Properties of Excited States of P^{31}. I. Gamma-Ray Spectra and Decay Schemes*

GALE I. HARRIS† AND L. W. SEAGONDOLLAR
The University of Kansas, Lawrence, Kansas
(Received May 17, 1962)

Gamma-ray decay schemes of the resonances in the reaction $Si^{30}(p,\gamma)P^{31}$ at $E_p=1177$, 1204, 1324, 1392, 1400, 1481, 1489, and 1509 keV are proposed from a study of pulse-height spectra and coincidence measurements. With some exceptions, the decay properties of intermediate states of P^{31} are found to be in agreement with previously existing information. The exceptions are (1) The 3.51-MeV level appears to decay to either the 1.27- or to the 2.23-MeV level (or both) in addition to the well-known decay to the ground state. (2) The 4.26-MeV level decays to the ground state and to the first excited state with the relative intensities 84% and 16%, respectively. (3) The 5.01-MeV level decays to the first excited state in addition to the previously reported decay to the ground state. Evidence is presented for the decay properties of the 4.63- and 4.78-MeV levels for which there is no previous information. Evidence is presented for the existence of four bound states between 5- and 7-MeV excitation which have not previously been reported. A relatively strong and well isolated transition to the 3.41-MeV level was found at the 1509-keV resonance. This transition is of considerable interest because it can be used to determine the spin and parity of the 3.41-MeV level for which no assignments exist.

INTRODUCTION

CONSIDERABLE attention has been given recently to the question of the applicability of the collective model to nuclei in the region $A\approx 25$. A particular point of interest is the range of A over which the model is valid. Broude, Green, and Willmott[1] (hereafter referred to as BGW) have studied the properties of energy levels of P^{31} in an attempt to decide whether the Nilsson model is applicable to this nucleus. They made a study of the gamma rays emitted at ten resonances below one-MeV proton energy in the reaction $Si^{30}(p,\gamma)P^{31}$ and then presented a detailed comparison of the experimental data with the predictions of the Nilsson model. Their conclusions were that the energies, spins, and parities of the lower levels are well accounted for by the model, but their decay properties, however, are not. Recently, it has been pointed out by Baart *et al.*[2] and by Green *et al.*,[3] however, that the neglect of the Kerman mixing of Nilsson's bands 9 and 11 in BGW's calculations is not justified and that the method of obtaining the values of the decoupling and rotation-particle coupling factors are questionable. The agreement with the experimental results of the calculations based on the Nilsson wave functions are therefore not expected to be as good as implied in the paper of BGW. The model (as applied by BGW) predicts that the level in P^{31} at 3.41 MeV should be $1/2^+$ and suggests that one of the levels between 4 and 5 MeV should be $7/2^+$.

Thankappan and Pandya[4] have attempted to explain the energy levels below 4-MeV excitation of P^{31} and their decay properties in terms of the vibrational model. Fairly satisfactory agreement was found. They stress, however, the serious difficulty in the predictions of this model that a $7/2^+$ level must lie below about 4 MeV. No such level has been observed and the only remaining candidate is the level at 3.41 MeV. The 3.41-MeV level is reported by BGW to have a decay mode to the $1/2^+$ ground state and therefore it appears unlikely that this level could be $7/2^+$. It appears that, on the basis of the available experimental information, the rotational and the vibrational model are about equally successful (or unsuccessful) in explaining the properties of the energy levels of P^{31}. It is clear that further detailed information on the properties of excited states of P^{31} may be of value to further theoretical interpretation.

This paper is the first in a series of two papers which will present the results of a study of gamma rays emitted from eight resonances between one and two MeV in the $Si^{30}(p,\gamma)P^{31}$ reaction. The first paper presents gamma-ray energies, intensities, and decay schemes for the resonances, and the second paper will present the results of angular correlation measurements at each of the resonances.

EXPERIMENTAL TECHNIQUE

The reaction under study was initiated by a beam of protons from the University of Kansas Van de Graaff generator. The ion beam emerging from the generator was separated into its various components by a magnetic momentum analyzer. The proton component then passed through a strong-focusing lens, beam-defining apertures, a liquid-nitrogen-cooled aperture, and then entered the target chamber. Simultaneously, the diatomic component of the beam passed through a precision, 127°, one-meter-radius, electrostatic analyzer, the exit slit of which was arranged to provide a signal for control of the accelerating voltage. For most of the work, the entrance and exit slits of the electrostatic analyzer were set for about 1-keV beam resolution.

* Supported by the National Science Foundation and the Office of Naval Research.
† Now at the Aeronautical Research Laboratories, Wright-Patterson Air Force Base, Ohio.
[1] C. Broude, L. L. Green, and J. C. Willmott, Proc. Phys. Soc. (London) **72**, 1097 (1958); (Parts I, II, and III).
[2] E. E. Baart, L. L. Green, and J. C. Willmott, Proc. Phys. Soc. (London) **79**, 237 (1962).
[3] L. L. Green, G. Kaye, and J. C. Willmott, Nuclear Phys. **25**, 278 (1961).
[4] V. K. Thankappan and S. P. Pandya, Nuclear Phys. **19**, 303 (1960).

Proton energies up to 1830 keV and proton currents between about 2 and 10 μA were used.

Thin targets of natural silicon and enriched (78.4%)Si^{30} in metallic form[5] were prepared by evaporation onto 10-mil tantalum backings which had been previously cleaned by heating to incandescence in vacuum.[6] The targets were mounted in a target chamber which was designed in such a way that the target backing was in thermal contact with the end of the chamber. Target cooling was accomplished by passing a blast of air over the end of the chamber. The liquid nitrogen cold trap, which was located about 18 in. from the target, served to minimize fluorine and carbon buildup during bombardment. Gamma rays were detected with 3- × 3-in. or 5- × 5-in. NaI(Tl) crystals optically coupled to 3-in. photomultipliers. The output pulses were amplified, shaped, and then analyzed by an RCL 256-channel pulse height analyzer.

The yield of gamma rays from a thin ($\approx$3 keV) natural silicon target was observed with a 3- × 3-in. detector with its cylindrical axis located at 0° with respect to the direction of the proton beam. The front face of the crystal was $\frac{1}{2}$ in. from the target. A discriminator bias was set such that only those pulses with amplitude greater than that corresponding to the full energy pulse of a 2.8-MeV gamma ray were accepted. This bias setting was high enough to exclude those pulses arising from the relatively intense 1.46- and 2.60-MeV room-background gamma rays. The bombarding energy was varied between 975 and 1830 keV in steps of 1.4 keV in regions of no resonance structure, and in steps of 0.7 keV where resonances appeared in the yield. Target depreciation was monitored during the course of the run by periodically checking the yield of the prominent resonance at $E_p=1204$ keV. By the end of the run a 25% reduction in yield from that in the beginning was observed. The energy scale for the bombarding protons was calibrated by means of the $Al^{27}(p,\gamma)Si^{28}$ resonance at 992.0±0.5 keV.[7] The linearity of the electrostatic analyzer voltage scale was checked by observing the voltages corresponding to other resonances[8] in the $Al^{27}(p,\gamma)Si^{28}$ reaction. No nonlinearity was observed.

The spectrum of gamma rays originating from each of the eight resonances at $E_p=1177$, 1204, 1324, 1392, 1400, 1481, 1489, and 1509 keV was observed with a 5- × 5-in. detector with its cylindrical axis located at 55° with respect to the incoming proton beam direction. The 55° position was used to minimize the effect of the $P_2(\cos\theta)$ term in the angular distributions of the emitted radiation. Higher terms were not, of course, thereby eliminated, but they are infrequent and usually of small magnitude in this reaction. A 2.5-in.-thick lead collimator was interposed between the target and the detector. The collimator was designed in such a way that only a conical region subtended from the target by the 5-in.-diam backface of the crystal was exposed to direct radiation from the target.

Each of the eight resonances above was identified by its spectrum as arising from Si^{30}. Preliminary studies of all resonances, except those at 1177 and 1509 keV, were made using natural silicon targets. Enriched Si^{30} targets, obtained later, were used to study the spectra from the 1177- and 1509-keV resonances and, in addition, to check the natural silicon data.

Correction for natural background pulses and pulses arising from the bombardment of impurities in the target and target backing was done by observing the spectrum at an energy a few keV off resonance and then subtracting these (appropriately normalized) data from the "on-resonance" data. In addition to the natural background peaks at 1.46 and 2.60 MeV, gamma rays at 4.43 and 6.13 MeV due to the $N^{15}(p,\alpha\gamma)C^{12}$ and $F^{19}(p,\alpha\gamma)O^{16}$ reactions were always present in the "off-resonance" runs. Fortunately, the variations in intensity of the contaminant gamma rays between "on-resonance" and nearby "off-resonance" runs were so small that they could be ignored. These four gamma rays mentioned above were found to be very convenient as calibration points for the energy scale of the pulse height distributions. After cross checks with Na^{22} and Co^{60} sources, and the $B^{11}(p,\gamma)C^{12}$ and $C^{13}(p,\gamma)N^{14}$ reactions to be sure of their identity these gamma rays were used exclusively to calibrate the energy scales.

