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Resonance Model for Photoproduction of K Mesons on Nucleons

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An approximate one-dimensional dispersion relation of the Cini-Fubini type is assumed for the invariant amplitudes of the photoproduction of K mesons on nucleons on the supposition that among the various intermediate states in the three related channels only the single-particle states and the resonant states make appreciable contributions. The resonant states that are taken into account are the following: the three-pion-nucleon resonances, the pion-kaon resonance, and the three pion-hyperon resonances. The expressions for the spectral functions are derived by applying the unitarity conditions to each of the three related channels. By the conservation of angular momentum and parity only certain multipole amplitudes determined by the spin and parity of the resonances contribute to the spectral functions, and for these nonvanishing multipole amplitudes the Breit-Wigner formula is assumed.

1. INTRODUCTION

THE application of dispersion relations to problems involving strange particles generally involves considerable complication because of the number of channels coupled to the process of interest both in the physical and unphysical regions. The discussion of photoproduction of K mesons from nucleons is no exception; certain simplifying assumptions must be made in order to obtain any predictions.

In this paper we assume the validity of the Mandelstam representation, but use the (approximate) one-dimensional representation obtained by keeping only single particle and resonant intermediate states in the unitarity condition. The resonances we retain are the pion-nucleon resonances N^*, the hyperon resonances Y_0^*, Y_0^{**}, and Y_1^*, and the K-π resonance K^*. Thus, in addition to the usual coupling constants and parities, the various resonance energies, widths, and spins are the parameters of the model.[1] In principle, these resonance parameters can be determined from the analysis of other experiments; pion-nucleon scattering, photoproduction of pions, K^* production, and associated production. We thus have a model which correlates the predictions of photoproduction of K mesons in terms of the parameters obtained from these other experiments. Here we present the formal results of these considerations, while numerical results (dependent on the Λ-Σ parity and the spin of the K^*) will be presented in a subsequent paper. Even if our model is not completely successful in fitting the experimental results, the contributions we consider enter in any dispersion treatment and thus these results should suggest possible refinements.

[1] Similar models were proposed for associated production by M. Gourdin and M. Rimpault, Nuovo cimento **20**, 1166 (1961), and by T. Tsuchida, T. Sakuma, and S. Furui, Progr. Theoret. Phys. (Kyoto) **26**, 1005 (1961). See also M. Gourdin and M. Rimpault (to be published).

2. KINEMATICS AND DISPERSION RELATIONS

Although kinematical relations were given previously by Okubo[2] and by Fayyazuddin,[3] we will give a summary of these relations since somewhat different notation and assumptions are used in this paper. We will denote the four-momenta of the incoming photon and nucleon by k and p_1, and those of the outgoing K meson and hyperon by q and p_2 respectively. By the conservation of momentum and energy, the invariant variables defined by

$$s=-(p_1+k)^2=-(p_2+q)^2, \tag{2.1a}$$

$$u=-(p_1-q)^2=-(p_2-k)^2, \tag{2.1b}$$

$$t=-(p_1-p_2)^2=-(q-k)^2, \tag{2.1c}$$

satisfy the following relation

$$s+t+u=M^2+M_Y^2+m_K^2, \tag{2.2}$$

where M, M_Y, and m_K are the masses of nucleon, hyperon, and K meson, respectively.

These invariants are the square of the total energy in the barycentric system of the following reactions designated by channels I, II, and III, respectively:

$$\gamma+N\rightarrow K+Y, \tag{2.3}$$

$$\gamma+Y\rightarrow \bar{K}+N, \tag{2.4}$$

$$\gamma+\bar{K}\rightarrow Y+\bar{N}. \tag{2.5}$$

The intrinsic parity of the Λ-K system with respect to the nucleon is established to be negative,[4] but the relative Λ-Σ parity is not yet experimentally decided al-

[2] S. Okubo, Progr. Theoret. Phys. (Kyoto) **19**, 43 (1958).
[3] Fayyazuddin, Phys. Rev. **123**, 1882 (1961). See also references 6 and 7.
[4] M. M. Block, E. B. Brucker, J. S. Hugh, T. Kikuchi, C. Meltzer, F. Anderson, A. Pevsner, E. M. Harth, J. Leitner, and H. O. Cohn, Phys. Rev. Letters **3**, 291 (1960).

though there is some indications that it is even.[5] Therefore, we will give formalisms for both even and odd K-Y parity so that we can deduce some information on the K-Σ parity from the K-meson photoproduction experiment.

The T matrix, defined by

$$S_{fi}=[-i/(2\pi)^2]\delta^{(4)}(p_1+k-p_2-q)\times(MM_Y/4E_1E_2k\omega)^{\frac{1}{2}}\bar{u}(p_2)T(p_2,q;p_1,k)u(p_1), \quad (2.6)$$

can be represented as a linear combination of four Lorentz- and gauge-invariant operators:

$$T=\sum_{i=1}^{4}A_i(s,t,u)O_i, \quad (2.7)$$

with

$$O_1=i\gamma_5\gamma\cdot\epsilon\gamma\cdot k, \quad (2.8a)$$

$$O_2=2i\gamma_5(\epsilon\cdot Pk\cdot q-\epsilon\cdot qk\cdot P), \quad (2.8b)$$

$$O_3=\gamma_5[\gamma\cdot\epsilon q\cdot k-\gamma\cdot kq\cdot\epsilon-i(M-M_Y)\gamma\cdot\epsilon\gamma\cdot k], \quad (2.8c)$$

$$O_4=2\gamma_5\left[\gamma\cdot\epsilon P\cdot k-\gamma\cdot kP\cdot\epsilon-\frac{i}{2}(M+M_Y)\gamma\cdot\epsilon\gamma\cdot k\right], \quad (2.8d)$$

for odd K-Y parity. Here ϵ is the polarization vector of the photon and $P=\frac{1}{2}(p_1+p_2)$. The terms proportional to $i\gamma\cdot\epsilon\gamma\cdot k$ in O_3 and O_4, which are added for later convenience, differs slightly from those in reference 3 and reduces to those used by Chew, Goldberger, Low, and Nambu (CGLN)[6] when the mass difference between a nucleon and a hyperon is neglected.

