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Approximate Symmetries in the Two-Neutrino Theory*

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It is shown that if there are two neutrinos, the Lagrangian of the electromagnetic interactions and the leptonic weak interactions which preserve strangeness are invariant under three-dimensional rotations in an internal "leptonic spin" space. These rotations transform ν_1 into ν_2 and e into μ. Only the μ-e mass difference does not satisfy this invariance. Some consequences of the symmetry for lepton scattering experiments are discussed. Extensions of the symmetry to other leptonic interactions are considered. In the "conventional" two-neutrino theory, all weak interactions obey the symmetry, and the baryons and mesons are scalars under the rotations. Alternative possibilities in which the baryons form multiplets in leptonic spin space are also presented and their relation to the suggestion of "neutrino flip" interactions is discussed.

I. INTRODUCTION

THE possible existence of two distinct neutrinos, ν_1 and ν_2, with the same helicity has been conjectured in order to account for the nonexistence of muon decays into electrons without neutrinos. In this note we discuss a symmetry of the weak and electromagnetic interactions which is allowed by the two-neutrino theory. The symmetry involves a three parameter rotation group in which the electron rotates into the muon, and the ν_1 rotates into the ν_2. We find that the strangeness-conserving weak interactions are invariant under such a group in the two-neutrino theory, providing that the leptons occur in them through a current

$$J_\lambda = \bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_2 + \bar{e}\gamma_\lambda(1+\gamma_5)\nu_1. \tag{1}$$

This structure for the strangness-conserving weak interactions is dictated by experiment, once two neutrinos are assumed. If we further assume that the same lepton current occurs in strangeness-changing interactions, then all known leptonic interactions will be invariant under the group.

The invariance is only broken by the large muon-electron mass difference, which reduces the symmetry to a one-parameter gauge group. This remaining symmetry leads to the conservation of muon charge,[1] which was the original motivation for considering two neutrinos. Insofar as the mass difference can be neglected, it is possible to deduce relations among various leptonic processes from the three-dimensional symmetry, in the same way as meson-baryon processes are related by using isotopic spin invariance. It seems reasonable that in the very high energy region, where in principle the contribution of weak interactions to lepton cross sections becomes greatest, the mass difference can, in fact, be neglected. A detailed discussion of how symmetries of interactions, broken by mass differences, could manifest themselves in high-energy experiments has been given by Gell-Mann and Zachariasen.[2] We also expect that the mass difference can be neglected in the computation of higher order corrections to the weak interactions. For the purpose of our note, we assume the existence of some domain of validity for the approximation of neglecting the μ-e mass difference.

It should be pointed out that insofar as one treats weak interactions in first-order perturbation theory, symmetry considerations are unnecessary, since explicit expressions for cross sections may be obtained from the assumed interaction. However, it may well be that higher order contributions are important in the high-energy region, especially for lepton processes. Indeed, unless some new length is introduced into the weak interactions, as in the intermediate boson theories, the higher order contributions will necessarily be important at high energies, since they must provide the damping implied by unitarity.[3] Furthermore, some of the processes we consider occur only through higher order effects in the weak interactions. In view of the difficulty of carrying out higher order calculations, any statement can be made about physical processes on the basis of symmetry principles may be regarded as useful.

It is also our opinion that the existence of a new symmetry group for leptons may be of some significance for the understanding of the group theoretical structure of the other elementary particles.

In Sec. II, it is shown that the conjectured Lagrangian can be rewritten in terms of a neutrino doublet ψ_ν and an electron-muon doublet ψ_l in a form which is invariant under unitary transformations on the doublets in a "leptonic spin" space. In Sec. III we consider various leptonic processes and derive relations among them implied by leptonic spin invariance at high energies

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[1] B. Pontecorvo, Zhur. Eksptl. i Teoret. Fiz. **37**, 1751 (1959) [translation: Soviet Phys.—JETP **10**, 1236 (1960)]. G. Feinberg and S. Weinberg, Phys. Rev. Letters **6**, 381 (1961).

[2] M. Gell-Mann and F. Zachariasen, Phys. Rev. **123**, 1065 (1961).

[3] T. D. Lee, CERN lectures, Summer, 1961 (unpublished).

when the electron-muon mass difference is neglected. Section IV is devoted to estimates of cross sections for lepton-lepton scattering and some comments on the validity of neglecting the μ-e mass difference. In Sec. V, we consider the possibility that the baryons also transform under the leptonic spin group. A possible consequence of this would be the phenomenon of "neutrino flip" decays.[4] In the Appendices a 2×2 matrix formalism is developed and it is shown that the two-neutrino theory leads to a Lagrangian involving a four-component neutrino of a type previously discussed by Pauli.[5] The equivalence of the leptonic spin rotation with the Pauli transformation of this field is also demonstrated.

II. LEPTONIC SPIN INVARIANCE

We assume two left-handed neutrinos, ν_1 and ν_2, each having two components and we group them into a neutrino doublet ψ_ν. In the same way, we regard the electron e and the muon μ as forming a doublet ψ_l in the same internal space. That is, we write

$$\psi_\nu=\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix},\quad \psi_l=\begin{pmatrix}e\\ \mu\end{pmatrix}. \tag{2.1}$$

We also assume, for the present, that baryons, mesons, and photons are singlets in this internal space, which we shall refer to as leptonic spin space. This makes the strong interactions trivially invariant under the rotations that act on ψ_ν and ψ_l. These transformations are determined so that ψ_ν, ψ_l act as spinors, and thus are of the form

$$\psi_\nu'=\exp(\tfrac{1}{2}i\boldsymbol{\rho}\cdot\boldsymbol{\omega})\psi_\nu,\quad \psi_l'=\exp(\tfrac{1}{2}i\boldsymbol{\rho}\cdot\boldsymbol{\omega})\psi_l. \tag{2.2}$$

Here the components of $\boldsymbol{\omega}$ are rotation parameters and those of $\boldsymbol{\rho}$ are Pauli matrices acting in leptonic spin space, so the transformations are a three-dimensional rotation group.