The pulses of amplitude greater than those corresponding to 2.8 MeV were recorded by using a separate amplifier and a scaler. Possible effects on the data due to target depreciation were eliminated by terminating each run at a predetermined number of counts of this scaler. The "live-time" indicated by the pulse height analyzer at the end of each run was recorded so that corrections to the data for natural background effects could be properly made if necessary.

Analysis of the rather complicated spectra obtained in this work requires an elaborate and tedious "stripping" process. An accurate knowledge of the gamma-ray line shapes is required at all relevant energies. It was found that the "standard" line shapes obtained from well-isolated gamma rays of various energies between 1.28 and 8.7 MeV were quite dependent on slight changes in detector-collimator geometry, and on slight changes in detector resolution. For this reason each new setup of the experiments required a complete new line-shape calibration. The 1.28-MeV gamma ray from Na^{22}, the 4.43-MeV gamma ray from the $B^{11}(p,\gamma)C^{12}$ reaction, the 6.13-MeV gamma ray from the $F^{19}(p,\alpha\gamma)O^{16}$ reaction, and the well-isolated 7.4- and 8.7-MeV gamma rays from the 1400- and 1481-keV resonances in Si^{30} were usually used for the line-shape calibrations. Line-shapes for other required energies were interpolated

[5] The separated Si^{30} was supplied by the Isotopes Division of the Oak Ridge National Laboratory.

[6] R. A. Moore, L. W. Seagondollar, and R. B. Smith, Rev. Sci. Instr. **30**, 837 (1959).

[7] Jerry B. Marion, Revs. Modern Phys. **33**, 139 (1961).

[8] S. L. Anderson, H. Bö, T. Holtebekk, O. Lönsjö, and R. Tangen, Nuclear Phys. **9**, 509 (1959).

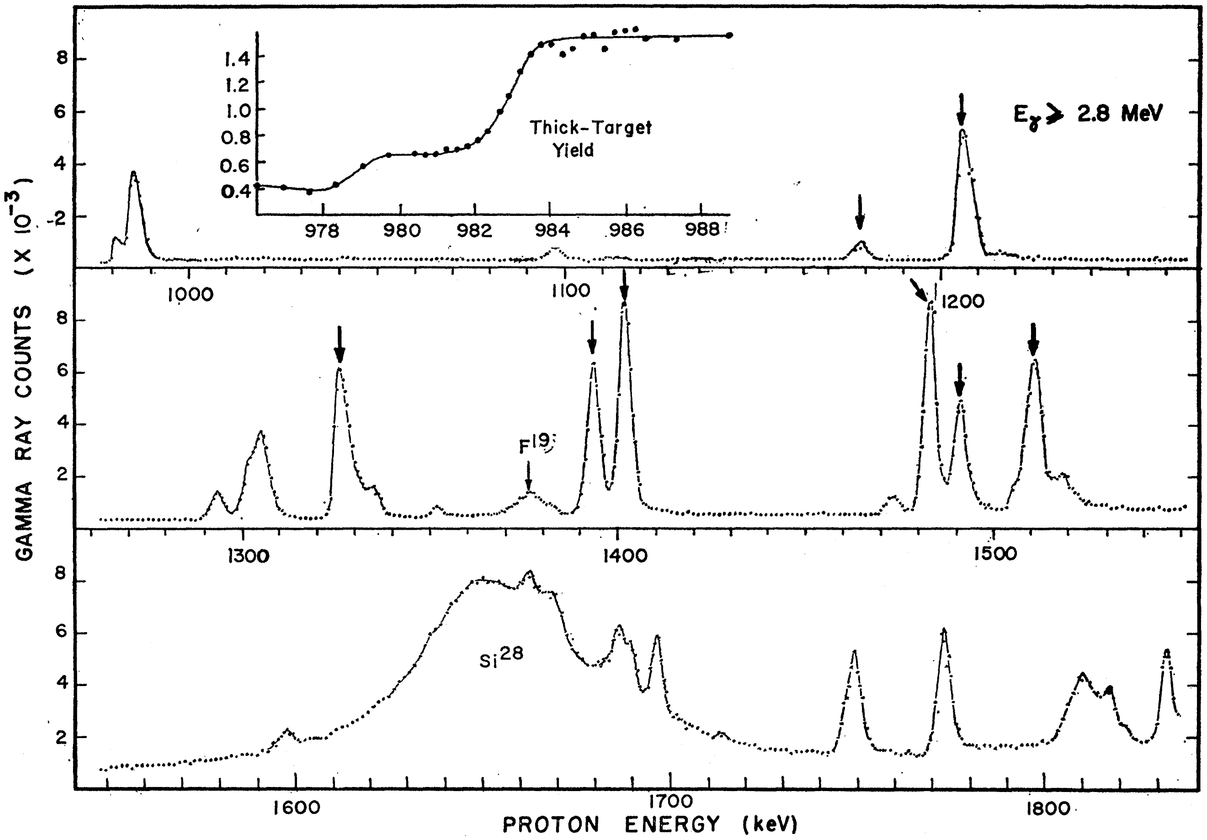

FIG. 1. Excitation curve for gamma-ray yield from the proton bombardment of natural silicon. A target of about 3-keV thickness was used and the detector was located at 0° relative to the incoming proton beam. Those resonances studied in detail are indicated by the arrows.

from the above data. The relative intensities of the gamma rays in the spectra were obtained by measuring the entire area, including the Compton tail, under each component line shape and then correcting for the appropriate detector efficiency. The detector efficiency factors were computed from the tabulations of Wolicki *et al.*[9]

The energies and intensities of the gamma rays at each resonance were used to construct a preliminary decay scheme which was then checked, where practicable, by coincidence measurements. For the coincidence measurements a 3- × 3-in. crystal located at 90° to the beam was used to detect one gamma ray and the 5- × 5-in. crystal at 55° was used to obtain the coincidence spectrum. Collimation of the crystals was dispensed with in order to obtain a reasonable counting rate. A fast-slow coincidence system with a resolving time of about 10^{-8} sec was used. With this system the chance-count rate was negligible. The coincidence spectra were obtained by feeding the pulses from the 5-in. crystal into the pulse-height analyzer in such a way that the analyzer recorded a pulse only if another pulse occurred at the same time in the 3-in. crystal and fell within a particular region of the gamma-ray spectrum defined by a single-channel analyzer.

RESULTS

Excitation Function

The gamma-ray yield from proton bombardment of an approximately 3-keV-thick natural silicon target (92.17% Si^{28}, 4.71% Si^{29}, 3.12% Si^{30}) is shown in Fig. 1 for the region of bombarding energies between 975 and 1830 keV. The inset shows a thick-target yield curve in the neighborhood of the pair of resonances near 980 keV. A careful comparison of the voltage scale of this pair of resonances with that of the $Al^{27}(p,\gamma)Si^{28}$ resonance at 992±0.5 keV gives 978.8±0.5 and 982.8±0.5 keV for the resonance energies. These resonances are to be identified with the resonances referred to by BGW at 995 and 1000 keV. (BGW stated they had no absolute energy calibration for their yield curve.) The broad (≈50-keV width) resonance at 1651 keV is presumed to be the known[10–12] resonance of this width and energy

[9] E. A. Wolicki, R. Jastrow, and F. Brooks, U. S. Naval Research Laboratory NRL Report 4833, 1956 (unpublished).

[10] J. O. Newton, Nuclear Phys. **21**, 529 (1960).

[11] K. J. Van Oostrum, N. Hazewindus, A. H. Wapstra, J. W. Olness, and J. L. Parker, Nuclear Phys. **25**, 409 (1961).

[12] H. Ohmura, H. Ejiri, Y Nakajima, K. Horie, K. Etoh, A. Ohuchi, and Y. Nogami, J. Phys. Soc. Japan **16**, 593 (1961).

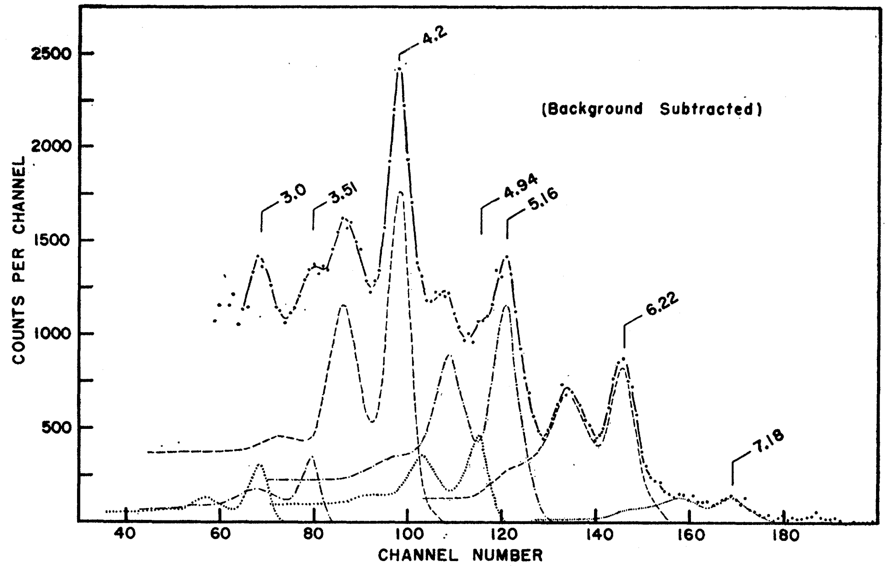

FIG. 2. Pulse-height spectrum for the 1204-keV resonance. The various components into which the spectrum has been analyzed by the stripping procedure are shown. This spectrum includes only the higher-energy gamma rays from the resonance. The lower-energy gamma rays were observed in a similar spectrum obtained with a higher amplifier gain. The gamma-ray energies, in MeV, are indicated.

in the $Si^{28}(p,\gamma)P^{29}$ reaction. This assumption is well supported by the observation of a 4.32-MeV gamma ray as a "contaminant" in the spectra of Si^{30} resonances which lie on the tail of the 1651-keV resonance. The small peak at 1375 keV is at least partly due to the $F^{19}(p,\alpha\gamma)O^{16}$ resonance at this energy and partly due to the $Si^{29}(p,\gamma)$ reaction. Most of the remaining resonances have been identified elsewhere[12–17] as arising from Si^{29} or from Si^{30}.

