi rST\sigma}{\alpha A(M/2\pi RT)^{1/2}(\partial P/\partial T)L}. \quad (7)$$

At 1.2°K, this gives $\Delta T\approx 3\times 10^{-7}$°K, which corresponds to a pressure head of about 1.8×10^{-4} cm of liquid helium. This consideration makes it highly improbable that the explanation given by Eselson and Lazarew[3] suffices.

APPARATUS

The experiments reported here were performed in a closed chamber submerged in a bath of liquid helium. The chamber consisted of a 2-in i.d. Pyrex tube, 7 in. long, with two indium O-ring seals.[8] This arrangement isolated the experiment from the effects of pumping and allowed considerable stability from temperature fluctuations. The temperature of the liquid helium in the main helium Dewar was kept constant by a temperature-control unit which consisted of a heater and a thermometer whose design was originated by Sommers.[9] Under optimum conditions below 1.3°K, it was possible to maintain the bath temperature in the closed system constant within less than 10^{-5}°K. With care this constancy could be maintained for 2 or 3 h.

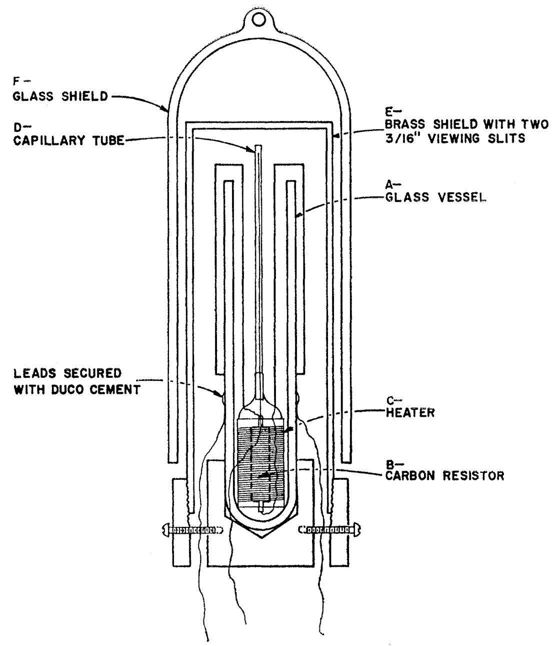

FIG. 3. Experimental apparatus.

The essential part of the apparatus is shown in Fig. 3 and was arranged so that it could be raised or lowered by a pulley at the top of the cryostat. In essence, it consisted of a 1-cm i.d. test tube, A, with a thermometer, B, heater, C, and a capillary tube, D, inside. The thermometer was one of a matched pair of carbon resistance thermometers, the other being in the bath. The four leads, two for the thermometer and two for the heater, were of No. 38 enameled copper wire and mounted so that they did not touch the glass wall of the tube above the liquid surface in order not to perturb the film. These were secured to terminals in the bath where they connected to manganin leads from the top of the cryostat. The capillary tube was inserted to facilitate the observation of changes in the inner level. The meniscus in the capillary, which was raised by surface tension, could be clearly viewed even near the equilibrium position when the inner level could not be seen because of the bath level. The capillary had the effect, also, of averaging out the surface ripples

[7] K. R. Atkins, B. Rosenbaum, and H. Seki, Phys. Rev. **113**, 751 (1959).
[8] H. Seki, Rev. Sci. Instr. **23**, 213 (1959).
[9] H. S. Sommers, Rev. Sci. Instr. **25**, 793 (1954).

due to small vibrations in the system. Illumination for observing the meniscus was provided by a neon bulb.

A detailed analysis[10] indicates that the pressure in the capillary tube at the liquid level should be corrected for the increased vapor pressure due to the increased temperature. Hence, in evaluating the effective pressure it was necessary to correct Eq. (4) to

$$P_{eff}=(S/g)\Delta T-(\partial P/\partial T)\Delta T-\Delta h \quad \text{cm of liquid helium,} \quad (8)$$

where $\partial P/\partial T$ is the slope of the vapor pressure curve in cm of liquid helium/°K.

The Kapitza boundary effect[11] and the thermal conductivity of glass reduced the heat leak through the test tube wall to less than 3% of the total heat input into the tube which was considered negligible.

The brass shield, E, was to keep to a minimum the stray heat input due to radiation. The carbon resistance thermometers are extremely sensitive to radiation and those in the bath were also shielded.

