M Meson and a Generalization of the Pomeranchuk Relations*

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Under the hypothesis that a K-π resonance is vector, we examine its role in the associated production of Λ by π and in Λ production by $\bar{K}$. We shall demonstrate the existence of a new symmetry between two reaction amplitudes. This symmetry may be regarded as a generalization of Pomeranchuk's relations and should appear at high energies and low momentum transfers when both amplitudes are dominated by the same pole or pseudopole, as is to be expected according to the Regge pole hypothesis. Specifically, we find, in considering the details of the role of a strange vector meson in the processes $\pi+N\rightarrow\Lambda+K$ and $\bar{K}+N\rightarrow\Lambda+\pi$, that the associated production amplitude in the forward direction (for the K) at high energies is asymptotically equal to the negative of the amplitude characterizing Λ production by a $\bar{K}$. The contribution of the dominant pole terms in these amplitudes is constructed for the high-energy limit and the energy and momentum transfer dependences are compared for the alternative hypothesis of composite or elementary particle behavior of a pole term. We discuss experiments which are needed to supply data for a test of the Regge pole hypothesis. The results of these experiments, which are feasible with the new large accelerators, will be most important as guides for the construction of theories of the strong interactions.

I. INTRODUCTION

TWO resonances have been found in the $K-\pi$ system, one at 884 MeV[1] and another at 730 MeV.[2] It is quite possible that one of these resonances belongs to the octet of vector mesons predicted by Gell-Mann and Ne'eman; adopting the notation of Gell-Mann, we call it the M meson, or simply the M. There are two important questions to be raised regarding this object: (1) How strongly is it coupled to other particles? (2) Does the assumption that the M contribution dominates a given amplitude enable us to understand any important features of reactions in which it is exchanged?

Both questions are considered in this paper. In Sec. II, the strength of the coupling of the M to the $K\pi$ system is related to the width of the resonance. The same coupling constant is involved in the production of the M in the reaction $K^-+p\rightarrow M^-+p$, which we also investigate. In Sec. III, the contribution of the M to the associated production amplitude is studied. We treat the M according to the Regge pole hypothesis there, and discuss how experiments at beam energies within the range of existing accelerators can be used to decide whether the M behaves as predicted by the Regge pole hypothesis. The crossed hyperon production reaction, $\bar{K}^0+p\rightarrow\Lambda+\pi^+$, is examined in the same spirit in Sec. IV. Finally, the existence of a new class of symmetries in asymptotic amplitudes, which are generalized Pomeranchuk relations, is illustrated in Sec. V.

II. PROPERTIES OF THE M MESON

There is, at 884 MeV, an object which appears as an $I=1/2$ resonance[1] in the $K\pi$ system. Assuming it to be a vector particle, we define the coupling constant $\gamma_{MK\pi}$ so that the matrix element for the decay $M^+\rightarrow K^+ +\pi^0$ is

$$T=\gamma_{MK\pi}e^M\cdot(p_\pi-p_K), \tag{II1}$$

where e^M is the polarization four-vector of the M, and p_π, p_K are the four-momenta of the decay products. The rate for the decay is

$$\Gamma(M^+\rightarrow K^++\pi^0)=\gamma_{MK\pi}{}^2k^3/6\pi m_M{}^2, \tag{II2}$$

where

$$4m_M{}^2k^2=[m_M{}^2-(m_K-m_\pi)^2][m_M{}^2-(m_K+m_\pi)^2]. \tag{II3}$$

Since the M has $I=1/2$, the charged M decays more often into a charged pion and neutral K meson; the branching ratio is two. Neglecting other decay modes, which certainly have much smaller widths, the decay rate for the M meson is

$$\Gamma_M=(\gamma_{MK\pi}{}^2/4\pi)\times 58\text{ MeV}. \tag{II4}$$

The width of the M is quoted to be 60 MeV,[2] so that

$$\gamma_{MK\pi}{}^2/4\pi=1.03. \tag{II5}$$

According to the unitary symmetry scheme,[3] this number should be comparable to the coupling of the ρ meson to the two-pion system, which is $\gamma_{\rho\pi\pi}{}^2/4\pi=0.50$ if we assume 100 MeV for the ρ width.[4]

The coupling constant $\gamma_{MK\pi}$ enters also in the pion pole approximation to the M production amplitude in

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[1] M. Alston, L. Alvarez, P. Eberhard, M. Good, W. Graziano, H. Ticho, and S. Wojcicki, Phys. Rev. Letters **6**, 300 (1961).

[2] G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. A. Smith, Phys. Rev. Letters **8**, 447 (1962).

[3] M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report CTSL-20 (1961) (unpublished); Phys. Rev. **125**, 1067 (1962).