Those Si^{30} resonances studied in detail in this work are indicated in Fig. 1 by arrows. Their resonance maxima occur at 1177, 1204, 1324, 1392, 1400, 1481, 1489, and 1509 keV. The resonances will be referred to in this paper by the resonance maxima energies. The energies stated are believed to be within ± 2 keV of the true resonance energies. The observed widths of the resonances with the exception of the Si^{28} resonance, are experimental and arise primarily from the target thickness.

Gamma-Ray Spectra and Decay Schemes

The decay schemes are discussed in an order which is most convenient for establishing decay properties of intermediate levels. Those transitions which are obvious from their energies and relative intensities are not always referred to explicitly in the discussions but are shown in the diagrams. In almost all cases the measured gamma-ray energies were in agreement, within the experimental uncertainties, with energies calculated from the binding energy of 7.286 MeV[18], the beam energy, and the level-energy measurements of Endt and Paris.[19] The gamma rays are therefore referred to, where possible, by the calculated energy rather than the measured energy unless, either a discrepancy is suspected, or the gamma ray is complex.

There is evidence that each of the levels reported by Endt and Paris has now been excited by the capture reaction. There is also evidence in this work that 5 levels above 5 MeV, which is above the range studied by Endt and Paris, have been excited. Their locations are, of course, subject to the inherent inaccuracy of gamma-ray energy measurements and, also, in most cases, to the fact that they are weakly populated. Such levels are shown as dashed lines in Figs. 3–10 and 12. Those transitions which have been checked by coincidence measurements are designated in the diagrams by a dot placed on the level from which transition originates. The decay schemes shown in Figs. 3–10 show only those levels involved in the decay of the resonance being discussed. A decay scheme which shows all the intermediate levels observed in the present work is shown in Fig. 12.

1204-keV Resonance

A pulse-height spectrum of the gamma rays emitted at this resonance is shown in Fig. 2. The spectrum is typical of those obtained at all resonances, and illustrates the stripping procedure used. The spectra will not, therefore, be shown at all resonances. The energies of the gamma rays observed at this resonance are listed in Table I along with their measured intensities and the

[13] A. K. Val'ter, S. P. Tsytko, Yu. P. Antuf'ev, E. G. Kopanets, and A. N. L'vow, Izvest. Akad. Nauk. S.S.S.R., Ser. Fiz. **25**, 854 (1961).
[14] A. E. Litherland, E. B. Paul, G. A. Bartholomew, and H. E. Gove, Can. J. Phys. **37**, 53 (1959).
[15] A. C. L. Barnard, S. Bashkin, C. Broude, and C. E. Hornback, Nuclear Phys. **23**, 327 (1961).
[16] Unpublished data from the M. S. theses of R. F. Wiseman and N. K. Green, U. S. Naval Postgraduate School, 1957.
[17] Unpublished data from the Master's thesis of Urs Wild, the University of Kansas, 1962.
[18] *Nuclear Data Sheets*, National Academy of Science (National Research Council, Washington, 1960).
[19] P. M. Endt and C. H. Paris, Phys. Rev. **106**, 764 (1957).

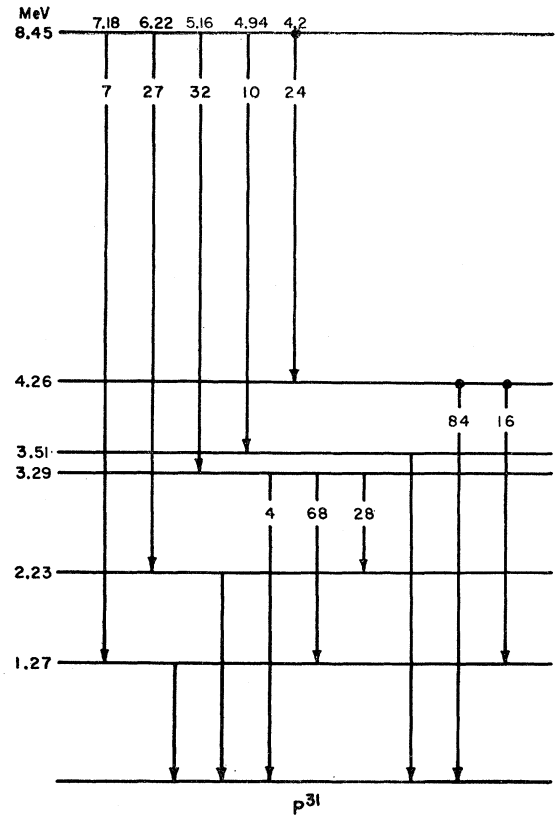

FIG. 3. Proposed decay scheme for the 1204-keV resonance.

intensities used in deriving a decay scheme (D. S.). The units for the intensities in the tables are arbitrary, but those shown in the decay figures are normalized such that the total intensity of the gamma rays which arise from a given level is 100. No attempt has been made to assign explicit errors to the measured intensities. An indication of the accuracy of the intensity measurements can, however, be obtained by comparing the measured values with those required to derive a consistent decay scheme. The agreement usually is, and should be, close since significant $P_4(\cos\theta)$ terms in the angular distribution are found to be rare in this reaction.

The energies and intensities of all except the 4.2- and 3.0-MeV gamma rays are reasonably well accounted

TABLE I. 1204-keV resonance.

	Relative intensities			Relative intensities	
E_γ(MeV)	Measured	D. S.[a]	E_γ(MeV)	Measured	D. S.
7.18	7.6	11	3.0	6.2	6
6.22	39.4	42	2.23	58.7	56
5.16	52.9	50	2.03	31.1	34
4.94	20.1	15	1.27	53.5	51
4.2	68.0	68	1.06	13.4	14
3.51	8.9	15			

[a] Decay scheme.

for by cascades through the 1.27-, 2.23-, 3.29-, and 3.51-MeV levels. (See Fig. 3.) Use has been made of the fact that BGW have shown the 3.29-MeV level decays to the ground state, the 1.27-MeV level, and 2.23-MeV level. This decay system has also been verified by coincidence measurements at other resonances in this work. It has also been established,[1,14] and is verified here, that the 2.23-MeV level decays only to the ground state. The intensity discrepancy of the 4.94- and 3.51-MeV gamma rays is probably caused by their proximity to the full-energy peak of the 5.16-MeV and to the first-escape peak of the 4.2-MeV gamma rays, respectively. The latter gamma rays are relatively very intense and thus make intensity measurement of neighboring weak gamma rays difficult.

It was noted that the energy of the 4.2-MeV gamma ray is nearly half the excitation energy of 8.45 MeV for this resonance. This suggested that it represents the two members of a cascade through a level at approximately 4.2-MeV excitation. In a coincidence spectrum obtained by gating the analyzer with pulses from the full energy peak of the 4.2-MeV gamma ray, a strong 4.2-MeV and a weaker 3.0-MeV gamma ray were observed in addition to those expected to be in coincidence with pulses from the Compton tails of high-energy gamma rays. This result verifies the existence of the cascade and also indicates that the 3.0-MeV gamma ray probably represents a transition from the 4.2-MeV level to the first excited state at 1.27 MeV. The possibility is not excluded, however, that the 3.0-MeV gamma ray is from a transition of the 3.13-MeV level to the ground state. The 3.13-MeV level could be excited by a weak primary obscured by the intense 5.16-MeV gamma ray. This possibility is considered unlikely because: (1) The energy of the gamma ray appears definitely lower than 3.13 MeV; (2) Its intensity in the coincidence spectrum appears too strong for it to be explained as a gamma ray in coincidence with pulses in the Compton tail of a weak 5.32-MeV gamma ray; (3) Its angular distribution, which will be discussed in Part II, is not consistent with this possibility. The later argument will exclude also the possibility that the 3.0-MeV gamma ray is actually 2.92 MeV and arises from a transition from the 4.19-MeV level to the 1.27-MeV level.