The glass cover, F, was added to insure that the vapor around the test tube was in equilibrium with the bath. This was found necessary since whenever there was a net heat flow from the closed chamber to the cryostat bath, the temperature of the chamber wall above the liquid level in the chamber was generally colder than the chamber liquid by an amount depending on the value of the heat flow. It is believed that a type of convection took place in which liquid evaporated from the bath, condensed onto the wall and ran down the wall in the form of a film. As a result, the vapor pressure inside the chamber was somewhat below the vapor pressure corresponding to the bath temperature. Reproducible measurements could not be obtained with the steady-state method until this glass cover was attached.

It is quite possible that the lack of complete equilibrium between the vapor phase and the bath may be one of the factors which have contributed to the discrepancies in other measurements concerning the helium film, even when they were made in closed systems. As can be seen from the measurements on unsaturated helium films, the thickness and flow rate are strongly varying functions of the relative pressure P/P_s just below the saturated vapor pressure P_s.[12]

The matched pair of thermometers was used to measure the temperature difference between the liquid in the tube and the chamber bath. They were selected from a number of 1-W, Allen Bradley 12-Ω carbon resistors whose Bakelite coating had been removed and then coated with a thin layer of glyptal. The two thermometers each in series with a resistance box formed two adjacent arms of a Wheatstone bridge. The remaining two arms were kept constant and equal. A dc bridge voltage was used with a dc amplifier[13] as a detector. The actual bridge circuit was arranged so that it could be switched to an ordinary Wheatstone-bridge circuit for each thermometer since it was necessary to calibrate the thermometers for each experiment under identical bridge conditions.

All precautions necessary for low-voltage measurements were taken such as minimization of thermal emf's maintenance of steady temperatures for all electric circuits, the use of low noise switches regulated power supply, and proper shielding. It was difficult to shield against rf noise, but, following Ambler and Plumb,[14] it was found that putting 0.05 μF capacitors in parallel with the resistance thermometers eliminated a good part of the fluctuations.

The capillary meniscus level was observed through a cathetometer whose roter for the moving hair line was coupled to a potentiometer, so that the level readings could be recorded continuously. During the steady state, measurements were accurate to better than ±0.001 cm. The limiting factor was the lack of definition of the meniscus, which was somewhat blurred by uneveness in the cryostat glass and convection current in the liquid-nitrogen bath.

During the experiment, the readings of bath temperature, temperature difference, and cathetometer were cycled through a multigang time switch onto a recorder. The cycle had a period of 15 sec, and in this way a semicontinuous recording was made simultaneously for the three measurements.

The temperature difference measured in the experiment was of the order of 10^{-5} °K with an accuracy of better than $\pm 5\times 10^{-7}$ °K, which corresponds to about 2×10^{-4} cm of liquid helium at 1.2°K. Although the general temperature fluctuation was of the order of 10^{-5} °K with an average period of a few seconds, the thermal coupling between the liquid in the tube and bath through distillation seemed very effective and the temperature difference during a steady-state measurement remained constant to better than 5×10^{-7} °K.

A conservative estimate of the accuracy in the measurements of the effective pressure is $\pm 1\times 10^{-3}$ cm of liquid helium.

EXPERIMENTS

The experiments were difficult and took a long time, so only a few reliable results were obtained. The measurements reported here were all taken in the vicinity of 1.2°K.

Prior to precooling the cryostat, the inner chamber was evacuated to about 2×10^{-5} mm Hg. After pre-

[10] H. Seki, Ph.D. thesis, University of Pennsylvania, (unpublished).

[11] P. L. Kapitza, J. Phys. (U.S.S.R.) 4, 181 (1941).

[12] R. Bowers, D. F. Brewer, and K. Mendelssohn, Phil. Mag. **42**, 1445 (1951); D. F. Brewer and K. Mendelssohn, Proc. Roy. Soc. (London) **A260**, 1 (1961); E. A. Long and L. Meyer, Phys. Rev. **87**, 153 (1952); *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1953), Vol. 2, p. 1. Phys. Rev. **98**, 1616 (1955).

[13] Beckman Model 14 dc breaker amplifier with 5 Ω input impedance.

[14] E. Ambler and H. Plumb, Rev. Sci. Instr. **31**, 656 (1960).

TABLE I. Results of steady-state measurements.

