Decay Mode $\omega \rightarrow 2\pi+\gamma$*

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The width of the decay mode $\omega \rightarrow 2\pi+\gamma$ is calculated, assuming that it proceeds through a $(\rho\pi)$ intermediate state. The γ energy spectrum, which happens to be strongly asymmetric, is also presented. The ratio $(\omega \rightarrow 2\pi+\gamma)/(\omega \rightarrow 3\pi)$ is evaluated by using two different theoretical models and it is concluded that the $2\pi^0+\gamma$ mode is appreciably less frequent than the $\pi^0+\gamma$ mode. Finally, the experimental implications are discussed.

I. INTRODUCTION

IN the past two years several two- and three-pion resonant systems have been found. For at least two of them, the 750-MeV (throughout this paper we use units in which $\hbar=c=1$) isovector two-pion resonance[1] (ρ meson) and the 780-MeV isoscalar three-pion resonance[2] (ω meson), the quantum numbers have been quite well established. Both of them are vector mesons and of opposite G parity (positive for the ρ meson).

There is, nowadays, an understandable increasing interest in both the theoretical and the experimental aspects of the decays of these "new" mesons. This article is devoted to a discussion of the first order (in the "fine structure" constant $\alpha=1/137$) electromagnetic decays of the ω meson, with special emphasis on the $2\pi+\gamma$ mode.

The main decay mode of the ω meson is into 3 pions $(\pi^+\pi^-\pi^0)$, presumably via strong interactions. The most probable first-order electromagnetic decays are $\omega^0 \rightarrow \pi^0+\gamma$ and $\omega^0 \rightarrow \pi^+ + \pi^-$(or $2\pi^0$)$+\gamma$.[3] While the $\omega \rightarrow \pi^0+\gamma$ decay proceeds as a magnetic dipole transition, the lowest configuration for the $2\pi+\gamma$ decay is realized by an electric dipole transition with the two pions in a relative angular momentum S state. The next possible configuration, obtained by a magnetic dipole transition with the two pions in a P state, cannot occur in the decay $\omega \rightarrow 2\pi+\gamma$; obviously, the two neutral pions cannot be in a P state while the charge conjugation invariance of the electromagnetic decay forbids the appearance of the $\pi^+\pi^-$ pair in a P state (with $C=-1$).[4] One is tempted to anticipate, therefore, that the $2\pi+\gamma$ decay of the ω would not be much less frequent than the $\pi+\gamma$ mode, as the final-state configuration does not appear to be drastically reduced by the available phase space. However, a detailed analysis of the mechanism of this decay will show that sensibly different conclusions can be reached by use of specific models.

The ratio $r=(\omega \rightarrow \pi^0+\gamma)/(\omega \rightarrow \pi^+ + \pi^- + \pi^0)$ has been recently evaluated by several methods. In one of these approaches it is assumed, in the sense of dispersion theory, that the most important intermediate states in the ω decay are the states of lowest mass. Brown and Singer[5] assume the decay proceeding through the chain $\omega \rightarrow \pi^0+\pi^++\pi^- \rightarrow \pi^0+\gamma$ and by using a cutoff at the nucleon mass for the logarithmic divergent integral which appears when integrating over the momentum of the intermediate pion loop, they obtain $r\simeq 1\%$. Gell-Mann, Sharp, and Wagner[6] assume that the relevant intermediate state is $(\rho+\pi)$ so that the decay proceeds $\omega \rightarrow \rho^0+\pi^0 \rightarrow \gamma+\pi^0$ and they get $r\simeq 17\%$, using a certain approximation for the $\rho^0 \rightarrow \gamma$ coupling (and a ρ width of 100 MeV to determine the coupling which appears in the expression for $\omega \rightarrow \pi^+ + \pi^- + \pi^0$). In both these works the strong coupling constant $f_{\omega\rho\pi}$ or $f_{\omega 3\pi}$ cancels in taking the ratio r. There is one experimental value already reported for the ratio of the neutral-to-charged modes of the ω, namely, $r \leqslant (3\pm 4)\%$.[7] There-

* Research supported by the National Science Foundation.