The relevant part of the Lagrangian involving leptons is

$$L=L_e+L_w+L_f, \tag{2.3}$$

where

$$L_e=e\bar{\psi}_l\gamma_\rho\psi_lA_\rho=e(\bar{\psi}_e\gamma_\rho\psi_e+\psi_\mu\gamma_\rho\psi_\mu)A_\rho \tag{2.4}$$

is the electromagnetic interaction, L_w stands for the weak interaction, and L_f is the free lepton Lagrangian. Clearly L_e is invariant under the transformations (2.2).

L_w has the general structure

$$L_w=(G_1/\sqrt{2})J_\lambda{}^\dagger J_\lambda+(G_2/\sqrt{2})(\bar{J}_\lambda{}^\dagger J_\lambda+J_\lambda{}^\dagger\bar{J}_\lambda)+G'(J_\lambda{}'^\dagger S_\lambda+S_\lambda{}^\dagger J_\lambda{}'), \tag{2.5}$$

where

$$J_\lambda=\bar{e}\gamma_\lambda(1+\gamma_5)\nu_1+\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_2=\bar{\psi}_l\gamma_\lambda(1+\gamma_5)\psi_\nu \tag{2.6}$$

is the charged lepton current that appears in μ decay, in β decay, and in μ capture. Note that J_λ is a scalar under the transformations (2.2).

$$\bar{J}_\lambda=\bar{n}\gamma_\lambda(1+a\gamma_5)p+\cdots \tag{2.7}$$

is the strangeness-conserving strong charged current S_λ is the strangeness-changing charged current, and $J_\lambda{}'$ is the lepton current occurring in the strangeness-changing part of L, which is not very well known. The universality of strangeness-conserving weak interactions says that $G_1=G_2$, which is consistent with a current $\times$current interaction of the form $(G_1/\sqrt{2})\mathcal{J}_\lambda{}^\dagger\mathcal{J}_\lambda$, where $\mathcal{J}_\lambda=J_\lambda+\bar{J}_\lambda$, for the strangeness conserving interactions. It has been generally assumed that $J_\lambda{}'=J_\lambda$ in strangeness-changing decays. If this is so, there is only one lepton current in L_w, given in (2.6), and since this has been written as a scalar under the transformations (2.2), it follows that L_w is also invariant under these transformations. The Lagrangian L is further invariant under the lepton number gauge transformations

$$\psi_l\to e^{i\lambda}\psi_l,\quad \psi_\nu\to e^{i\lambda}\psi_\nu, \tag{2.8}$$

which, together with (2.2), generate the two-dimensional unitary group.

It must be emphasized that once the hypothesis of two neutrinos is granted, the form of J_λ is essentially determined by experiment, for $\Delta S=0$ processes. Lepton conservation forbids the replacement of ν_1 or ν_2 by $\nu_1{}^c$ or $\nu_2{}^c$. The replacement of $1+\gamma_5$ by $1-\gamma_5$ in the $\mu\nu_2$ term of the current would lead to the wrong helicity of muons in π decay,[6] or to the wrong Michel parameter.[7] The equality of coupling constants for e and μ terms follows from the branching ratio of $\pi e\nu$ decay to $\pi\mu\nu$ decay, at least to 10% accuracy.

Turning to L_f, we have in the doublet notation

$$L_f=\bar{\psi}_\nu\gamma_\rho\partial_\rho\psi_\nu+\bar{\psi}_l\gamma_\rho\partial_\rho\psi_l+\bar{\psi}_l[\tfrac{1}{2}(m_\mu+m_e)+\tfrac{1}{2}(m_\mu-m_e)\rho_3]\psi_l. \tag{2.9}$$

Thus, the only part of L not invariant under the full rotation group (2.2) is the term in L_f proportional to the $\mu-e$ mass difference. The complete Lagrangian is invariant under rotations around the third axis in leptonic spin space, in addition to charge and lepton number gauge transformations. The lepton number and the third component of leptonic spin can be combined to give the muon charge transformation recently discussed[1]

$$\psi_\nu\to\exp[\tfrac{1}{2}i(1-\rho_3)\alpha]\psi_\nu,\ \psi_l\to\exp[\tfrac{1}{2}i(1-\rho_3)\alpha]\psi_l, \tag{2.10}$$

which leads to an additive selection rule forbidding $\mu\to e\gamma$ to all orders. In the one neutrino theory, there is no invariance under ρ_3 rotations, leaving only the conservation of electric charge and lepton number. In this theory, if the $e-\mu$ mass difference is neglected, then L admits the discrete substitution invariance

[4] G. Feinberg, F. Gürsey, and A. Pais, Phys. Rev. Letters **7**, 208 (1961).

[5] W. Pauli, Nuovo cimento **6**, 204 (1957).

[6] M. Bardon, P. Franzini, and J. Lee, Phys. Rev. Letters **7**, 23 (1961).

[7] R. Plano, Phys. Rev. **119**, 1400 (1960).

$e \leftrightarrow \mu$, instead of the group (2.2) of the two-neutrino theory.

Assuming the validity of the Lagrangian (2.3) we may regard the leptonic spin group (2.2) as a "broken symmetry" of L. This will lead to relations among cross sections for leptonic processes, whenever the $e-\mu$ mass difference can be neglected in comparison with some important parameter. We discuss such relations in the next section.

It may be noted that leptonic spin rotations act on multiplets containing particles with a common charge, and are therefore quite different from an extension of isotopic spin rotations to leptons, which has been considered by some authors.[8] Such an extension may be attempted by regarding $(e\nu_1)$ and $(\mu\nu_2)$ as doublets. In this case a rotational invariance would require the existence of neutral lepton currents, and so be violated by electromagnetism as well as by the lepton mass terms.