[4] B. Maglić, L. Alvarez, A. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters **7**, 421 (1961).

the reaction $K+N\rightarrow M+N$. One finds

$$\gamma_{MK\pi}^2/4\pi=\lim_{t\rightarrow m_\pi^2}\left\{4(g_{\pi NN}^2/4\pi)^{-1}\times\frac{(p_K/p_M)s(t-m_\pi^2)^2m_M^2}{(-t)[(m_M^2+m_K^2-t)-4m_M^2m_K^2]}\frac{d\sigma}{d\Omega}\right\},\quad\text{(II6)}$$

where s is the square of the total energy in the center-of-mass system, and

$$4sp_M^2=[s-(m_N-m_M)^2][s-(m_N+m_M)^2],\quad\text{(II7)}$$

$$4sp_K^2=[s-(m_N-m_K)^2][s-(m_N+m_K)^2],\quad\text{(II8)}$$

$$2s^{1/2}E_M=s+m_M^2-m_N^2,\quad\text{(II9)}$$

$$2s^{1/2}E_K=s+m_K^2-m_N^2,\quad\text{(II10)}$$

$$t=(E_M-E_K)^2-p_M^2-p_K^2+2p_Mp_K\cos\theta.\quad\text{(II11)}$$

Angular distributions for this reaction are not yet available, so that the coupling constant cannot be determined by this extrapolation procedure. However, Bég and DeCelles[5] and Chan[6] have proposed that existing experimental data on the total production cross section be fitted in the pion pole approximation. Alston *et al.*[1] state that at $s=3.48$ GeV^2, the total cross section for M^- production is 1.4 ± 0.3 mb. If it is assumed that the pion pole dominates the amplitude, this leads to a value of $(0.21\pm0.05)\%$ for $\gamma_{MK\pi}^2/4\pi$, which is not in agreement with the value obtained from the M width. Theoretically, however, we have no reason to expect the pion pole to dominate the total cross section at such low energies, and one suspects strongly that any agreement would be fortuitous. That it, indeed, must be so has recently been demonstrated by a measurement of the total cross section for M^0 production by the Alston group.[7] Their value of 0.7 mb is $\frac{1}{8}$ of what should be expected if the pion pole dominates. It is, thus, apparent that angular distributions at considerably higher energies are needed to test the correlation expected between M production and its decay width.

If the 730-MeV resonance is to be identified with the M meson, then Eq. (II4) becomes

$$\Gamma_M=(\gamma_{MK\pi}^2/4\pi)\times(14\text{ MeV}).\quad\text{(II12)}$$

III. ASSOCIATED PRODUCTION

In considering the amplitude for associated production by pions we shall treat the reaction:

$$\pi^-+p\rightarrow\Lambda+K^0.$$

All other amplitudes can be obtained from it, since when Λ's are produced, the reaction is in a pure $I=1/2$ state. The amplitude contains only two independent functions of the relativistic invariants, and can be written as

$$T=\bar{u}_\Lambda\{A(s,t)-iB(s,t)(\boldsymbol{q}+\boldsymbol{r})/2\}u_p,\quad\text{(III1)}$$

since the relative $(K\Lambda N)$ parity is almost certainly negative. In our work we designate the four-momentum of the N, Λ, π, K by p, p', q, r, respectively, and we adhere to the convention that

$$s=-(p+q)^2,\quad t=-(p'-p)^2,\quad u=-(r-p)^2.\quad\text{(III2)}$$

The subsidiary condition,

$$s+t+u=m_N^2+m_\Lambda^2+m_\pi^2+m_K^2,\quad\text{(III3)}$$

expresses the well-known fact that there are only two relativistically invariant variables in the problem.

Let us proceed by analyzing the t-exchange channel. In this channel only a system with unit hypercharge, zero baryonic charge, and $I=1/2$ can be exchanged. By developing the $\pi+\bar{K}\rightarrow\Lambda+\bar{N}$ amplitude in partial waves, as is done in Appendix A, it can be shown that the exchange of a state of spin J, which must necessarily have parity $(-)^J$, gives the following expressions for the invariant functions A and B:

$$A(s,t)\rightarrow C_J^{(1)}(t)s^J,\quad\text{(III4)}$$

$$B(s,t)\rightarrow JC_J^{(2)}(t)s^{J-1}.\quad\text{(III5)}$$

Only the term which dominates at high energies has been retained.

If the Regge pole hypothesis[8] is correct, the functions A and B will be dominated at high energies in the forward direction, i.e., $s\rightarrow\infty$, and t small, by a term associated with the exchange of a vector meson with one unit of hypercharge—the M meson, which in the numerical formulas we assume to have a mass of 884 MeV. The asymptotic form of these functions at high energies will be:

$$A(s,t)\underset{s\rightarrow\infty}{\longrightarrow}\frac{1-e^{-i\pi\alpha_M(t)}}{2\sin\pi\alpha_M(t)}\left(\frac{s}{s_0}\right)^{\alpha_M(t)}\frac{2s_0}{m_N+m_\Lambda}\times[-b_{\Lambda NMK\pi}^{(1)}+\alpha_M(t)b_{\Lambda NMK\pi}^{(2)}],\quad\text{(III6)}$$

$$B(s,t)\underset{s\rightarrow\infty}{\longrightarrow}-\frac{1-e^{-i\pi\alpha_M(t)}}{2\sin\pi\alpha_M(t)}2\alpha_M(t)\left(\frac{s}{s_0}\right)^{\alpha_M(t)-1}\times b_{\Lambda NMK\pi}^{(2)}(t).\quad\text{(III7)}$$

The signature of the Regge trajectory is negative, since the resonance has $J=1$, and the two functions $b^{(i)}(t)$ are independent. The constant s_0 is arbitrary, and

[5] M. Baqi Bég and P. DeCelles, Phys. Rev. Letters 6, 145, 428 (1961).
[6] C. Chan, Phys. Rev. Letters 6, 383 (1961).
[7] M. Alston, L. Alvarez, P. Eberhard, M. Good, W. Graziano, H. Ticho, and S. Wojcicki, Bull. Am. Phys. Soc. 6, 435 (1961).
[8] S. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).

should be chosen so that the functions $b(t)$ vary as slowly as possible.