There are three levels, at 4.19, 4.26, and 4.43 MeV, observed in the inelastic scattering experiments, each of which could be the "4.2-MeV level" in question. The latter level would require, however, that the "4.2-MeV gamma ray" in the spectrum be considerably broader and higher in energy than observed. This possibility is thus excluded. BGW have presented convincing evidence that the 4.19-MeV level decays to the 1.27- and 2.23-MeV levels, and not to the ground state. The only remaining possibility is the 4.26-MeV level. The primary and secondary gamma rays would, in this case, have energies of 4.19 and 4.26 MeV which are in excellent agreement with the narrowness and

TABLE II. 1177-keV resonance.

E_γ(MeV)	Relative intensities Measured	D. S.	E_γ(MeV)	Relative intensities Measured	D. S.
6.19	85.0	85	2.92	12	6
5.13	60.3	61	2.2	174	164
4.6	3.2	[a]	2.0	101	113
4.23	8.8	9	1.27	82.9	66
3.83	9.5	10	1.06	16.3	16
3.29	4.5	5			

[a] Not accounted for.

energy of the observed "4.2-MeV gamma ray." This result is in disagreement with the conclusion of BGW that the 4.26-MeV level decays to the 2.23-MeV level. They point out, however, that in their work this level was observed as a member of the cascade from the resonance levels through a level at 6.41 MeV. The order of the second and third gamma rays in the cascade was not definitely fixed and thus the intermediate level could be at 4.26 or 4.43 MeV. The results obtained here indicate it must be the 4.43-MeV level which is involved in the cascades through the 6.41-MeV level proposed by BGW. Use is made of this cascade order in the discussion of the 1177-keV resonance which follows next.

1177-keV Resonance

The gamma rays observed at this resonance are shown in Table II. The dominant modes of decay are those to the 2.23- and 3.29-MeV levels. If one subtracts from the measured intensities the contributions due to cascades following the 6.19- and 5.13-MeV gamma rays, there remain gamma rays with residual relative intensities as follows:

E (MeV)	1.06	1.27	2.0	2.2	2.92	3.83	4.23
Res. int.	2	37	55	75	12	9.5	8.8

A coincidence experiment shows the 1.06-, 1.27-, 2.0-, 2.2-, (4.23)-, 5.13-, and 6.9-MeV gamma rays to be in coincidence with the 2.2-MeV gamma ray. (Parentheses indicate gamma ray is weak in the coincidence spectra.) The 1.06-, 5.13-, and 6.19-MeV gamma rays are expected from the cascades through the 2.23- and 3.24-MeV levels. Part of the 2.0-MeV gamma ray and all of the 4.23-MeV gamma ray would arise from a cascade through the 4.19-MeV level. Such a cascade should also give rise to a 2.92-MeV gamma ray, which is observed.

Another coincidence experiment shows (1.06)-, 1.27-, 2.0-, (2.2)-, (2.92)-, (3.83)-, (4.23)-, 5.13-, and (6.19)-MeV gamma rays in coincidence with the 1.27-MeV gamma ray. The presence here of the 2.92- and 4.23-MeV gamma rays verifies the cascade through the 4.19-MeV level. The observation that the 1.27-MeV gamma ray is in rather strong coincidence with itself suggests there must be a transition of this energy between higher lying levels. A similar situation was found by BGW in their 980-keV resonance. They obtained consistent results by proposing that a level at about 4.55 MeV decays to the 3.29-MeV level by a 1.26-MeV gamma ray. Similarly, consistent results are obtained here by proposing a 3.83-MeV primary transition to the 4.59-MeV level observed by Endt and Paris, which then decays to the 3.29-MeV level. These cascades are shown in Fig. 4.

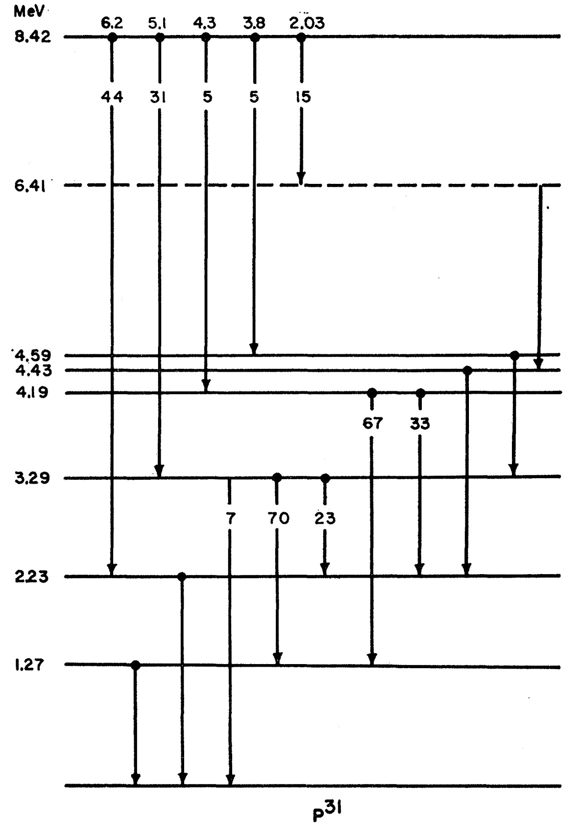

FIG. 4. Proposed decay scheme for the 1177-keV resonance.

The above discussion does not account for all the intensity of the 2.0- and 2.2-MeV gamma rays. The residual intensities can, however, be accounted for by the four-step cascade (which would consist entirely of 2.0- and 2.2-MeV gamma rays) through the 6.41-, 4.43-, and 2.23-MeV levels mentioned in the discussion of the 1204-keV resonance. This cascade would be in agreement with the observation that the 2.0- and the 2.2-MeV gamma ray itself, was in strong coincidence with the 2.2-MeV gamma ray. The weak 4.6-MeV gamma ray is regarded as too uncertain to include in the decay scheme. Angular distribution measurements at this resonance have shown rather strong $P_4(\cos\theta)$ terms in the 2.0- and 2.2-MeV gamma rays. The intensities given must therefore be regarded with due reservation.

1400-keV Resonance

For all resonances other than the 1400- and 1489-keV resonances, the thickness of the target was not critical

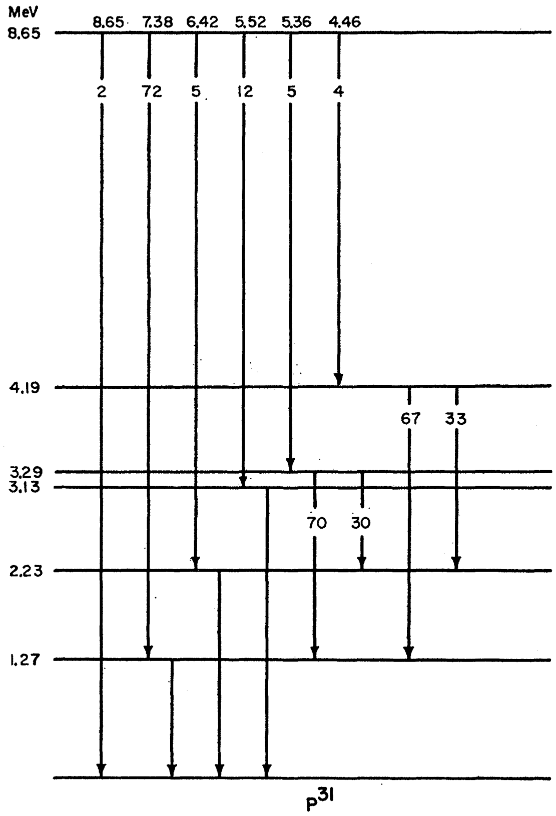

FIG. 5. Proposed decay scheme for the 1400-keV resonance.

and targets of approximately 3-keV thickness were used. A target of about 1-keV thickness was used in studying the 1400-keV resonance in order to insure that the resonance at 1392 keV was not simultaneously excited. Close comparison of the spectra from the two resonances leads to no doubt that they were adequately resolved.

The gamma rays observed at this resonance are listed in Table III. They are all well explained by cascades through the 1.27-, 2.23-, 3.13-, 3.29-, and 4.19-MeV levels. The proposed decay scheme is shown in Fig. 5. The transition from the 3.29-MeV level to the ground state was not observed, but this is not disturbing since the primary to the 3.29-MeV level is weak. No evidence is found for decay of the 3.13-MeV level to either the 1.27- or 2.23-MeV levels (in agreement with BGW who place an upper limit of 5% on such transitions). The intensity discrepancy of the 1.06-MeV gamma ray is almost certainly due to its proximity to the intense 1.27-MeV gamma ray.

1489-keV Resonance

The dominant feature of the spectrum at this resonance is a slightly broadened gamma-ray peak at 5.6 MeV which is interpreted as a combination of the 5.61- and 5.55-MeV primary transition to the 3.13- and

TABLE III. 1400-keV resonance.

E_γ(MeV)	Relative intensities Measured	D. S.	E_γ(MeV)	Relative intensities Measured	D. S.
8.65	4	4	3.13	16.0	17
7.38	98.7	105	2.92	3.8	4
6.42	10.1	7	2.23	8.5	11
5.52	18.9	17	2.0	6.9	5
5.36	6.9	7	1.27	143	114
4.46	6.5	6	1.06	10	2

3.29-MeV levels. Another important feature is a rather strong gamma ray at 3.75 MeV. These are shown along with the rest of the observed gamma rays in Table IV.