III. RELATIONS AMONG HIGH-ENERGY LEPTONIC CROSS SECTIONS

In the following we will consider the consequences of the conservation of the square of the leptonic spin, since the selection rules implied by the conservation of the third component are equivalent to those implied by the conservation of muon charge, which have been discussed elsewhere.[1]

We first consider lepton-lepton scattering. Let the particles in the order they are written have incoming momenta p_1 and p_2 and outgoing momenta p_3 and p_4. The physically distinguishable charged lepton processes are

$$\begin{aligned}
&\text{(a)}\quad e^-+e^- \to e^-+e^-,\\
&\text{(b)}\quad e^-+\mu^- \to e^-+\mu^-,\\
&\text{(c)}\quad e^-+\mu^- \to \mu^-+e^-,\\
&\text{(d)}\quad \mu^-+\mu^- \to \mu^-+\mu^-,
\end{aligned}\tag{3.1}$$

and the charge conjugate processes, when the initial state has lepton number ± 2. Note that $\mu^-+\mu^- \to e^- + e^-$ is forbidden by muon charge conservation. If the initial state has lepton number zero, the allowed lepton scatterings are

$$\begin{aligned}
&\text{(a}'\text{)}\quad e^-+\mu^+ \to e^-+\mu^+,\\
&\text{(b}'\text{)}\quad e^-+e^+ \to \mu^-+\mu^+,\\
&\text{(c}'\text{)}\quad e^-+e^+ \to e^-+e^+,\\
&\text{(d}'\text{)}\quad \mu^-+\mu^+ \to \mu^-+\mu^+,\\
&\text{(e}'\text{)}\quad \mu^-+\mu^+ \to e^-+e^+,
\end{aligned}\tag{3.2}$$

and the charge-conjugate processes. These will all occur through electromagnetic interactions and as second order weak interactions for the Lagrangian (2.5).

Now whenever the $\mu-e$ mass difference can be ignored (or corrected for kinematically in the initial and final states), we have[9]

$$\sigma_a=\sigma_d,\quad \sigma_{c'}=\sigma_{d'},\quad \sigma_{b'}=\sigma_{e'},\tag{3.3}$$

which follows from the particular rotation

$$\begin{aligned}
&e\to\mu, &&\nu_1\to\nu_2\\
&\mu\to -e, &&\nu_2\to -\nu_1,
\end{aligned}\tag{3.4}$$

or from the substitution invariance of the one-neutrino theory.

In the two-neutrino theory, the additional rotational invariance of the Lagrangian implies that the three processes (a), (b), and (c) depend only on two independent amplitudes. In order to discuss the general case, we consider the scattering of two charged leptons with spins $\boldsymbol{\sigma}^{(1)}$ and $\boldsymbol{\sigma}^{(2)}$ and the leptonic spins $\boldsymbol{\varrho}^{(1)}$ and $\boldsymbol{\varrho}^{(2)}$. The scattering is described by a product matrix referring to spin space and leptonic spin spaces. If we neglect the $\mu-e$ mass difference, then for a given incident momentum $\mathbf{p}$ and outgoing momentum $\mathbf{p}'$ in the center-of-mass system, the scattering matrix is

$$M=M_1P_1+M_0P_0,\tag{3.5}$$

where M_1 and M_0 are matrices in spin space and P_1, P_0 are, respectively, the triplet and singlet leptonic spin projection operators, so that

$$P_1=\tfrac{1}{4}(3+\boldsymbol{\varrho}^{(1)}\cdot\boldsymbol{\varrho}^{(2)}),\quad P_0=\tfrac{1}{4}(1-\boldsymbol{\varrho}^{(1)}\cdot\boldsymbol{\varrho}^{(2)}).\tag{3.6}$$

We note that in process (b), the outgoing electron has momentum $+\mathbf{p}'$ (and the muon momentum is $-\mathbf{p}'$) while in process (c) it is the muon that comes out with momentum $+\mathbf{p}'$. Taking matrix elements of M between states with two charged leptons, we obtain the spin density matrices

$$\begin{aligned}
M_a&=\langle ee|M|ee\rangle=M_1,\\
M_b&=\langle e\mu|M|e\mu\rangle=\tfrac{1}{2}(M_1+M_0),\\
M_c&=\langle e\mu|M|\mu e\rangle=\tfrac{1}{2}(M_1-M_0).
\end{aligned}\tag{3.7}$$

Similarly, taking matrix elements between initial and final states with one charged lepton and one charged antilepton, we obtain

$$\begin{aligned}
M_{a'}&=\langle\bar{e}\mu|M|\bar{e}\mu\rangle=M_1',\\
M_{b'}&=\langle\bar{e}e|M|\bar{\mu}\mu\rangle=\tfrac{1}{2}(M_1'+M_0'),\\
M_{c'}&=\langle\bar{e}e|M|\bar{e}e\rangle=\tfrac{1}{2}(M_1'-M_0').
\end{aligned}\tag{3.8}$$

Here we have used the fact that the leptonic spin doublet of antileptons is $\begin{pmatrix}-\mu^c\\ e^c\end{pmatrix}$.

If we consider a given state, then M_1, $\frac{1}{2}(M_1+M_0)$

[8] S. Bludman, Nuovo cimento **9**, 433 (1958).

