Stability of the Graviton Against Radiative Decay

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Dimensional and covariance considerations, as well as an examination of the linearized Einstein-Maxwell theory, show that a graviton cannot decay directly into a pair of photons.

CONSERVATION laws allow for the direct radiative decay of the graviton, described in symbols by

$$\Gamma \to \gamma + \gamma. \tag{1}$$

Here, Γ represents a graviton of four-momentum $k_\mu = (-k,\mathbf{k})$, $k = |\mathbf{k}|$, and the two γ's represent photons of four-momenta αk_μ and $(1-\alpha)k_\mu$ with the number α greater than zero but less than unity. The two photons necessarily have their four-momenta parallel to k_μ in order to conserve the energy and momentum of the graviton. Furthermore, the helicities of the particles in (1) can be related in an obvious way in order to conserve the spin of the graviton. However, the decay reaction (1) is not a natural process, as shown by the simple arguments which follow. The stability of the graviton against radiative decay is suggested by dimensional and covariance considerations and confirmed by an examination of the linearized Einstein-Maxwell theory.

Let us first consider the general form of an expression for the rate of the decay reaction (1). By dimensional considerations[1] the rate (integrated with respect to α) is given by $r = k\phi(G^{\frac{1}{2}}k)$, where ϕ is a universal function of the dimensionless argument $G^{\frac{1}{2}}k$. In the limit of vanishing gravitational coupling $G \to 0$, physical considerations dictate that the rate of decay vanishes, and hence $\phi(0) = 0$. Now a Lorentz transformation in the direction of the graviton momentum (or the photon momenta) induces the changes $r \to \xi r$ and $k \to \xi k$ where the positive number ξ depends on the velocity of the Lorentz transformation. Thus, with respect to the new coordinate frame, the rate expression is $r = k\phi(\xi G^{\frac{1}{2}}k)$. By comparing this form with the original expression for r, it follows that ϕ must be independent of $G^{\frac{1}{2}}k$ and therefore equal to a numerical constant. But since $\phi(0) = 0$, we must have ϕ identically equal to zero. Hence, r vanishes identically and the graviton is stable against *direct* radiative decay into a pair of photons.[2]

The fact that the graviton is stable against radiative decay is corroborated by considering the interaction of the gravitational and source-free electromagnetic fields, according to linearized Einstein-Maxwell theory. The action of the Einstein-Maxwell theory is

$$I = \frac{1}{16\pi}\int (G^{-1}R - g^{\mu\rho}g^{\nu\sigma}f_{\mu\nu}f_{\rho\sigma})(-g)^{\frac{1}{2}}d^4x, \tag{2}$$

where the tensor indices range from 0 to 3, the signature of the metric is $+2$, and the electromagnetic field tensor $f_{\mu\nu} = -f_{\nu\mu}$ satisfies the subsidiary conditions

$$f_{\mu\nu,\rho} + f_{\nu\rho,\mu} + f_{\rho\mu,\nu} = 0. \tag{3}$$

For practical computations the gravitational components in (2) are expanded in powers of $G^{\frac{1}{2}}$ by writing the metric tensor as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + 2G^{\frac{1}{2}}h_{\mu\nu}, \tag{4}$$

in which $\bar{g}_{\mu\nu} \equiv \mathrm{diag}[-1, 1, 1, 1]$ denotes the Lorentz metric tensor. Expressed in terms of $h_{\mu\nu}$, the curvature scalar density is

$$R(-g)^{\frac{1}{2}} = -Gh^{\mu\nu,\rho}h_{\mu\nu,\rho} + \tfrac{1}{2}Gh^{,\mu}h_{,\mu} + (\text{a complete four-divergence}) + (\text{terms at least cubic in } G^{\frac{1}{2}}h_{\mu\nu}), \quad [h \equiv h^{\mu}{}_{\mu} \equiv \bar{g}^{\mu\nu}h_{\mu\nu}], \tag{5}$$

provided the space-time coordinates are compatible with the conditions

$$(h^{\mu\nu} - \tfrac{1}{2}h\bar{g}^{\mu\nu})_{,\nu} = 0. \tag{6}$$

In (5), (6), and in all subsequent equations, tensor indices are raised with the Lorentz metric. By putting (4) and (5) into (2), discarding a surface integral, and dropping some small terms, one obtains the action for the linearized theory as

$$I = \frac{1}{16\pi}\int (-h^{\mu\nu,\rho}h_{\mu\nu,\rho} + \tfrac{1}{2}h^{,\mu}h_{,\mu} - f^{\mu\nu}f_{\mu\nu} + G^{\frac{1}{2}}h^{\mu\nu}t_{\mu\nu})d^4x, \tag{7}$$

where

$$t_{\mu\nu} \equiv 4f_{\mu}{}^{\rho}f_{\nu\rho} - f^{\rho\sigma}f_{\rho\sigma}\bar{g}_{\mu\nu}. \tag{8}$$