For even K-Y parity, in which case the amplitudes and operators will be distinguished by a bar, $i\gamma_5$ should be replaced by 1, and also the sign of M_Y has to be changed. Thus we write

$$\bar{O}_1=\gamma\cdot\epsilon\gamma\cdot k, \quad (2.9a)$$

$$\bar{O}_2=2\{\epsilon\cdot Pk\cdot q-\epsilon\cdot qk\cdot P\}, \quad (2.9b)$$

$$\bar{O}_3=-i\{\gamma\cdot\epsilon q\cdot k-\gamma\cdot kq\cdot\epsilon-i(M+M_Y)\gamma\cdot\epsilon\gamma\cdot k\}, \quad (2.9c)$$

$$\bar{O}_4=-2i\{\gamma\cdot\epsilon P\cdot k-\gamma\cdot kP\cdot\epsilon-i\tfrac{1}{2}(M-M_Y)\gamma\cdot\epsilon\gamma\cdot k\}. \quad (2.9d)$$

The scalar amplitudes A_i (or $\bar{A}_i$ for even K-Y parity) are matrices in isotopic spin space, and their dependence on the isotopic spin can be expressed by means of the following matrices:

$$A_i=\sum_j \mathcal{J}^{(j)}A_i^{(j)}, \quad (2.10)$$

$$\mathcal{J}^{(S)}=1, \quad (2.11a)$$

$$\mathcal{J}^{(V)}=\tau_3 \quad (2.11b)$$

for Λ production, and

$$\mathcal{J}^{(+)}=\delta_{\alpha 3}, \quad (2.12a)$$

$$\mathcal{J}^{(-)}=\tfrac{1}{2}[\tau_\alpha,\tau_3], \quad (2.12b)$$

$$\mathcal{J}^{(0)}=\boldsymbol{\tau}_\alpha \quad (2.12c)$$

for Σ production, α being the isotopic spin index of the Σ hyperon.

It should be noted that $A_i^{(S)}$ and $A_i^{(V)}$ represent contributions from isotopic scalar and vector photon respectively, both leading to a final state with total isotopic spin $T=1/2$. For Σ production, $A_i^{(0)}$ represents the absorption of an isotopic scalar photon with $T=1/2$ final state, while $A_i^{(+)}$ and $A_i^{(-)}$ are the contributions from an isotopic vector photon leading to a final state with $T=1/2$ or $3/2$, whose eigenamplitudes can be expressed in terms of $A_i^{(+)}$ and $A_i^{(-)}$ as follows:

$$A^{(\frac{3}{2})}=A^{(+)}-A^{(-)}, \quad (2.13a)$$

$$A^{(\frac{1}{2})}=A^{(+)}+2A^{(-)}. \quad (2.13b)$$

As was shown by Ball,[7] the choice of O_i as given by Eq. (2.8), does not introduce any kinematical singularities, and the Mandelstam representations for the A_i are

$$A_i(s,t,u)=\frac{R_{Ni}}{M^2-s}+\frac{R_{\Lambda i}}{M_\Lambda^2-u}+\frac{R_{\Sigma i}}{M_\Sigma^2-u}+\frac{1}{\pi}\int ds'\frac{\rho_1^i(s')}{s'-s}+\frac{1}{\pi}\int dt'\frac{\rho_2^i(t')}{t'-t}+\frac{1}{\pi}\int du'\frac{\rho_3^i(u')}{u'-u}$$
$$+\frac{1}{\pi^2}\int ds'\int dt'\frac{\rho_{12}^i(s',t')}{(s'-s)(t'-t)}+\frac{1}{\pi^2}\int dt'\int du'\frac{\rho_{23}^i(t',u')}{(t'-t)(u'-u)}+\frac{1}{\pi^2}\int ds'\int du'\frac{\rho_{13}^i(s',u')}{(s'-s)(u'-u)}. \quad (2.14)$$

The three poles represent the lowest order perturbation theory.[8] For odd K-Y parity, the residues of the poles are

$$R_{N1}=-\tfrac{1}{2}eg_\Lambda(\mathcal{J}^{(S)}+\mathcal{J}^{(V)}),$$

$$R_{N2}=\frac{eg_\Lambda}{t-m_K^2}(\mathcal{J}^{(S)}+\mathcal{J}^{(V)}),$$

$$R_{N3}=R_{N4}=[\tfrac{1}{2}(\mu_p+\mu_n)\mathcal{J}^{(S)}+\tfrac{1}{2}(\mu_p-\mu_n)\mathcal{J}^{(V)}]g_\Lambda, \quad (2.15)$$

$$R_{\Lambda 1}=R_{\Lambda 2}=0, \quad R_{\Lambda 3}=-R_{\Lambda 4}=-\mu_\Lambda g_\Lambda\mathcal{J}^{(S)},$$

$$R_{\Sigma 1}=\mu_T g_\Sigma(M_\Lambda-M_\Sigma)\mathcal{J}^{(V)}, \quad R_{\Sigma 2}=0,$$

$$R_{\Sigma 3}=-R_{\Sigma 4}=-\mu_T g_\Sigma\mathcal{J}^{(V)}$$

[5] R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi: Phys. Rev. Letters **8**, 175 (1962).

[6] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[7] J. S. Ball, Phys. Rev. **124**, 2014 (1961).

[8] M. Kawaguchi and M. J. Moravcsik, Phys. Rev. **107**, 563 (1957), A. Fujii and R. E. Marshak, Phys. Rev. **107**, 570 (1957).

for Λ-production, and

$$R_{N1}=-\tfrac{1}{2}eg_\Sigma(\mathcal{G}^{(+)}+\mathcal{G}^{(-)}+\mathcal{G}^{(0)}),$$

$$R_{N2}=\frac{eg_\Sigma}{t-m_K^2}(\mathcal{G}^{(+)}+\mathcal{G}^{(-)}+\mathcal{G}^{(0)}),$$

$$R_{N3}=R_{N4}=\tfrac{1}{2}\{(\mu_p+\mu_n)\mathcal{G}^{(0)}+(\mu_p-\mu_n)(\mathcal{G}^{(+)}+\mathcal{G}^{(-)})\}g_\Sigma,$$

$$R_{\Sigma 1}=-eg_\Sigma\mathcal{G}^{(-)},\quad R_{\Sigma 2}=\frac{2eg_\Sigma}{t-m_K^2}\mathcal{G}^{(-)}, \tag{2.16}$$

$$R_{\Sigma 3}=-R_{\Sigma 4}=-g_\Sigma\{\mu_0\mathcal{G}^{(0)}+\tfrac{1}{2}(\mu_+-\mu_-)\mathcal{G}^{(-)}\},$$

$$R_{\Lambda 1}=\mu_T g_\Lambda(M_\Sigma-M_\Lambda)\mathcal{G}^{(+)},\quad R_{\Lambda 2}=0,$$

$$R_{\Lambda 3}=-R_{\Lambda 4}=-\mu_T g_\Lambda\mathcal{G}^{(+)},$$

for Σ production.

In these expressions, μ with suffices p, n, Λ, +, −, and 0 represent anomalous magnetic moments of proton, neutron, Λ hyperon, and Σ hyperon with the corresponding charges, respectively. Due to charge independence, the magnetic moments of the Σ hyperons satisfy the relation[9] $\mu_0=\frac{1}{2}(\mu_++\mu_-)$. μ_T is the magnetic moment related to the transition between the Λ and Σ particles defined by

$$\langle\Sigma|j_\mu A_\mu|\Lambda\rangle=\tfrac{1}{2}\mu_T\bar{u}_\Sigma\sigma_{\mu\nu}F_{\mu\nu}u_\Lambda. \tag{2.17}$$

g_Λ and g_Σ are the coupling constants at the $N\Lambda K$ and $N\Sigma K$ vertices,

$$\langle Y|J|N\rangle=g_Y\bar{u}_Y\gamma_5 u_N, \tag{2.18}$$

J being the K-meson current.