[9] In the discussion of reference 2, it is shown that relations such as our (3.3) will be modified by Z factors for the muon and electron. In the theory we consider, we expect that the leading term in these Z factors will be the same for muon and electron, and the terms which are different will be of order $Gm^2\sim 10^{-7}$. See the discussion of Sec. IV.

and $\frac{1}{2}(M_1-M_0)$ are complex numbers forming a triangle in the complex plane. The corresponding cross sections for processes (a), (b), and (c) are given by

$$\sigma_a \propto |M_1|^2, \quad \sigma_b \propto \tfrac{1}{4}|M_1+M_0|^2, \quad \sigma_c \propto \tfrac{1}{4}|M_1-M_0|^2, \tag{3.9}$$

where it is understood that kinematic corrections for the $\mu-e$ mass difference have been made. We therefore obtain, in complete analogy with the analyses of nucleon-nucleon[10] and nucleon-antinucleon[11] scattering, the triangular inequalities:

$$\sigma_a^{\frac{1}{2}}+\sigma_b^{\frac{1}{2}} \geq \sigma_c^{\frac{1}{2}} \geq \sigma_a^{\frac{1}{2}}-\sigma_b^{\frac{1}{2}}, \tag{3.10}$$

and their cyclic permutations. There is a similar set for the processes (3.2) with (a), (b), (c) replaced by (a′), (b′), (c′).

We note that if weak interactions remain much smaller than electromagnetic interactions at all energies, then the relations (3.10) follow just from the invariance of L_e under our group, provided that the $\mu-e$ mass difference can be neglected in intermediate states. Such an approximation is probably not valid for higher order electrodynamic corrections, because of the renormalizability of quantum electrodynamics. However, because of this very renormalizability, we expect that these higher order corrections will be small for energies which are not exponentially large.

Turning to neutrino-lepton scattering, we consider the reactions

$$\begin{aligned} &\text{(a)}\ \nu_1+e \to \nu_1+e,\\ &\text{(b)}\ \nu_1+\mu \to \nu_1+\mu,\\ &\text{(c)}\ \nu_1+\mu \to \nu_2+e. \end{aligned} \tag{3.11}$$

These will also be related by triangular inequalities of the form (3.10). As reactions (b) and (c) are probably experimentally inaccessible, we replace them by the following reactions, whose cross sections are equal by invariance under (3.4)

$$\begin{aligned} &\text{(b}_1\text{)}\ \nu_2+e \to \nu_2+e,\\ &\text{(c}_1\text{)}\ \nu_2+e \to \nu_1+\mu. \end{aligned} \tag{3.12}$$

The triangular inequalities will then hold for (a), (b_1), and (c_1).

Similarly, for antineutrino-lepton scattering, we consider

$$\begin{aligned} &\text{(a)}\ \bar{\nu}_2+e \to \bar{\nu}_2+e,\\ &\text{(b)}\ \bar{\nu}_1+e \to \bar{\nu}_1+e,\\ &\text{(c)}\ \bar{\nu}_1+e \to \bar{\nu}_2+\mu, \end{aligned} \tag{3.13}$$

and the corresponding amplitudes again satisfy (3.10).

Let us now consider processes in which leptons are produced by high-energy neutrinos. For example, we consider neutrinos incident on a target nucleus

$$\nu_1+\text{target} \to \text{target}+2\ \text{leptons}+1\ \text{antilepton}. \tag{3.14}$$

[10] D. Feldman, Phys. Rev. **89**, 1159 (1953).

[11] D. Amati and B. Vitale, Nuovo cimento **4**, 145 (1956).

If two of the leptons in the final state are charged, then the reaction must have the form

$$\nu_1+\text{target} \to \text{target}+\nu+l+\bar{l}', \tag{3.15}$$

where l, l' may or may not be different. The outgoing leptons must have a total leptonic spin of $\frac{1}{2}$. The charged leptons l, l' may be combined into a singlet with wave function $(1/\sqrt{2})(\bar{e}e+\bar{\mu}\mu)$ or a triplet with components $e\bar{\mu}$, $(1/\sqrt{2})(\bar{e}e-\bar{\mu}\mu)$, and $\bar{\mu}e$. Hence the final-state wave function will be proportional to

$$A(\bar{e}e+\bar{\mu}\mu)\nu_1+B((\tfrac{2}{3})^{\frac{1}{2}}\bar{\mu}e\nu_2-(\tfrac{1}{6})^{\frac{1}{2}}[\bar{e}e-\bar{\mu}\mu]\nu_1), \tag{3.16}$$

where A, B are functions of momenta and are matrices in spin space. Then

$$\begin{aligned} \langle\nu_1|S|\nu_1\bar{e}e\rangle &= A-(\tfrac{1}{6})^{\frac{1}{2}}B,\\ \langle\nu_1|S|\nu_1\bar{\mu}\mu\rangle &= A+(\tfrac{1}{6})^{\frac{1}{2}}B,\\ \langle\nu_1|S|\nu_2\bar{\mu}e\rangle &= (\tfrac{2}{3})^{\frac{1}{2}}B. \end{aligned} \tag{3.17}$$

These matrix elements correspond to the processes

$$\begin{aligned} &\text{(a)}\ \nu_1+\text{target} \to \nu_2+\bar{\mu}+e+\text{target},\\ &\text{(b)}\ \nu_1+\text{target} \to \nu_1+\bar{e}+e+\text{target},\\ &\text{(c)}\ \nu_1+\text{target} \to \nu_1+\bar{\mu}+\mu+\text{target}, \end{aligned} \tag{3.18}$$

which then also satisfy the triangular relations (3.10). It can be seen that the same set of relations are obtained when any other pair of the leptons of the final state are combined into singlet or triplet.

For ν_2 incident on a target nucleus the 3-component of the lepton spin of initial and final state is $-\frac{1}{2}$ so that the processes satisfying triangular inequalities are

$$\begin{aligned} &\text{(a)}\ \nu_2+\text{target} \to \nu_1+\bar{e}+\mu+\text{target},\\ &\text{(b)}\ \nu_2+\text{target} \to \nu_2+\bar{\mu}+\mu+\text{target},\\ &\text{(c)}\ \nu_2+\text{target} \to \nu_2+\bar{e}+e+\text{target}. \end{aligned} \tag{3.19}$$

Similar relations hold for electron scattering. The processes that satisfy (3.10) are now

$$\begin{aligned} &\text{(a)}\ e+T \to \mu+\bar{\mu}+e+T,\\ &\text{(b)}\ e+T \to e+\bar{e}+e+T,\\ &\text{(c)}\ e+T \to e+\bar{\mu}+\mu+T \end{aligned} \tag{3.20}$$

($T\equiv$target nucleus).

