Statistical Theory Predictions for 5- to 11-MeV (p,n) and (p,p') Nuclear Reactions in V^{51}, Co^{59}, Cu^{63}, Cu^{65}, and Rh^{103}†

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(Received May 31, 1962)

Absolute (p,n) cross sections for proton energies between 5 and 11 MeV have been measured for V^{51}, Co^{59}, Cu^{63}, Cu^{65}, and $Rh^{1\ 3}$ using a "long counter." The experimental values are compared with the predictions of the statistical theory. An analysis is made of the use of different level density functions in the calculations of the cross sections. The choice of parameters—the nuclear radius r_0, the level density parameter a, and pairing energies, $P(Z,N)$—has been made with the idea of fitting not only these measured (p,n) cross sections but also other available experimental information such as the proton-reaction cross section, $\sigma(p,p')$, the differential energy spectrum of the neutrons resulting from these proton reactions, and some of the proton spectra from the (p,p') reaction for the copper isotopes. It is shown that the "measured" value of a changes drastically from approximately $A/20$ or $A/30$ to approximately $A/8$ to $A/6$ depending upon whether $\exp\{2[a(U+P(Z,N))]^{\frac{1}{2}}\}$ or $[U+P(Z,N)]^{-2}\exp\{2[a(U+P(Z,N))]^{\frac{1}{2}}\}$ is used for the level density. However, the (p,n) and (p,p') cross sections obtained with these level density functions when the correspondence a is introduced are quite close. In most of the cases it is not possible to make a definite choice if the accuracy of the measured cross sections is not better than 5%.

INTRODUCTION

THERE are two processes that compete in the interpretation of nuclear reactions: a direct reaction process, and a compound nucleus process. Several criteria, such as angular distribution, energy spectrum, polarization, and gamma correlation of the emitted particle, help to decide in favor of either mechanism. In the energy region of the present experiment and for the elements here studied, Anderson *et al.*[1] have measured the differential (p,n) cross section and angular distribution of the emitted neutrons. The angular distributions are quite isotropic and the energy spectrum of the neutrons can be fitted with a Maxwellian distribution. These two characteristics show the absence of direct interaction in (p,n) reactions, so the analysis of the data has been done unambiguously considering the compound nucleus model.

The neutron energy spectra predicted by this model differ according to the level density function $\omega(E)$ used in the calculations. It has been felt for some time[2–5] that the level density corresponding to a degenerate Fermi gas as given by Weisskopf[6] is inadequate to reproduce the experimental measured cross sections. A more complete treatment of the nuclear process, taking into account particle interaction in the potential well and angular momentum restrictions, had been suggested in the literature.[7–10] These modified level density functions have been used to fit the experimental results in this work.

Several parameters are involved in these calculations, such as the nuclear radius parameter r_0, the level density parameter a, and the pairing energies $P(Z,N)$.

It will be shown that the level density parameter a, obtained by fitting a given energy density function to the neutron energy spectrum and to some of the proton energy spectra coming from (p,n) and (p,p') reactions, respectively, have quite different values, according to the level density function used. Furthermore, once a function for the level density has been chosen and a parameter a obtained, variations in a do not change appreciably the (p,n) cross sections but have a much more important effect on the (p,p') and (p,α) cross sections.

If one is interested in obtaining a "good" set of parameters that will fit (p,n) cross sections as well as (p,p') cross sections at a given energy, it is necessary to know the behavior of (p,p') and (p,α) cross sections at these energies. For example, in the 5- to 11-MeV energy region of this experiment, there is evidence of a direct interaction process in (p,p') cross sections which is sometimes as large as 25% of the total measured (p,p') cross section, so these cross sections must be corrected to obtain the contribution due to compound nucleus formation. The simultaneous fitting of (p,n) and (p,p') cross sections has proved quite useful in deciding on the choice of pairing-energy parameters.

EXPERIMENTAL METHOD

The measurements of the (p,n) cross sections were done using a "long counter"[11] technique. The proton beams were obtained from the Livermore 90-in. variable energy cyclotron. The neutrons were detected with a BF_3 long counter set at 90°. The counter efficiency as a function of neutron energy was determined using

† Work performed under the auspices of the U. S. Atomic Energy Commission.

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neutron sources of different average neutron energy. Pu-Be, Po-Be, mock fission, and Sn-Be sources whose average neutron energies are 4.5 MeV, 4.2 MeV, 1.6 MeV, and 24 keV, respectively, were used. The sources were calibrated in a $MnSO_4$ bath.[12] From the measured efficiency and following Allen,[13] a correction of 1.08 $\pm 0.05\%$ was made for the ratio of 1-MeV neutrons to 4.2-MeV neutrons (Po-Be source).

The targets used were all free foils of thickness varying between 1.5 and 8 mg/cm^2. All the foils were carefully checked for thickness uniformity, weight, and chemical purity. For most of the elements two or three targets of different thickness were made. The values of the cross sections obtained for a given element from these targets were quite consistent.

The targets were mounted in a 24-port target changer, remotely controlled. The centered position of the proton beam on the target was checked on a television screen.

The absolute values for the (p,n) cross sections were obtained by calibrating the neutron yields obtained with a standard Po-Be source calibrated by the National Bureau of Standards. The Po-Be source was chosen because it is spherical, which assures an isotropic flux of neutrons. It is interesting to point out that cylindrical sources such as Pu-Be or Sn-Be are quite anisotropic with variations in the neutron flux as large as 30%, according to the position of the source. Inasmuch as the total calibrated flux corresponds to a 4π geometry ($MnSO_4$ bath), one must be careful to interpret the efficiency factor obtained from them.