The presence of several gamma rays near 5 MeV makes intensity measurements difficult in this region. Their existence and energies were not hard to establish, however. The relative intensities of the 3.13-, 2.03- and 1.06-MeV gamma rays indicate that 25% of the 5.6-MeV gamma ray is a primary to the 3.13-MeV level and 75% a primary to the 3.29-MeV level. A coincidence measurement shows the 3.13- 2.03-, and 1.06-MeV gamma rays in coincidence with the 5.6-MeV gamma ray combination as expected. The weak ground-state gamma ray from the 3.29-MeV level is obscured by the 3.13-MeV gamma ray. A slight indication of the

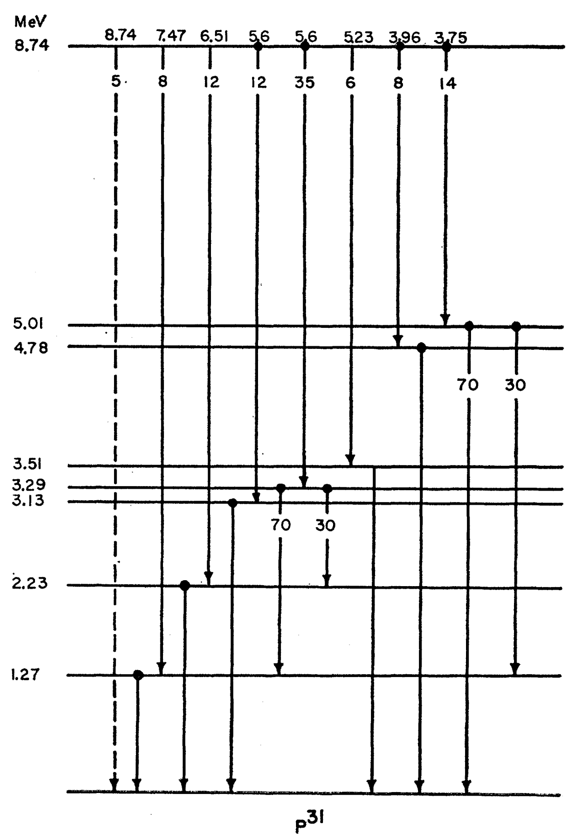

FIG. 6. Proposed decay scheme for the 1489-keV resonance.

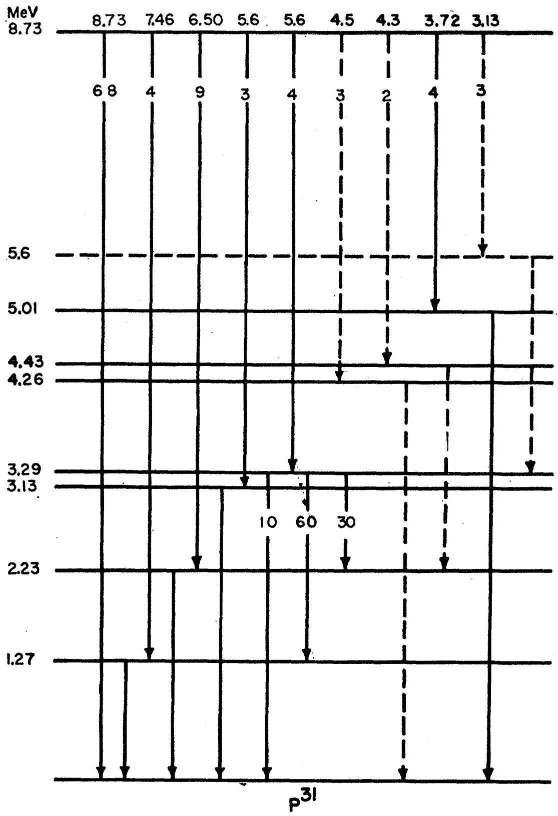

FIG. 7. Proposed decay scheme for the 1481-keV resonance.

3.29-MeV gamma ray is seen, however, in the coincidence spectrum. It is not shown in the proposed decay scheme for this resonance in Fig. 6.

Gamma rays of energies 1.27, 3.75, (4.8), and 5.01 MeV were found to be in coincidence with pulses from the full-energy peak of the 3.75-MeV gamma ray. The 1.27-, 3.75-, and 5.01-MeV gamma rays can thus be explained by a cascade through the 5.01-MeV state which decays by about 70% to the ground state and 30% to the first excited state. BGW have already shown that the 5.01-MeV level decays to the ground state. They could easily have missed the weaker decays to the first excited state since the 5.01-MeV level was weakly excited at all resonances studied by them. It is very likely that the 4.8-MeV gamma ray in the coincidence spectrum is, in fact, in coincidence with pulses from the 3.96-MeV gamma ray which was necessarily included in the window over the 3.75-MeV gamma ray. A cascade through the 4.78-MeV state observed by Endt and Paris is thereby indicated. This is the first-known indication that this state has been excited by the capture reaction.

TABLE IV. 1489-keV resonance

E_γ(MeV)	Relative intensities Meas-ured	D. S.	E_γ(MeV)	Relative intensities Meas-ured	D. S.
8.74	7.1	7	3.75	29.9	27
7.47	11.8	11	3.51	4.8	8
6.51	17.4	18	3.13	17.1	17
5.6	66.1	67	2.23	35.1	33
5.23	3[a]	8	2.03	39.8	35
5.01	14.6	15	1.27	48.8	52
4.78	10[a]	12	1.06	10.2	15
3.96	16.4	12			

[a] Very uncertain

TABLE V. 1481-keV resonance

E_γ(MeV)	Relative intensities Meas-ured	D. S.	E_γ(MeV)	Relative intensities Meas-ured	D. S.
8.73	124	124	3.72	9.3	8
7.46	6.6	7	3.29	3.7	1
6.50	20.0	17	3.13	11.1	11
5.6	13.6	14	2.23	30.7	32
5.01	7.3	8	2.03	7.8	8
4.5	4.6	5	1.27	14.6	15
4.3	8	8	1.06	4.8	4

A situation similar to that of the 1400-keV resonance exists for the 1489-keV resonance in that it lies only 8 keV above a strong resonance at 1481 keV. Again a 1-keV target was used in order to resolve the resonances. As will be shown later, the 1481-keV resonance exhibits a strong ground-state transition with all other cascades being relatively weak. If one assumes all the observed intensity of the weak 8.74-MeV gamma ray in the 1489-keV resonance is due to interference from the 1481-keV resonance, an upper limit is thus obtained for interference for all other gamma rays in the spectrum. Such analysis shows the interference to be insignificant for all gamma rays except perhaps for the 8.74-MeV gamma ray itself. For this reason the relative intensity of the 8.74-MeV gamma ray is shown in Fig. 6 only as $\leq 5\%$. It will be shown in Part II that the angular distribution of this gamma ray is consistent only with its arising from the 1481-keV resonance. The 1489-keV resonance, therefore, probably does not have a significant ground-state decay mode.

1481-keV Resonance

As has been recently reported,[13,20] the 1481-keV resonance decays mainly to the ground state. The gamma rays observed at this resonance[20a] are shown in Table V. The 5.6-MeV gamma ray is interpreted as a combination of primary transitions to the 3.13- and

[20] G. I. Harris and L. W. Seagondollar, Bull. Am. Phys. Soc. 6, 440 (1961).

[20a] *Note added in proof*. Using a thin, highly enriched Si^{30} target, H. van Rinsvelt and P. B. Smith have recently found that the 1481-keV "resonance" is a doublet with two equally strong components at a separation of 1.5 keV (P. M. Endt, private communication). This has been verified by the authors, who find that both members of the doublet exhibit strong ground-state transitions, and that both have $J=3/2$. The doublet will be discussed at greater length in Part II.

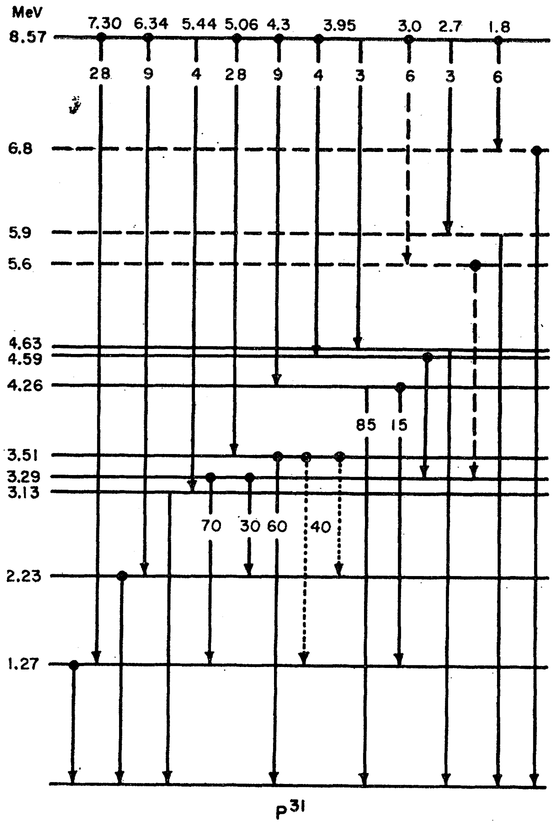

FIG. 8. Proposed decay scheme for the 1324-keV resonance.