Finally, we may consider "charge exchange" events of the type

$$\bar{\nu}+p \to l+\bar{l}+\bar{l}'+n,$$

or

$$\bar{\nu}+p \to \nu+\bar{\nu}+\bar{l}+n. \tag{3.21}$$

For the first type of event, which involves only charged leptons in the final state, the amplitudes which satisfy the triangular inequalities are

$$\begin{aligned} &\text{(a)}\ \bar{\nu}_2+p \to \bar{\mu}+e+\bar{e}+n,\\ &\text{(b)}\ \bar{\nu}_2+p \to \bar{e}+e+\bar{\mu}+n,\\ &\text{(c)}\ \bar{\nu}_2+p \to \bar{e}+e+\bar{e}+n. \end{aligned} \tag{3.22}$$

Here (c) may be replaced by the symmetric process

$$(c') \ \bar{\nu}_1+p\rightarrow \bar{e}+e+\bar{e}+n. \qquad (3.23)$$

If one does not distinguish between processes (a) and (b) of (3.22), as in the total cross section, one obtains

$$\sigma_a+\sigma_b\geq \tfrac{1}{2}\sigma_c.$$

IV. LEPTON INTERACTIONS AT HIGH ENERGY

In this section we try to estimate the cross section for a typical lepton-lepton scattering process such as $\bar{\nu}_1+e^-\rightarrow \bar{\nu}_2+\mu^-$ at very high energies.

(a) Fermi Theory

In the Fermi theory, the matrix element for $\bar{\nu}_1+e^-\rightarrow \bar{\nu}_2+\mu^-$ is, by definition, constant with energy, and involves a finite number of partial waves. It is therefore necessary that some damping mechanism should exist at high energy in order to make the cross sections consistent with the limits imposed by unitarity. This necessarily involves going beyond the Fermi theory. In this model we therefore can only assume that the cross sections will satisfy the Fermi theory for energies below the "unitarity limit." We then find that

$$\sigma(\bar{\nu}+e^-\rightarrow \nu_2+\mu)\sim G_1^2 m_e E_\nu \sim 10^{-41}(E_\nu/m_p)\ \text{cm}^2, \qquad (4.1)$$

where E_ν is the neutrino energy in the lab system, and we assume $E_\nu \gg m_e$, m_μ. We expect that this will be the approximate size for all lepton-lepton scattering cross sections, which are allowed in first order by the Lagrangian L_w.

When the cross section of (4.1) becomes comparable to the "unitarity limit," which we may take as $1/m_eE_\nu = 1/(E_\nu^{\text{c.m.}})^2$, then the damping must become important and we expect that the cross section will never rise above the value given by equating (4.1) to this limit. This corresponds to a ν energy of $10^8 m_p$ and a cross section $\sim G=10^{-33}$ cm^2. This may be verified by using a particular model for the damping, that of the Heitler theory, in which the Fermi matrix element is taken to be the K matrix, and the T matrix is made unitary in the standard way. In this theory the cross section corresponding to (4.1) becomes

$$\sigma\sim G^2 m_e E_\nu/(1+\pi^2 G^2 m_e^2 E_\nu^2),$$

which approaches zero as $E_\nu\rightarrow\infty$, (4.2)

with a maximum value of G/π at $E_\nu=1/\pi G m_e \sim 10^8$ BeV. The prospects for measurement at such neutrino energies are not very bright at present.

(b) Intermediate Boson Theory

A second model theory we consider is that in which the Fermi interactions are two-step processes involving the emission and absorption of a heavy vector meson. In such a theory, the lowest order matrix element for lepton scattering will have the typical form

$$M\sim G/(1+q^2/\mu^2), \qquad (4.3)$$

where q is the momentum transfer and μ is the meson mass. The total cross section resulting from the matrix element will be

$$\sigma\sim\frac{G^2\mu^2}{1+\mu^2/4m_eE_\nu}. \qquad (4.4)$$

Since now all partial waves now occur in M, the cross section need not go to zero as $E_\nu\rightarrow\infty$ to satisfy unitarity. However, upon projecting out the separate partial wave amplitudes, it is found that as $E_\nu\rightarrow\infty$ they go as $\ln E_\nu/(E_\nu)^{\frac{1}{2}}$ instead of $1/(E_\nu)^{\frac{1}{2}}$ as required. Thus even the lowest approximation to the intermediate boson theory does not give a unitary theory, and it is necessary to use a better approximation, such as the Heitler damping theory. In this case, it is found that as $E_\nu\rightarrow\infty$, the cross section now approaches

$$\sigma\sim(G\mu^2/m_eE_\nu)\tan^{-1}(1/g^2), \quad g^2=G\mu^2, \qquad (4.5)$$

and the partial-wave amplitudes indeed decrease as $1/(E_\nu)^{\frac{1}{2}}$ as required.

The above considerations indicate that the observation of the phenomena we have considered will not be simple (lepton-lepton scattering) in the foreseeable future, since even at cosmic-ray energies, the cross sections are unlikely to become greater than $G\sim10^{-33}$ cm^2.