On comparing these asymptotic expressions with those resulting from the exchange of the M meson in the pole approximation, which are derived in Appendix B, one can identify various quantities at $t=m_M^2$. First of all, since the resonance occurs in p-wave $K-\pi$ scattering, we have $\mathrm{Re}\alpha_M(m_M^2)=1$. The width of the resonance is proportional to $\mathrm{Im}\alpha_M(m_M^2)=I_M$, and inversely proportional to the slope of the Regge trajectory, $\epsilon_M=\mathrm{Re}(d\alpha/dt)_{t=m_M^2}$:

$$\Gamma_M=I_M/m_M\epsilon_M. \quad \text{(III8)}$$

Finally (leaving off some of the subscripts where their omission results in no ambiguity), we have

$$b^{(1)}(m_M^2)/\pi\epsilon_M=6^{1/2}\gamma_{MK\pi}\gamma_{\Lambda NM}, \quad \text{(III9)}$$

and

$$b^{(2)}(m_M^2)/\pi\epsilon_M = -6^{1/2}\gamma_{MK\pi}[\gamma_{\Lambda NM}+\mu_{\Lambda NM}(m_\Lambda+m_N)], \quad \text{(III10)}$$

where $\gamma_{MK\pi}$ and $\gamma_{\Lambda NM}$ are, respectively, the coupling constants of the M to the $K\pi$ and the ΛN currents, and $\mu_{\Lambda NM}$ is the anomalous magnetic moment in the ΛNM vertex.

To calculate the cross sections and polarizations, it is convenient to write T in a reduced form which is sandwiched between two-component spinors. Defining functions T' and T'' such that $T\to T'+iT''\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{r}/qr$, one finds

$$\{(E_\Lambda+m_\Lambda)(E_N+m_N)\}^{1/2}T' = (E_\Lambda+m_\Lambda)(E_N+m_N)[A+B(E_\pi+E_K)/2] +B[(E_\Lambda+m_\Lambda)q^2+(E_N+m_N)r^2]/2 +qr\cos\theta[-A+\tfrac{1}{2}B(2s^{1/2}+m_\Lambda+m_N)], \quad \text{(III11)}$$

$$[(E_\Lambda+m_\Lambda)(E_N+m_N)]^{1/2}T'' = qr\sin\theta[A-\tfrac{1}{2}B(2s^{1/2}+m_\Lambda+m_N)]. \quad \text{(III12)}$$

In these reduced expressions, E refers to the energy of the particle and q, r the magnitude of the three-momentum in the center-of-mass system. The Λ's produced will be partially polarized in the $\mathbf{q}\times\mathbf{p}' = -\mathbf{q}\times\mathbf{r}$ direction; the degree of polarization, P, is easily shown to be

$$P=2\,\mathrm{Im}T'^*T''\sin\theta/[|T'|^2+|T''|^2\sin^2\theta]. \quad \text{(III13)}$$

The cross section for associated production is then

$$d\sigma/d\Omega=(1/64\pi^2s)(r/q)\{|T'|^2+|T''|^2\sin^2\theta\}, \quad \text{(III14)}$$

where

$$(r/q)^2=[s-(m_\Lambda-m_K)^2][s-(m_\Lambda+m_K)^2]/[s-(m_N-m_\pi)^2][s-(m_N+m_\pi)^2].$$

This may be rewritten in terms of the functions A and B, in which case one finds

$$d\sigma/dt=\{16\pi[s-(m_N-m_\pi)^2]\times[s-(m_N+m_\pi)^2]\}^{-1}\times\tfrac{1}{2}\Sigma|T|^2,$$

where

$$\tfrac{1}{2}\Sigma|T|^2=|A|^2[(m_N+m_\Lambda)^2-t]+\mathrm{Re}A^*B[(m_\Lambda+m_N)\times(2s+t-m_\Lambda^2-m_N^2)-2m_Nm_K^2-2m_\Lambda m_\pi^2] +|B|^2[s^2-s(m_\Lambda^2+m_N^2+m_K^2+m_\pi^2-t) -\tfrac{1}{4}t(m_\Lambda+m_N)^2+\tfrac{1}{4}(m_\Lambda^2+m_N^2)^2 +m_K^2m_\pi^2+m_\Lambda m_N(m_K^2+m_\pi^2)]. \quad \text{(III15)}$$

In units of an energy of 1 GeV, Eq. (III15) becomes

$$16\pi(s-0.636)(s-1.161)d\sigma/dt =|A|^2(4.22-t)+\mathrm{Re}A^*B(4.10s+2.05t-4.88) +|B|^2(s^2-2.39s+st-1.055t+1.415). \quad \text{(III16)}$$

At high energies, the cross section in the backward (Λ) direction will approach

$$\frac{d\sigma}{dt}\to\frac{1}{16\pi}\left|\frac{1-e^{-i\pi\alpha_M(t)}}{2\sin\pi\alpha_M(t)}\right|^2\left(\frac{s}{s_0}\right)^{2\alpha_M(t)-2}\left\{|b^{(1)}(t)|^2 -\frac{t}{(m_\Lambda+m_N)^2}|b^{(1)}(t)-\alpha_M(t)b^{(2)}(t)|^2\right\}. \quad \text{(III17)}$$