The experimental data given in this paper were obtained from six different runs at the cyclotron, where cross sections were measured for all the targets at each run. The errors quoted in these measurements are all $\pm 7\%$ except for Cu^{65}, where the error is $\pm 10\%$ due to chemical impurities.

THEORY AND CALCULATIONS

The compound nucleus model cross section for the process $A(p,x)B$ is given by the following expression:

$$\sigma(p,x)=\sigma_c(E_p')F_x/\textstyle\sum_i F_i, \qquad (1)$$

where $\sigma_c(E_p')$ is the cross section for the formation of a compound nucleus by a proton of energy E_p' incident on the nucleus A, in the center-of-mass system. F_i is the probability of emission of a particle i by the compound system $A+a$. In the energy region of 5 to 11 MeV only the emissions of protons, neutrons, and alpha particles are important, so the summation $\sum_i$ extends to only these cases.

The probability F_i is given by

$$F_i=2M_i(2I_i+1)\int_{E_{\min}}^{E_{\max}}\sigma_c(\epsilon_i)\epsilon_i\omega(U_B)d\epsilon_i, \qquad (2)$$

where M_i and I_i are the reduced mass and the spin of particle i. $\sigma_c(\epsilon_i)$ is the inverse cross section for the formation of a compound nucleus by particle i incident on nucleus B. $\omega(U_B)$ is the level density of the residual nucleus B, which we will discuss in detail further on. U_B is the excitation energy of nucleus B and is given by $U_B=E_p'+Q-\epsilon_i$, where Q is the "Q" value of the reaction $A(p,x)B$.

The cross section for the formation of the compound nucleus or capture cross section, σ_c, used in these calculations was taken from the work of Dostrowsky *et al.*[3] For neutrons the capture cross section is given by

$$\sigma_c=\sigma_g\alpha(1+\beta/E), \qquad (3)$$

where σ_g is the geometrical cross section and α and β are functions of the atomic number A.

For charged particles the capture cross section is given by

$$\sigma_c=\sigma_g(1+c_{Zj})(1-k_{Zj}V_j/E), \quad j=p,\alpha \qquad (4)$$

where c_{Zj} and k_{Zj} are parameter functions of the atomic number Z chosen by Dostrowsky *et al.* to give a good fit to the compound nucleus cross section calculated by Shapiro.[14] V_j is the Coulomb barrier, and $k_{Zj}V_j$ represents an effective barrier to account for barrier penetration.

The use of expressions (3) and (4) in the calculation of the emission probability given by (2) makes the integral analytic, which facilitates the calculation and coding of the problem. The integral was evaluated between the following limits:

$E_{\min}=0$ for neutrons,
$\quad =k_jV_j$ for charged particles,
$E_{\max}=E_p'+Q,$
$\quad =E_p'+Q+P(N,Z),$ if pairing energies are taken into account in the level density function.

LEVEL DENSITY FUNCTION

The first level density function used in these calculations was that corresponding to the Fermi gas model of the nucleus, given by Weisskopf[6] as

$$\omega(U_B)=C\exp[2(aU_B)^{\frac{1}{2}}], \qquad (5)$$

where C is considered a constant having the same value for all final nuclei B, so that in the calculation of (1) its value is eliminated. The level density parameter a is proportional to the atomic weight A, and can be measured from the slope of the line formed by plotting $\log_{10}[N(\epsilon)/\epsilon\sigma_c(\epsilon)]$ vs $U_B^{\frac{1}{2}}$.

From measurements[15] of the energy spectrum of the neutrons emitted in (p,n) reactions for the nuclei here studied, an average value of $A/13$ was obtained for a, in agreement with similar fittings done by previous

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workers.[16–18] However, when expression (5) with the level density parameter of $A/13$ was used in the calculations of the cross sections given by reference 15, the fit was poor. For some of the nuclei, the dependence of the cross section as a function of the incoming proton energy is not reproduced, and, in general, (p,α) cross sections are overestimated as much as a factor of 10; also the ratio $(p,n)/(p,p')$ is too large.

The next level density function used to fit the measured (p,n) cross section was the one suggested by Hurwitz and Bethe.[7] They postulated that the excitation energy U_B must not be measured from the ground state, but from a corrected ground state or a characteristic level which would depend in a smooth way on the number of protons and neutrons in the nucleus, and not be affected by shell effects. Their hypothesis was introduced to explain the large neutron-capture cross section in odd-odd nuclei. The level density function in this case is given by

$$\omega(U_B)=C\exp\{2[a(U_B-E_c)]^{\frac{1}{2}}\}, \quad (6)$$

where E_c is the binding energy of the characteristic level and corresponds to the pairing energy of the nucleons in the final nucleus:

$$-E_c=P(Z)+P(N)=P(Z,N). \quad (7)$$

The pairing energies $P(Z)$ and $P(N)$ are zero for odd values of Z and N, respectively, and are negative for even values. Pairing energy values have been calculated by Cameron[19] from a comparison of his semiempirical mass formula and measured atomic masses, and by Dostrowsky *et al.*,[3] who have taken into consideration shell correction.

The value of the parameter a obtained from the experimental neutron spectra[1] was between $A/20$ and $A/30$. The fit of the proton spectra[13] from (p,p') reactions in Cu^{63} and Cu^{65} also gave an a value of $A/20$.

However, it is necessary to point out that the plot of $\log_{10}[N(\epsilon_n)/\epsilon_n\sigma_c(\epsilon_n)]$ vs $[U_B+P_B(Z,N)]^{\frac{1}{2}}$ was not a straight line through all the points and presented a slight downward curvature toward large values of ϵ_n.