[1] A. C. Anderson, V. X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Phys. Rev. Letters **6**, 365 (1961); D. Stonehill, C. Baltay, H. Courant, *et al. ibid.* **6**, 624 (1961); A. R. Erwin, R. March, W. D. Walker, and E. West, *ibid.* **6**, 628 (1961); E. Pickup, D. K. Robinson, and E. O. Salant, *ibid.* **7**, 192 (1962).

[2] B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters **7**, 178 (1961); M. L. Stevenson, L. W. Alvarez, B. C. Maglić, and A. H. Rosenfeld, Phys. Rev. **125**, 687, 2208 (1962); N. H. Xuong and G. R. Lynch, Phys. Rev. Letters **7**, 327 (1961); M. Meer, R. Kraemer, L. Madansky, M. Nussbaum, *et al.*, Report to the High-Energy Nuclear Physics Geneva Conference, July, 1962 (to be published).

[3] The charge conjugation quantum number for the ω meson is $C=-1$, so that the decay $\omega \rightarrow n\pi+\gamma$ is electromagnetically allowed with n limited by energy conservation only. One would reasonably expect phase space to reduce appreciably the modes with $n \geqslant 3$. However, if it happens that the $\omega \rightarrow 2\pi+\gamma$ decay has a very small width (as will be shown to be predicted by a certain specific model, the "pole approximation"), the $\omega \rightarrow 3\pi+\gamma$ decay may reveal itself as competing favorably with it. In contradistinction to the $(\pi+\gamma)$ and $(2\pi+\gamma)$ decays of the ω, the $(3\pi+\gamma)$ decay has two kinds of contributions: (a) the "direct emission" which reflects the structure of the interactions responsible for the ω decay and belongs to the same class as the one and two-pion radiative decays, and (b) "internal bremsstrahlung" which arises as radiation from the charged pions emitted in the strong 3π decay of ω. The type (a) is probably negligible, but the internal bremsstrahlung may well compete favorably with a very small direct $(2\pi+\gamma)$ mode. In this case, the $3\pi+\gamma$ mode will exhibit a characteristic bremsstrahlung spectrum favoring low-energy photons, while the $2\pi+\gamma$ direct decay gives predominantly high-energy photons, as is derived in this article.

[4] More generally, only electric transitions can occur in the $\omega \rightarrow 2\pi+\gamma$ decay because of parity and charge conjugation invariance. See also G. Feinberg and A. Pais, Phys. Rev. Letters **9**, 45 (1962) for a summary of the rules governing the mesonic electromagnetic decays.

[5] L. M. Brown and P. Singer, Phys. Rev. Letters **8**, 155, 353 (1962).

[6] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[7] R. Strand, R. Kraemer, M. Meer, *et al.*, Report to the High-Energy Nuclear Physics Geneva Conference, July, 1962 (to be published).

fore, a theoretical estimate for the $(\omega\to 2\pi^0+\gamma)/(\omega\to 3\pi)$ is needed for a reliable interpretation of this experiment.[8]

In this work we are talking about ω only, as we assume that it is the only $T=0$, 1^{--} existing meson (1^{--} stands for spin 1, parity and G parity negative). The other early candidate for these quantum numbers, the 550-MeV three-pion resonance (the η meson), seems more probably to be a 0^{-+} meson. Of course, our analysis can be numerically adjusted to any other possibly existing $T=0$, 1^{--} meson.

