Decay Rates of Neutral Mesons*

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We re-examine the model of Gell-Mann, Sharp, and Wagner for relating the $\omega \to \pi^+ + \pi^- + \pi^0$ decay rate to the π^0 lifetime. It is shown that, if present experimental results, finding large structure effects in the π^0 decay form factor, are confirmed by more precise data, then Gell-Mann *et al.* have considerably under-estimated the $\omega \to 3\pi$ decay rate. The branching ratio, $\Gamma(\rho \to \pi + \gamma)/\Gamma(\rho \to 2\pi)$ is also greatly enhanced, becoming a percent or more.

The possibility that the η is a 1^{--} meson is discussed briefly. We find that, in terms of the model, the 1^{--} assignment of quantum numbers for the η does not appear likely.

SINCE the observation of resonances in the production of two- and three-pion systems, there has been a considerable amount of speculation about the decay modes and decay rates of these various "mesons." Perhaps the most ambitious attempt to estimate the rates of the decay modes of the ω meson has been made by Gell-Mann, Sharp, and Wagner.[1] Using an idea developed by Gell-Mann and Zachariasen,[2] the former authors related the $\omega \to \pi^+ + \pi^- + \pi^0$ decay rate to the lifetime of the π^0. It is the purpose of this note to point out that, if the present experimental results on the π^0 decay form factor[3,4] are confirmed by more precise data, then Gell-Mann *et al.* considerably under estimated the $\omega \to 3\pi$ decay rate. Furthermore, the observed structure effects in the π^0 decay form factor cast doubt on the assumption that ω decays are dominated by the $\omega\rho\pi$ vertex, making their estimate of the branching ratio, $\Gamma(\omega \to \pi^0 + \gamma)/\Gamma(\omega \to 3\pi)$, unreliable.

In the discussion that follows, we shall assume that the η meson is not 1^{--} and, therefore, does not contribute to the π^0 decay amplitude. We include, however, a brief discussion based on the conjecture that the η is a vector particle analogous to the ω.

There are two experimental quantities associated with π^0 decay that have been measured. These are the lifetime and "mean square radius" of the decay form factor. We can relate these numbers to the covariant amplitude, T_μ, for the decay of a π^0 into a real photon of polarization ϵ_ν' and four-momentum k_λ' and a virtual photon with four-momentum k_τ (i.e, $\cdots k^2 = k_0^2 - \mathbf{k}^2 \neq 0$),

$$T_\mu = \epsilon_{\mu\nu\lambda\tau}\epsilon_\nu' k_\lambda' k_\tau G(k^2)/m_\pi. \quad (1)$$

$G(k^2)$ is the dimensionless form factor for the decay of the π^0 into one real photon and one virtual photon of mass $(k^2)^{1/2}$. $G(0)$ is determined by the decay rate of the π^0 into two photons:

$$\Gamma(\pi^0 \to 2\gamma) = ([G(0)]^2/64\pi)m_\pi. \quad (2)$$

For a π^0 lifetime of $\sim 2\times10^{-16}$ sec, $[G(0)]^2/4\pi \sim 4\times10^{-7}$. G is of order α, the fine structure constant, so that if we write $[G(0)]^2/4\pi = \alpha^2 f_\gamma^2/4\pi$, we find $f_\gamma^2/4\pi \sim 0.7\times10^{-2}$. This number is surprisingly small and is related to the fact that the π^0 decay rate is slower than had been expected from earlier theoretical estimates.[5] It is the relatively small value for $G(0)$ that is largely responsible for the narrow ω width obtained by Gell-Mann *et al.*

The "mean square radius" of the π^0 decay form factor can be measured by a careful study of the distribution of Dalitz pairs produced by the reaction, $\pi^0 \to \gamma + e^+ + e^-$.[6] Since this process restricts k^2 to $0 < k^2 \leqslant m_\pi^2$, we can approximate $G(k^2)$ in this range by

$$G(k^2) = G(0)[1 + ak^2/m_\pi^2], \quad (3)$$

Two measurements of a have been made, yielding

$$a = -0.24 \pm 0.16,^3 \qquad a = -0.3 \pm 0.2.^4 \quad (4)$$

We shall show that this relatively large and negative value of a implies that strong cancellations occur between various contributions to $G(k^2)$. This result is attractive because it is consistent with the observed, small value for $G(0)$. As pointed out by Gell-Mann and Zachariasen,[2] it also implies that only part of the π^0 decay amplitude is due to the process,

$$\pi^0 \to \rho + \omega, \quad \rho \to \gamma, \quad \omega \to \gamma. \quad (5)$$