The value found for a is lower by a factor of 1/2 to 1/3 than the one obtained with expression (5). This agrees with values found by el-Nadi and Walik,[20] Fong,[21] Porile,[2] and Dostrowsky *et al.*[22] using the level density function given by (6), but is lower than the one used by Kaufman[4] in fitting his experimental data and the one measured by Allan[5] in (n,p) reactions at 14 MeV.

The fact that no direct interaction is observed in these (p,n) reactions and yet a lower density parameter is measured seems to indicate that non-compound processes cannot be blamed[23] for lower values of a, but rather the explanation must lie in partial excitation of the nucleus[24] at these lower energies.

The cross sections using the level density given by (6) and values of a equal to $A/20$ and $A/30$ were calculated using pairing energies of Cameron and of Dostrowsky *et al.* The best agreement with the experimental results was found with Cameron's values. The choice between these two sets of values was made on the basis of fit to the (p,n) and (p,p') cross sections simultaneously, since the (p,n) reaction alone did not permit decision in favor of either.

The main criticism of Eq. (6) for the level density is that it eliminated all levels below the characteristic level, producing a sharp change in the level density at the energy of the level. Weinsberg and Blatt had suggested a function which approaches smoothly a constant value for energies smaller than the characteristic energy E_c, and approaches the value given by (6) for energies larger than E_c. They arrived at the following expression:

$$\omega(U_B)=C\exp\left\{2\left[\frac{a[U_B+P_B(Z,N)]}{1-\exp\{-[U_B+P_B(Z,N)]\}}\right]^{\frac{1}{2}}\right\}. \quad (8)$$

For the energy region of this experiment, this level density function gave almost the same results as expression (6). They begin to differ only for the high energy neutrons of the spectra (but not enough to overcome the experimental errors) in such a way that the a values measured from the neutron spectra are the same as those obtained with the level density given by (6).

The cross sections measured using the level density given by (8) increased with energy slightly faster than those calculated by (6), in such a way that only a very refined measurement of (p,n) cross sections will be really able to decide between them; the difference in the values of the cross section is no larger than 5%. Another reason for not observing larger differences between the two functions is that for the nuclei here studied, with the exception of Cu^{63}, the threshold for the (p,n) reaction is much lower than the lower energy of this experiment. It is, in fact, close to the threshold where one expects the difference between these two level density functions to be accentuated because of the levels existing between the ground state and the characteristic level, which is totally suppressed by (6) but not by (8).

The fact that neither level density function as given by Eqs. (6) and (8) gave a straight line when the neutron spectrum was plotted in the form $N(\epsilon)/\epsilon\sigma_c(\epsilon)$ vs $[U_B+P_B(Z,N)]^{\frac{1}{2}}$ made us go a step further in the choice of level density. If angular momentum consider-

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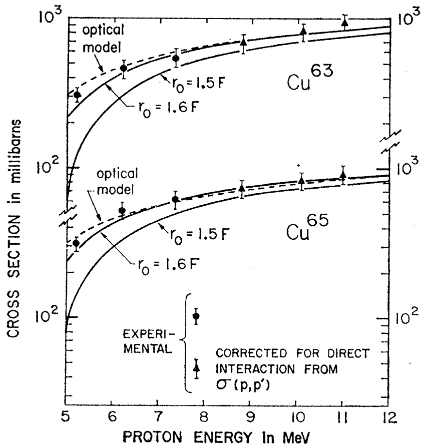

FIG. 1. Cross section for the formation of a compound nucleus by proton bombardment of Cu^{63} and Cu^{65}. The theoretical curves for $r_0=1.5$ F and 1.6 F are calculated using Dostrowsky's (reference 3) empirical formula for the reaction cross section.

ations are taken into account, such as the low orbital angular momentum of the emitted particles and the random combination of the angular momenta of the nucleons in the final nucleus, the level density is given by

$$\omega(U_B)=CU_B^{-2}\exp[2(aU_B)^{\frac{1}{2}}]. \tag{9}$$

If even-odd effects are also considered and pairing energies introduced in the calculations, the level density will be given by

$$\omega(U_B+P_B(Z,N))=C[U+P_B(Z,N)]^{-2}\times\exp\{2[a(U_B+P_B(Z,N))]^{\frac{1}{2}}\}. \tag{10}$$

The level density parameter obtained from the neutron spectra was between $A/8$ and $A/6$ and corresponds to the slope of a very good straight line through the data. These values of a agree well with the ones found by Thomson[25] and Lang[26] using expression (10).

COMPARISON WITH EXPERIMENT

The first parameter determined from the experimental data was the nucleon radius parameter r_0, for the proton interaction. For this purpose, to the measured (p,n) cross sections were added the respective (p,p') and (p,α) cross sections to obtain the proton reaction cross section for the formation of the compound nucleus.

For Cu^{63} and Cu^{65} these charged particle emission cross sections were taken from the work of Benveniste *et al.*[27] Some of the values of (p,p') were corrected for the presence of direct interaction,[28] which was not observed in the (p,α) cross sections. Figure 1 shows the experimental proton-reaction cross sections and the results of the theoretical calculation according to expression (4) for r_0 equal to 1.5 F and 1.6 F. As one can see, $r_0=1.6$ F gives a good fitting to the experimental points.