We may recall that at high energy in the center-of-mass system,

$$t=m_\Lambda^2+m_N^2-2E_\Lambda E_N+2p'p\cos\theta \to -\tfrac{1}{2}(s-m_\Lambda^2-m_N^2-m_K^2-m_\pi^2)(1-\cos\theta), \quad \text{(III18)}$$

or, in units of $(\mathrm{GeV})^2$,

$$t\to-\tfrac{1}{2}(s-2.39)(1-\cos\theta).$$

Data on this reaction at high energies are not yet available. The best one has at this time are those of Eisler *et al.*[9] at a pion lab momentum of 1.43 GeV/c, which corresponds to $s=3.58$ GeV^2. This is certainly not a large enough energy for us to suggest that the Regge pole on the M trajectory must dominate the associated production amplitude; at ten times this energy, which is now possible with the CERN and Brookhaven machines, we would expect the dominance of this Regge pole in the forward (K^0) direction. The Λ production is backward-peaked even at these low energies; however, the degree of peaking should be very much greater at the higher energies. The peak should be exponential, considered as a function of t, with a width that falls off logarithmically with the pion lab energy. This latter statement is made on the assumption that the M meson behaves as a composite particle; we

[9] F. Eisler, R. Plano, A. Prodwell, N. Samios, M. Schwartz, and J. Steinberger, Nuovo cimento **10**, 468 (1958). See also review of J. Steinberger, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1958), p. 147.

expect ϵ_M to be of the order 1 $(\text{GeV})^{-2}$. In contrast, if the M contributes in the fashion of an elementary particle, the trajectory degenerates to a point, and the amplitude will not drop off exponentially in the momentum transfer.

It is very important that the angular distributions at small angles and high energies be measured in order to determine the character of the M pole. The formulas we have derived will be useful in interpreting such data.

IV. HYPERON PRODUCTION IN $\bar{K}N$ SCATTERING

As a specific example of the $I=1$ reaction $\bar{K}+N\rightarrow\Lambda+\pi$, let us consider $\bar{K}^0+p\rightarrow\Lambda+\pi^-$, which corresponds to the u-channel of the associated production reaction studied in the preceding section. If q and r again denote the pion and K meson four-momentum, respectively, then the amplitude for this $\bar{K}N$ inelastic scattering process is given by

$$T=\bar{u}_\Lambda\{A(s,t)+\tfrac{1}{2}iB(s,t)(\mathbf{q}+\mathbf{r})\}u_p, \qquad \text{(IV1)}$$

where

$$\begin{aligned} s&=-(p-q)^2=u_u,\\ t&=-(p'-p)^2=t_u,\\ u&=-(p+r)^2=s_u, \end{aligned} \qquad \text{(IV2)}$$

and the functions A and B are analytic continuations of those in the preceding section.

According to the Regge hypothesis, at high energies in the forward direction, $(s_u\rightarrow\infty,\ u_u\rightarrow-\infty,\ t_u \text{ small})$, the functions A and B are again dominated by the pole associated with the M meson. In fact, all our results regarding the asymptotic form of the functions and cross sections for the reaction $\pi+N\rightarrow\Lambda+K$ apply as well to this inelastic $\bar{K}N$ scattering process. In particular, the following asymptotic relations will be valid as $u\rightarrow\infty$, t small:

$$A(s,t)\underset{u\rightarrow\infty}{\longrightarrow}-\frac{1-e^{-i\pi\alpha_M(t)}}{2\sin\pi\alpha_M(t)}\left(\frac{u}{s_0}\right)^{\alpha_M(t)}\frac{2s_0}{m_\Lambda+m_N}\times\{-b^{(1)\prime}(t)+\alpha_M(t)b^{(2)\prime}(t)\}, \qquad \text{(IV3)}$$

$$B(s,t)\underset{u\rightarrow\infty}{\longrightarrow}-\frac{1-e^{-i\pi\alpha_M(t)}}{2\sin\pi\alpha_M(t)}\left(\frac{u}{s_0}\right)^{\alpha_M(t)-1}2\alpha_M(t)b^{(2)\prime}(t). \qquad \text{(IV4)}$$

But we can go further than this. The functions b and b' are characteristic of the crossed channel, i.e., the t channel, which is the same for both the associated production and the $\bar{K}N$ reactions. Therefore, the functions b and b' are essentially one and the same, provided only that we put in the angular functions in a consistent fashion. This latter requirement is easily fulfilled simply by continuing to write $x_t=\cos\theta_t$ as $(s+2E_NE_\pi-m_N^2-m_\pi^2)/(2q_tp_t)$. On going from the s channel to the u channel in the asymptotic region, the only change is that of the sign of x_t. But such an interchange gives back the same amplitude except for the factor σ, where σ is the signature of the Regge pole. Accordingly, we see that

$$b_{\Lambda NMK\pi}{}^{(1)\prime}(t)=+b_{\Lambda NMK\pi}{}^{(1)}(t), \qquad \text{(IV5)}$$

$$b_{\Lambda NMK\pi}{}^{(2)\prime}(t)=+b_{\Lambda NMK\pi}{}^{(2)}(t). \qquad \text{(IV6)}$$

[The sign change coming from $(\mathbf{q}+\mathbf{r})$ on going from the s to the u channel is responsible for the extra minus sign in Eq. (IV3) as compared to Eq. (IV4).] At a given center-of-mass energy in the asymptotic region and at a given small momentum transfer, the amplitudes for $\pi+N\rightarrow\Lambda+K$ and $\bar{K}+N\rightarrow\Lambda+\pi$ are related by a minus sign, and thus the differential cross sections, polarizations, etc., will be the same for the two processes.