Since a large fraction of the contribution due to (5) is canceled out by other processes, the strength of the $\pi\rho\omega$ vertex is much larger than would be estimated from the π^0 lifetime alone. Finally, since π^0 decay is not dominated by process (5), it is doubtful that ω decay is due entirely to the corresponding process,

$$\omega \to \rho + \pi, \quad \begin{cases} \rho \to 2\pi \\ \rho \to \gamma \end{cases}. \quad (6)$$

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[1] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).
[2] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).
[3] N. Samios, Phys. Rev. **121**, 275 (1961).
[4] H. Kobrak, Nuovo cimento **20**, 1115 (1961).
[5] See, for example, M. A. Goldberger, and S. B. Treiman, Nuovo cimento **9**, 451 (1958).
[6] S. M. Berman and D. A. Geffen, Nuovo cimento **18**, 1192 (1960).

We calculate $G(k^2)$ by means of a dispersion relation.[6,7] To simplify the discussion slightly, we take an unsubtracted relation although our results follow as well if we make a subtraction at $k^2=0$, provided all of the contributions from high-mass intermediate states can be included into the subtraction constant. The lowest mass intermediate states that can be formed from an incident pion and photon and which can decay into a virtual photon of mass k^2 are states of two and three pions. If we replace these states by ρ and ω mesons, respectively, we can, hopefully, decompose the contributions to $\mathrm{Im}G(k^2)$ into three parts, as shown in Fig. 1(a), (b), and (c). Figure 1(c) represents the sum of contributions from high-mass intermediate states and contributes a constant to $G(k^2)$ for small k^2. In Fig. 1(a) and 1(b), we have indicated the various couplings taken between $\pi\gamma\rho$, $\rho\gamma$, $\pi\gamma\omega$, and $\omega\gamma$. We are using the notation of Gell-Mann and Zachariasen[2] and we assume $\gamma_\rho\sim\gamma_{\rho\pi\pi}$, where $\gamma_{\rho\pi\pi}$ is related to the decay rate of the ρ meson:

$$\Gamma_\rho=(\gamma_{\rho\pi\pi}{}^2/12\pi)(1-4m_\pi{}^2/m_\rho{}^2)^{3/2}m_\rho.$$

This assumption is equivalent to assuming that the ρ resonance dominates the charged pion electromagnetic form factor[1] and that the form factor does not have a zero anywhere near the resonance (i.e., the "bare mass" of the ρ meson is much greater than the observed ρ mass,[2] m_ρ). Unitary symmetry predicts $\gamma_\omega\sim\gamma_\rho$;[8] otherwise, γ_ω is not well known.

The expression for $G(k^2)$ becomes

$$G(k^2)=\frac{e}{2\gamma_\rho}f_{\rho\pi\gamma}\frac{m_\rho{}^2}{m_\rho{}^2-k^2}+\frac{e}{2\sqrt{3}\gamma_\omega}f_{\omega\pi\gamma}\frac{m_\omega{}^2}{m_\omega{}^2-k^2}+\frac{e}{\gamma_\rho}\xi f_{\rho\pi\gamma}. \quad (7)$$

This is Eq. (5.6) of Gell-Mann and Zachariasen,[2] except for the additional constant, $(e/\gamma_\rho)\xi f_{\rho\pi\gamma}$, arising from higher mass contributions to $\mathrm{Im}G(k^2)$ (for a subtracted dispersion relation, this term comes from the subtraction constant). ξ is a constant with a magnitude of order unity. We have obtained this result using a dispersion relation, and keeping to the lowest order in the ρ and ω widths. No assumptions have been made about the ρ and ω currents.

If we assume that $f_{\rho\omega\gamma}$ and $f_{\omega\pi\gamma}$ can be calculated by means of unsubtracted dispersion relations in the photon mass, it is easy to see that the lowest mass intermediate state contributing to both coupling constants arises from the $\rho\pi\omega$ vertex.[2] If only the lowest mass term is retained, the result of Gell-Mann and Zachariasen[2] is obtained:

$$f_{\rho\pi\gamma}=(e/2\sqrt{3}\gamma_\omega)f_{\rho\pi\omega},\quad f_{\omega\pi\gamma}=(e/2\gamma_\rho)f_{\rho\pi\omega}. \quad (8)$$

[7] How-Sen Wong, Phys. Rev. **121**, 289 (1961).
[8] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