$\dot{Q}$ (μW)	σ (10^{-5} cc/sec cm)	ΔT (10^{-5} °K)	$P_{\Delta T}$ (cm liquid/cm²)	P_h (cm liquid/cm²)	$P_{\Delta T}$[a] (cm liquid/cm²)	P_{eff}[b] (cm liquid/cm²)
206	2.24	1.55	0.0080	0.0076	0.0080	0.0004
322	3.51	2.49	0.0129	0.0128	0.0126	0.0002
390	4.24	2.93	0.0151	0.0152	0.0152	0.0001
465	5.05	3.50	0.0181	0.0178	0.0181	0.0003
503	5.47	3.88	0.0198	0.0191	0.0198	0.0007
545	5.93	4.09	0.0211	0.0200	0.0215	0.0015
588	6.39	4.33	0.0224	0.0205	0.0230	0.0025
632	6.87	4.74	0.0245	0.0218	0.0247	0.0029
678	7.37	5.18	0.0268	0.0224	0.0266	0.0042
726	7.89	5.43	0.0281	0.0190	0.0284	0.0094
745	8.10	5.57	0.0285	0.0163	0.0122	0.0125
765	8.31	5.79	0.0299	0.0116	0.0299	0.0183
785	8.53	5.73[b]				

[a] This column represents the corrected value read from the straight line drawn through the measured points of $P_{\Delta T}$ in the fourth column. Having confirmed the linear relation of $P_{\Delta T}$ and $\dot{Q}$, fluctuation errors are eliminated and P_{eff} was calculated from the values in this column.

[b] The meniscus was falling at the rate of 2×10^{-6} cm/sec which corresponds to 0.05×10^{-5} cc/cm sec film flow. This was at $P_{eff}\approx0.496$.

cooling to liquid-nitrogen temperature, the inner chamber was closed off from its pumping system and the cryostate was filled with liquid helium. It was believed that most of the small amount of residual gas would first condense onto the walls of the chamber leaving the test tube relatively free from contamination. The chamber was then filled to the desired level by opening the needle valve connecting it to the cryostat bath.

The general procedure for flow measurements was to first balance the difference thermometer bridge while the tube was completely submerged. The tube was then raised to the desired height and the initial rate of emptying was measured. The stationary steady-state measurements were made subsequently, keeping the tube stationary. In between some of these measurements, transient flow rates were measured by creating an initial level difference by a suitable heat input. In this way, steady-state measurements could be compared

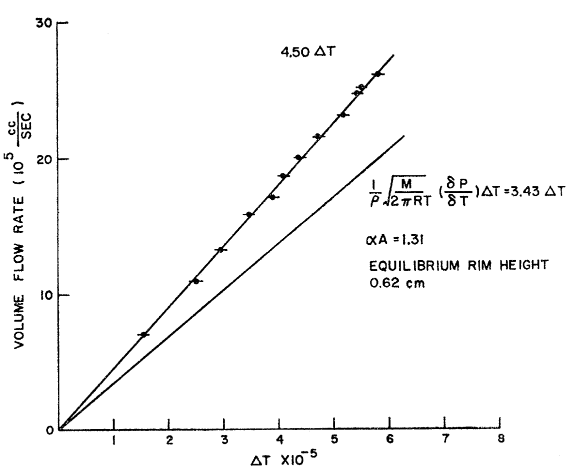

FIG. 4. Liquid volume flow rate, $\dot{Q}/\rho(L+ST)$ (which is equal to the evaporation rate) vs temperature difference. $\alpha A=1.31$ cm^2$=1.67\ \pi_r^2$.

with transient measurements when they were performed under the nearest possible conditions. Temperature calibration was made using the $T_{L_{55}}$ vapor pressure scale.[15]

In all transient flow measurements, when there was no appreciably heat input into the test tube, no temperature difference could be detected between the bath and the liquid in the tube. Since the expected temperature difference is less than 3×10^{-7} °K according to Eq. (7), and this was about the limit of sensitivity of the difference thermometer, this result seems reasonable.

The transient flow process was generally followed by an oscillation of the meniscus level with an initial amplitude of a few thousandths of a cm. The duration of the oscillation varied considerably and this is thought to be due to vibrations in the cryostat system.

When the initial emptying had ended and equilibrium had been established, the equilibrium level of the capillary meniscus was determined. Since the oscillation sometimes persisted for a long time, or could be started by a slight disturbance, the equilibrium level had to be determined carefully over a time greater than the period of oscillation, which was of the order of a minute. The voltage was applied to the heater in the test tube, after which a transient state followed before a steady state set in.

The steady-state height of the meniscus increased linearly with the heat input at first and then, for high heat inputs, started to decline rapidly and the duration of the transient state increased, indicating that the pressure-head dependence of the flow rate was decreasing for the high flow rates. As a result, the time necessary to determine whether a steady state had been achieved or not became exceedingly long. Measurements were discontinued when the transient time exceeded an hour. At these heat inputs the steady-state level was near the bath level or lower.