The changes in the cross-section formulas are very slight and may be obtained by the interchanges, $m_K\leftrightarrow m_\pi$, $s\leftrightarrow s_u=u$. For example, from Eqs. (III15) and (III16) we find the differential cross section for $\bar{K}^0+p\rightarrow\Lambda+\pi^+$:

$$\frac{d\sigma}{dt}=\frac{\{|A|^2(4.22-t)-\text{Re}A^*B(4.10s_u+2.05t-4.96)+|B|^2[s_u^2-s_u(2.39-t)-1.055t+1.415]\}}{16\pi[s-0.195][s-2.06]}, \qquad \text{(IV7)}$$

in units of an energy of 1 GeV.

The CERN and Brookhaven machines produce meson beams with energies in the 10–15 GeV regions; however, experiments with these beams have so far not been designed to measure the two-body inelastic processes. It is essential that such experiments be undertaken because of the greater simplicity in the analysis of these reactions.

V. GENERALIZATION OF THE POMERANCHUK RELATION

In the course of this work we have found a new set of relationships between asymptotic cross sections,[10] which may be regarded as generalizations of the Pomeranchuk relations.[11] Our basic result, that Regge pole dominance implies that the two asymptotic amplitudes in the s and u channels are equal to each other for small values of t (except possibly for a sign) is quite general for the case of scalar particles. In our problem, we saw that going from one channel to the other in the asymptotic region amounts to changing the sign of $\cos\theta_t$. This change of sign results in the factor σ, which is the orbital parity or "signature" of the Regge pole in the t channel.

[10] See the forthcoming report by M. Gell-Mann in the *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (to be published).

[11] I. Ia. Pomeranchuk, J. Exptl. Theoret. Phys. (USSR) 34, 725 (1958) [translation: Soviet Phys.—JETP 7, 499 (1958)].

There is another way to obtain the foregoing result. In the diagrammatic representation of amplitudes, two channels of a scattering process are related to each other by the reversal of two external lines. If the lines to be reversed involve scalar bosons, the effect is unambiguous and simple. We are dealing with a three-point vertex representing the coupling of two spinless particles to an intermediate boson of spin J, which we take to be integral for the purpose of formulating the rule. Such an intermediate boson may be represented by a tensor field of rank J, which is symmetric and divergenceless in all indices, and traceless in any pair. The vertex must also be a completely symmetric tensor of rank J constructed from the four-momenta r_u and r_u' of the two bosons. It is more convenient, however, to consider the linear combinations, $\Sigma_\mu = r_\mu + r_\mu'$ and $q_\mu = r_\mu - r_\mu'$, which have simple transformation properties under line reversal when the energies are high enough that any mass differences may be neglected, namely, $\Sigma_\mu \to -\Sigma_\mu$, $q_\mu \to +q_\mu$. We also note that the tensor must be constructed solely from Σ_μ and $\delta_{\mu\nu}$; the other vector q_μ and the antisymmetric tensor $\epsilon_{\mu\nu\lambda\sigma}$ are ineffective because they give zero contributions to the residue of the pole when they are dotted into the propagator of rank $2J$ representing the intermediate state of spin J. Since the tensor is a sum of terms, each of which is the direct product of $(J-2n)\Sigma_\mu$'s and $n\delta_{\mu\nu}$'s (n being an integer), under line reversal we get the factor $(-)^J$, which is the signature of the intermediate state.

For the reversal of baryon lines, we must consider as well the transformation of the Dirac matrices. This transformation is the same as for particle-antiparticle conjugation, under which

$$1 \to 1,\ \gamma_5 \to \gamma_5,\ \gamma_\mu\gamma_5 \to \gamma_\mu\gamma_5,\ \gamma_\mu \to -\gamma_\mu,\ \sigma_{\mu\nu} \to -\sigma_{\mu\nu}.$$

A $\sigma_{\mu\nu}$ term will always appear here in the combination $\sigma_{\mu\nu}q_\nu$ which is a vector and is odd under line reversal. In vertex tensors of rank J involving the pseudovector Dirac matrices, the operation of line reversal results in the factor $-(-)^J$, the negative of the signature. It is apparent that the result of line reversal is thus the factor $\sigma\epsilon$, where σ is the signature, and $\epsilon = -1$ if the Dirac matrix $\gamma_\mu\gamma_5$ is involved and $\epsilon = +1$ for the other Dirac matrices. The factor ϵ is always $+1$, whenever (signature)(parity)$=+1$, which is the case for the exchange of the K^*, the ρ, and the ω.