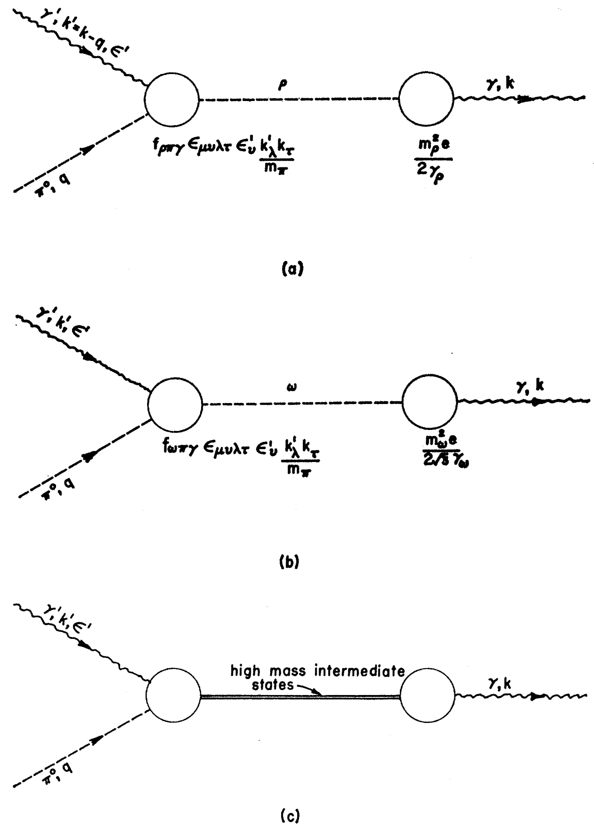

FIG. 1. Feynman graphs representing the three contributions to a dispersion relation for the $\pi^0\to\gamma'$(real)$+\gamma$(virtual) decay form factor, $G(k^2)$.

This approximation, however, will appear less reasonable if we cannot neglect the higher mass contribution in Eq. (7). Let us assume Eq. (8), take $m_\rho{}^2=m_\omega{}^2$, and write $G(k^2)$ as,

$$G(k^2)=\frac{e^2}{2\sqrt{3}\gamma_\rho\gamma_\omega}f_{\rho\pi\omega}\left[\frac{m_\rho{}^2}{m_\rho{}^2-k^2}+\xi\right]. \quad (9)$$

Comparing this equation with Eq. (3) leads immediately to the result

$$G(0)=(e^2/2\sqrt{3}\gamma_\rho\gamma_\omega)(1+\xi)f_{\rho\pi\omega},\qquad a=(m_\pi{}^2/m_\rho{}^2)(1+\xi)^{-1}. \quad (10)$$

For the purpose of illustration we choose a to be -0.2. It is clear how to make the corrections for other values of a. With $a=-0.2$, we find $\xi=-1.15$ and $1+\xi=-0.15$. This magnitude for a makes $f_{\rho\pi\omega}{}^2$ 45 times greater than the value obtained by Gell-Mann, Sharp, and Wagner,[1] assuming $\xi=0$. This increases the estimate of the $\omega\to\pi^++\pi^-+\pi^0$ decay width, made by these authors, from 0.4 to 18 MeV.[9,10]

[9] This result is based on the assumptions that ω decays primarily through the $\omega\to\rho+\pi$ vertex and that Eq. (8) is valid. Indeed, just as in the case of π^0 decay, other contributions to ω decay could cancel out a large part of the amplitude obtained from

Aside from the assumptions made in arriving at Eq. (10), a guess must be made for the coupling constants γ_ρ and γ_ω. We have taken $\gamma_\omega^2/4\pi \sim \gamma_\rho^2/4\pi \sim \gamma_{\rho\pi\pi}^2/4\pi \sim \frac{1}{2}$. Since the ω width is sensitive to the values of these coupling constants, moderate changes in their values can produce a much larger change in the width. Our estimate of the ω decay rate is also sensitive to the value of a. Since the present value for a, given by Eq. (4), is so uncertain, a more precise measurement of this quantity is very desirable.

The decay rates for $\rho \to \pi+\gamma$ and $\omega \to \pi^0+\gamma$ are determined by the coupling constants $f_{\rho\pi\gamma}$ and $f_{\omega\pi\gamma}$, respectively. These decay rates can be estimated, therefore, using Eq. (7), without resorting to the assumptions made to obtain $f_{\rho\pi\gamma}$ and $f_{\omega\pi\gamma}$ in terms of $f_{\rho\pi\omega}$. If we take $f_{\rho\pi\gamma}$ and $f_{\omega\pi\gamma}$ to be comparable, we find, for $a=-0.2$,

$$\Gamma(\omega \to \pi^0+\gamma) \cong 5(\gamma_\omega^2/4\pi)\ \text{MeV}.$$

For reasonable values of γ_ω (unitary symmetry predicts $\gamma_\omega \sim \gamma_\rho$), the width for this decay mode is several MeV. Since it is known from experiment that the total width of the ω is less than 30 MeV, this estimate predicts a branching ratio, $\Gamma(\omega \to \pi^0+\gamma)/\Gamma(\omega \to 3\pi)$, of at least 10%.

