Reaction $V^{51}(n,d)Ti^{50}$ at 14.4 MeV

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(Received July 26, 1962)

Measurements have been made of the differential cross sections for the $V^{51}(n,d)$ reaction at 14.4 MeV, exciting the 0^+, 2^+, 4^+, and 6^+ states in Ti^{50}. A distorted-wave Born approximation (DWBA) analysis of the results shows them to be consistent with the predictions of the simple shell model, that is, pure $(f_{7/2})^3$ configuration for V^{51} ground state, with seniority a good quantum number, within the uncertainties of both the experiment and the analysis. In particular, the data are consistent with pure $l=3$ transitions, although small amounts of $l=1$ pickup to the 2^+ and 4^+ states cannot be excluded. The uncertainties in the DWBA analysis, and their effects on quantitative determination of the spectroscopic factors, are fully discussed.

I. INTRODUCTION

Although pickup reactions induced by charged particles have been relatively well studied, angular distributions of deuterons from (n,d) reactions on medium nuclei are scarce[1–3] owing to the need for an intense monoenergetic beam of neutrons. These reactions are of interest for they are probing the proton configurations in nuclei whereas the more commonly studied reactions (p,d) and (d,t), deal with neutron configurations.

It is well known that stripping and pickup reactions constitute a powerful tool in the study of nuclear states.[4] The measurement of the shape of the angular distribution should give the orbital angular momentum l of the particle removed or added and hence information on the shell structure of the nuclear states. A measurement of the absolute cross section in principle would give the fraction V_j^2 by which the l,j shell is filled (where j is the total angular momentum of the transferred nucleon). Interest in this type of reaction for nuclei with one closed shell has increased recently, since the values of V_j^2 obtained by experiment may be directly compared with the theoretical predictions of the pairing calculations of Kisslinger and Sorenson.[5] However, difficulty still exists in extracting the nuclear structural information from the experimentally measured quantities. So far, analysis of stripping and pickup reactions has been performed mainly using either the Butler theory[6] or a semiempirical procedure using ratios of cross sections such that the kinematic factors approximately cancel out.[4,7] We believe that distorted-wave Born approximation (DWBA) calculations,[8–10] which have become more accessible in recent years, will allow us to make a more reliable analysis.

This paper describes measurements made of the differential cross sections for the $V^{51}(n,d)Ti^{50}$ reaction at 14.4 MeV and presents an attempt to analyze the results using DWBA calculations. In the course of this and other work it has become clear that although DWBA theory is much more realistic than the simple Butler theory, there remain still ambiguities in both absolute magnitudes and the shapes of the angular distributions which coupled with experimental uncertainties prevent us from making a completely quantitative analysis. Consequently, the nuclear structural information we can obtain is also subject to uncertainties. A detailed discussion of the DWBA calculations is given in Sec. III, and the implications for nuclear structure discussed in Sec. IV.

II. EXPERIMENTAL PROCEDURE

The deuteron spectra have been measured using a $(dE/dx)-E$ counter telescope described by Kuo, Petravić and Turko.[11] Briefly, it consists of a CsI(Tl) scintillation counter and two proportional counters in coincidence and another proportional counter in anticoincidence. The main feature of this counter telescope is its low background counting rate so that relatively small cross sections in the region of tenths of a millibarn/steradian may be measured. The deuterons were separated from other particles by displaying dE/dx and E pulses on an X-Y oscilloscope. A specially constructed mask allows only light spots produced on the oscillo-

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scope by deuterons to reach a photomultiplier which gates a 100-channel ultrasonic memory analyzer. We refer the reader to reference 11 for the details of the circuitry used in this experiment. Figure 1 shows a sample of the deuteron spectra obtained at 40°. The corresponding background is also shown. Beside the background correction, the leakage of protons produced mainly by $V^{51}(n,p)Ti^{51}$ through the mask on the oscilloscope has also been corrected. This has been done by measuring the percentage of protons leaking over using a polythene target at regular intervals during the run. Later the number of protons from $V^{51}(n,p)Ti^{51}$ were determined with a mask for protons on the oscilloscope, and the number of protons leaking over were determined. As may be seen from Fig. 1, the number of leakage protons is highest at the high-energy end of the spectra where the Landau curves for energy loss for protons and deuterons are least well separated. At lower energies, the corrections introduced are very small. The deuteron peak at 3.2 MeV which is very pronounced at forward angles both in the "run" and the "background" curves is most likely due to the reaction $O^{16}(n,d)N^{15}$, $Q=-9.88$ MeV, from CO_2 in the proportional counter.