Since in the odd Λ-Σ parity case, the K-Λ parity is assumed negative, the residue of the poles at $s=M^2$ and $u=M_\Lambda^2$ of the Λ-production amplitude are equal to those occurring in Eq. (2.15). The only term that has to be modified is the pole at $u=M_\Sigma^2$, i.e., a contribution from the transition magnetic moment, which is now defined by

$$\langle\Sigma|j_\mu A_\mu|\Lambda\rangle=\tfrac{1}{2}\mu_T\bar{u}_\Sigma\sigma_{\mu\nu}\gamma_5F_{\mu\nu}u_\Lambda. \tag{2.19}$$

Thus, we have the following expressions for the residues at $u=M_\Sigma^2$:

$$\begin{aligned}R_{\Sigma 1}&=-(M_\Lambda+M_\Sigma)\mu_T g_\Sigma\mathcal{G}^{(V)},\quad R_{\Sigma 2}=0,\\ R_{\Sigma 3}&=-R_{\Sigma 4}=\mu_T g_\Sigma\mathcal{G}^{(V)}.\end{aligned} \tag{2.20}$$

For Σ production, we have, for even K-Σ parity:

$$\begin{aligned}R_{N3}&=-R_{N4}=-\tfrac{1}{2}\{(\mu_p+\mu_n)\mathcal{G}^{(0)}\\&\qquad+(\mu_p-\mu_n)(\mathcal{G}^{(+)}+\mathcal{G}^{(-)})\}g_\Sigma,\\ R_{\Sigma 3}&=-R_{\Sigma 4}=g_\Sigma\{\mu_0\mathcal{G}^{(-)}+\tfrac{1}{2}(\mu_+-\mu_-)\mathcal{G}^{(0)}\},\\ R_{\Lambda 1}&=(M_\Lambda+M_\Sigma)g_\Lambda\mu_T\mathcal{G}^{(0)},\\ R_{\Lambda 3}&=-R_{\Lambda 4}=g_\Lambda\mu_T\mathcal{G}^{(0)},\end{aligned} \tag{2.21}$$

while the other R's are the same as the corresponding ones in Eq. (2.16).

From Eq. (2.14), we can find the location of the singularities of the partial wave amplitudes,[10] and set up integral equations for them, but the equations thus obtained are too complicated to give any physically interesting solution. Therefore, in accordance with the plan stated in the introduction, we will assume the following one-dimensional representation:

$$A_i(s,t,u)=\text{Born terms}+\frac{1}{\pi}\int\frac{a_i(s',t)}{s'-s}ds' +\frac{1}{\pi}\int\frac{b_i(u',t)}{u'-u}du'+\frac{1}{\pi}\int\frac{c_i(t',s)}{t'-t}dt'. \tag{2.22}$$

The spectral functions a_i, b_i, and c_i can be obtained as imaginary parts of the amplitudes of the reactions in channels I, II, and III, respectively. In the next three sections, the expressions for these spectral functions will be derived on the assumption that only resonances in each channel make appreciable contributions.

3. π-N RESONANCES

(i) Odd K-Y Parity

In the center-of-mass system of channel I, we introduce the following three-momenta and energies:

$$\begin{aligned}p_1&=(-\mathbf{k},E_1),\quad k=(\mathbf{k},\kappa),\\ p_2&=(-\mathbf{q},E_2),\quad q=(\mathbf{q},\omega).\end{aligned} \tag{3.1}$$

These are related to the total energy $W=\sqrt{s}$ as follows:

$$\begin{aligned}&E_1=(W^2+M^2)/2W,\quad E_2=(W^2+M_Y^2-m_K^2)/2W,\\ &\kappa=(W^2-M^2)/2W,\quad \omega=(W^2-M_Y^2+m_K^2)/2W,\\ &q=\{[(W-M_Y)^2-m_K^2][(W+M_Y)^2-m_K^2]\}^{\frac{1}{2}}/2W.\end{aligned} \tag{3.2}$$

Using the solutions of the Dirac equation of the form

$$u(p)=\frac{-i\gamma\cdot p+M}{[2M(E+M)]^{\frac{1}{2}}}u(0), \tag{3.3}$$

with

$$u(0)=\begin{pmatrix}\chi\\0\end{pmatrix}, \tag{3.4}$$

where χ is a two-component spinor, the T matrix can be reduced to a matrix in Pauli spin space as follows:

$$\bar{u}(p_2)Tu(p_1)=4\pi[W/(MM_Y)^{\frac{1}{2}}]\chi_f F\chi_i, \tag{3.5}$$

with

$$F=i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}F_1+(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})/qk)F_2 +(i\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}\cdot\boldsymbol{\varepsilon}/qk)F_3+(i\boldsymbol{\sigma}\cdot\mathbf{q}\mathbf{q}\cdot\boldsymbol{\varepsilon}/q^2)F_4. \tag{3.6}$$

[9] E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. **104**, 267 (1956).

[10] What is called a branch cut in reference 3 is actually a locus of branch points. Although branch cuts can be chosen to coincide with this curve, this is possible only when the path of integration over cosθ is distorted into a complex region. If the integral over cosθ is taken on a real axis, the branch cuts form a set of quartic curves. For a more detailed discussion on this point see J. Dreitlein and B. W. Lee, Phys. Rev. **124**, 1274 (1961).

The relation between the amplitudes F_i and the invariant amplitudes A_i will be written in a matrix form:

$$A_i(s,t,u)=\xi_{ij}(s,x)F_j(s,x), \tag{3.7}$$

where the dependence of the matrix ξ_{ij} on the total energy $W=\sqrt{s}$ and the scattering angle $x=\cos\theta$ has been explicitly indicated. The form of ξ_{ij} can be expressed conveniently as a product of two matrices: a matrix η_{ij} which depends on both s and x and a diagonal matrix $\zeta_{ij}\equiv\zeta_i\delta_{ij}$ which depends only on the energy variable s. Thus we have

$$\xi_{ij}(s,x)=\eta_{ik}(s,x)\zeta_{kj}(s), \tag{3.8}$$

with

$$\eta_{ij}=\frac{1}{2W}\begin{bmatrix} W+M & -(W-M) & -(M_Y-M)(W+M)+\dfrac{2Mq\cdot k}{W-M} & -(M_Y-M)(W-M)+\dfrac{2Mq\cdot k}{W+M} \\ 0 & 0 & 1 & -1 \\ 1 & 1 & W+M+\dfrac{q\cdot k}{W-M} & W-M+\dfrac{q\cdot k}{W+M} \\ 1 & 1 & \dfrac{q\cdot k}{W-M} & \dfrac{q\cdot k}{W+M} \end{bmatrix}, \tag{3.9}$$

$$\zeta_i=\frac{8\pi W}{W-M}[(E_1+M)(E_2+M_Y)]^{-\frac{1}{2}}\left(1,\frac{E_2+M_Y}{q},\frac{1}{q},\frac{E_2+M_Y}{q^2}\right). \tag{3.10}$$