Gell-Mann *et al.*,[1] by assuming Eq. (8), predicted

$$\Gamma(\omega \to \pi^0+\gamma)/\Gamma(\omega \to \pi^++\pi^-+\pi^0) \cong 0.04(\gamma_\rho^2/4\pi)^{-2}. \quad (11)$$

This result is independent of the value of $f_{\rho\pi\omega}$ and, therefore, independent of the value of a. A reasonable upper bound for the ρ width, $\Gamma_\rho \leqslant 120$ MeV, restricts γ_ρ to $\gamma_\rho^2/4\pi \leqslant 0.6$. Using this upper bound on $\gamma_\rho^2/4\pi$, Eq. (11) predicts a *lower* bound for this branching ratio of 11%.

On the other hand, recent experimental evidence indicates that $\Gamma(\omega \to \pi^0+\gamma)/\Gamma(\omega \to \pi^++\pi^-+\pi^0)$ may be considerably less than the values obtained by the estimates made above.[11] This fact, if true, would further support the notion that other intermediate states contribute to the π^0 and ω decay processes. If the $\omega \to \pi^0+\gamma$ width is much less than the value we obtained assuming $f_{\omega\pi\gamma} \sim \sqrt{3} f_{\rho\pi\gamma}$ [the $\sqrt{3}$ factor gives the ρ and ω terms equal contributions to $G(k^2)$], then it is interesting to make the assumption

$$f_{\omega\pi\gamma} \ll \sqrt{3} f_{\rho\pi\gamma}. \quad (12)$$

Taking again $a=-0.2$, assumption (12) implies $\Gamma(\rho \to \pi+\gamma) \cong 7\gamma_\rho^2/4\pi$ MeV and

$$\Gamma(\rho \to \pi+\gamma)/\Gamma(\rho \to 2\pi) \cong 0.035. \quad (13)$$

(Assuming $f_{\omega\pi\gamma} \sim \sqrt{3} f_{\rho\pi\gamma}$ reduces this branching ratio to about 1%.) The estimate given by Eq. (13) is at least an order of magnitude larger than the predicted branching ratio would be if $a \sim 0$. Consequently, a measurement of $\Gamma(\rho \to \pi+\gamma)/\Gamma(\rho \to 2\pi)$ would be an important test of the model.

The earlier estimates[12,13,1] of $\omega \to \pi^++\pi^-$, $\omega \to \mu^+ +\mu^-$, and $\omega \to e^++e^-$ are unaffected by the π^0 form factor. Our results, therefore, reduce considerably the estimated branching ratio for these decay modes.

The possibility has not been excluded that the η meson has quantum numbers 1^{--}. This would mean that we must include the η intermediate state in the calculation of $G(k^2)$. If we introduce a new coupling constant $f_{\eta\pi\gamma}$ and make the obvious extension of Eq. (8), $f_{\eta\pi\gamma}=(e/2\gamma_\rho)f_{\rho\pi\eta}$, we can estimate the branching ratio of neutral to charged decay modes of the η:

$$\Gamma(\eta \to \pi^0+\gamma)/\Gamma(\eta \to \pi^++\pi^-+\pi^0).$$

For reasonable values of γ_ρ, this branching ratio is calculated to be much greater than 3, the observed experimental value. Interestingly enough, we could explain the discrepancy by taking $f_{\eta\pi\gamma} \ll f_{\rho\pi\gamma}$ to suppress $\eta \to \pi^0+\gamma$ just as $\omega \to \pi^0+\gamma$ was suppressed. We are still faced, however, with the problem that the decay $\rho \to \eta+\pi$ has not been seen. We are not allowed to reduce $f_{\rho\pi\eta}$ as we might reduce $f_{\eta\pi\gamma}$ because this would make the neutral to charged branching ratio too large again. But this means that $f_{\rho\pi\eta}$ and, therefore, $\rho \to \eta +\pi$, is enhanced by a large value of a. Restricting $\Gamma(\rho \to \eta+\pi)/\Gamma(\rho \to 2\pi)$ to less than 1% is completely inconsistent with a value of a of the order of -0.2. Since it is possible that the $\eta \to \pi^++\pi^-+\pi^0$ decay mode does not progress primarily by means of $\eta \to \rho+\pi$, these results are not conclusive evidence against a 1^{--} assignment to the η. In terms of this model, however, the spin 1 assignment does not appear likely. Unfortunately, there are corresponding problems in explaining the η decay branching ratios with a spin 0 assignment as well[14]