II. CALCULATIONS

In order to calculate the width of the $2\pi+\gamma$ decay we will follow the same general approach as in references 5 and 6. We assume that the most important intermediate states are the states of lowest mass, which in this case ought to be $\omega\to\pi^++\pi^-+\pi^0\to 2\pi+\gamma$. Hence, the knowledge of the amplitude for $\pi^{(1)}+\pi^{(2)}\to\pi^{(3)}+\gamma$ is necessary. The transition amplitude, T, may be written using only general principles of Lorentz and gauge invariance as

$$T=(2\pi)^4\delta(p^{(1)}+p^{(2)}+p^{(3)}+k)\times\lambda\epsilon_{\mu\nu\rho\sigma}\frac{p_\mu{}^{(1)}p_\nu{}^{(2)}p_\rho{}^{(3)}\epsilon_\sigma}{4(p_0{}^{(1)}p_0{}^{(2)}p_0{}^{(3)}k_0)^{1/2}}F(s_1,s_2,s_3), \quad (1)$$

where $p^{(i)}$ are the momenta of the pions, ϵ is the photon polarization vector, and F is a scalar function of the three scalar invariants which can be formed from the momenta of the four particles in the customary notation. $\epsilon_{\mu\nu\rho\sigma}$ is the fourth-order complete antisymmetric tensor. λ is an "effective coupling constant" for the process.

There is no direct way of obtaining amplitude (1), consequently, different authors have considered other processes, like photoproduction of pions on nucleons, Compton scattering on nucleons, where this amplitude is expected to be significant.[9] There is still no detailed knowledge of this amplitude and it seems that a successful approach is to use a "bi-pion" model, e.g., to assume that the process $\pi^{(1)}+\pi^{(2)}\to\pi^{(3)}+\gamma$ is proceeding through a metastable vector particle of spin 1. In fact, the decay under study here could be a useful tool (if frequent enough experimentally), for investigating the $\pi^{(1)}+\pi^{(2)}\to\pi^{(3)}+\gamma$ amplitude. In the ω decay, $m_\pi^2\leqslant s_1\leqslant\frac{1}{2}m_\omega^2-m_\pi^2$ $(=14.5m_\pi^2)$, where s_1 is the square of the total energy in the center-of-momentum system of the two intermediate pions. To this end we have, of course, to assume the knowledge of the $\omega\to 3\pi$ amplitude.

We proceed by using, instead of the exact $\pi\pi\to\pi\gamma$ amplitude, the "ρ approximation,"[6] i.e., we assume that the ω dissociates into a π and a virtual ρ (instead of $\pi^{(1)}\pi^{(2)}$, $J=T=1$ state), which decays either into 2π or into $\pi+\gamma$, the final states relevant for our purpose. We do not expect the "ρ approximation" to induce any significant error in the ratio $(\omega\to 2\pi+\gamma)/(\omega\to 3\pi)$ or in the γ spectrum. As it will be seen later, the $\rho\pi\gamma$ vertex is the sensitive one in this context. With this model, as the $f_{\omega\rho\pi}$ coupling constant cancels from the ratio, we will be able to express the ratio under investigation in terms of two measurable quantities, the partial widths of ρ for 2π and $\pi\gamma$ decays.[10]

In using the "ρ-dominant" model we tacitly assume that the ρ is the only 2π resonance with 1^{-+} quantum numbers. If it happens that the other reported[11] two-pion resonance, the ζ meson has the same quantum numbers (which is improbable) our analysis should be modified.

We will treat the ρ meson as a vector boson of zero width. The inclusion of the width only complicates the calculations without introducing significant changes.[12] Accordingly, we take for its propagator

$$D_{\mu\nu}{}^{(\rho)}=(g_{\mu\nu}-p_\mu{}^{(\rho)}p_\nu{}^{(\rho)}/m_\rho{}^2)[1/(p^{(\rho)2}-m_\rho{}^2)].$$

The following Lorentz- and gauge-invariant expressions for the vertices occurring in the $\omega\to 3\pi$ and $2\pi+\gamma$ decays are used:

$$\omega\rho\pi \text{ vertex:}\quad \frac{f_{\omega\rho\pi}}{m_\pi}\epsilon_{\alpha\beta\gamma\delta}p_\alpha{}^{(\omega)}\epsilon_\beta{}^{(\omega)}p_\gamma{}^{(\rho)}\epsilon_\delta{}^{(\rho)}, \quad (2)$$

$$\rho\pi\pi \text{ vertex:}\quad 2f_{\rho\pi\pi}\epsilon_\mu{}^{(\rho)}(p_\mu{}^{(+)}-p_\mu{}^{(-)}), \quad (3)$$

$$\rho\pi\gamma \text{ vertex:}\quad \frac{f_{\rho\pi\gamma}}{m_\pi}\epsilon_{\nu\sigma\tau\xi}p_\nu{}^{(\rho)}\epsilon_\sigma{}^{(\rho)}k_\tau\epsilon_\xi{}^{(\gamma)}. \quad (4)$$