For V^{51} and Co^{59} the only available information on (p,q) cross sections was the $\sigma(p,p')$ from 4 to 6.5 MeV given by Taketani *et al.*,[29] the $\sigma(p,q)$ at 7.5 MeV for V^{51} given by Shore *et al.*,[30] the $\sigma(p,q)$ at 9.85 MeV given by Meyer and Hintz,[31] and the $\sigma(p,\alpha)$ from 8 to 23 MeV given by Fulmer *et al.*[32]

Figure 2 shows the proton reaction cross sections for V^{51} and Co^{59}. In the case of V^{51} only the experimental point at 7.5 MeV falls on the curve calculated with $r_0=1.6$ F. At 9.85 MeV one expects that $r_0=1.6$ F will be a better fitting, since the reported $\sigma(p,q)$[31] is supposed to be larger than 165 mb. For Co^{59}, the experimental proton-reaction cross section at 9.85 MeV fits the one calculated with $r_0=1.6$ F, and at the lower energies it falls between the two theoretical curves calculated with $r_0=1.6$ and 1.5 F.

It is true that at low energies $\sigma(p,\alpha)$ has not been included for these two nuclei, but according to Fulmer[32] $\sigma(p,\alpha)$ is about 3 mb for V^{51} and 15 mb for Co^{59} at 10 MeV. The exclusion of these small values of the (p,α) cross sections will not account for the use of a smaller r_0.

On the other hand, if corrections for the presence of direct interactions in the (p,p') cross sections had to be made, these would tend to lower the values of the proton-capture cross sections, favoring the choice of a lower value for r_0.

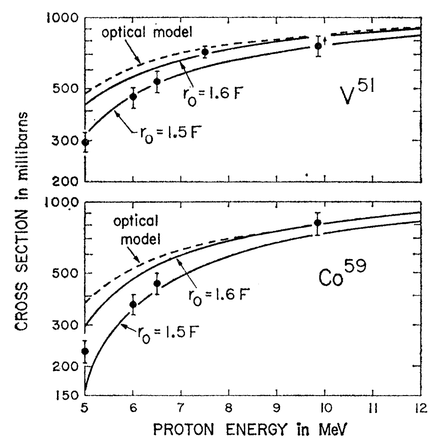

FIG. 2. Cross section for the formation of a compound nucleus by proton bombardment of V^{51} and Co^{59}. The theoretical curves for $r_0=1.5$ F and 1.6 F are calculated from Dostrowsky's (reference 3) empirical formula for the reaction cross sections.

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[32] C. B. Fulmer and D. D. Goodman, Phys. Rev. **117**, 1339 (1960).

TABLE I. Level density parameters.

Nucleus	E_p (MeV)	a [MeV^{-1}] [Eq. (6)]	a [MeV^{-1}] [Eq. (10)]
V^{51}	11.12	2.29	6.70
	10.79	2.48	7.63
	9.93	2.46	7.70
	9.07	2.38	8.48
	8.80	1.49	9.94
	7.0[a]		10.10
Co^{59}	10.80	2.03	8.28
	9.10		7.69
	7.0[a]		8.80
Cu^{63}	11.15	2.89	7.33
Cu^{65}	11.12	3.35(p,n)	7.97
	8.80	3.36(p,p')	
Cu	7.0[a]		12.4
	5.0[a]		11.5
Rh	9.2	4.50	11.3
	10.0	4.46	12.1

[a] See reference 27.

Now, one can try to find arguments to use a smaller value of r_0 in the case of Cu^{63} and Cu^{65}, for example, once the cross sections below 7.5 MeV are corrected for direct interaction they will give a better fitting to the theoretical curve calculated with $r_0=1.5$ F. However, it can be shown that even if the worst case is considered [i.e., assuming that all the (p,p') cross section is due to direct interaction] the experimental cross sections are still too large. For example, at 5.2 MeV the (p,q) cross section is 49 ± 10 mb for Cu^{65} and 173 ± 20 mb for Cu^{63}. Once these values are corrected for direct interaction, a better fit to the theoretical curve calculated with $r_0=1.6$ F will be obtained.

It was also thought that the lack of a better fitting for all these nuclei with a single value of r_0 was due somehow to the simplicity of the model for the proton-reaction cross section used in these calculations, where a square well had been taken for the nuclear potential. Figures 1 and 2 show the calculated proton-reaction cross sections using the optical model of Bjorklund and Fernbach, where a Gaussian well is used with a radius of 1.25 F and a half-width of 1.2 F. The reaction cross sections obtained, however, agree very well with those calculated with the square well potential for $r_0=1.6$ F.

It is difficult to believe that different nuclear radius parameters are needed to fit V^{51} and Co^{59}. It seems sensible to assume that the experimental proton-reaction cross sections below 6.5 MeV are underestimated. For example, if compound elastic scattering at these lower energies is larger than is generally thought (20 to 40 mb) for V^{51} and Co^{59}, its cross section added to $\sigma(p,n)$ and $\sigma(p,q)$ will raise the proton-reaction cross-section values.

Figure 3 shows the proton-capture cross section for Rh^{103}, where again $r_0=1.6$ F, gives the best fit.

For the neutron-reaction cross section needed to calculate the neutron emission probability given by Eq. (2), a nuclear radius parameter of 1.5 F was used. The values obtained with this radius agree well with

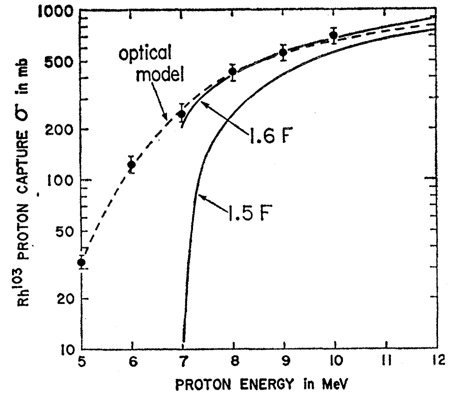

FIG. 3. Cross section for the formation of a compound nucleus by proton bombardment on Rh^{103}.

the experimental neutron cross sections obtained by Howerton[33] and Hughes *et al.*,[34] for all these nuclei.