Linearized Einstein-Maxwell equations are derived by equating to zero the variation of (7) with respect to $f_{\mu\nu}$ subject to the subsidiary conditions (3) and the variation of (7) with respect to $h_{\mu\nu}$. The coordinate conditions (6) are relaxed for the variation of (7) with respect to $h_{\mu\nu}$, but applied later for the determination of admissible solutions to the field equations. Dropping some small terms which involve $h_{\mu\nu}$ in the set of equa-

[1] Units are such that $\hbar$ and c equal unity, so that the square root of Newton's gravitational constant $G^{\frac{1}{2}}$ equals 1.6×10^{-33} cm.

[2] Of course, *indirect* radiative decay of the graviton involving virtual π^0's and other fields with nonzero mass cannot be ruled out by this kind of an argument. A dependence of the rate of decay on a practical cutoff length (necessitated by closed loop divergences) is ignored, for the physical rate must be insensitive to such a cutoff length.

tions for $f_{\mu\nu}$, one obtains

$$f^{\mu\nu}{}_{,\nu}=0, \tag{9}$$

$$(2h_{\mu\nu}-h\bar{g}_{\mu\nu}){,}^{\rho}{}_{\rho}=-G^{\frac{1}{2}}t_{\mu\nu}. \tag{10}$$

By virtue of the form of the coupling term $G^{\frac{1}{2}}h^{\mu\nu}t_{\mu\nu}$ in (7), there is no interaction between gravitational radiation and electromagnetic radiation if the two radiation fields propagate in the same direction. This is made clear by considering a gravitational wave which propagates in the x_1 direction together with a linearly polarized electromagnetic wave which propagates in the same direction. To within an infinitesimal coordinate transformation, the relevant solution to Eqs. (3), (6), (9), and (10) is

$$\begin{aligned} &h_{00}=-h_{01}=-h_{10}=h_{11}=\tfrac{1}{2}(x_0+x_1)w', \\ &h_{22}=-w+u, \quad h_{23}=v=h_{32}, \quad h_{33}=-w-u, \\ &f_{02}=-f_{20}=-f_{12}=f_{21}=G^{-\frac{1}{4}}(w'')^{\frac{1}{2}}, \end{aligned} \tag{11}$$

with the other components of $h_{\mu\nu}$ and $f_{\mu\nu}$ equal to zero. In the solution (11) u, v, and w are independent functions of (x_0-x_1). These functions are small compared to $1/G^{\frac{1}{2}}$ but otherwise arbitrary, except for a weak condition on the second derivative of w with respect to (x_0-x_1), namely the condition $w''\geqslant 0$. Gravitational radiation is associated only with the functions u and v, the so-called *traceless transverse* degrees of freedom of the gravitational field.[3,4] The other degrees of freedom of the gravitational field which depend on w do not represent gravitational radiation and are just a concomitant of the electromagnetic field which depends exclusively on w. Since u, v, and w appear in (11) as independent functions, it follows that the two forms of radiation do not interact. Alternatively, one may observe that if the coupling term in (7) $G^{\frac{1}{2}}h^{\mu\nu}t_{\mu\nu}$ is evaluated for the solution (11), the gravitational radiation functions u and v disappear in the result, again showing there is no interaction between gravitational radiation and electromagnetic radiation if the two fields propagate in the same direction. It is easy to verify that this conclusion holds for an arbitrary polarization of the electromagnetic wave. Similarly, in the quantum theory the matrix element for the process (1) vanishes, as seen most readily by writing the matrix element in the formalism of a Feynman quantization[5–7] with the action (7). Therefore, the decay rate for the process (1) vanishes (at least to first order in G) according to quantum perturbation theory based on the Einstein-Maxwell equations.[8]

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[3] P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1942) pp. 187–189.

[4] R. Arnowitt and S. Deser, Phys. Rev. **113**, 745 (1959).

[5] B. E. Laurent, Nuovo cimento **4**, 1445 (1956).

[6] C. W. Misner, Revs. Modern Phys. **29**, 497 (1957).

[7] G. Rosen, thesis, Princeton University, 1959 (unpublished).

[8] It has been pointed out that dimensional-invariance considerations can be applied to the S-matrix element, in order to show that the decay rate vanishes to all orders in G. One notes that the matrix element must be a function of the scalars formed from k_μ, the polarization four-vectors of the photons, and the polarization tensor of the graviton. But the only scalars which do not vanish are formed from the polarization four-vectors and the polarization tensor, scalars which are independent of k_μ and therefore dimensionless. Thus the matrix element cannot depend on G and so vanishes, since it must vanish as $G\rightarrow 0$.