The p's are the four-momenta and the ϵ's the polarization vectors of the designated particles, k_τ is the photon four-momentum, and $f_{\rho\pi\gamma}$, $f_{\rho\pi\pi}$, $f_{\omega\rho\pi}$ as defined here are dimensionless coupling constants. The width of the $\omega\to\pi^++\pi^-+\pi^0$ decay with this model was calculated in reference 6 and is given by

$$\Gamma_\omega(3\pi)=\frac{f_{\omega\rho\pi}{}^2}{4\pi}\frac{f_{\rho\pi\pi}{}^2}{4\pi}\frac{(m_\omega-3m_\pi)^4m_\omega}{3^{3/2}(m_\rho{}^2-4m_\pi{}^2)^2}\times 3.5. \quad (5)$$

For the decay $\omega\to\pi^++\pi^-+\gamma$ we obtain, using (2) and (4),

$$\Gamma_\omega(\pi^+\pi^-\gamma)=\frac{f_{\omega\rho\pi}{}^2}{4\pi}\frac{f_{\rho\pi\gamma}{}^2}{4\pi}\frac{m_\omega{}^5}{24\pi m_\pi{}^4}\int_0^{x_m}I(x)dx, \quad (6)$$

[8] Note that the decay $\omega\to 3\pi^0$ is strongly and electromagnetically forbidden by charge conjugation invariance.

[9] See M. Gourdin, "Lectures given at the School of Theoretical Physics in Yugoslavia," Summer, 1961 (unpublished), for an ample discussion of the photoproduction of pions on pions and for further references.

[10] The chain $\omega\to\rho+\gamma\to\pi^++\pi^-+\gamma$ is obviously forbidden by charge conjugation invariance applied at the first vertex.

[11] R. Barloutaud, J. Heughehaert, A. Leveque, J. Meyer, and R. Omnes, Phys. Rev. Letters 8, 32 (1962); B. Sechi Zorn, Phys. Rev. Letters 8, 282, 386 (1962).

[12] H. Chew (to be published) has analyzed the decay $K_2{}^0\to\pi^++\pi^-+\gamma$ with the intermediate step $K_2{}^0\to\zeta+\gamma$ and has shown that the photon spectrum is very insensitive to the ζ width (varied between 0–40 MeV).

where $x = k/m_\omega$ and $x_m = k_{\max}/m_\omega$ with

$$k_{\max} = m_\omega/2 - 2m_\pi^2/m_\omega. \tag{7}$$

The photon energy spectrum is given by

$$I(k) = A(k)\beta(k) + B(k)\ \ln\frac{\alpha(k)+\beta(k)}{\alpha(k)-\beta(k)}, \tag{8}$$

where

$$\alpha(k) = m_\rho^2 - m_\pi^2 - m_\omega k, \tag{9a}$$

$$\beta(k) = m_\omega k\left(\frac{k_{\max}-k}{\frac{1}{2}m_\omega - k}\right)^{1/2}, \tag{9b}$$

and

$$\begin{aligned} A(k) = {} & \alpha^2 + m_\omega(\tfrac{1}{2}m_\omega - k)[m_\omega(k_{\max}-k) + 2(\alpha + 2m_\pi^2)] \\ & + \tfrac{1}{2}(m_\rho^2 - m_\pi^2)[(m_\omega^2 + m_\rho^2 - m_\pi^2)^2 - 4m_\omega^2 m_\rho^2] \\ & \times (\alpha^2 - \beta^2)^{-1}, \end{aligned} \tag{10a}$$