The multipole expansion of the amplitudes F_i are given by CGLN,[6] and

$$\begin{aligned} F_1&=\sum(lM_{l+}+E_{l+})P_{l+1}'(x) \\ &\quad+\sum[(l+1)M_{l-}+E_{l-}]P_{l-1}'(x), \\ F_2&=\sum[(l+1)M_{l+}+lM_{l-}]P_l'(x), \\ F_3&=\sum(E_{l+}-M_{l+})P_{l+1}''(x)+\sum(E_{l-}+M_{l-})P_{l-1}''(x), \\ F_4&=\sum(M_{l+}-E_{l+}-M_{l-}-E_{l-})P_l''(x). \end{aligned} \tag{3.11}$$

In our model, the contributions to a_i from states in channel I are assumed to come from the following pion-nucleon resonances: the $P_{3/2}$ state with $T=3/2$ at $W=1238$ MeV, the $D_{3/2}$ state with $T=1/2$ at $W=1510$ MeV, and the $F_{5/2}$ state with $T=1/2$ at $W=1680$ MeV. From unitarity and conservation of parity and angular momentum, we see that these resonances contribute to the spectral functions $a_i(s,t)$ through the amplitudes M_{1+}, E_{1+}; E_{2-}, M_{2-}; E_{3-}, M_{3-}, respectively. In the case of the photoproduction of pion on nucleon[6] the amplitude M_{1+} is much larger than E_{1+}. We assume that the same thing holds for the photoproduction of K-meson and only the multipole amplitudes with the lower order, i.e., M_{1+}, E_{2-}, and E_{3-} contribute to $a_i(s,t)$. We will represent these amplitudes by φ_i with $i=1$, 2, and 3, respectively. Then the relevant multipole expansion of F_i can be expressed in a matrix form as follows:

$$F_i=\alpha_{ij}(x)\varphi_i, \tag{3.12}$$

with

$$\alpha_{ij}=\begin{bmatrix} 3x & 1 & 3x \\ 2 & 0 & 0 \\ -3 & 0 & 3 \\ 0 & -3 & -15x \end{bmatrix}. \tag{3.13}$$

Further, from the isotopic spin of the resonances, we have the following conditions on the isotopic spin components of φ_i:

$$\begin{aligned} &M_{1+}^{(S)}=M_{1+}^{(V)}=0, \\ &M_{1+}^{(+)}=-2M_{1+}^{(-)}, \quad M_{1+}^{(0)}=0, \\ &E_{2-}^{(+)}=E_{2-}^{(-)}, \\ &E_{3-}^{(+)}=E_{3-}^{(-)}. \end{aligned} \tag{3.14}$$

Now putting together Eqs. (3.7) and (3.12), we have an expression for $a_i(s,t)$:

$$a_i(s,t)=\xi_{ij}(s,x)\alpha_{jk}(x)\,\mathrm{Im}\,\varphi_k(s), \tag{3.15}$$

and as contributions to K-meson photoproduction from the π-N resonances, we have

$$A_i^{\mathrm{I}}=\frac{1}{\pi}\int\frac{\xi_{ij}(s',x')\alpha_{jk}(x')\,\mathrm{Im}\,\varphi_k(s')}{s'-s}ds'. \tag{3.16}$$

Here x' should be expressed in terms of s' and t as

$$x'=(1/2k'q')(t-m_K^2+2k'\omega'), \tag{3.17}$$

and the integration over s' should be performed at a fixed t.

For $\mathrm{Im}\,\varphi_k(s)$ we assume a relativistic generalization of the Breit-Wigner formula:

$$\mathrm{Im}\,\varphi_k(s)=(1/4W)\{\Gamma_k(\Gamma_{ki}\Gamma_{kf})^{\frac{1}{2}}/[(W-W_r)^2+\tfrac{1}{4}\Gamma_k^2]\}, \tag{3.18}$$

where Γ=total width of π-N resonance, Γ_i=a partial width for $\gamma+N\rightarrow N^*$, and Γ_f=a partial width for $K+Y\rightarrow N^*$. Γ_i and Γ_f depends on the energy as follows:

$$\Gamma_i=\gamma_i k^{2L},\qquad \Gamma_f=\gamma_f q^{2l}, \tag{3.19}$$

where L and l are the orbital angular momenta of a photon and a K meson, and γ_i and γ_f are slowly varying functions of energy.

For the resonances below the threshold of K-Y production, the denominator $s'-s$ in Eq. (3.16) does not vanish in the physical region of channel I; therefore, we can make a zero width approximation for them when integrating over s'. Then Eq. (3.18) reduces to

$$\mathrm{Im}\,\varphi_k=(\pi/2W)(\Gamma_{ki}\Gamma_{kf})^{\frac{1}{2}}\delta(W-W_k)\equiv\pi\varphi_{k0}\delta(W^2-W_k{}^2), \tag{3.20}$$

and the integral in Eq. (3.16) reduces to

$$A_i{}^{\mathrm{I}}=\frac{1}{s_k-s}\xi_{ij}(s_k,x_k)\alpha_{jk}(x_k)\varphi_{k0}(s_k), \tag{3.21}$$

where s_k and x_k are the values of s and x at $W=W_k$, the energy of the kth resonance.

(ii) Even K-Y Parity Case

The T matrix, defined by

$$T=\sum_{i=1}^{4}\bar{A}_i(s,t,u)\bar{O}_i, \tag{3.22}$$

can be reduced to the following $\bar{F}$ matrix:

$$\bar{F}=(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\boldsymbol{\sigma}\cdot\mathbf{k}/k)\bar{F}_1+(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}/q)\bar{F}_2+(\mathbf{q}\cdot\boldsymbol{\varepsilon}/q)\bar{F}_3+(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\varepsilon}\cdot\mathbf{q}/q^2k)\bar{F}_4. \tag{3.23}$$

The matrix $\bar{\xi}_{ij}(s,x)$ that relates $\bar{A}_i$ to $\bar{F}_i$, as in Eq. (3.7), is represented as a product of a matrix $\zeta_{ij}(s)$ defined by Eq. (3.10) and the following matrix $\bar{\eta}_{ij}(s,x)$:

$$\bar{\xi}_{ij}(s,x)=\bar{\eta}_{ik}(s,x)\zeta_{kj}(s), \tag{3.24}$$

$$\bar{\eta}_{ij}=\frac{1}{2W}\begin{bmatrix} W-M & -(W+M) & -(M+M_K)(W-M)-\dfrac{2Mk\cdot q}{W+M} & -(M+M_Y)(W+M)-\dfrac{2Mk\cdot q}{W-M} \\ 0 & 0 & 1 & -1 \\ -1 & -1 & -(W-M)-\dfrac{k\cdot q}{W+M} & -(W+M)-\dfrac{k\cdot q}{W-M} \\ -1 & -1 & -\dfrac{k\cdot q}{W+M} & -\dfrac{k\cdot q}{W-M} \end{bmatrix}. \tag{3.25}$$