While it is commonly argued that a 1^{--} η meson would simplify the problem of nucleon electromagnetic form factors, we do not share this view for several reasons. If one attempts to fit the data with a Clementel-Villi form, using the ρ, ω, and η pole terms, then the observed small values for F_{1n},[15] even at large momentum

$\omega \to \rho+\pi$, thereby reducing the ω width considerably. This alternative is too depressing to consider seriously so that we at least hope that our estimate of $f_{\rho\pi\omega}$ yields the correct order of magnitude for the $\omega \to \pi^++\pi^-+\pi^0$ decay rate.

[10] Recent experiments indicate that the ω width may be of the order of 20 MeV. Results reported in a talk by A. Pevsner, Bull. Am. Phys. Soc. 7, 351 (1962).

[11] Northwestern-John Hopkins Universities Bubble Chamber group. In the preliminary analysis of their data, they find no evidence for a neutral ω decay mode and set an upper limit of 8% for the branching ratio. The author thanks M. M. Block for communicating this result to him. (private communication).

[12] S. Glashow, Phys. Rev. Letters 7, 469 (1961).

[13] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 8, 79 (1962).

[14] See, for example, an interesting explanation for the η decay branching ratios, assuming 0^{-+} quantum numbers for the η, by L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

[15] C. de Vries, R. Hofstadter, and R. Herman, Phys. Rev. Letters 8, 381 (1962). These Stanford data predict a significantly smaller F_{1n} than is obtained by the electron scattering group at Cornell. A fit to the Cornell data has been made recently by

transfer, immediately implies that the η contributes little to the form factors. Furthermore, since is is unlikely that the η, ω, ρ contribute to the form factors in a symmetric way, it is hard to understand why the various terms should just combine to give a vanishing neutron charge radius. Accidents do happen in physics, but they are not esthetically pleasing, especially in this case when a more likely explanation seems possible if we include only a ρ and ω meson with almost identical masses and which couple in a symmetric way to the nucleon. Finally, it is by no means clear that the electron-nucleon scattering data cannot be fit without a low-mass vector meson. After all, the Clementel-Villi form is only an approximation and, therefore, we should not expect perfect agreement. One possibility is that the core terms, usually treated as constant, can contain contributions from low enough mass states (i.e., $\rho^+\rho^-$) to produce an observable change in the magnitude of these core terms for large momentum transfers.

No discussion of neutral mesons can be complete without stressing the importance of the decay modes,

$$\rho\rightarrow\begin{cases}\mu^++\mu^-\\ e^++e^-\end{cases},\quad \omega\rightarrow\begin{cases}\mu^++\mu^-\\ e^++e^-\end{cases},\quad \eta\rightarrow\begin{cases}\mu^++\mu^-\\ e^++e^-\end{cases}\quad (?)$$

as a means of verifying the vector character of these particles.[13,16] This is particularly true for the η since its spin is least certain and its branching ratio for direct decay into a lepton pair would be the largest of the three mesons, if the η were spin 1.[17] The absence of the lepton decay modes for the η would be conclusive evidence that it is not a vector meson.

An interesting example where these decay modes would appear is the photoproduction of μ (or electron) pairs at BeV energies, i.e., $\gamma+p\rightarrow p+\mu^++\mu^-$. One should observe resonant peaks in the cross section as a function of the center-of-mass energy of the lepton pair system. The relative heights of these peaks depend, of course, on the production cross sections for the ρ, ω, and $\eta(?)$. It is almost certain, however, that the peaks should rise well above the background due to the usual electrodynamic pair production mechanism.

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J. S. Levinger, Bull. Am. Phys. Soc. **7**, 326 (1962), using the observed values of the ρ, ω, and η masses. He finds a large contribution from the η but the vanishing neutron charge radius occurs as an accident. An interesting result that he obtains is that the η and ω couple to the photon (taking γ_ω, γ_η positive as a convention) with opposite sign. (See also reference 13.) If this were true, it would mean that cancellations would occur, in the π^0 form factor $G(k^2)$, among the ρ, ω and η terms. This would require a very high degree of cancellation, however, in order to produce an a of the order of -0.2.

[16] D. A. Geffen, Phys. Rev. **125**, 1745, (1962).

[17] If we ignore the problem of $\rho\rightarrow\eta+\pi$, an $a=-0.2$ predicts an η width of a few tenths of an MeV. This leads to an expected $\eta\rightarrow e^++e^-$ $(\mu^++\mu^-)$ branching ratio of close to 1%.