$$\begin{aligned} B(k) = {} & \alpha[m_\omega^2 m_\pi^2 - (m_\rho^2 - m_\pi^2)^2] \\ & - 2m_\rho^2 m_\omega[m_\omega(\tfrac{1}{2}m_\omega - k) - \alpha k] + \tfrac{1}{2}m_\omega^4 m_\pi^2 \\ & + \alpha^{-1}\{\tfrac{1}{4}(m_\rho^2 - m_\pi^2)^2[(m_\rho^2 - m_\pi^2)^2 \\ & - 2m_\omega k(2m_\rho^2 - m_\omega^2) - 2m_\omega^2 m_\pi^2] \\ & + m_\omega^2 k[2m_\rho^2(m_\rho^2 - \alpha)(\tfrac{1}{2}m_\omega - k) \\ & + m_\omega m_\pi^2(2m_\rho^2 - m_\pi^2)]\}. \end{aligned} \tag{10b}$$

Integration over the photon energy distribution function gives

$$\Gamma_\omega(\pi^+\pi^-\gamma) = \frac{f_{\omega\rho\pi}^2}{4\pi}\,\frac{f_{\rho\pi\gamma}^2}{4\pi}\,\frac{m_\omega^5}{24\pi m_\pi^4} \times 1.92\times10^{-3}, \tag{11}$$

and, consequently, from (5) and (11)

$$R = \Gamma_\omega(\pi^+\pi^-\gamma)/\Gamma_\omega(\pi^+\pi^-\pi^0) = 0.99(f_{\rho\pi\gamma}^2/4\pi), \tag{12}$$

where we used $m_\rho = 750$ MeV, $m_\omega = 780$ MeV, $m_\pi = 140$ MeV and we have taken an experimental width of 100 MeV for the ρ to calculate $f_{\rho\pi\pi}$. For the neutral decay one has

$$\Gamma_\omega(\pi^0\pi^0\gamma) = \tfrac{1}{2}\Gamma_\omega(\pi^+\pi^-\gamma). \tag{13}$$

Practically, the $2\pi^0+\gamma$ mode is slightly more frequent than stated in (13) due to the phase-space correction for the $\pi^{+,-} - \pi^0$ mass difference.

Now we have in (12) the ratio R still expressed in one unknown, but measurable quantity which is related to the ρ decay. The width for the electromagnetic decay of ρ is given, using (4), by

$$\Gamma_{\rho^{+,-,0}}(\pi^{+,-,0}\gamma) = \frac{f_{\rho\pi\gamma}^2}{4\pi}\,\frac{(m_\rho^2 - m_\pi^2)^3}{24 m_\pi^2 m_\rho^3}, \tag{14}$$

so that

$$f_{\rho\pi\gamma}^2/4\pi = 1.24\times10^{-3}\Gamma_\rho(\pi\gamma), \tag{15}$$

with $\Gamma_\rho(\pi\gamma)$ expressed in MeV.

Before discussing the implications of Eq. (12) we would like to mention briefly the energy spectrum of the γ ray from the decay $\omega \to 2\pi+\gamma$ given in Fig. 1. Interestingly enough, the distribution is highly asymmetric with a great probability for the photon of being emitted with an energy nearly equal to the maximal one allowed by energy and momentum conservation ($k_{\max} = 0.44 m_\omega = 345$ MeV). This feature of the spectrum is easily understood remembering that the pions are emitted in an S state while a relative angular momentum P state is forbidden. The next possibility is an electric dipole transition with the pions in a D state. This state is appreciably reduced by angular momentum barrier compared to the S state. Obviously, the S state is realized by the two pions emerging preferentially with nearly equal momenta so that in the decay of ω at rest the opening angle between the two-pion tracks is predominantly small.