A flux of 1.5×10^9 neutrons/sec in 4π was obtained from the reaction $T(d,n)He^4$ using the 200-keV Cockcroft-Walton accelerator at the Institut Ruder Bošković.[12] The neutrons were monitored by counting the associated α particles with a thin CsI(Tl) crystal polished to 6 mg/cm². The absolute cross sections were determined by normalization to n-p elastic scattering

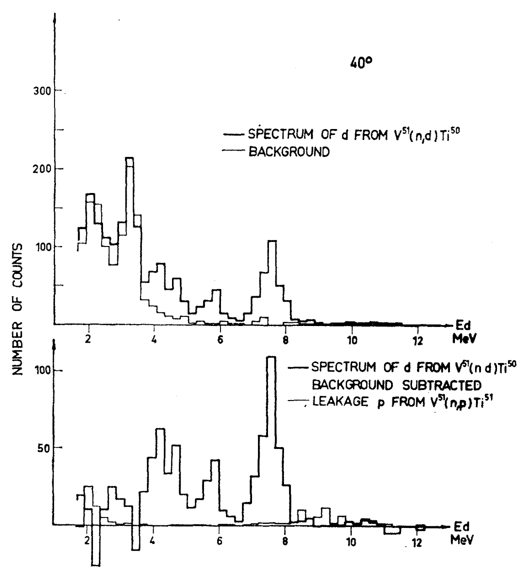
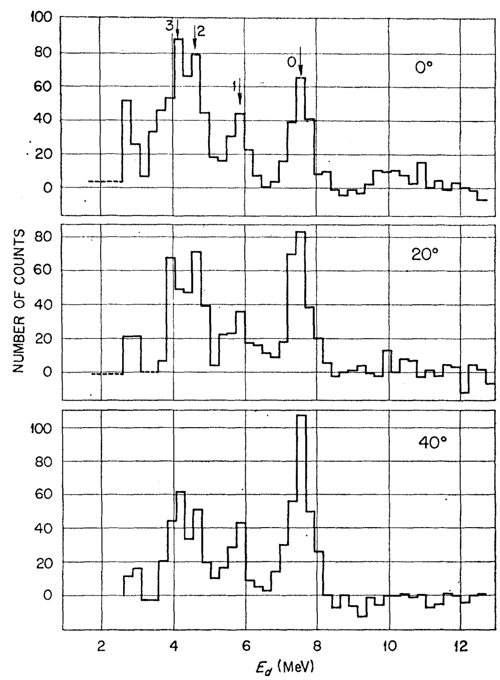
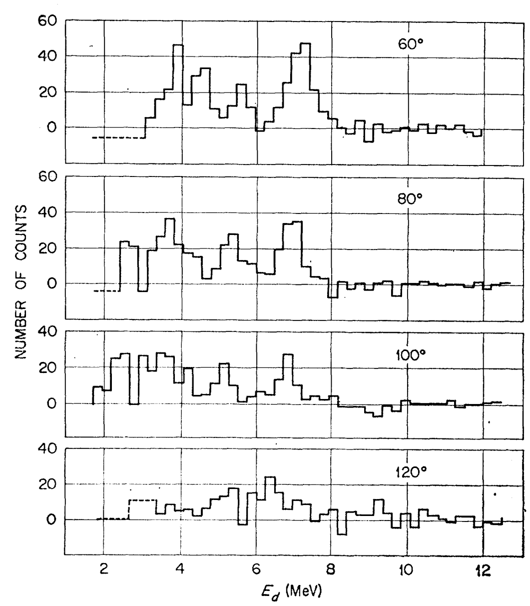


FIG. 1. Spectrum of deuterons from $V^{51}(n,d)Ti^{50}$ at the laboratory angle of 40°. The background and leakage proton counts, normalized to the same number of incident neutrons, are also shown.