[15] H. Van Dijk and M. Durieux, Kammerlingh Onnes Laboratory, Leiden, Holland, 1955 (unpublished).

EXPERIMENTAL RESULTS

Measurements of the transient flow of the film indicated phenomena similar to those observed by Eselson and Lazarew[3] and recently by Allen,[5] where the film is observed to flow at two rather distinct flow rates depending on the history of the film. The high flow rate generally occurred in the initial emptying after the tube had been raised from complete submergence and subsequent transient measurements, when the level was initially changed as a result of emptying or filling through the film, gave the lower rate. The two rates differed by about 20% but there were cases when the lower rates occurred in the initial emptying and also where the high rate continued to occur in the measurements when the tube was filled through the film. Once the lower flow rate occurred, the higher flow rate never reappeared unless the tube was again completely immersed in the liquid. The highest flow rate measured by the steady-state method agreed with the lower transient flow rate.

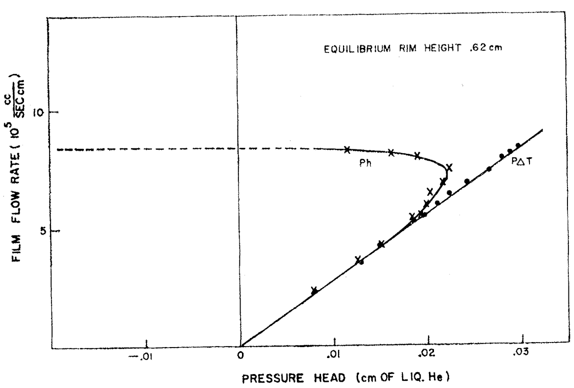

FIG. 5. $P_{\Delta T}=[S/g-(1/\rho g)(\partial P\partial T)]\Delta T$ and P_h vs film flow rate.

The result of a detailed steady-state measurement made at 1.210°K for an equilibrium rim height of 0.62 cm is given in Table I. Figure 4 is a plot of the evaporation rate $\dot{Q}/\rho(L+ST)$ against the temperature difference. The fact that the plot gives a linear relation indicates that αA is constant with respect to ΔT, and its value is 1.67 πr^2 in this case. The value of αA, however, varied from experiment to experiment between about 1.4 πr^2 and 1.7 πr^2. Since the value of α cannot exceed unity, this result indicates that the effective area of evaporation is greater than the cross-sectional area of the tube.

A possible contribution to the extra area may very well be the lower part of the film near the bulk liquid. This might be expected since the thickness of the film is known to be greater as it approaches the liquid level, and it is quite reasonable to believe that the temperature of the film would correspond to the tube liquid temperature up to some height. However, it is surprising that αA should vary so much from experiment to experiment. Another possible contribution to the area is most likely due to the surface waves on the meniscus. These could vary from experiment to experiment due to the variation in the mechanical vibrations of the building from day to day.

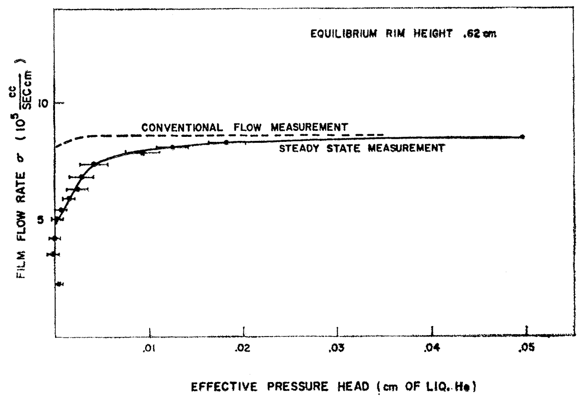

FIG. 6. Film flow rate vs effective pressure head, $P_{\Delta T}-P_h$.

In Fig. 5, $P_{\Delta T}=[(S/g)-(1/\rho g)(\partial P/\partial T)]\Delta T$ and P_h are plotted against the film flow rate, and in Fig. 6 the flow rate is plotted against the effective pressure, $P_{\Delta T}-P_h$. As can be seen for low flow rates, $P_{\Delta T}$ and $P_{\Delta h}$ both lie on the same straight line indicating a true superfluid flow with no effective pressure head within the experimental accuracy of ± 0.001 cm of liquid helium. Then at a flow rate of 4.8×10^{-5} cc/sec cm, which may be considered to be the critical flow rate, there seems to be a linear development of a pressure head, after which the pressure head develops rapidly and nonlinearly as the flow-rate curve approaches the value of 8.6×10^{-5} cc/sec cm which is the transient flow rate shown on the graph by the dashed line.