We turn to the question of the effect of the $\mu-e$ mass difference on radiative corrections to the weak interactions, which is important for the question of whether our symmetry has any domain of validity, Here the crucial point is that the weak interactions. whether Fermi interactions, or mediated by a vector meson, are unrenormalizable by the conventional method. It follows from this that if we find some other method for extracting finite answers from the higher order corrections to weak interactions, the leading terms of these corrections will indeed be independent of the mass difference. To see this concretely, suppose we introduce a cutoff momentum Λ into all integrals over virtual momenta. It is easy to see that in either the Fermi theory or the intermediate boson theory, the leading terms in powers of Λ for any process will involve a series of powers, $(G\Lambda^2)^n$, and be independent of the mass of the intermediate particles. There will indeed be other terms going as $(G\Lambda^2)^n(Gm^2)$. However, these will be smaller by factors of 10^7 than the leading terms, so that if the higher order effects are important at all, they will be essentially the same for muons as for electrons. This conclusion can be explicitly tested in a theory with a prescription for extracting the finite part of a perturbation series which would diverge term by term in the absence of a cutoff, such as the prescription suggested in the intermediate boson theory by Lee and Yang.[3] It is indeed found that the leading corrections are of magnitude comparable to the lowest order term, the neglect of the lepton mass difference is justified, and our symmetry is useful in the sense that it predicts interesting properties of the main terms of the whole perturbation series, rather than a trivial consequence of the lowest order term.

The electromagnetic corrections do not satisfy the above criteria, because the quantum electrodymamics of leptons is renormalizable, so that the leading corrections go as $(\alpha \ln m^2)^n$. However, these corrections are likely in any case to be only a few percent, so that we can neglect them, as we do for the electromagnetic corrections to strong interactions.

We close this section by emphasizing that in our opinion, the need for a theory which gives finite answers for the radiative corrections to weak interactions is as pressing as was the need in 1947 for a theory which gave finite answers in quantum electrodynamics.

V. BARYONS AS DOUBLETS IN LEPTONIC SPIN SPACE

In this section, we discuss the possibility of giving strongly interacting particles nontrivial transformation properties under the leptonic spin group. If baryons do not transform like scalars under this group, they could form doublets ($l=\frac{1}{2}$) or triplets ($l=1$), limiting ourselves to $l\leq 1$ for simplicity. The transformation properties of mesons follow, if, for instance, they are regarded as bound states of baryons and antibaryons.

We have already seen that leptons may be regarded as neutral or charged doublets in leptonic spin space. In generalizing leptonic spin to strongly interacting particles, we retain the property that fermion number and charge commute with leptonic spin. Hence, we must try to group baryons into multiplets with a common charge. Considering only baryons that are stable against strong interactions, it is obvious that we cannot have triplets with a common charge and baryon number, whereas such doublets can be defined. Just as the electron and muon are grouped into a doublet provided their mass difference is neglected, the grouping of baryons into doublets implies the possibility of having physical situations such as the high-energy region, where the effect of mass differences is minimized.

Now, if the doublet approximation makes sense as a possible symmetry shared by weak and strong interactions, then one possibility is to group the four doublets N_1, N_2, N_3, N_4 into two leptonic spin doublets:

$$\zeta=\begin{pmatrix}N_1\\N_2\end{pmatrix},\quad \eta=\begin{pmatrix}N_3\\N_4\end{pmatrix}, \tag{5.1}$$

or alternatively,

$$\zeta'=\begin{pmatrix}N_1\\N_2\end{pmatrix},\quad \eta'=\begin{pmatrix}N_4\\N_3\end{pmatrix}. \tag{5.2}$$

A general Fermi type baryon-lepton interaction invariant under leptonic spin rotation (omitting Dirac operators) will have the form

$$\begin{aligned}L=&G\bar{\psi}_l\psi_\nu[\kappa_0\bar{\zeta}\tau^+\zeta+\kappa_1\bar{\eta}\tau^+\eta+\kappa_2\sqrt{2}\bar{\zeta}\eta]/\sqrt{2}\\&+G\bar{\psi}_l\boldsymbol{\varrho}\psi_\nu[\lambda_0\bar{\zeta}\tau^+\boldsymbol{\varrho}\zeta+\lambda_1\bar{\eta}\tau^+\boldsymbol{\varrho}\eta+\lambda_2\sqrt{2}\bar{\zeta}\boldsymbol{\varrho}\eta]/\sqrt{2}.\end{aligned} \tag{5.3}$$

Here $\boldsymbol{\tau}$ and $\boldsymbol{\varrho}$ operate in the isospin space and leptonic spin space, respectively. κ and λ are dimensionless constants. For the choice (5.2) for the baryon doublets we have an interaction of the same form that we call L', where λ', κ', ζ', η' are substituted for λ, κ, ζ, η in L.

The universality between e and μ in nucleon-lepton interactions gives the restriction

$$\kappa_0=1,\quad \lambda_0=0, \tag{5.4a}$$

or

$$\kappa_0=0,\quad \lambda_0=1. \tag{5.4b}$$

If, until proof of the contrary, we assume that $\Delta S=2$ leptonic decays are absent, we obtain the condition $\lambda_2=0$. A further simplification is obtained if a symmetry between the β decay of nucleons and cascade particles is assumed. This is expressed by

$$\kappa_0=\kappa_1,\quad \lambda_0=\lambda_1. \tag{5.5}$$

The simplified Lagrangians with the doublet choice (5.1) take the form

$$\begin{aligned}L_a=G(\bar{e}\nu_1+\bar{\mu}\nu_2)[&(\bar{p}n+\bar{\Sigma}^+Y^0+Z^0\Sigma^-+\bar{\Xi}^0\Xi^-)/\sqrt{2}\\&+\kappa_2(\bar{p}Z^0+\bar{n}\Sigma^-+\bar{\Sigma}^+\Xi^0+\bar{Y}^0\Xi^-)],\end{aligned} \tag{5.6a}$$

in case (5.4a) is satisfied, or

$$\begin{aligned}L_b=&G\kappa_2(\bar{e}\nu_1+\bar{\mu}\nu_2)(\bar{p}Z^0+\bar{n}\Sigma^-+\bar{\Sigma}^+\Xi^0+\bar{Y}^0\Xi^-)\\&+G(\bar{e}\nu_1-\bar{\mu}\nu_2)[(\bar{p}n-\bar{\Sigma}^+Y^0)/\sqrt{2}-(\bar{\Xi}^0\Xi^--\bar{Z}^0\Sigma^-)/\sqrt{2}]\\&+\sqrt{2}G\bar{e}\nu_2(\bar{\Sigma}^+n+\bar{\Xi}^0\Sigma^-)+\sqrt{2}G\bar{\mu}\nu_1(\bar{p}Y^0+\bar{Z}^0\Xi^-),\end{aligned} \tag{5.6b}$$

in case (5.4b) is satisfied.