In some cases, the existence of particular symmetries among the baryons provides an alternate rule. These symmetries obviously must be such as to prohibit the mixing of the two pseudovector forms $\Sigma_\mu\gamma_5$ and $\gamma_\mu\gamma_5$. Charge conjugations C and more generally the isoparity operation G, yield the desired selection rules when the object being exchanged has a definite value of C and/or G. Reversals of baryon lines in the same isotopic multiplet introduce the factor $(-)^I G$, which is C for the neutral objects. However, we must stress that the result of line reversal is fundamentally determined by the Lorentz transformation properties of the couplings. Only when the particles being reversed belong to the same isotopic multiplet is the factor the same as $(-)^I G$ or C. It is easy to imagine possible couplings where this last rule would fail. As an example, one may consider the coupling of π and η to a fictitious particle with quantum numbers J^P, $I^G = 1^-$, 1^-.

It is interesting to note that our above results, when applied to the relation between that part of the interaction between N and $\bar{N}$ due to the exchange of pions and the corresponding part in the NN interaction, yield a conclusion differing from that usually quoted.[12]

Apparently, no symmetry exists if the exchanged object has half-integral spin.

We may close this article by listing, in Table I, some quadruplets of asymptotic amplitudes which should be equal, except for the "signature" factor, on the basis of the Regge pole hypothesis. We note that our results about the asymptotic equality of (5a) and (5b) are actually quite weak, since by charge independence we know that the two amplitudes are negatives of each other at any energy and angle. Similarly for (5c) and (5d). Also, G conjugation is sufficient to guarantee strict equality between (5a) and (5c), (5b) and (5d), (6a) and (6d), and (6b) and (6c). We may also remark

TABLE I. Some quadruplets of asymptotic amplitudes.

Pole	Asymptotic amplitudes	Number
M	$T(\pi+N \to \Lambda+K)$	(1a)
	$-T(\bar{K}+N \to \Lambda+\pi)$	(1b)
	$-T(\pi+\bar{\Lambda} \to \bar{N}+K)$	(1c)
	$T(\bar{K}+\bar{\Lambda} \to \bar{N}+\pi)$	(1d)
M	$T(\pi+N \to \Sigma+K)$	(2a)
	$-T(\bar{K}+N \to \Sigma+\pi)$	(2b)
	$-T(\pi+\bar{\Sigma} \to \bar{N}+K)$	(2c)
	$T(\bar{K}+\bar{\Sigma} \to \bar{N}+\pi)$	(2d)
M	$T(\bar{N}+N \to \bar{\Lambda}+\Lambda)$	(3a)
	$-T(\Lambda+N \to \Lambda+N)$	(3b)
	$-T(\bar{N}+\bar{\Lambda} \to \bar{N}+\bar{\Lambda})$	(3c)
	$T(\Lambda+\bar{\Lambda} \to \bar{N}+N)$	(3d)
M	$T(\bar{N}+N \to \bar{\Sigma}+\Sigma)$	(4a)
	$-T(\Sigma+N \to \Sigma+N)$	(4b)
	$-T(\bar{N}+\bar{\Sigma} \to \bar{N}+\bar{\Sigma})$	(4c)
	$T(\Sigma+\bar{\Sigma} \to \bar{N}+N)$	(4d)
ρ	$T(\pi^0+n \to \pi^-+p)$	(5a)
	$-T(\pi^++n \to \pi^0+p)$	(5b)
	$-T(\pi^0+\bar{p} \to \pi^-+\bar{n})$	(5c)
	$T(\pi^++\bar{p} \to \pi^0+\bar{n})$	(5d)
ρ	$T(p+n \to p+n)$	(6a)
	$-T(\bar{n}+n \to p+\bar{p})$	(6b)
	$-T(p+\bar{p} \to \bar{n}+n)$	(6c)
	$T(\bar{n}+\bar{p} \to \bar{n}+\bar{p})$	(6d)

[12] See, for example, James S. Ball and Geoffrey F. Chew, Phys. Rev. **109**, 1385 (1958); and G. F. Chew, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1958), p. 109.

that the Pomeranchuk relations hold when the amplitudes are dominated by the Pomeranchuk pole,[8,13] for which $C=+1$.

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APPENDIX A

Partial Wave Decomposition for $\pi+\bar{K}\rightarrow\Lambda+\bar{N}$

In the center-of-mass system, choose coordinates such that

$$\begin{aligned}\bar{p}&=-p=(0,0,-p_t,iE_N),\\ p'&=(0,0,p_t,iE_\Lambda),\\ q&=(-q_t\sin\theta_t,0,-q_t\cos\theta_t,iE_\pi),\\ \bar{r}&=-r=(q_t\sin\theta_t,0,q_t\cos\theta_t,iE_K).\end{aligned}\tag{A1}$$

In terms of relativistic invariants, the momenta and energies are given by

$$4tp_t^2=[t-(m_\Lambda-m_N)^2][t-(m_\Lambda+m_N)^2],\tag{A2}$$

$$4tq_t^2=[t-(m_K-m_\pi)^2][t-(m_K+m_\pi)^2],\tag{A3}$$

$$2t^{1/2}E_N=t+m_N^2-m_\Lambda^2,\tag{A4}$$

$$2t^{1/2}E_\Lambda=t+m_\Lambda^2-m_N^2,\tag{A5}$$

$$2t^{1/2}E_K=t+m_K^2-m_\pi^2,\tag{A6}$$

$$2t^{1/2}E_\pi=t+m_\pi^2-m_K^2,\tag{A7}$$

and

$$x_t=\cos\theta_t=(s+2E_NE_\pi-m_N^2-m_\pi^2)/(2q_tp_t).\tag{A8}$$

The first step is to evaluate the helicity amplitudes in terms of the functions A and B appearing in Eq. (III1). The computation is straightforward, and so only the results will be given here. If the helicity states are denoted by $(\lambda\bar{\lambda})$, the amplitudes are