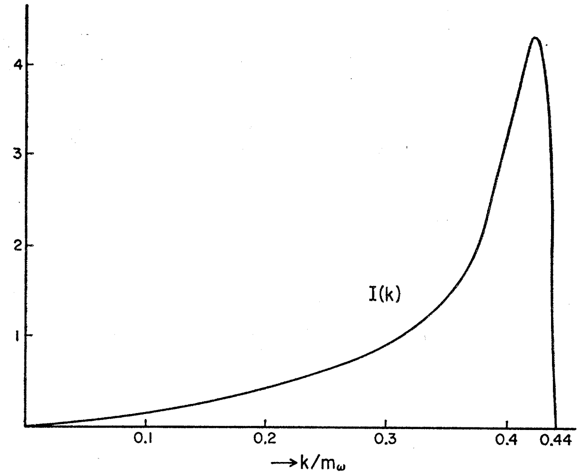

FIG. 1. The energy spectrum $I(k)$, (8), for the photon with energy k from the decay $\omega \to 2\pi+\gamma$, in units of the ω mass (780 MeV). The ordinate is in arbitrary units.

III. RESULTS AND DISCUSSION

The straightforward way to obtain the ratio R is to use the experimental width for the $\pi+\gamma$ decay of the ρ meson. However, no definite measurement exists at present.[13] Instead, we shall use different theoretical models which give an estimate for $f_{\rho\pi\gamma}$. The observed rate of the $2\pi+\gamma$ decay mode could provide an additional test for these models.

Gell-Mann and Zachariasen[14] have related $f_{\rho\pi\gamma}$ to $f_{\omega\rho\pi}$, implying that the electromagnetic ρ decay proceeds $\rho \to \omega+\pi \to \gamma+\pi$. Then

$$f_{\rho\pi\gamma} = (e/2\sqrt{3}f_\omega)f_{\omega\rho\pi}, \tag{16}$$

where f_ω gives the effective coupling $\omega \leftrightarrow \gamma$. $f_{\omega\rho\pi}$ can be calculated from the measured π^0 lifetime assuming[14] that the π^0 decay is dominated by the $(\rho\omega)$ intermediate state. Then[6]

$$\Gamma_{\pi^0}(2\gamma) = \alpha^2\left(\frac{f_\rho^2}{4\pi}\right)^{-1}\left(\frac{f_\omega^2}{4\pi}\right)^{-1}(192)^{-1}m_\pi\left(\frac{f_{\omega\rho\pi}^2}{4\pi}\right), \tag{17}$$

[13] A preliminary result reported in reference 7 gives $[\rho \to \text{neutrals (presumably } \pi^0+\gamma)]/(\rho \to \pi^+ + \pi^-) \leqslant (6\pm35)\%$.

[14] M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).

TABLE I. Calculated ratios for electromagnetic decays of ρ and ω.

Decay	"Pole approximation"[a]	"Higher mass contributions"[b]
$\Gamma_\rho(\pi\gamma)/\Gamma_\rho(2\pi)$	0.016%	3%
$\Gamma_\omega(\pi^+\pi^-\gamma)/\Gamma_\omega(3\pi)$	0.002%	0.4%
$\Gamma_\omega(\pi^0\pi^0\gamma)/\Gamma_\omega(3\pi)$	0.001%	0.2%

[a] See reference 14.
[b] See referenec 17.

where f_ρ stands for the effective coupling $\rho \leftrightarrow \gamma$. Unitary symmetry[15] predicts $f_\omega \simeq f_\rho$ and arguing that the ρ meson dominates the pion form factor[6] $f_{\rho\pi\pi} \simeq f_\rho \simeq f_\omega$. If we take $\tau_{\pi^0} = 2.7\times 10^{-16}$ sec,[16] then

$$f_{\omega\rho\pi}^2/4\pi = 1.6\times 10^{-2}; \quad f_{\rho\pi\gamma}^2/4\pi = 1.9\times 10^{-5}. \quad (18)$$

Hence, this model predicts a very low yield for the $\rho \to \pi+\gamma$ mode, only about 0.02% compared to the main decay mode. This, in turn, implies within the model used here to calculate $\omega \to 2\pi+\gamma$, a very low frequency for this decay.