The values of the level density parameter a measured from the neutron spectra resulting from the (p,n) reactions in these nuclei and some proton spectra of the copper isotopes are given in Table I.

The values of a in the first column of Table I obtained using the level density function given by Eq. (6) can be represented in terms of the atomic number as $A/20$ to $A/30$, while the values of a in the second column obtained from the level density given in Eq. (10) can be expressed as $A/6$ to $A/8$.

Figure 4 is a typical example of the difference in the plots of the neutron spectra when $\log_{10}\{N(\epsilon)/\epsilon\sigma_c(\epsilon)\}$ was plotted vs $[U_B+P_B(Z,N)]^{\frac{1}{2}}$ and when $\log_{10}\{[U_B+P_B(Z,N)]^2 N(\epsilon)/\epsilon\sigma(\epsilon)\}$ was plotted vs

$$[U_B+P_B(Z,N)]^{\frac{1}{2}}.$$

An interpretation of the lack of linearity in the first curve would be to assume that a fraction of the high-energy neutrons seen in the spectrum are due to direct interaction, so that the differential cross sections resulting from the compound nucleus process have been overestimated. However, the fact that a more refined function for the level density, as in the second curve, is able to give a good fit to a Maxwellian distribution for the neutron energies seems to confirm the absence of direct interaction in these (p,n) reactions.

With the measured values of r_0 and a, the (p,n) and (p,p') cross sections were calculated for the level densities given by Eqs. (6), (8), and (10), which will be called from now on, w_{II}, w_{III}, and w_{IV}. The calculations were done for the pairing energies given by Cameron[21] and Dostrowsky *et al.*[3] The general characteristics of the calculated cross sections are as follows:

(a) For a given set of values of r_0, a, and $P(Z,N)$, the cross sections calculated with any of these three

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level density functions are quite similar. The (p,n) cross sections differ by less than 10% and the $\sigma(p,p')$ by no more than 20%.

(b) The values obtained with w_{III} are intermediate between the cross sections calculated with w_{II} and w_{IV}.

(c) Variations in the density parameter a from $A/20$ to $A/30$ make the (p,n) cross sections decrease less than 5%, while making the $\sigma(p,p')$ increase as much as 30%. The same effect is obtained with the level density given by w_{IV} when a changes from $A/8$ to $A/6$.

(d) The shell corrections in the values of the pairing energies introduced by Dostrowsky *et al.*[3] make their values of $P(Z,N)$ larger in absolute value than those of Cameron for the (p,p') reactions in these nuclei. These larger values of $P(Z,N)$ make the calculated $\sigma(p,p')$ cross sections smaller than the measured ones.

(e) The calculated (p,p') seems to work for proton energies larger than 7 MeV. This is due to the empirical form for the proton-reaction cross section[3] used in the calculations, which match the Shapiro[14] calculations for protons at around 3 MeV for these middle nuclei ($Z=30$). Below this energy the ratio σ_p/σ_g obtained from Dostrowsky's expression falls down sharply to zero. In the calculation of the emission probability for the process $A(p,p')B$ due to Coulomb barrier and pairing energies, which are both negative, the reaction cross section for the inverse reaction, corresponding to 3-MeV protons incident on nucleus B, occurs for protons of around 7 MeV incident on A. As a result, the cross sections below 7 MeV are underestimated for (p,p') and overestimated by the same amount for the (p,n) process.

Cu^{63} and Cu^{65} (p,n) and (p,p') Cross Sections

Figures 5 and 6 show the (p,n) and (p,p') cross sections for Cu^{63} and Cu^{65}, respectively. The theoretical

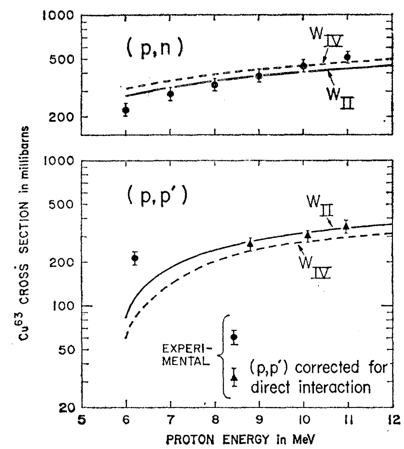

FIG. 5. $Cu^{63}(p,n)Zn^{63}$ and $Cu^{63}(p,p\ )Cu^{63}$ cross sections. The theoretical curves are calculated with $r_0=1.6$ F for the proton reaction cross section and for the level densities

$$w_{\mathrm{II}}=\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}$$

and

$$w_{\mathrm{IV}}=[U_B+P_B(Z,N)]^{-2}\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}.$$

curves shown were calculated with $r_0=1.6$ F, $a=A/30$, and $a=A/6$ for the level densities given by w_{II} and w_{IV}, respectively. Cameron pairing energies were used. The curves correspond to the best fit to the experimental data.

Since only (p,n) and (p,p') cross sections were fitted, the experimental values[29] of the $\sigma(p,\alpha)$ were subtracted from the calculated proton-reaction cross section, before calculating Eq. (1).