$$\begin{aligned}T(++)=T(--)=&[(E_\Lambda+m_\Lambda)(E_N+m_N)]^{-1/2}\\ &\times\{-Ap_t(E_\Lambda+E_N+m_\Lambda+m_N)\\ &+Bq_t(E_N+m_N)(E_\Lambda+m_\Lambda-E_N+m_N)\cos\theta_t\\ &+\tfrac{1}{2}Bp_t(E_K-E_\pi)(E_\Lambda+m_\Lambda-E_N-m_N)\},\end{aligned}\tag{A9}$$

$$\begin{aligned}T(+-)=-T(-+)=&-Bq_t[(E_N+m_N)/(E_\Lambda+m_\Lambda)]^{1/2}\\ &\times(E_\Lambda+E_N+m_\Lambda-m_N)\sin\theta_t.\end{aligned}\tag{A10}$$

Secondly, we must determine the form of the helicity amplitudes for a partial wave with angular momentum J and parity $(-)^J$. With our sign conventions, the following helicity combinations are eigenstates of S, and S_z,

(S,S_z)	$(\Lambda,\bar{N})$	$(\lambda\bar{\lambda})$
(0,0)	$2^{-1/2}[(\uparrow\downarrow)-(\downarrow\uparrow)]$	$2^{-1/2}[(++)+(--)]$
(1,1)	$(\uparrow\uparrow)$	$-(+-)$
(1,0)	$2^{-1/2}[(\uparrow\downarrow)+(\downarrow\uparrow)]$	$2^{-1/2}[(++)-(--)]$
(1, −1)	$(\downarrow\downarrow)$	$(-+)$

The projection of a partial wave amplitude onto a given helicity state is given by

$$\langle(\lambda\bar{\lambda})|J,0\rangle=\sum_{S_z}\sum_{S}\sum_{l}\sum_{m}\langle(\lambda\bar{\lambda})|S,S_z;l,m\rangle\times\langle S,S_z;l,m|J,0;l;S\rangle,$$

where

$$\langle(\lambda\bar{\lambda})|S,S_z;l,m\rangle=\langle(\lambda\bar{\lambda})|S,S_z\rangle Y_l^m(\theta,\phi),$$

and $\langle S,S_z;l,m|J,0;l;S\rangle$ are the Clebsch-Gordan vector coupling coefficients. For a given J, there are, in general, four elements of the T matrix, corresponding to $S=0$, $l=J$; and $S=1$, $l=J$, $J+1$, $J-1$. (For $J=0$, of course, there are but three.) If we choose to label them in the following way:

$$\tau(J,0,t)=[(2J+1)/8\pi]^{1/2}\langle l=J,S=0\|T(t)\|J\rangle,$$

$$\tau(J,1,t)=[(2J+1)/8\pi J(J+1)]^{1/2}\langle l=J,S=1|T(t)\|J\rangle,$$

$$\tau(J,1^+,t)=[(J+1)/8\pi]^{1/2}\langle l=J+1,S=1|T(t)\|J\rangle,$$

and

$$\tau(J,1^-,t)=[J/8\pi]^{1/2}\langle l=J-1,S=1\|T(t)\|J\rangle,$$

the helicity amplitudes can be written in a rather simple form:

$$\begin{aligned}T(++)=&\tau(J,0,t)P_J(x_t)-\tau(J,1^+,t)P_{J+1}(x_t)\\ &+\tau(J,1^-,t)P_{J-1}(x_t),\end{aligned}\tag{A11}$$

$$\begin{aligned}T(--)=&\tau(J,0,t)P_J(x_t)+\tau(J,1^+,t)P_{J+1}(x_t)\\ &-\tau(J,1^-,t)P_{J-1}(x_t),\end{aligned}\tag{A12}$$

$$\begin{aligned}T(+-)/\sin\theta_t=&\tau(J,1,t)P_J'(x_t)\\ &-\tau(J,1^+,t)P_{J+1}'(x_t)/(J+1)\\ &-\tau(J,1^-,t)P_{J-1}'(x_t)/J,\end{aligned}\tag{A13}$$

$$\begin{aligned}T(-+)/\sin\theta_t=&-\tau(J,1,t)P_J'(x_t)\\ &-\tau(J,1^+,t)P_{J+1}'(x_t)/(J+1)\\ &-\tau(J,1^-,t)P_{J-1}'(x_t)/J.\end{aligned}\tag{A14}$$

Only two of these amplitudes $\tau(J,S,t)$ occur if parity is conserved. In our case, $\tau(J,1\pm,t)=0$, since the parity of the system is $(-)^J$.