For the choice (5.2) for leptonic spin doublets, $\Delta S=2$ transitions are eliminated by taking $\kappa_2'=\lambda_2'=0$. In this case, according as $\lambda_0'=0$ or $\kappa_0'=0$ we obtain

$$L_a'=G(\bar{e}\nu_1+\bar{\mu}\nu_2)(\bar{p}n+\bar{\Sigma}^+Y^0+Z^0\Sigma^-+\bar{\Xi}^0\Xi^-)/\sqrt{2}, \tag{5.7a}$$

or

$$\begin{aligned}L_b'=&G(\bar{e}\nu_1-\bar{\mu}\nu_2)(\bar{p}n-\bar{\Sigma}^+Y^0+\bar{\Xi}^0\Xi^--Z^0\Sigma^-)/\sqrt{2}\\&+\sqrt{2}G\bar{e}\nu_2(\bar{p}Y^0+\bar{\Xi}^0\Sigma^-)+\sqrt{2}G\bar{\mu}\nu_1(\bar{\Sigma}^+n+Z^0\Xi^-).\end{aligned} \tag{5.7b}$$

The models obtained are interesting in several respects. The Lagrangian L_a shows that $\Delta S=1$ leptonic decays can occur with a strength different from $\Delta S=0$ decays and still exhibit leptonic spin invariance. Furthermore, the rule $\Delta S/\Delta Q=+1$ is valid. The lepton pairings (e with ν_1 and μ with ν_2) are the same for $\Delta S=0$ and $\Delta S=1$ processes.

L_b has rather complicated properties. Again the strengths of $\Delta S=0$ and $\Delta S=1$ decays are different. The lepton pairing for $\Delta S=1$ are the same as for $\Delta S=0$ in some cases and different in others. The $\Delta S/\Delta Q=+1$ rule is valid for muonic decays but not for electron decays. L_a' forbids $\Delta S=1$ leptonic decays in the doublet approximation for strong interactions. Such decays can then only occur through violation of doublet symmetry, hence they are strongly suppressed compared with $\Delta S=0$ leptonic decays.

L_b' exhibits a "neutrino flip" for strangeness-changing decays[12] since lepton pairings are opposite for $\Delta S=0$ and $\Delta S=1$ processes. For $\Delta S=1$ we have $\Delta S/\Delta Q=\pm 1$. Should two different neutrinos be shown to exist and the phenomenon of neutrino flip to occur in strangeness-changing decays and should experiment definitely establish the existence of $\Delta S/\Delta Q=-1$ transistions,[13] then the Lagrangian L_b' might be of great interest.

In conclusion to this section we would like to add the following remark: Although invariance with respect to third axis rotations in leptonic spin space is preserved for the above Lagrangian when mass difference are introduced, this invariance should also be valid for nonleptonic decays in order to ensure the absence of the radiative decay of the muon to all orders. The charged current that occurs in L_a is a scalar in leptonic spin space and, if coupled to itself, will give rise to nonleptonic decays without destroying leptonic spin invariance. This condition seems difficult to fulfill in the other models. We are also leaving aside the discussion of compatibility of the $\Delta T=\frac{1}{2}$ rule with leptonic spin invariance. *Note added in proof.* The existence of distinct neutrons ν_1, ν_2 has recently been confirmed.[14]

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APPENDIX I. FORMULATION OF THE THEORY IN TERMS OF A FOUR-COMPONENT NEUTRINO

The two-neutrino theory discussed in this paper is equivalent to a four-component neutrino theory if we define a four component neutrino spinor by

$$\xi=\begin{pmatrix} \nu_1 \\ i\sigma_2\nu_2^* \end{pmatrix}, \tag{AI.1}$$

where ν_1 and ν_2 are two-component neutrinos, and we use the representation with γ_5 diagonal. The charge-conjugate neutrino operator is given by

$$\xi^c=\gamma_2\xi^*=\rho_2\sigma_2\xi^*=\begin{pmatrix} \nu_2 \\ i\sigma_2\nu_1^* \end{pmatrix}, \tag{AI.2}$$

so that the transformation

$$\xi\leftrightarrow\xi^c \tag{AI.3}$$

induces the exchange of ν_1 and ν_2 as in (2.2).

The two neutrinos can be extracted from the ξ-component field by application of suitable projection operators. We have

$$\nu_1=\tfrac{1}{2}(1+\gamma_5)\xi, \tag{AI.4}$$

and

$$\nu_2=\tfrac{1}{2}(1+\gamma_5)\xi^c,$$

showing that both neutrinos are left-handed.

The lepton current (2.6) now reads

$$J_\lambda=\bar{e}\gamma_\lambda(1+\gamma_5)\xi+\bar{\mu}\gamma_\lambda(1+\gamma_5)\xi^c, \tag{AI.5}$$

a form which leads to a weak interaction Lagrangian of the type discussed by Pauli,[5] involving a four-component neutrino field as well as its charge conjugate. It can be verified that the Pauli transformations

$$\xi\rightarrow a\xi+b\gamma_5\xi^c, \tag{AI.6}$$

with $|a|^2+|b|^2=1$, induce the leptonic spin rotations on the two neutrinos, namely

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}\rightarrow\begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \tag{AI.7}$$

which is of the form (2.2) provided we define

$$\exp(\tfrac{1}{2}i\boldsymbol{\varrho}\cdot\boldsymbol{\omega})=\begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}. \tag{AI.8}$$

Hence the Pauli transformations of the four-component neutrino, together with a μ-e leptonic spin rotation with the same parameters (AI.8), leave the Lagrangian invariant, with the exception of the term involving the μ-e mass difference.