The multipole expansion for $\bar{F}_i$ reads as follows:

$$\begin{aligned}
\bar{F}_1&=\sum[(l+1)\bar{M}_{l+}P_{l+1}'(x)-l\bar{M}_{l-}P_{l-1}'(x)],\\
\bar{F}_2&=\sum[(l+2)\bar{M}_{l+}-(l-1)\bar{M}_{l-}-\bar{E}_{l+}+\bar{E}_{l-}]P_l'(x),\\
\bar{F}_3&=\sum[(\bar{E}_{l+}-\bar{M}_{l+})P_{l+1}''(x)-(\bar{M}_{l-}+\bar{E}_{l-})P_{l-1}''(x)],\\
\bar{F}_4&-\sum[\bar{M}_{l+}+\bar{M}_{l-}-\bar{E}_{l+}+\bar{E}_{l-}]P_l''(x).
\end{aligned} \tag{3.26}$$

From the selection rules, we see that the first, second and the third π-N resonances contribute to $\bar{a}_i$ through $\bar{M}_{2-}$, $\bar{E}_{1+}$, and $\bar{E}_{2+}$, respectively, if only the lower multipole amplitudes are retained. We will denote these amplitudes by $\bar{\varphi}_i$ with $i=1$, 2 and 3 respectively, then we have

$$\bar{F}_i=\bar{\alpha}_{ij}\bar{\varphi}_j, \tag{3.27}$$

with

$$\bar{\alpha}_{ij}=\begin{bmatrix} -2 & 0 & 0 \\ -3x & -1 & -3x \\ 0 & 3 & 15x \\ 3 & 0 & -3 \end{bmatrix}. \tag{3.28}$$

Of course, the isotopic spin dependence remains the same as the previous case.

From Eqs. (3.7) and (3.27), we have as contributions from πN resonances, an equation similar to Eq. (3.16) with bars on ξ, α, and φ.

4. π-Y RESONANCE

(i) Even K-Y Parity

In order to derive the contribution from the π-Y resonances, we introduce the S-matrix for the reaction in channel II, $\gamma+\bar{Y}\rightarrow\bar{N}+K$, which by the invariance under charge conjugation is equivalent to $\gamma+Y\rightarrow N+\bar{K}$. By the substitution rule, we can obtain the S-matrix element for $\gamma+\bar{Y}\rightarrow K+\bar{N}$ from that of $\gamma+N\rightarrow K+Y$ by replacing p_1, p_2, $u(p_1)$ and $u(p_2)$ by $-p_1'$, $-p_2'$, $v(p_1')$ and $\bar{v}(p_2')$ respectively. Further, we express the wave function v in terms of u as follows:

$$v(p_1')=C\bar{u}^T(p_1'),\qquad \bar{v}(p_2')=u^T(p_2')C^{-1T}, \tag{4.1}$$

where C is the matrix with the following properties:

$$C\gamma_\mu{}^TC^{-1}=-\gamma_\mu,\qquad C^T=-C. \tag{4.2}$$

Then we have

$$S_{fi}{}^{\mathrm{II}}=-\frac{i}{(2\pi)^2}\left(\frac{MM_Y}{4E_1'E_2'k\omega}\right)^{\frac{1}{2}}\bar{u}(p_1')Tu(p_2'), \quad (4.3)$$

with

$$T=\sum_{i=1}^{4} A_i'O_i'. \quad (4.4)$$

The matrices O_i' are the same as O_i except that p_1 and p_2 are replaced by p_1' and p_2'. The A_i' are related to A_i as follows:

$$A_i=\beta_{ij}A_j', \quad (4.5)$$

with

$$\beta_{ij}=\begin{bmatrix}1 & 0 & 2(M_Y-M) & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}. \quad (4.6)$$

In the barycentric system, we denote the energy and three-momenta of each particle as follows:

$$\begin{aligned} k&=(\mathbf{k},\kappa), & p_2'&=(-\mathbf{k},E_2'),\\ q&=(\mathbf{q},\omega), & p_1'&=(-\mathbf{q},E_1'), \end{aligned} \quad (4.7)$$

where the energies and the magnitudes of the momenta are related to the total energy of channel II by the following formula:

$$\begin{aligned} E_1'&=(W^2+M^2-m_K{}^2)/2W, & \omega&=(W^2+m_K{}^2-M^2)/2W,\\ E_2'&=(W^2+M_Y{}^2)/2W, & \kappa&=(W^2-M_Y{}^2)/2W, \quad (4.8)\\ q&=\{[(W-M)^2-m_K{}^2][(W+M)^2-m_K{}^2]\}^{\frac{1}{2}}/2W. \end{aligned}$$

The T matrix (4.4) is the same as the corresponding Eq. (3.5) in channel I, except that p_1 and p_2 are interchanged, therefore, it can be reduced to the F' matrix which is formally identical to the F matrix defined by Eq. (3.6) for channel I, and the matrix $\xi_{ij}'(u,y)$ that relates A_i' and F_i' can be obtained from Eqs. (3.8) to (3.10) by interchanging E_1 and M for E_2 and M_Y, except for those M and M_Y appearing explicitly in the definition of O_3 and O_4 given by Eqs. (2.8c) and (2.8d). Thus we have

$$A_i'=\xi_{ij}'(u,y)F_j', \quad (4.9)$$

with

$$\xi_{ij}'(u,y)=\eta_{ik}'(u,y)\zeta_{kj}'(u),$$

$$\eta_{ij}'=\frac{1}{2W}\begin{bmatrix} W-M_Y+2M & -(W+M_Y-2M) & -(M_Y-M)(W+M_Y)+\dfrac{2Mq\cdot k}{W-M_Y} & -(M_Y-M)(W-M_Y)+\dfrac{2Mq\cdot k}{W+M_Y}\\ 0 & 0 & 1 & -1\\ 1 & 1 & W+M_Y+\dfrac{q\cdot k}{W-M_Y} & W-M_Y+\dfrac{q\cdot k}{W+M_Y}\\ 1 & 1 & \dfrac{q\cdot k}{W-M_Y} & \dfrac{q\cdot k}{W+M_Y} \end{bmatrix}, \quad (4.10)$$

and

$$\zeta_j'=\frac{8\pi W}{W-M_Y}[(E_1+M)(E_2+M_Y)]^{-\frac{1}{2}}\left(1,\frac{E_1+M}{q},\frac{1}{q},\frac{E_1+M}{q^2}\right), \quad (4.11)$$

where $y=\cos\theta'$, θ' being a scattering angle in the c.m. system in channel II. The multipole expansion of F_i' is identical to that of F_i in channel I.