3.29-MeV levels. The cascade through the 5.01-MeV level also appears at this resonance. The system is too weak, however, to confirm the existence of decay of the 5.01-MeV level to the first excited state. The presence of the 4.5- and 4.3-MeV gamma rays, and the fact that the intensity of the 2.23-MeV gamma ray is not entirely accounted for by the cascades through the 2.23- and 3.29-MeV levels, suggests the cascades through the 4.26- and 4.43-MeV levels as shown in Fig. 7. There still remain residual intensities for the 2.23- and 3.13-MeV gamma rays. To account for these intensities a new level is proposed at 5.6 MeV which decays to the 3.29-MeV level. Because of their uncertain nature, these cascades are indicated by broken lines in Fig. 7. They are too weak to be conclusively verified by coincidence measurements.

1324-keV Resonance

This resonance exhibits the most complicated decay pattern of any studied in this work. It has been possible, however, to derive a consistent decay scheme with the aid of several coincidence experiments. The observed gamma rays are listed in Table VI. The 5.06-MeV gamma ray represents a transition to the 3.51-MeV level, and is interesting because it appears to be the first observation of a strong cascade through the

TABLE VI. 1324-keV resonance.

E_γ(MeV)	Relative intensities Meas-ured	D. S.	E_γ(MeV)	Relative intensities Meas-ured	D. S.
7.30	43.6	44	3.51	24.3	26
6.8	9.9	10	3.13	4.3	5.5
6.34	13.8	14	3.0	11.8	12
5.9	3.7	5	2.7	6.3	5
5.44	6.9	5.5	2.23	47.7	47
5.06	44.1	44	2.03	6.8	11
4.7	5.1	5	1.8	9.9	10
4.3	25.4	25	1.27	76.5	81
3.95	11.0	11	1.06	2.8	5

3.51-MeV level which is not also mixed with cascades through the closely lying 3.41- and 3.29-MeV levels. Thus a good opportunity is provided to check the mode of decay of the 3.51-MeV level.

If the only mode of decay of the level is that to the ground state, one would expect to observe the 3.51-MeV gamma ray, and relativley very weak 1.27-, 2.23-, and 3.13-MeV gamma rays caused by the analyzer being gated by pulses from the weak Compton tails of the 7.30-, 6.34-, and 5.44-MeV gamma rays, in coincidence with the 5.06-MeV gamma ray. Such a coincidence spectrum contained a strong 3.51-MeV gamma ray and a very weak, if existent, 3.13-MeV gamma ray as expected. It also contained, however, 1.27- and 2.23-MeV gamma rays with intensities comparable to that of the 3.51-MeV gamma ray. The energy measurements and resolution were sufficient to exclude the possibility that the "2.23-MeV" gamma ray is, instead, the 2.15-MeV transition known to exist between the 3.41- and 1.27-MeV levels. There must exist, therefore, a mode for decay of the 3.51-MeV level through either the 1.27- or the 2.23-MeV levels, or perhaps both. (The energies are such that it is not possible to choose between the two.) Further support is that the intensity of the 3.51-MeV gamma ray only accounts for about 60% of that of the 5.06-MeV gamma ray. Additional checks showed the 2.23-MeV gamma ray in strong coincidence with the 1.27-MeV gamma ray, and conversely, the 1.27-MeV gamma ray is strong in a spectrum in coincidence with the 2.23-MeV gamma ray. Both possible modes of decay are shown in Fig. 8 where it is understood that either one or both exist. (Dotted lines are used to stress the ambiguity involved.)

This resonance exhibits, and thereby confirms, the cascade through the 4.26-MeV level discussed at the 1204-keV resonance. The 3.0- and 4.3-MeV gamma rays were found to be in coincidence with the 1.27-MeV gamma ray as expected. Indications from intensity considerations suggest the presence here of the cascade through the 5.6-MeV level proposed at the 1481-keV resonance. Again, however, the results are not conclusive. The presence of 2.7- and 5.9-MeV gamma rays of comparable intensity, and having a total energy ap-

TABLE VII. 1392-keV resonance.

E_γ(MeV)	Relative intensities Meas-ured	D. S.	E_γ(MeV)	Relative intensities Meas-ured	D. S.
7.37	103	100	3.51	13.9	13
6.41	32.6	30	3.29	4	3
5.7	6	8	2.9	13.7	14
5.35	25.3	25	2.2	56.7	63
5.13	12	13	2.0	19.2	16
4.45	8	9	1.27	105	111
4.21	14.3	11	1.06	9.5	8
3.9	9	[a]			

[a] Not accounted for.

proximately equal to the resonance excitation energy, leads to the proposal of a level at 5.9-MeV which decays to the ground state. Similarly, the presence of 1.8- and 6.8-MeV gamma rays of equal intensity leads to the proposal of a level at 6.8-MeV which decays to the ground state. The results were, in this case, confirmed by a coincidence measurement. The 4.7-MeV gamma ray is best accounted for by assuming that the "3.95-MeV gamma ray" is partially a primary to the 4.63-MeV level observed by Endt and Paris. The proposed decay scheme (Fig. 8), though complicated, accounts well for all the observed gamma ray energies and intensities.

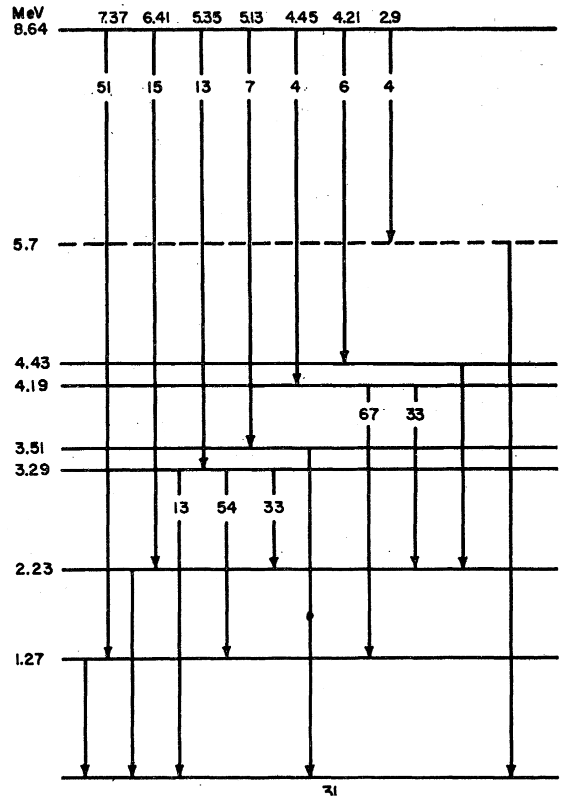

FIG. 9. Proposed decay scheme for the 1392-keV resonance.

1392-keV Resonance

The gamma rays observed at this resonance are shown in Table VII. All energies and intensities, except those for the 2.9-, 3.9-, and 5.7-MeV gamma rays, are well explained by cascades through levels already discussed. (See Fig. 9.) The residual 2.9-MeV gamma-ray intensity is nearly equal to the intensity of the 5.7-MeV gamma ray and their total energy is nearly that of the excitation energy of the resonance. Hence, a level is proposed at 5.7 MeV which decays to the ground state. The results could not be conclusively verified by a coincidence measurement because of the weakness of the cascade. No explanation is given for the weak 3.9-MeV gamma ray.

1509-keV Resonance

This resonance exhibits a relatively simple decay system and is important because, in the quartet of levels between 3.13 and 3.51 MeV, it decays only to the 3.41-MeV level, and strongly so. The observed gamma rays are shown in Table VIII.

The "2.2-MeV gamma ray" appears in the spectrum with its maximum located at 2.18 MeV and it has a "hump" on the high-energy side in good agreement with its being comprised of the 2.15-MeV gamma ray from a transition between the 3.41- and 1.27-MeV levels, and the 2.23-MeV gamma ray from the second-excited state. A coincidence spectrum was obtained by gating the analyzer with pulses from the full-energy peak of the 5.34-MeV gamma ray. The spectrum so obtained is shown in Fig. 10. The spectrum shows a 3.41-, a 2.15- (with the hump now almost gone), and a 1.27-MeV gamma ray. In addition, the 1.27- and 5.34-MeV gamma rays were quite strong in a spectrum in coincidence with the "2.2-MeV gamma ray." The cascade through the 3.41-MeV level is thus well established. The 2.0- and part of the residual intensity of the 2.2-MeV gamma ray can be explained by a cascade through the 6.41-MeV level. This latter system is indicated by broken lines in the proposed decay scheme (Fig. 11) to emphasize its uncertainty. The very weak 3.2- and 2.9-MeV gamma rays are not included.

TABLE VIII. 1509-keV resonance.

E_γ(MeV)	Relative intensities Meas-ured	D. S.	E_γ(MeV)	Relative intensities Meas-ured	D. S.
7.48	120	115	3.2	4	[a]
6.52	18.4	20	2.9	3	[a]
5.34	59.3	59	2.2	107	103
4.32	9.1	9	2.0	7	4
3.41	6.2	6	1.27	152	168

[a] Not accounted for.