V^{51} and Co^{59} (p,n) and (p,p') Cross Sections

For these two nuclei, the cross sections were calculated for $r_0=1.5$ F and 1.6 F, because from the proton-reaction cross sections shown in Fig. 2, it is not clear

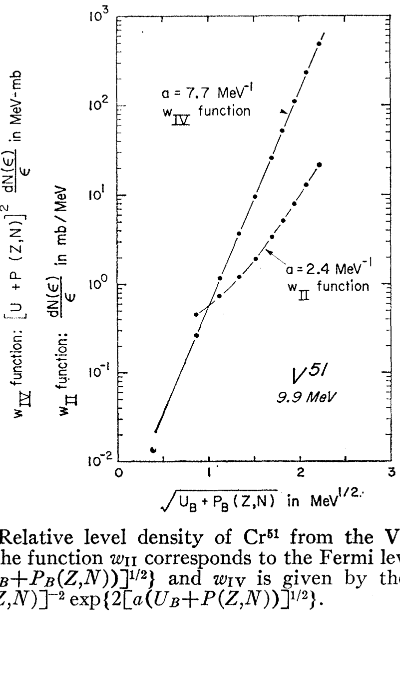

FIG. 4. Relative level density of Cr^{51} from the $V^{51}(p,n)Cr^{51}$ reaction. The function w_{II} corresponds to the Fermi level density $\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}$ and w_{IV} is given by the function $[U_B+P_B(Z,N)]^{-2}\exp\{2[a(U_B+P(Z,N))]^{1/2}\}$.

TABLE II. $\sigma(p,n)$ and $\sigma(p,p')$ for V^{51} calculated with different level densities given by w_{II}, w_{III}, and w_{IV}.

E (MeV)	w_{II}	w_{III}	w_{IV}	Exp
		$\sigma(p,n)$ in V^{51} (mb)		
5	316	316	316	275±28
6	452	454	462	380±38
7	530	532	537	475±48
8	580	584	588	535±54
9	614	620	628	555±56
10	638	645	659	590±59
11	655	662	685	595±60
12	669	675	705	
		$\sigma(p,p')$ in V^{51} (mb)		
5	...	...	...	18±1.8
6	15	13	5	70±7
7	44	42	37	
7.5				134
8	75	71	67	
9	104	98	90	
10	130	123	109	>165±10
11	152	147	124	
12	174	168	138	

which is the value of the nuclear radius parameter to be used.

The general feature of the calculations done with $r_0=1.5$ F is that the ratios $\sigma(p,n)$ to $\sigma(p,p')$ are overestimated. As an example, Table II shows the values of the cross sections for V^{51}, calculated with a level density parameter of $A/30$ for the level density given by w_{II} and w_{III} and $A/6$ for the level density given by w_{IV}. Cameron's pairing energies were used. The table shows, as pointed out previously, that the cross sections calculated in these three different ways are very similar.

Figures 7 and 8 show the (p,n) and (p,p') cross sections for V^{51} and Co^{59}, where the proton-reaction cross section was calculated with $r_0=1.5$ F and $r_0=1.6$ F. A better fit is obtained with the larger value for the nuclear radius, mainly for the (p,p') cross sections. The (p,n) cross sections, however, are still

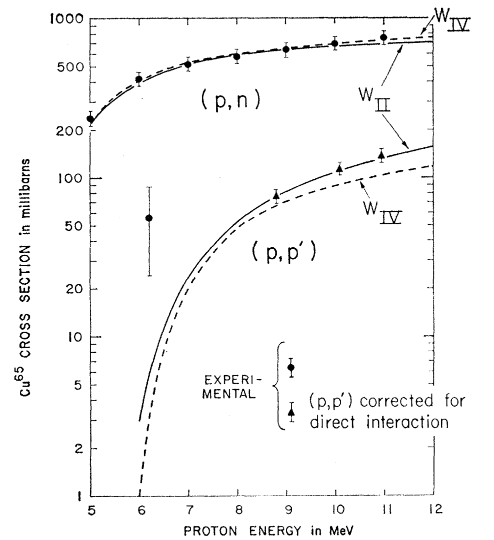

FIG. 6. $Cu^{65}(p,n)Zn^{65}$ and $Cu^{65}(p,p')Cu^{65}$ cross sections. The theoretical curves are calculated using $r_0=1.6$ F for the proton reaction cross section and for the level densities

$$w_{II}=\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}$$

and

$$w_{IV}=[U_B+P_B(Z,N)]^{-2}\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}.$$

overestimated by as much as 30% at 5 MeV and 10% at 10 MeV. This results from the fact that the proton-reaction cross sections calculated with $r_0=1.6$ F are larger than the experimental values, especially at the lower energies (Fig. 2). If $r_0=1.5$ F is used, good agreement for both (p,n) and (p,p') cross sections can be obtained if the ratio of the emission probabilities for neutron and proton emission matches the experimental ratio of these two cross sections. The previous calculations suggest that a larger value for the proton-emission probability is needed to account for the experimental results. Such an effect will result if the inverse-reaction cross section in Eq. (2) is larger than the reaction cross section for the incident proton for a

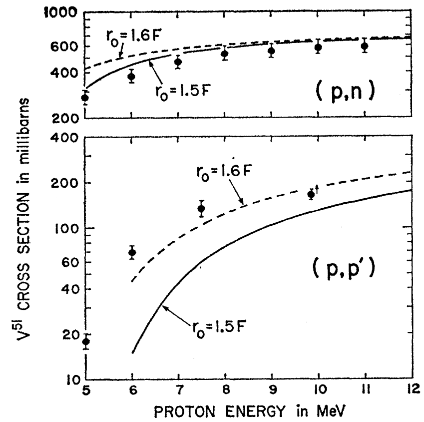

FIG. 7. $V^{51}(p,n)Cr^{51}$ and $V^{51}(p,p')V^{51}$ cross sections. The theoretical curves are calculated using $r_0=1.5$ F and $r_0=1.6$ F for the proton reaction cross sections. The level density used is $w_{II}=\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}$.

given proton energy. Fulmer and Cohen[35] and others[36,37] have suggested that this is so, because in the process $A(p,p')B$ the inverse-reaction cross section is calculated for the interaction of the outgoing proton p' and the excited nucleus B, which has a lower Coulomb barrier than A due to nuclear surface waves.[38]

This argument of a lower Coulomb barrier for an excited nucleus has been challenged by Lane and Parker[39] especially at these lower excitation energies. However, to show the effect of this correction, a 10% increase in the nuclear radius of the excited nucleus was arbitrarily assumed.