In another measurement at 1.217°K, steady-state measurements were made at two different equilibrium rim heights. The dependence of flow rate on the effective pressure head is plotted on Fig. 7. The equilibrium rim heights were 1.2 and 0.3 cm and it is seen that, although the critical velocity was the same for both

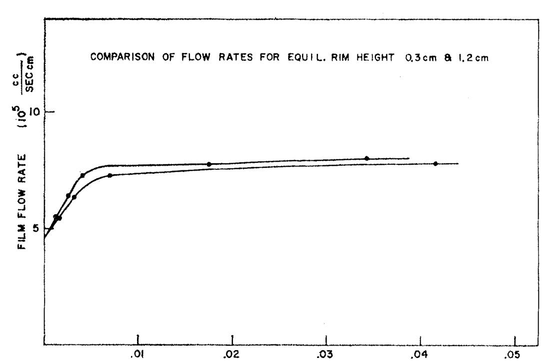

FIG. 7. Film flow rate vs effective pressure head. Top curve is for equilibrium rim height 0.3 cm, and lower curve is for equilibrium rim height 1.2 cm.

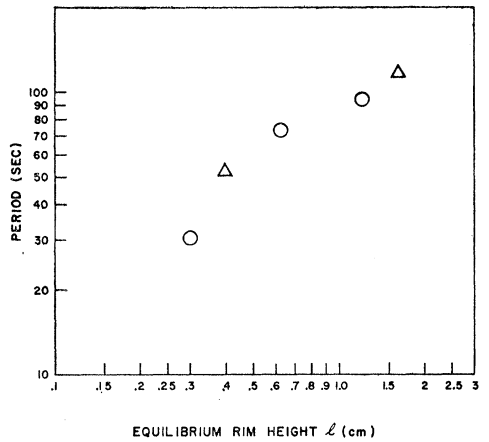

FIG. 8. Period of film oscillation vs equilibrium rim height.

cases, above the critical velocity the pressure head is greater for the case of higher rim height.

In Fig. 8, the period of film oscillation observed after transient flow is plotted against the equilibrium rim height for a number of experiments. The period of the cases just discussed has been circled.

CONCLUDING REMARKS

It seems that the results of these measurements bear out the need to distinguish between the critical rate and what might be called the limiting rate in discussing the flow of films. The critical flow rate is the value below which there is no measurable pressure head. This was found to be a little less than 5×10^{-5} cc/sec cm. The limiting flow rate is what is usually measured in the transient state measurement and often has been called the critical flow rate. The limiting flow rate for the steady-state measurement agrees with the lower of the two possible flow rates observed by Eselson and Lazarew[3] and Allen.[16]

The fact that as the flow rate exceeds the critical rate, the pressure head seems to develop linearly at first and then increases rapidly, suggesting that more than one type of dissipative mechanism is present. If the dissipation is due to some type of elementary excitations, as suggested by Landau,[17] the linear region must represent a region where the interaction between the excitations are still negligible. As soon as the interaction becomes important, a sudden rise in the dissipation occurs resulting in a limiting flow rate. The two possible limiting flow rates observed by Eselson and Lazarew and Allen would undoubtedly be related to this latter process.

There is obviously room for more measurements using the steady-state method described here. It may be pointed out that this method is applicable to any other purely superfluid flow process such as the flow through very narrow channels.

In particular, careful measurements in an apparatus designed to be free of vibration should be interesting in determining the excess area of evaporation. Should it turn out that parts of the film participate in the distillation process, it would be the first experimental measurement of a temperature gradient in the helium film.

In light of the results reported, a careful measurement of the oscillation of the film flow would also be interesting. The initial amplitude after an emptying or a filling should be close to

$$h=(2m/\rho gr)^{1/2}\sigma_s,$$

where

$$m=\frac{\rho^2}{\rho_s}\left[\left(1+\frac{r}{R}\right)\int_0^l\frac{dH}{d}\right].$$

σ_s is the limiting flow rate, and should then damp down to an amplitude of

$$h=(2m/\rho gr)^{1/2}\sigma_s,$$

where σ_s is the critical flow rate. At this amplitude the oscillation would be expected to continue indefinitely.

ACKNOWLEDGMENT

The author is indebted to Dr. K. R. Atkins for his unfailing encouragement and many helpful discussions.

[16] J. F. Allen, Nature **185**, 831 (1960).

[17] L. D. Landau, J. Phys. (U.S.S.R.) **5**, 71 (1941).