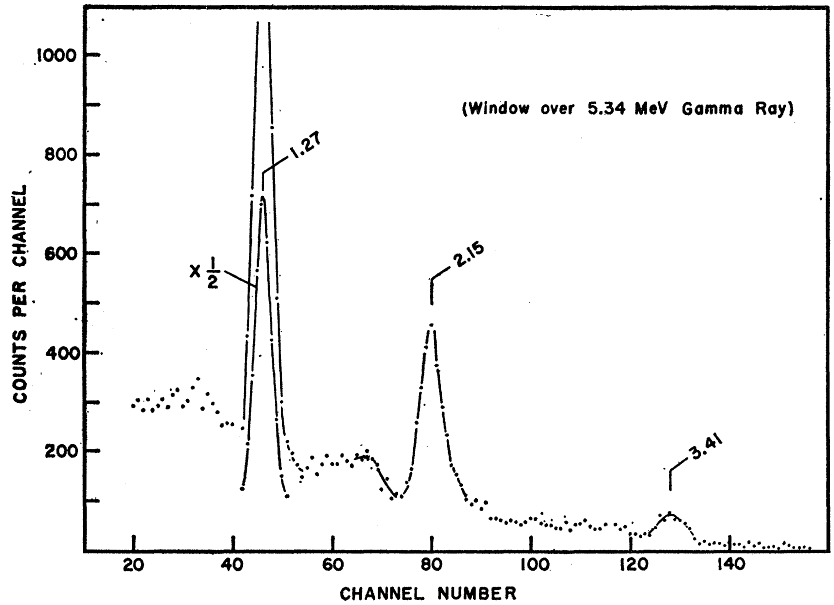

FIG. 10. Proposed decay scheme for the 1509-keV resonance.

Decay Properties of Intermediate States

This section summarizes the information on the decay properties of intermediate bound states through which the eight resonance levels studied in this work decay. Eighteen bound, excited states were required to explain the observed gamma rays. Thirteen of these were observed by Endt and Paris in inelastic scattering experiments. The 6.41-MeV level was proposed by BGW. The remaining four have not been previously reported.

1.27- and 2.23-MeV Levels

These levels are excited at all eight resonances and decay to the ground state. All results are in agreement with BGW and Litherland *et al.*[14] who place upper limits of 5% and 3%, respectively, on the decay of the 2.23-MeV level to the first excited state.

3.13-MeV Level

This level is observed at the 1324-, 1400-, 1481-, and 1489-keV resonances. It is observed to decay only to the ground state in agreement with BGW who estimate an upper limit of 5% on the possible transitions to the first and second excited states.

3.29-MeV Level

This level is excited at all resonances except the one at 1509 keV. As mentioned by BGW, the proximity of neighboring levels, the weakness of transitions, and the possible presence of $P_4(\cos\theta)$ terms make measurement of branching ratios difficult. The level was sufficiently isolated and was strongly enough excited at the 1204- and 1392-keV resonances, however, to establish with fair certainty its modes of decay. At the 1204-keV resonance 32% of the radiation from the resonance level feeds the 3.29-MeV level. The adopted average values for the branching ratios, taken from results at all resonances, but heavily weighted by those at the 1204- and 1392-keV resonances, are 10% for the mode to the ground state, 60% for that to the 1.27-MeV level, and 30% to the 2.23-MeV level.

3.41-MeV Level

This level is observed only at 1509 keV where it is excited by 28% of the transition from the resonance level. At this resonance, the transitions to the 3.41-MeV level are not accompanied by transitions to nearby levels and thus the data should lead to reliable determination of its decay modes. It is found to decay to the ground-state and to the first excited state with the relative intensities 10% and 90%, respectively. The possible decay to the second excited state, which would result in a 1.18-MeV gamma ray, is estimated from the coincidence spectra to consist of $\leq 3\%$ of the radiation from the level.

3.51-MeV Level

This level is observed at the 1204-, 1324-, 1392-, and 1489-keV resonances, and in each case exhibited the well-known decay mode to the ground state. The level is more strongly excited at the 1324-keV resonance than at any other resonance in this work or in that of BGW. At this resonance 28% of the radiation from the resonance level is due to a transition to the 3.51-MeV level.[21] This resonance affords, therefore, a good oppor-

[21] It is not possible to entirely exclude the possibility that, in this case, there is a competing cascade through the 5.01-MeV level which would, of course, affect these results. Intensity measurements of the 3.51- and 5.06-MeV gamma rays make this possibility appear unlikely.

tunity to check the decay modes of the 3.51-MeV level. As shown in the discussion of the 1324-keV resonance, there must exist a mode of decay either to the 1.27- or to the 2.23-MeV level, or both. It will be discussed in Part II, by no means conclusively, that the 3.51-MeV level may be a closely spaced doublet. If this is so, the appearance of new decay modes not observed by BGW could be explained by different relative populations of the two members of the doublet, each of which have different decay properties.

4.19-MeV Level

The 1177-, 1392-, and 1400-keV resonances each decay weakly to the 4.19-MeV level. In each case the results are consistent with the 2 to 1 ratio for decay of the 4.19-MeV level to the 1.27- and 2.23-MeV levels given by BGW. No evidence for decay to the ground state was found.

4.26-MeV Level

This level is excited at the 1204-, 1324-, and possibly at the 1481-keV resonances. The most reliable determination of the decay properties of the level comes from the data at the 1204-keV resonance where 24% of the radiation from the resonance level results from transition to the 4.26-MeV level. The values adopted from this data for the branching ratios are 84% for decay to the ground state and 16% for decay to the first excited state. The possible transition to the 2.23-MeV level would have an energy of 2.03 MeV which means this transition can only be ruled out from intensity considerations. It is estimated on this basis that this decay mode must account for less than 10% of the total radiation from the 4.26-MeV level. There is no evidence for other modes of decay.

4.43-MeV Level

This level appears as a member of the cascade through the 6.41-MeV level at the 1177-keV resonance, and there is good evidence for direct primaries to the 4.43-MeV level at the 1392-, 1481-, and 1509-keV resonances. In all cases the level was found to decay to the second excited state. Weaker modes of decay to the ground and first excited states can not be excluded by the data. No evidence is found here for decay reported by BGW to the 3.29-MeV level. Such a transition would have, however, an energy of 1.14 MeV and thus, in each case, it would be obscured by the intense 1.27-MeV gamma ray in the spectrum

4.59-MeV Level

Evidence for direct primary transitions to this level is found at the 1177- and 1324-keV resonances. In both cases, intensity considerations and coincidence measurements support the existence of a decay mode to the

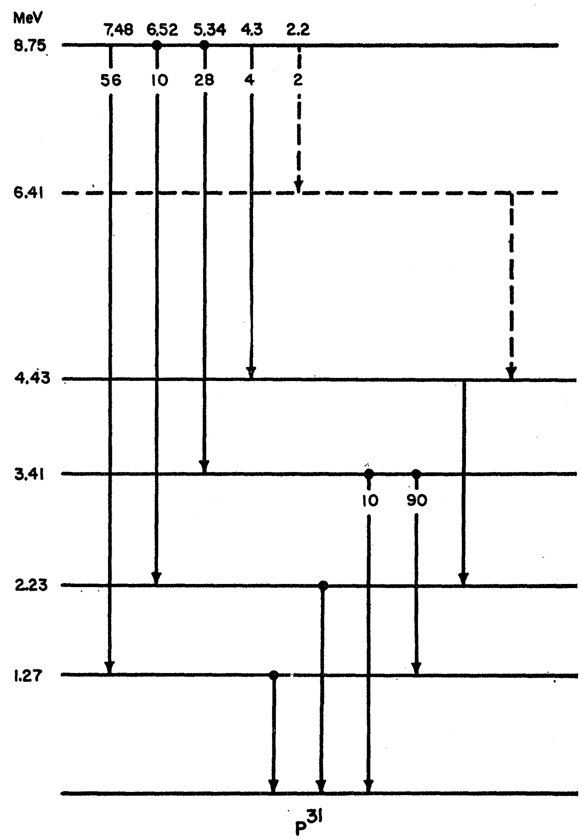

FIG. 11. Spectrum of gamma rays in coincidence with pulses from the full-energy peak of the 5.34-MeV gamma ray in the 1509-keV resonance.

3.29-MeV level. BGW postulate a level at 4.55 MeV which decays to the 3.29-MeV level. The same level is probably involved in both cases.

4.63- and 4.78-MeV Levels

At the 1324-keV resonance there appears a 4.67-MeV gamma ray which is considered to be, from the energy measurement, a ground-state transition from the 4.63-MeV level. In the 1489-keV resonance decay a 4.0-MeV gamma ray in coincidence with a 4.8-MeV gamma-ray of about the same intensity suggests that the 4.78-MeV level is being excited. No evidence for modes of decay other than those to the ground state is found. Since both levels are weakly excited, the results are not considered conclusive.[22]

5.01-MeV Level

This level is observed at the 1481- and 1489-keV resonances, and at the latter, the 5.01-MeV level is

[22] *Note added in proof.* An intense 1.65-MeV gamma ray has been observed which would correspond to a 4.78- to 3.13-MeV transition in the $P^{31}(p,p')P^{31}$ reaction. [See Wakatasuki, *et al.*, *Proceedings of the Kingston Conference* (University of Toronto Press, Toronto, Canada, 1960), p. 971.