From these formulas, we can read off the form of the functions A and B resulting from the exchange of a pure J state with parity $(-)^J$ in the t channel:

$$\begin{aligned}B(s,t)=&-\tau(J,1,t)P_J'[x_t(s,t)]/\{q_t(E_\Lambda+E_N+m_\Lambda-m_N)\\ &\times[(E_N+m_N)/(E_\Lambda+m_\Lambda)]^{1/2}\},\end{aligned}\tag{A15}$$

$$\begin{aligned}A(s,t)=&-\tau(J,0,t)P_J(x_t)[(E_\Lambda+m_\Lambda)(E_N+m_N)]^{1/2}/\\ &\{p_t(E_\Lambda+E_N+m_\Lambda+m_N)\}-\tau(J,1,t)P_J'(x_t)\\ &\times[(E_\Lambda+m_\Lambda)/(E_N+m_N)]^{1/2}\\ &\times(E_\Lambda+E_N+m_\Lambda+m_N)^{-1}(E_\Lambda+E_N+m_\Lambda-m_N)^{-1}\\ &\times\{(E_K-E_\pi)(E_\Lambda+m_\Lambda-E_N-m_N)/q_t\\ &-x_t(E_N+m_N)(E_\Lambda+m_\Lambda-E_N+m_N)/p_t\},\end{aligned}\tag{A16}$$

[13] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961).

where we recall that as $s\rightarrow\infty$, $x_t\rightarrow s/2q_tp_t$. We may put these formulas into a convenient relativistic form by introducing

$$F_J^{(1)}(t)=-\tau(J,0,t)[(E_\Lambda+m_\Lambda)(E_N+m_N)]^{1/2}/\{(2q_tp_t)^Jp_t(E_\Lambda+E_N+m_\Lambda+m_N)\},\quad\text{(A17)}$$

and

$$F_J^{(2)}(t)=-\tau(J,1,t)[(E_\Lambda+m_\Lambda)/(E_N+m_N)]^{1/2}/\{(2q_tp_t)^{J-1}q_t(E_\Lambda+E_N+m_\Lambda-m_N)\},\quad\text{(A18)}$$

in terms of which we have our final result for the functions A and B resulting from a pure J state:

$$B(s,t)=F_J^{(2)}(t)\{(2p_tq_t)^{J-1}P_J'\times[(s+2E_NE_\pi-m_N^2-m_\pi^2)/2p_tq_t]\},\quad\text{(A19)}$$

$$\begin{aligned}A(s,t)=&F_J^{(1)}(t)\{(2p_tq_t)^JP_J\times[(s+2E_NE_\pi-m_N^2-m_\pi^2)/2p_tq_t]\}\\&+F_J^{(2)}(t)\{(2q_tp_t)^{J-1}P_J'\}\{(m_\Lambda+m_N)\times(s+2E_NE_\pi-m_N^2-m_\pi^2)/[t-(m_\Lambda+m_N)^2]\\&+(m_\Lambda-m_N)(m_K^2-m_\pi^2)/2t\}.\quad\text{(A20)}\end{aligned}$$

APPENDIX B

Contribution of the *M* Pole to the Amplitudes

Using Feynman's rules as in perturbation theory, we can compute from the diagram in Fig. 1 the "pole" in the amplitude at $t=m_M^2$ due to the exchange of the M meson. (We write "pole" since this pole lies off the physical sheet because of the instability of the M.) Near the pole, the amplitude for associated production is given by

$$(-i)(m_M^2-t)^{-1}6^{1/2}\gamma_{MK\pi}(q+r)_\alpha\times[\delta_{\alpha\beta}+(r-q)_\alpha(r-q)_\beta/m_N^2]\bar{u}_\Lambda X_\beta u_p,\quad\text{(B1)}$$

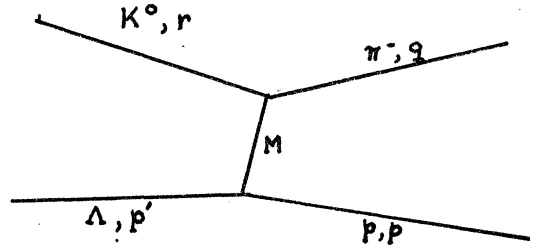

FIG. 1. Feynman diagram for contribution due to exchange of *M* meson.

where

$$X_\beta=\gamma_{\Lambda NM}\gamma_\beta-\mu_{\Lambda NM}\sigma_{\beta\nu}(p'-p)_\nu+i\mu_{\Lambda NM}'(p'-p)_\beta.\quad\text{(B2)}$$

$\mu_{\Lambda MN}$ is the anomalous magnetic moment term in the coupling of the M to ΛN. $\mu_{\Lambda MN}'$ is an additional term which is seldom encountered since this type of coupling is ruled out in electrodynamics and in some other theories by a certain class of symmetries having to do with the existence of mirror diagrams.

By using various formulas for the spinor matrix elements, one can show that the pole contributions to the functions A and B are:

$$B(s,t)=[-6^{1/2}/(t-m_M^2)]\gamma_{MK\pi}2\{\gamma_{\Lambda NM}+\mu_{\Lambda NM}(m_\Lambda+m_N)\};\quad\text{(B3)}$$

$$\begin{aligned}A(s,t)=&[-6^{1/2}/(t-m_M^2)]\gamma_{MK\pi}\\&\times\{(m_\Lambda-m_N)(m_K^2-m_\pi^2)\gamma_{\Lambda NM}/m_M^2\\&+\mu_{\Lambda NM}(m_\Lambda^2+m_N^2+m_K^2+m_\pi^2-2s-t)\\&-\mu_{\Lambda NM}'(t-m_M^2)(m_K^2-m_\pi^2)/m_M^2\}.\quad\text{(B4)}\end{aligned}$$

We note that $\mu_{\Lambda NM}'$ does not contribute a singular term to the amplitude, and, consequently, it can be eliminated from consideration near the pole.