[12] M. Paić, K. Prelec, P. Tomaš, M. Varićak, and B. Vosicki, Glasnik Mat. Fiz. i. Astr. **12**, 269 (1957).



(a)



(b)

FIG. 2. The complete set of spectra of deuterons from $V^{51}(n,d)Ti^{50}$ after background and leakage protons have been subtracted. The arrows marked with 0, 1, 2, and 3 indicate the expected energies of deuterons leading to the ground state of Ti^{50} ($Q=-5.82$ MeV) and the excited states at 1.59, 2.76, and 3.27 MeV, respectively.

using a polythene target. The vanadium target was in the form of a metallic foil 2.8 cm in diameter and 9 mg/cm² thick. The counter was placed with the vanadium target at 10-cm distance from the tritium target. The resulting geometry was such that the width

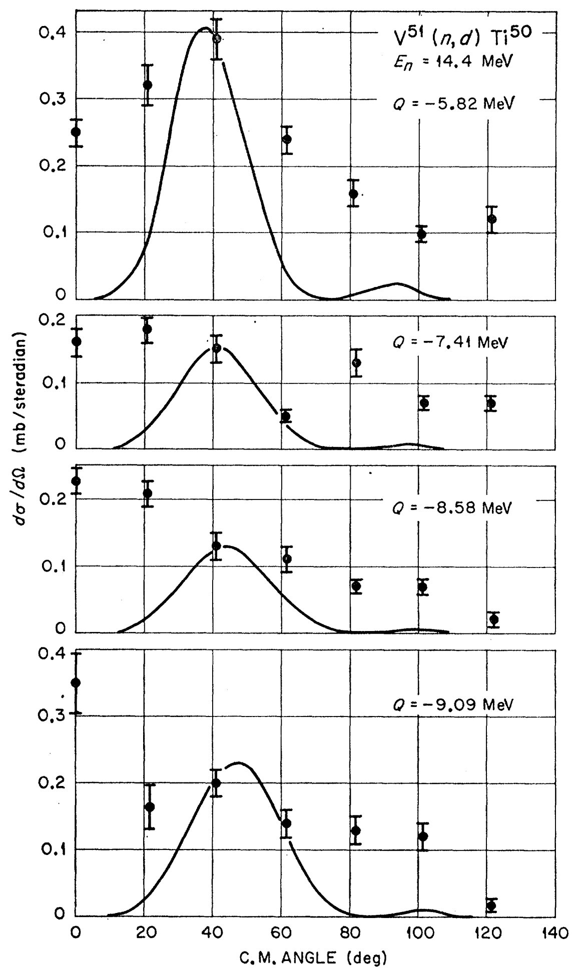


FIG. 3. Angular distributions of deuterons leading to the ground and excited states of Ti^{50}. The curves are Butler $l=3$ curves calculated by using $R_0=6.15$ F.

of the window function at half maximum at 60°, say, was 5°. The time required to measure each point in the angular distribution ranged from 50 to 100 h.

III. RESULTS AND COMPARISON WITH DWBA

Figure 2 shows the complete set of spectra of deuterons after background and leakage protons have been subtracted. Four deuteron groups leading to the ground ($Q=-5.82$ MeV) and excited states of Ti^{50} may be discerned. The low-lying excited states of Ti^{50} have been studied by Morinaga[13] through the decay of Sc^{50}. The positions, spins, and parities of the first three levels are given as: 1.59 MeV (2^+), 2.76 MeV (4^+), and 3.27 MeV (6^+), with no levels in between, although the assignment of 6^+ to the level of 3.27 MeV is uncertain.[14] The energies of the deuteron groups observed, as well as they may be determined, are not in conflict with the assumption that they lead to the ground and the first three excited states of Ti^{50} given by Morinaga.