When the Lagrangian is written with ξ instead of ν_1, ν_2, the lepton number gauge transformations are

$$e\rightarrow e^{i\lambda}e,\quad \mu\rightarrow e^{i\lambda}\mu,\quad \xi\rightarrow e^{i\gamma_5\lambda}\xi, \tag{AI.9}$$

which generate (2.7). If we take the subgroup of the Pauli transformations with $b=0$, we obtain the group

$$e\rightarrow e^{i\lambda}e,\quad \mu\rightarrow e^{-i\lambda}\mu,\quad \xi\rightarrow e^{i\lambda}\xi. \tag{AI.10}$$

The existence of these two independent groups has led to some confusion of terminology as some authors have referred to (AI.10) rather than (AI.9) as the lepton number gauge group. The terminology is of course a matter of convention when two independent groups exist. Theories in which only one of the two groups exists run into difficulty with experiment in at least one place.

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[13] R. P. Ely, W. M. Powell, H. White, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, O. Fabbri, F. Farini, C. Filippi, H. Huzita, G. Miari, U. Camerini, W. F. Fry, and S. Natali, Phys. Rev. Letters **8**, 132 (1962).

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APPENDIX II. RELATION OF THE LEPTONIC SPIN GROUP TO A LARGER SYMMETRY GROUP

If intermediate bosons exist which mediate weak interactions, and if they are related to gauge groups, it is clear that the part of the weak interaction Lagrangian we have considered is not generated by a boson associated with either (2.2) or (2.8) but rather with the analog of isotopic spin rotations which mix charged leptons with neutrinos. A compact formalism which enables such transformations to be considered simultaneously with those already discussed in the following. We consider the 2×2 matrix

$$\Psi=\begin{pmatrix} e & -\nu_1 \\ \mu & -\nu_2 \end{pmatrix}, \tag{AII.1}$$

and the seven-parameter transformation

$$\Psi \rightarrow e^{i\lambda}\exp(\tfrac{1}{2}i\boldsymbol{\rho}\cdot\boldsymbol{\omega})\Psi\exp(-\tfrac{1}{2}i\boldsymbol{\rho}\cdot\boldsymbol{\omega}) \tag{AII.2}$$

where as before, the $\boldsymbol{\rho}$ are Pauli matrices. The groups parameterized by λ and $\boldsymbol{\omega}$ are the same as (2.8) and (2.2), whereas the rotation group with parameters $\boldsymbol{\omega}'$ commutes with the leptonic spin group and corresponds to the following rotation group of the doublets $(\nu_1 e)$ and $(\nu_2\mu)$:

$$\begin{pmatrix} \nu_1 \\ e \end{pmatrix} \rightarrow \exp(\tfrac{1}{2}i\boldsymbol{\rho}\cdot\boldsymbol{\omega}')\begin{pmatrix} \nu_1 \\ e \end{pmatrix}, \qquad \begin{pmatrix} \nu_2 \\ \mu \end{pmatrix} \rightarrow \exp(\tfrac{1}{2}i\boldsymbol{\rho}\cdot\boldsymbol{\omega}')\begin{pmatrix} \nu_2 \\ \mu \end{pmatrix}. \tag{AII.3}$$

A four-vector lepton current which is invariant under the lepton spin rotations and the lepton number gauge transformations and which transforms like a component of a vector under the group (AII.3) is

$$\mathbf{J}_\mu = \mathrm{Tr}(\Psi^\dagger\gamma_4\gamma_\mu(1+\gamma_5)\Psi\boldsymbol{\rho}). \tag{AII.4}$$

The charged part of this current coincides with the usual charged lepton current. There is also a neutral part

$$\begin{aligned} J_\mu^{(3)} &= \mathrm{Tr}(\Psi\gamma_4\gamma_\mu(1+\gamma_5)\Psi\rho_3) \\ &= \bar{e}\gamma_\mu(1+\gamma_5)e+\bar{\mu}\gamma_\mu(1+\gamma_5)\mu \\ &\quad -\bar{\nu}_1\gamma_\mu(1+\gamma_5)\nu_1-\bar{\nu}_2\gamma_\mu(1+\gamma_5)\nu_2. \end{aligned} \tag{AII.5}$$

In order to have full symmetry under $\boldsymbol{\omega}'$ rotations, it would be necessary that $J_\mu^{(3)}$ should also occur in the weak interactions, in a way simply related to the occurrence of $J_\mu^{(+)}$. This is surely not the case for the strangeness-changing interactions. The experimental situation for the strangeness-conserving reactions is not clear. There is however, no evidence for the occurrence of neutral lepton currents in any weak interactions.

The electric charge gauge group is a subgroup of the seven-parameter group (AII.2) since under this gauge group we have as one transformation

$$\Psi \rightarrow \Psi\exp\tfrac{1}{2}i(1+\rho_3)q \tag{AII.6}$$

obtained from (AII.2) by taking $2\lambda=-\omega_3'=q$. There are altogether three commuting subgroups of (AII.2), generated by λ, ω_3, and ω_3'. These groups give the exact conservation laws of lepton number muon charge and electric charge. The full rotation group $\boldsymbol{\omega}$ is violated by the e-μ mass difference, while the "lepton isospin" group $\boldsymbol{\omega}'$ is violated by the e and μ mass terms, by the electromagnetic interactions, and by the absence of neutral lepton currents. It therefore appears unlikely to us that a charged vector boson is associated with local gauge transformations with parameters ω_1' and ω_2'.