The spin of $Y_1{}^*$ was recently found[11] to be larger than 1/2 but its parity is not yet determined. For even Λ-Σ parity, the most reasonable assumption would be the values obtained from global symmetry, i.e., P wave and $J=3/2$. As for the other two resonances[12] $Y_0{}^*$ and $Y_0{}^{**}$ with $T=0$ at $W=1405$ MeV and $W=1520$ MeV, these will be tentatively assumed to be in the $S_{1/2}$ and $D_{3/2}$ states, respectively. These resonances contribute to the spectral function through the amplitudes M_{1+}, E_{0+}, and E_{2-}, respectively, if only the lower multipole amplitudes are retained. We will express F_i' as

$$F_i'=\alpha_{ij}'(y)\varphi_j', \quad (4.12)$$

with

$$\alpha_{ij}'(y)=\begin{bmatrix} 3y & 1 & 1\\ 2 & 0 & 0\\ -3 & 0 & 0\\ 0 & 0 & -3 \end{bmatrix} \quad (4.13)$$

and $\varphi_i'=M_{1+}, E_{0+}, E_{2-}$ for $i=1, 2, 3$.

Since $Y_1{}^*$ has an isotopic spin 1, the $T=0$ component

[11] R. P. Ely, S. Y. Fung, G. Gidal, Y. L. Pan, W. M. Powell, and H. S. White, Phys. Rev. Letters **7**, 461 (1961).

[12] M. H. Alston, L. W. Alvarez, P. Eberhand, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters **6**, 698 (1961); P. Bastien, M. Ferro-Luzzi, and A. H. Rosenfeld, *ibid.* **6**, 702 (1961); M. Ferro-Luzzi, R. D. Tripp, and M. B. Watson, *ibid.* **8**, 28 (1962).

of M_{1+} vanishes; therefore, we have:

$$M_{1+}^{(S)}=0 \quad \text{for } \Lambda \text{ production}, \tag{4.14a}$$

and

$$M_{1+}^{(+)}=0 \quad \text{for } \Sigma \text{ production}. \tag{4.14b}$$

For the contribution from the resonances with $T=0$, we have the following conditions:

$$\varphi_i^{(V)}=0 \quad (i=2,3) \text{ for } \Lambda \text{ production}, \tag{4.15a}$$

and

$$\varphi_i^{(-)}=\varphi_i^{(0)}=0 \quad (i=2,3) \text{ for } \Sigma \text{ production}. \tag{4.15b}$$

Putting together Eqs. (4.5), (4.9), and (4.12), we obtain the following formula as a contribution from a resonant state in channel II:

$$A_i^{\mathrm{II}}=\frac{1}{\pi}\int\frac{\beta_{ij}\xi_{jk}'(u',y')\alpha_{kr}'(y')\,\mathrm{Im}\,\varphi_r'(u')}{u'-u}du', \tag{4.16}$$

where

$$y'=(1/2k'q')(t-m_K^2+2k'\omega'), \tag{4.17}$$

and the integration is performed at a constant t.

In the physical region of channel I, the denominator in Eq. (4.16) does not vanish; therefore, we can make a zero-width approximation for $\mathrm{Im}\,\varphi_r'$ as follows:

$$\mathrm{Im}\,\varphi_r'=(\pi/2W)(\Gamma_{ri}'\Gamma_{rf}')^{\frac{1}{2}}\delta(W-W_r), \tag{4.18}$$

where Γ_i' = a partial width for $\gamma+Y\to Y^*$ and Γ_f' = a partial width for $\bar{K}+N\to Y^*$.

(ii) Even K-Y Parity

In this case, the crossing of baryon lines and the charge conjugation operation give the following T matrix for $\gamma+Y\to\bar{K}+N$:

$$\bar{T}=\sum_{i=1}^{4}\bar{A}_i'\bar{O}_i', \tag{4.19}$$

with

$$\bar{A}_i=\bar{\beta}_{ij}\bar{A}_j', \tag{4.20}$$

and

$$\bar{\beta}_{ij}=\begin{bmatrix}1&0&0&2(M+M_Y)\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}. \tag{4.21}$$

This T matrix can be reduced to the $\bar{F}'$ matrix defined by Eq. (3.23) with primes on $\bar{F}_i$ to denote that it is a quantity in channel II. The relation between the $\bar{A}_i'$ amplitudes and the $\bar{F}_i'$ amplitudes is given by

$$\bar{A}_i'=\bar{\xi}_{ij}'(u,y)\bar{F}_j'=\bar{\eta}_{ij}'(u,y)\bar{\zeta}_{jk}'(u)\bar{F}_k', \tag{4.22}$$

with

$$\bar{\eta}_{ij}'=\frac{1}{2W}\begin{bmatrix}W+M_Y-M & -(W-M_Y+2M) & -(W-M_Y)(M+M_Y)-\dfrac{2Mq\cdot k}{W+M_Y} & -(W+M_Y)(M+M_Y)-\dfrac{2Mq\cdot k}{W-M_Y}\\ 0&0&1&-1\\ -1&-1&-W+M_Y-\dfrac{q\cdot k}{W+M_Y}&-W+M_Y-\dfrac{q\cdot k}{W-M_Y}\\ -1&-1&-\dfrac{q\cdot k}{W+M_Y}&-\dfrac{q\cdot k}{W-M_Y}\end{bmatrix}, \tag{4.23}$$

and $\bar{\zeta}_{jk}'=\zeta_{jk}'$ in Eq. (4.11). The multipole expansion of $\bar{F}_i'$ is given by Eq. (3.26). In the even K-Y parity case, we will take into account the possibility of both a $P_{3/2}$ and a $D_{3/2}$ state for Y_1^*. Y_0^* and Y_0^{**} will be assumed to be in $S_{1/2}$ and $D_{3/2}$ states. If only the lower multipole amplitudes are retained, $P_{3/2}$, $S_{1/2}$, and $D_{3/2}$ states can contribute only to $\bar{M}_{2-}$, $\bar{E}_{1-}$, and $\bar{E}_{1+}$, which will be denoted by $\bar{\varphi}_i$ with $i=1,2,3$. Then we have

$$\bar{F}_i'=\bar{\alpha}_{ij}'(y)\bar{\varphi}_j'(u), \tag{4.24}$$

$$\bar{\alpha}_{ij}=\begin{bmatrix}-2&0&0\\-3y&1&-1\\0&0&3\\3&0&0\end{bmatrix}. \tag{4.25}$$

From Eqs. (4.20), (4.22), and (4.24), we have equations similar to (4.16) as contributions from the π-Y resonances in the even K-Y parity case.