Figure 3 shows four angular distributions of deuterons leading to the ground and excited states of Ti^{50}. In order to obtain information about the angular momentum l transferred and the spectroscopic factor[4] involved, a Butler or a DWBA analysis has to be performed. It is clear that no Butler curve with whatever value of R_0 and with whatever combination of l values would be able to give a good fit to any of the curves. Butler $l=3$ curves calculated for all four deuteron groups by using the cutoff radius $R_0=6.15$ F, are displayed also in Fig. 3.

The distorted-wave calculations were based upon the theory of Tobocman,[8] and carried out using the Oak Ridge IBM-7090 computer.[10] In principle it is necessary to have optical-model potential parameters which give a good account of the observed elastic scattering in both incident and exit channels at the same energies as the reaction being studied. In practice a compromise is necessary. No data are available for 14-MeV neutrons on V^{51}, but Fig. 4 compares the elastic scattering predicted by the neutron potential used in these calculations with the measured differential cross section[15] of Ti for neutrons of 14.0±0.3 MeV. This potential has a real part of Saxon shape, and surface absorption,

$$U(r)=-V(e^x+1)^{-1}+iW(d/dx')(e^{x'}+1)^{-1},$$
$$x=(r-r_0A^{1/3})/a,\quad x'=(r-r_0'A^{1/3})/a',$$

and the parameters take the values which give a good account of the elastic scattering in this mass region,[16] $V=43$ MeV, $W=40$ MeV, $r_0=1.27$ F, $a=0.63$ F,

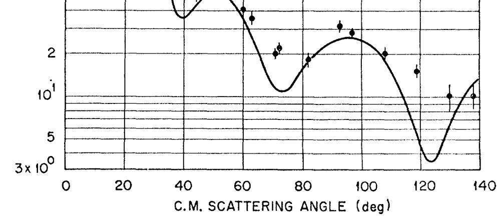


FIG. 4. Differential cross section for elastic scattering of 14.4-MeV neutrons from V^{51} predicted by the optical potential used in the DWBA calculations. Also shown are the measured cross sections[15] for neutrons of 14.0±0.3 MeV on Ti.

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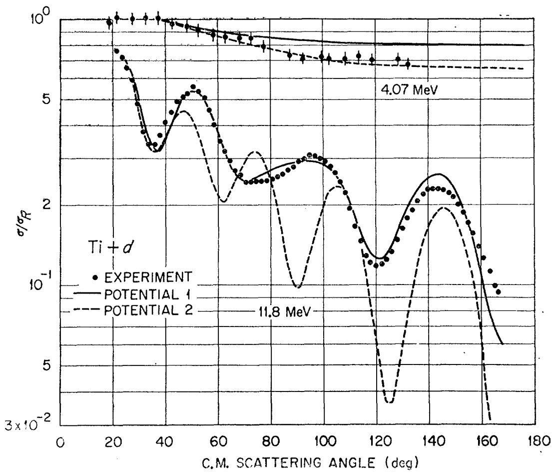

FIG. 5. Differential cross section for elastic scattering of deuterons on Ti. The experimental data were taken from reference 18 at 4.07 MeV, and reference 17 at 11.8 MeV.

$r_0'=1.24$ F and $a'=0.43$ F. Spin-orbit coupling was neglected in most of the DWBA calculations. Elastic scattering measurements with deuterons on Ti have been made at 11.8 MeV[17] and 4.07 MeV[18]. The former data have been fitted[19] with a Saxon potential with volume absorption,

$$U(r)=-V(e^x+1)^{-1}-iW(e^{x'}+1)^{-1},$$

and parameters $V=45$ MeV, $W=7.2$ MeV, $r_0=1.27$ F, $a=0.74$ F, $r_0'=1.81$ F, $a'=0.50$ F. This we call potential 1. A good fit to the lower energy data was obtained[18] with potential 2, with $V=55$ MeV, $W=15$ MeV, $r_0=r_0'=1.5$ F, and $a=a'=0.63$ F. A comparison between the predictions of these two potentials and the measured elastic scattering is shown in Fig. 5. Already we see some ambiguity; potential 1 is inadequate at the lower energy, and potential 2 is inadequate at the higher energy. Since the deuterons emitted in the present experiment have intermediate energies of 8.3, 6.7, 5.5, and 5.0 MeV, the DWBA calculations were made with both potentials.