Recently Geffen[17] has studied the implications of a possible failure of the above picture ("pole approximation") for the π^0 decay. The existing measurements of the "mean square radius" of the π^0-decay form factor indicate that additional intermediate states are important and strong cancellations occur between the various contributions to the form factor. By means of an unsubtracted dispersion relation for the π^0-decay form factor, Geffen estimates the contribution of the higher mass intermediate states using the experimental "mean square radius" and he shows that it implies a much higher rate for the electromagnetic ρ decay than predicted by the approach of Gell-Mann and Zachariasen. Assuming $f_{\omega\pi\gamma} = \sqrt{3} f_{\rho\pi\gamma}$ (unitary symmetry) or $f_{\omega\pi\gamma} \ll f_{\rho\pi\gamma}$ he obtains for $\Gamma_\rho(\pi\gamma)/\Gamma_\rho(\pi\pi)$ the values 1% or 3%, respectively.

In Table I we summarize the predictions of the use of the models of Gell-Mann and Zachariasen ("pole approximation") and Geffen ("higher mass contributions"—in the π^0 decay) for the decay rates of the ρ and ω mesons.

For the "pole approximation" the partial width for $\omega \to \pi^+\pi^-\gamma$ is significantly smaller than that for some second-order electromagnetic decays like $\omega \to \pi^+ + \pi^-$, $\mu^+ + \mu^-$, $e^+ + e^-$ (which were estimated in reference 6).

Experimentally, Stevenson *et al.*[2] report no evidence for $\omega \to \pi^+ + \pi^- + \gamma$ events in a sample in which some 100 ω's decaying through the mode $\pi^+ + \pi^- + \pi^0$ were observed. This upper limit is still consistent with both models used and bigger samples of ω's are needed before the theoretical predictions of the two models can be checked.

Moreover, measurements of both $(\rho \to \pi\gamma)/(\rho \to 2\pi)$ and $(\omega \to 2\pi\gamma)/(\omega \to 3\pi)$ would allow us to draw valuable conclusions on the validity of the approach which has been used elsewhere and here to estimate the decay rates of the "new" mesons. If (12) is verified, with $f_{\rho\pi\gamma}$ related to the measured electromagnetic width of ρ by (15), this would support the assumption that the ρ (or $\pi^{(1)}\pi^{(2)}$, $T=J=1) + \pi$ intermediate state is dominant in the ω decay.

As to the frequency of the $2\pi^0 + \gamma$ mode among the neutral decays of the ω we conclude that this mode represents only a small fraction whether we use the ratio predicted for $(\omega \to \pi^0 + \gamma)/(\omega \to \pi^+ + \pi^- + \pi^0)$ in reference 5 or that in reference 6. Consequently, an experimental measurement of the neutral-to-charge ratio for ω decay, being only very slightly "contaminated" by the $2\pi^0 + \gamma$ decay, could indicate which of the estimates of references 5 and 6 is more reliable.

Finally, it is amusing to point out that a simple estimate can be made to show that the $2\pi^0\gamma$ mode is definitely less frequent than the $\pi^0\gamma$ mode, without assuming a detailed model. As the $2\pi^0\gamma$ decay proceeds with the two pions coming preferentially "together," we can visualize it as a two-body decay into a scalar neutral particle having a mass of $(m_\omega - k_{\max})$ and a photon with the energy $k_{\max}$. Assuming that there is no appreciable difference in the rates because of the different dipole character of the γ transition, the ratio $\pi^0 + \gamma/2\pi^0 + \gamma$ is given by the respective invariant phase spaces. Including the indistinguishability of the two neutral pions, one obtains

$$(\omega \to \pi^0 + \gamma)/(\omega \to 2\pi^0 + \gamma)$$

$$= \frac{m_\omega^2 - m_\pi^2}{m_\pi} \Big/ \frac{1}{2} \frac{m_\omega^2 - (m_\omega - k_{\max})^2}{m_\omega - k_{\max}} \simeq 9.$$

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[15] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[16] A. W. Merrison, Suppl. Phil. Mag. **11**, 1 (1962).
[17] D. A. Geffen, Phys. Rev. **128**, 374 (1962).