5. K-π RESONANCE

(i) Odd K-Y Parity

In order to take into account the contributions from the π-K resonance, we have to consider the reaction in channel III, i.e., $\gamma+\bar{K}\to\bar{N}+Y$. The S matrix for this process can be obtained from that for the reaction in channel I by replacing p_1, q, and $u(p_1)$ by $-p_1''$, $-q''$, and $v(p_1'')$, respectively. Thus we have

$$S_{fi}^{\mathrm{III}}=-[i/(2\pi)^2](MM_Y/4E_1E_2k\omega)^{\frac{1}{2}}\delta(P_i-P_f)\times\bar{u}(p_2)T(p_2,-q'';-p_1'',k)v(p_1''). \quad (5.1)$$

In the center-of-mass system, we introduce the following three-momenta and energies:

$$\begin{aligned} k&=(\mathbf{k},\kappa), & q''&=(-\mathbf{k},\omega),\\ p_1''&=(-\mathbf{p},E_1), & p_2&=(\mathbf{p},E_2), \end{aligned} \quad (5.2)$$

and these can be expressed as functions of total energy $W=\sqrt{t}$ in channel III as follows:

$$\begin{aligned} \kappa&=(W^2-m_K^2)/2W, \qquad \omega=(W^2+m_K^2)/2W,\\ E_1&=(W^2+M^2-M_Y^2)/2W, \quad E_2=(W^2-M^2+M_Y^2)/2W,\\ p&=\{[(W-M)^2-M_Y^2][(W+M)^2-M_Y^2]\}^{\frac{1}{2}}/2W. \end{aligned} \quad (5.3)$$

We also define the following F'' matrix:

$$\bar{u}(p_2)Tv(p_1'')=4\pi\frac{W}{(MM_Y)^{\frac{1}{2}}}\chi_fF''\chi_i, \quad (5.4)$$

and

$$F''=[\boldsymbol{\varepsilon}\cdot\mathbf{p}/p]F_1''+[i\boldsymbol{\sigma}\cdot(\mathbf{p}\times\boldsymbol{\varepsilon})/p]F_2''+[i\boldsymbol{\sigma}\cdot\mathbf{pp}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})/p^2k]F_3''+[i\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})/k]F_4''. \quad (5.5)$$

Then the matrix $\xi_{ij}''(t)$ that expresses A_i in terms of F_i'' depends on only the energy variable t, and, as before, ξ_{ij}'' will be written as a product of two matrices:

$$A_i=\xi_{ij}''(t)F_j''=\eta_{ik}''(t)\zeta_{kj}''(t)F_j'', \quad (5.6)$$

with

$$\eta_{ij}''=\begin{bmatrix} 0 & \dfrac{\Delta}{W^2} & \dfrac{W^2-\Delta^2}{W(W+2\bar{M})} & \dfrac{W^2-\Delta^2}{W^2(W+2\bar{M})}\\ -\dfrac{1}{W^2-\Delta^2} & \dfrac{-\Delta}{W^2(W^2-\Delta^2)} & \dfrac{-1}{W(W+2\bar{M})} & \dfrac{-1}{W^2(W+2\bar{M})}\\ 0 & -\dfrac{1}{W^2} & \dfrac{\Delta}{W(W+2\bar{M})} & \dfrac{\Delta}{W^2(W+2\bar{M})}\\ 0 & 0 & \dfrac{2\bar{M}}{W(W+2\bar{M})} & \dfrac{-1}{W(W+2\bar{M})} \end{bmatrix} \quad (5.7)$$

and

$$\zeta_j''(t)=\frac{8\pi W[(E_1+M)(E_2+M_Y)]^{\frac{1}{2}}}{2p^2k(W+2\bar{M})}\left(2p,\frac{2p}{W},W+2\bar{M},W^2-4\bar{M}^2\right), \quad (5.8)$$

where the following abbreviations are used:

$$\bar{M}=\tfrac{1}{2}(M+M_Y),\quad \Delta=M_Y-M. \quad (5.9)$$

The multipole expansion of the amplitudes F_i'' is identical to the expansion of the amplitudes for $\gamma+\pi\rightarrow N+\bar{N}$ given by Ball[7]:

$$\begin{aligned} F_1''&=-\textstyle\sum(J+\frac{1}{2})E_{J0}P_J'(z),\\ F_2''&=-\textstyle\sum\{E_{J1}\frac{1}{2}[JP_{J+1}''(z)+(J+1)P_{J-1}''(z)] -(J+\frac{1}{2})M_{J1}P_J''(z)\},\\ F_3''&=\textstyle\sum\{\frac{1}{2}M_{J1}[JP_{J+1}''(z)+(J+1)P_{J-1}''(z)] -(J+\frac{1}{2})E_{J1}P_J''(z)-(J+\frac{1}{2})\mathfrak{M}_{J1}P_J'(z)\},\\ F_4''&=-\textstyle\sum\{\frac{1}{2}M_{J1}[JP_{J+1}''(z)+(J+1)P_{J-1}''(z)] -(J+\frac{1}{2})E_{J1}P_J''(z)\}, \end{aligned} \quad (5.10)$$

where M_{J1} and $\mathfrak{M}_{J1}$ (or E_{J1} and E_{J0}) represent magnetic (or electric) transitions, the first and second suffixes indicating the total angular momentum and total spin, respectively, of the antinucleon-hyperon pair in the final state. The parities of the triplet final states are $(-1)^J$ for M_{J1} and $\mathfrak{M}_{J1}$ and $(-1)^{J+1}$ for E_{J1}.

So far the spin of K-π resonance at total energy $W=884$ MeV is known only to the extent that it is either 0 or 1.[13] But since we cannot construct a gauge-invariant scalar out of a photon polarization vector ϵ and two independent four-momenta, the K-π resonance with $J=0$ can give no contribution to the reaction $\gamma+\bar{K}\rightarrow\bar{N}+Y$ and, hence, to the photoproduction of the K-Y pair either. Therefore, the K-π resonance will contribute only if it has $J=1$, and, in this case, contributions to c_i comes only from M_{11} and $\mathfrak{M}_{11}$, which will be denoted by φ_i'' with $i=1$ and 2. The relevant multipole expansion reduces to

$$F_i''=\alpha_{ij}''\varphi_j'', \quad (5.11)$$

with

$$\alpha_{ij}''=\begin{bmatrix} 0 & 0\\ 0 & 0\\ \frac{3}{2} & -\frac{3}{2}\\ -\frac{3}{2} & 0 \end{bmatrix}. \quad (5.12)$$

[13] M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. J. Wojcicki, Phys. Rev. Letters **6**, 300 (1961).

Since the isotopic spin of the K-π resonance is 1/2, only $T=1/2$ components of φ_i'' contributes.

Gathering Eqs. (5.6) and (5.11), we have the following contribution from the π-K resonance:

$$A_i^{\mathrm{III}}=\frac{1}{\pi}\int\frac{\xi_{ij}''(t')\alpha_{jk}''\,\mathrm{Im}\varphi_k''(t')}{t'-t}dt'. \tag{5.13}$$

Since $t'-t\neq0$ in the physical region of channel I, we can again use a zero-width formula for $\mathrm{Im}\varphi_k''$:

$$\mathrm{Im}\varphi_k''=(\pi/2W)(\Gamma_i''\Gamma_f'')^{\frac{1}{2}}\delta(W-W_r), \tag{5.14}$$

where $\Gamma_i''=$ a partial width for $\gamma+\bar{K}\to\bar{K}^*$ and $\Gamma_f''=$ a partial width for $\bar{N}+Y\to\bar{K}^*$.