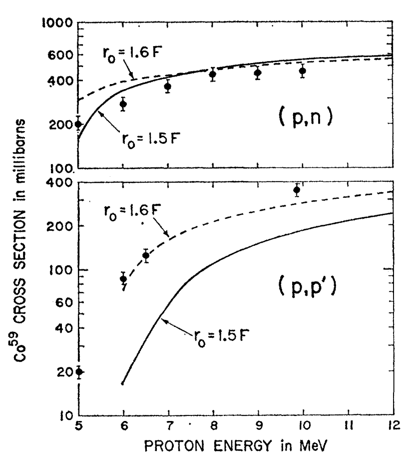

FIG. 8. $Co^{59}(p,n)Ni^{59}$ and $Co^{59}(p,p')Ni^{59}$ cross sections. The theoretical curves are calculated using $r_0=1.5$ F and $r_0=1.6$ F for the proton reaction cross sections. The level density used is $w_{III}=\exp\{2[a(U_B+P_B(Z,N))]^{1/2}[1-\exp(-U_B+P_B(Z,N))]^{-1/2}\}$.

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Figures 9 and 10 show the cross sections for V^{51} and Co^{59}. The reaction cross section for the incident proton was calculated with $r_0=1.5$ F and the Coulomb barrier of the final nucleus B with $r_0=1.65$ F.

The agreement with experiment in this case is very good, so as to encourage this line of thought. However, the present work does not permit a final decision in favor of a lower Coulomb barrier, since for that it will be necessary to have the spectrum of the emitted protons in the (p,p') reaction of these nuclei.

It is true that a larger proton-emission probability could also be obtained by means of a different level density function in Eq. (2), or with a higher nuclear temperature and the same level densities used here. Again, to confirm any of these hypotheses, the proton differential cross section is needed. However, it seems fair to assume that the level density and respective level density parameter measured from the neutron spectrum of the process $A(p,n)B$ is also valid for the proton spectrum of the $A(p,p')B$ reactions.

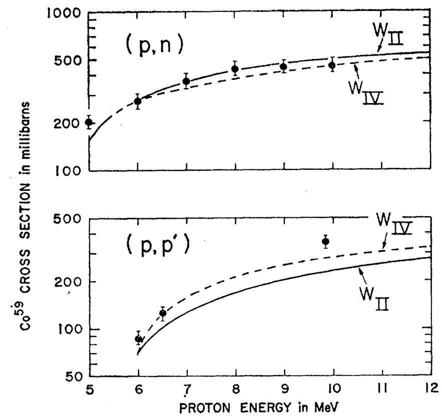

FIG. 10. $Co^{59}(p,n)Ni^{59}$ and $Co^{59}(p,p')Co^{59}$ cross sections. The theoretical curves are calculated using $r_0=1.5$ F for the reaction cross section of protons incident on Co^{59} and $r_0=1.65$ F for the Coulomb barrier of the final nucleus. The level densities used are $w_{II}=\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}$ and $w_{IV}=[U_B+P_B(Z,N)]^{-2}\times\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}$.

$R^{103}(p,n)Pd^{103}$ Cross Sections

For rhodium the empirical formula for the proton-reaction cross section given by Dostrowsky *et al.*[3] matches the continuous theory of Shapiro[14] for a proton kinetic energy of about 7 MeV. Below this energy, the ratio of the proton-reaction cross section with a given nucleus A to the geometrical cross section of the nucleus decreases much faster toward zero value than the ratio calculated with Shapiro theory.[1] Figure 3 shows that Dostrowsky's calculations fit the experimental point from 7 MeV up.

For rhodium, in the energy range of the present work, the predominant reaction initiated by protons is neutron emission. The (p,α)[33] reaction amounts to 1 or 2 mb and the (p,p') cross section must be about 30 to 50 mb for 10-MeV protons according to the figure given by Hintz for the (p,q) cross section in silver at 9.85 MeV.[31] Thus, the measured (p,n) cross sections at these energies are equivalent to the respective proton-reaction cross sections. The experimental points shown in Fig. 3 are the measured (p,n) cross sections.

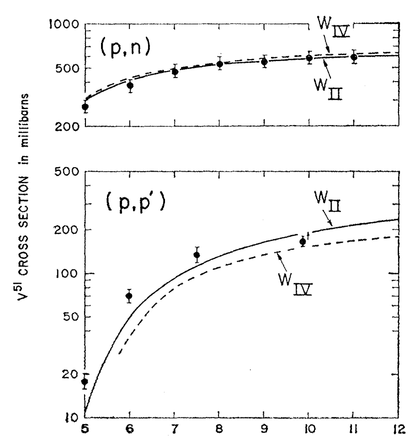

FIG. 9. $V^{51}(p,n)Cr^{51}$ and $V^{51}(p,p')V^{51}$ cross sections. The theoretical curves are calculated using $r_0=1.5$ F for the reaction cross section of protons incident on V^{51} and $r_0=1.65$ F for the Coulomb barrier of the final nucleus. The level densities used are $w_{II}=\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}$ and $w_{IV}=[U_B+P_B(Z,N)]^{-2}\times\exp\{2[a(U_B+P_B(Z,N))]^{1/2}\}$.