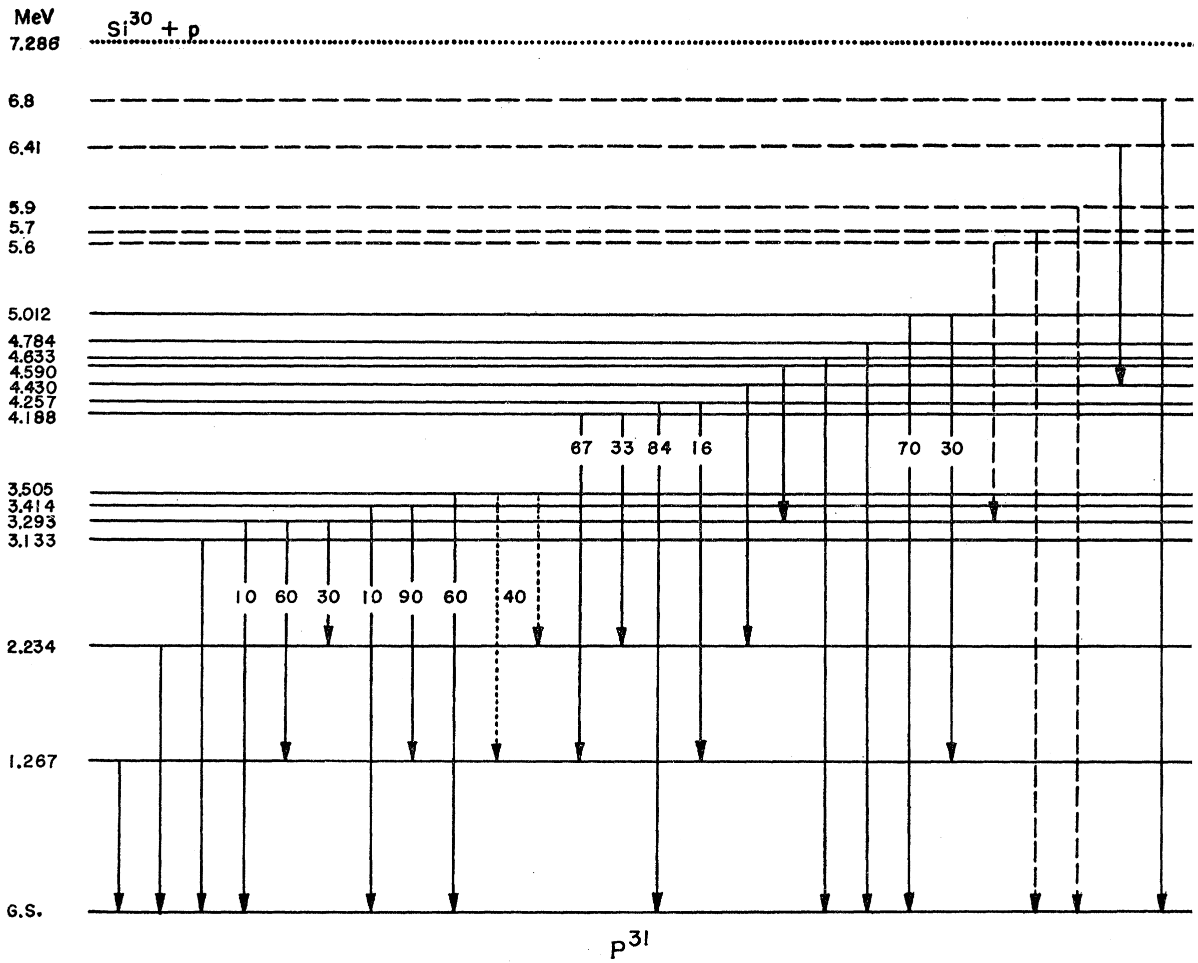

FIG. 12. Decay scheme summarizing the decay properties of intermediate states of P^{31}.

excited by 14% of the transitions from the resonance level. Evidence substantiated by coincidence measurements at the 1489-keV resonance shows the existence of the ground-state decay mode reported by BGW, and an additional decay mode to the first excited state. Intensity measurements show 70% of the radiation from the 5.01-MeV level arises from a ground-state transition and 30% from a transition to the first excited state.

5.6-MeV Level

A level at 5.6 or 6.3 MeV is required to explain the gamma rays in coincidence with the 1.27- and 2.2-MeV gamma rays at the 1324-keV resonance. The level decays to the 3.29-MeV state. The same level is postulated at the 1481-keV resonance from intensity considerations. The difference in energy of the primaries at the two resonances fixes the level at 5.6 MeV.

5.7- and 5.9-MeV Levels

Evidence for a level at 5.7 MeV which decays to the ground state is found at the 1392-keV resonance. Similarly, there is evidence at the 1324-keV resonance for a level at 5.9 MeV which also decays to the ground state. Conclusive coincidence checks were not possible at either resonance.

6.41-MeV Level

This level is observed at the 1177- and 1509-keV resonance and is supported by coincidence measurements at the former resonance. The level was originally proposed by BGW as a member of a cascade through the 4.26- or perhaps the 4.43-MeV level. Results at the 1204-keV resonance imply that the 6.41-MeV level must decay to the 4.43-MeV level, and not to the 4.26-MeV level.

6.8-MeV Level

This level is postulated to explain the 1.8- and 6.8-MeV gamma rays of about equal intensity in coincidence at the 1324-keV resonance.

SUMMARY

The decay properties of all intermediate levels observed in this study are summarized in a decay scheme in Fig. 12.

ACKNOWLEDGMENTS

The authors wish to express their appreciation for the assistance of Richard A. Moore, Denny Watson, Carl Peck, Robert Redding, and Urs Wild in machine operation and data analysis. Some of the equipment used was borrowed from the AEC-supported group here. One of us (G. I. H.) did much of the manuscript preparation at the Aeronautical Research Laboratories, Wright-Patterson Air Force Base, Ohio.

PHYSICAL REVIEW VOLUME 128, NUMBER 1 OCTOBER 1, 1962

Elastic Scattering of 6.7-MeV Neutrons from Silver and Indium†

P. R. Malmberg and S. C. Snowdon*
Nucleonics Division, U. S. Naval Research Laboratory, Washington, D. C.
(Received February 13, 1962)

The differential cross sections for the elastic scattering of 6.7-MeV neutrons by silver and indium have been measured over a wide angular range using a standard ring geometry. Improved detection and data treatment techniques were used in an attempt to minimize the errors in the results caused by the contributions from inelastically scattered neutrons. The corrections for finite volume of the scatterer and for double scattering were made using a simplified model that can be treated analytically. The experimental cross sections are, in general, somewhat smaller at the larger scattering angles than the theoretical curves calculated by Bjorklund and Fernbach.

INTRODUCTION

Since the proposal of the optical model of the nucleus,[1] many experiments have been performed to verify it and to determine the parameters involved in the various specific choices of the potential. An important class of experiments has been the measurement of the differential cross section for the elastic scattering of neutrons. Early survey experiments[2] emphasized collecting data for a large number of elements at a number of different energies. The present experiment is an attempt to obtain results with better statistical accuracy and less contamination from inelastically scattered neutrons. Measurements were made on silver and indium over a large angular range using 6.7-MeV neutrons.

EXPERIMENT

The neutron source, scatterer, and detector were arranged in a standard ring geometry as may be seen in Fig. 1. The scattering angle θ was varied by using various combinations of source-to-detector distance, ring diameter, and ring position.

Detector counts were taken under three conditions: (a) the arrangement using scatterer as shown in Fig. 1, (b) with scatterer removed to count background, and (c) with both scatterer and attenuator removed to count the direct beam. From these measurements, a "scattering ratio,"

$$S=\frac{\text{scattered counts}-\text{background counts}}{\text{direct counts}-\text{background counts}}, \tag{1}$$

may be computed. An uncorrected differential cross section, σ_A, may be obtained from this scattering ratio by using the expression

$$S=\sigma_A n\int_V Je^{-\sigma nl}dV, \tag{2}$$

where n is the number of atoms per unit volume, σ is the

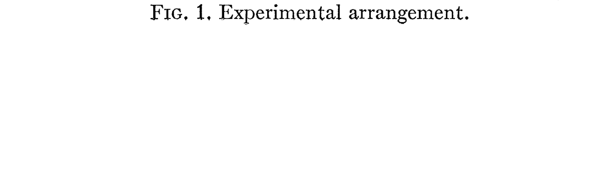

Fig. 1. Experimental arrangement.

† A preliminary report of this work has appeared in Bull. Am. Phys. Soc. **4**, 257 (1959).
* Present address: Midwestern Universities Research Association, Madison, Wisconsin.
[1] H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. **96**, 448 (1954).
[2] For example, J. R. Beyster, M. Walt, and E. W. Salmi, Phys. Rev. **104**, 1319 (1956).