It should be noted that Eq. (5.13) holds only when K^* has $J=1$ and, if it has spin 0, we have

$$A_i^{\mathrm{III}}=0. \tag{5.15}$$

(ii) Even K-Y Parity

For even K-Σ parity, the $\bar{F}_i''$ matrix corresponding to (5.5) in the previous subsection will be defined by

$$\bar{F}''=[i\mathbf{p}\cdot(\boldsymbol{\varepsilon}\times\mathbf{k})/pk]\bar{F}_1''+[\boldsymbol{\sigma}\cdot\{\mathbf{p}\times(\boldsymbol{\varepsilon}\times\mathbf{k})\}/pk]\bar{F}_2''+[\mathbf{p}\cdot\boldsymbol{\varepsilon}\boldsymbol{\sigma}\cdot\mathbf{p}/p^2]\bar{F}_3''+\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\bar{F}_4''. \tag{5.16}$$

Then we have the following relation between A_i and $\bar{F}_i''$:

$$A_i=\bar{\xi}_{ij}''(t)\bar{F}_j'', \tag{5.17}$$

with

$$\bar{\xi}_{ij}''(t)=\bar{\eta}_{ik}''(t)\bar{\zeta}_{kj}''(t) \tag{5.18}$$

$$\bar{\eta}'_{ij}''=\begin{pmatrix} -\dfrac{W^2-4\bar{M}^2}{W^2-\Delta^2} & \dfrac{\Delta(W^2-4\bar{M}^2)}{W^2(W^2-\Delta^2)} & 0 & -\dfrac{2\bar{M}}{W} \\ \dfrac{1}{W^2-\Delta^2} & \dfrac{-\Delta}{W^2(W^2-\Delta^2)} & \dfrac{-1}{W(W+2\bar{M})} & \dfrac{-1}{W^2(W+2\bar{M})} \\ \dfrac{2\bar{M}}{W^2-\Delta^2} & \dfrac{-2\Delta\bar{M}}{W^2(W^2-\Delta^2)} & 0 & \dfrac{-1}{W^2} \\ \dfrac{\Delta}{W^2-\Delta^2} & \dfrac{-1}{W^2-\Delta^2} & 0 & 0 \end{pmatrix}, \tag{5.19}$$

$$\bar{\zeta}_j''=\zeta_j''. \tag{5.20}$$

The multipole expansion of $\bar{F}_i''$ can be obtained from Eq. (5.10) for the odd K-Y parity case by exchanging the roles of electric and magnetic amplitudes, i.e., by replacing M_{J1} and $\mathfrak{M}_{J1}$ by $\bar{E}_{J1}$ and $\bar{\mathcal{E}}_{J1}$ and also replacing E_{J1} and E_{J0} by $\bar{M}_{J1}$ and $\bar{M}_{J0}$, respectively.

As in the odd K-Y parity case, there is no contribution from the K-π resonance unless its spin is 1, in which case only the $\bar{M}_{J0}$ and $\bar{M}_{J1}$ amplitudes can contribute to $\bar{c}_i$. Denoting these amplitudes by $\bar{\varphi}_i''$ with $i=1$ and 2 respectively, we have

$$\bar{F}_i''=\bar{\alpha}_{ij}''\bar{\varphi}_j'', \tag{5.21}$$

with

$$\bar{\alpha}_{ij}''=\begin{pmatrix} -\frac{3}{2} & 0 \\ 0 & -\frac{3}{2} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{5.22}$$

Combining Eqs. (5.17) and (5.21), we obtain an equation similar to Eq. (5.13) but "with bars" as a contribution from the K-π resonance for even K-Σ parity.

6. CROSS SECTION AND POLARIZATION

Now putting together the results of the three preceding sections, we have the following expressions for the K-meson photoproduction amplitude:

$$A_i=A_i^{\mathrm{I}}+A_i^{\mathrm{II}}+A_i^{\mathrm{III}}+A_i^{B}, \tag{6.1}$$

where A_i^{I}, A_i^{II}, and A_i^{III} are defined by Eqs. (3.16), (4.16), and (5.13), respectively (or the corresponding equations with bars for even K-Y parity).

For the calculation of cross sections and polarizations, it is more convenient to work with the F_i amplitudes defined by Eq. (3.5). By taking the inverse of Eq. (3.7), we have

$$F_i=(\xi^{-1})_{ij}A_j, \tag{6.2}$$

with

$$\xi^{-1}=\zeta^{-1}\eta^{-1}, \tag{6.3}$$

$$(\zeta^{-1})_i=\frac{W-M}{8\pi W}[(E_1+M)(E_2+M_Y)]^{\frac{1}{2}}\left(1,\frac{q}{E_2+M_Y},q,\frac{q^2}{E_2+M_Y}\right), \tag{6.4}$$

$$(\eta^{-1})_{ij}=\begin{bmatrix} 1 & 0 & M_Y-M-\dfrac{q\cdot k}{W-M} & W-M_Y+\dfrac{q\cdot k}{W-M} \\ -1 & 0 & M-M_Y-\dfrac{q\cdot k}{W+M} & W+M_Y+\dfrac{q\cdot k}{W+M} \\ 0 & W+M & 1 & -1 \\ 0 & -(W+M) & 1 & -1 \end{bmatrix} \tag{6.5}$$

for odd K-Y parity, and

$$\bar{\xi}^{-1}=\bar{\zeta}^{-1}\bar{\eta}^{-1}, \tag{6.6}$$

with

$$\bar{\zeta}^{-1}=\zeta^{-1} \tag{6.7}$$

and

$$(\bar{\eta}^{-1})_{ij}=\begin{bmatrix} 1 & 0 & -M_Y-M-\dfrac{q\cdot k}{W+M} & -W+M-\dfrac{q\cdot k}{W+M} \\ -1 & 0 & M_Y+M-\dfrac{q\cdot k}{W-M} & -W-M_Y-\dfrac{q\cdot k}{W-M} \\ 0 & W+M & -1 & 1 \\ 0 & -W+M & -1 & 1 \end{bmatrix} \tag{6.8}$$

for even K-Y parity.

The cross section $d\sigma/d\Omega$ can be expressed in terms of F_i as follows:

$$d\sigma/d\Omega=(q/k)\{|F_1|^2+|F_2|^2-2x\,\mathrm{Re}(F_1{}^*F_2) +(1-x^2)[\tfrac{1}{2}|F_3|^2+\tfrac{1}{2}|F_4|^2+\mathrm{Re}(F_4{}^*F_1+F_3{}^*F_2) +x\,\mathrm{Re}(F_3{}^*F_4)]\} \tag{6.9}$$

for both odd and even K-Y parity.

The polarization $\mathcal{P}$ of the produced hyperon is given by

$$(d\sigma/d\Omega)\mathcal{P}=(q/k)(1-x^2)^{\frac{1}{2}}\,\mathrm{Im}\{2F_1F_2{}^*+F_1F_3{}^*-F_2F_4{}^* +x(F_1F_4{}^*-F_2F_3{}^*)-(1-x^2)F_3F_4{}^*\} \tag{6.10}$$

for both odd and even K-Y parity.

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