CONCLUSIONS

The present work shows that the (p,n) cross sections in the energy range of 5 to 11 MeV are the result of the compound nucleus mechanism of the nuclear reactions studied.

The experimental values of the cross sections fit quite well with the predictions of the compound nucleus theory, without necessity of arbitrary adjustment of the parameters involved. This can be achieved if the values of these parameters are measured from other pertinent experimental data.

Although the different functions for the level density gave very close values among themselves for the (p,n) and (p,p') cross sections, we can conclude that the level density given by Eq. (10) is the one that better reflects the experimental results. It not only gives good values for the cross sections, but also gives a straight-line fit to the plot of

$$d\sigma/d\epsilon/\epsilon\sigma_c(\epsilon) \text{ vs } [U_B+P_B(Z,N)]^{\frac{1}{2}},$$

obtained from the differential neutron cross section for these nuclei.

Finally, the fit of the (p,p') cross sections could have been done more accurately if the proton spectra from the differential (p,p') cross sections had been known for these energies, especially in the case of V^{51} and Co^{59}. Only the proton spectra will establish whether the percentage of the (p,p') cross section due to direct interaction is large enough that, when subtracted from the total (p,p') cross section, it will fit with the calculations done for $r_0=1.5$ F, or if the inverse-reaction cross section was larger due to a lower Coulomb barrier for the excited nuclei.

ACKNOWLEDGMENTS

Our thanks go to Dr. J. Anderson and Dr. J. Benveniste for stimulating discussions and for permitting us to use experimental data previous to publication. We also thank Henry Catron, D. R. Rawles, and the crew of the 90-in. cyclotron for their cooperation during the running of the experiment, and Don Freedman who wrote the IBM programs.

PHYSICAL REVIEW VOLUME 128, NUMBER 1 OCTOBER 1, 1962

Level Structure in Ne^{22} and Si^{30} from the Reactions $O^{18}(\alpha,n)Ne^{21}$ and $Mg^{26}(\alpha,n)Si^{29}$

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(Received April 6, 1962; revised manuscript received June 20, 1962)

Compound states of high excitation in Ne^{22} and Si^{30} have been observed in the total neutron yield from the reaction $O^{18}(\alpha,n)Ne^{21}$ and $Mg^{26}(\alpha,n)Si^{29}$. In the case of Ne^{22}, twenty-five resonances were observed, varying in width from 5 to 150 keV for alpha bombarding energies from 2.5 to 5 MeV (excitation energy from 11.7 to 13.8 MeV). The reaction $Mg^{26}(\alpha,n)Si^{29}$ showed forty resonances varying in width from less than 10 to 60 keV for alpha energies from 3 to 5.3 MeV (excitation energy from 13.3 to 15.3 MeV). Absolute cross sections were measured for both reactions. A statistical analysis of the area under the excitation curves gives an alpha-particle strength function $\bar{S}_\alpha=0.02_6$ for $O^{18}+\alpha$ and $\bar{S}_\alpha=0.01_6$ for $Mg^{26}+\alpha$. Analysis of the individual resonances in $O^{18}+\alpha$ gives a value of $\langle\gamma_\alpha^2\rangle/D\leq0.04$, in agreement with the $\bar{S}_\alpha$ obtained by the statistical analysis. The strength functions are probably reliable to better than a factor of 2.

INTRODUCTION

COMPOUND states of high excitation in Ne^{22} have been studied by means of the reaction $O^{18}(\alpha,n)Ne^{21}$ by Roy *et al.*[1] using natural alpha particles, and by Bonner *et al.*[2] using accelerated particles. Roy's experiments suffered from the inherent lack of resolution typical of natural alpha sources. The work of the Rice group showed much structure; however, due to the target thickness (120 keV for 2-MeV alphas), most of the resonances were unresolved. Several investigators[3] have studied the $Mg^{26}(\alpha,n)Si^{29}$ reaction using natural alpha sources with poor energy resolution. No other experiments have been reported covering these high high excitation energies in the compound nuclei.

EXPERIMENTAL PROCEDURES

The $(He^4)^+$ beam of the ORNL 5.5-MV Van de Graaff was stripped to $(He^4)^{++}$ before entering the 90° analyzing magnet. After bending and energy analysis, it was allowed to bombard thin targets of high isotopic purity placed at the center of the graphite-sphere neutron detector.[4] Detector efficiency was checked with a radium-beryllium neutron source which had been calibrated by comparison with a Bureau of Standards source. For the neutron energies obtained here the response of the ball[4] is constant to within less than $\pm5\%$. Bombarding alpha energies were determined by calibrating the magnet with the $Li^7(p,n)Be^7$ reaction, utilizing the known proton calibration, and correcting the calibration constant with the known ratio of alpha-to-proton mass. It is estimated that the energy calibration is accurate to $\pm0.2\%$. Narrow resonances measured with several targets at various times repeated to within this figure.[5]

$O^{18}(\alpha,n)Ne^{21}$

The O^{18} targets used for the yield curves were prepared by anodizing tantalum blanks in water[6] whose oxygen was enriched to greater than 97% O^{18}. Since target thickness is proportional to the anodizing voltage, a series of targets could easily be prepared with reasonably well-known ratios of thicknesses. The thinnest target had a small energy loss compared to the narrowest resonance and the natural widths thus

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[6] Obtained from the Weizmann Institute.