The wave function for the bound state of the proton before pickup was calculated in a real Saxon potential of radius $1.27\,A^{1/3}$ F and diffuseness 0.63 F, whose depth was adjusted to give a binding energy of $|Q|+2.23$ MeV.

Calculated curves for $l=3$ transitions are compared with experiment in Fig. 6; in view of the uncertainties the agreement is quite good, and somewhat favors potential 2. Also shown for comparison are the Butler curves for $R_0=6.15$ F for the least and most energetic groups; the considerable improvement afforded by the DWBA is evident. In particular, the DWBA gives the right order of magnitude for the cross section (consistent

TABLE I. Spectroscopic factors deduced from analysis of the data, and predicted by the shell model, for $l=3$ pickup.

	Potential 1		Potential 2		Plane wave	Shell model	
$-Q$ (MeV)	S	$S/S(5.82)$	S	$S/S(5.82)$	$S/S(5.82)$	S	$S/S(5.82)$
5.82	0.19	1.0	0.46	1.0	1.0	0.75	1.0
7.41	0.16	0.84	0.29	0.63	0.47	0.42	0.56
8.58	0.18	0.95	0.39	0.85	0.51	0.75	1.0
9.09	0.39	2.05	0.70	1.52	0.77	1.08	1.44
Sum	0.92	5.84	1.84	4.00	2.75	3.00	4.00

with spectroscopic factors of order unity). The Butler curves shown in Figs. 3 and 6 require a "single particle width" [4] $\theta_0^2\approx 0.01$, while the proton bound-state function used in the DWBA calculations gives $\theta_0^2\approx 0.16$ at $R_0=6.15$ F, for the $1f$ orbit. Thus, the Butler theory overestimates the cross section by a factor of 16.

The spectroscopic factors S obtained from the fits shown in Fig. 6 are given in Table I, together with the shell-model predictions for a $(1f_{7/2})^3$ configuration. The

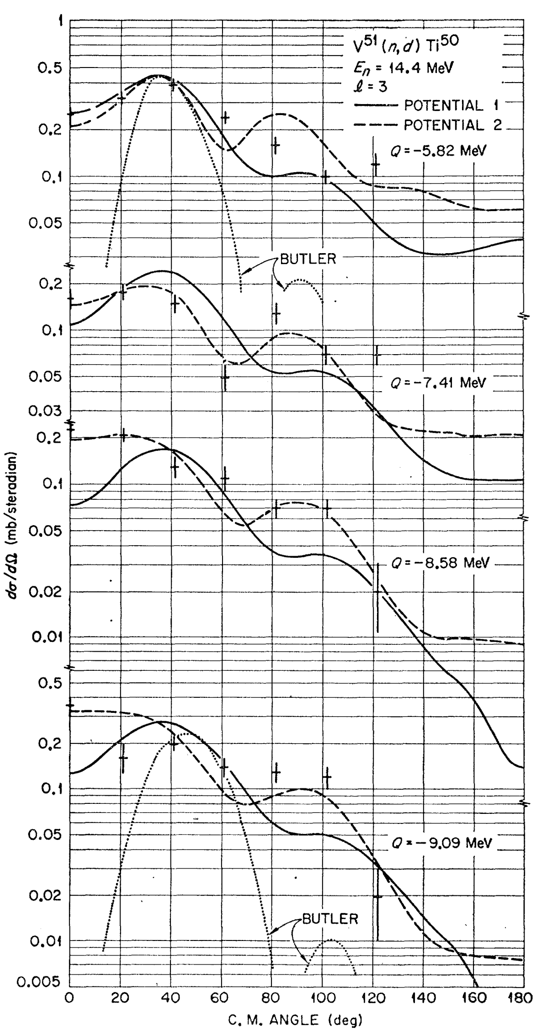

FIG. 6. Comparison of experimental differential cross sections for $V^{51}(nd)$ with the DWBA predictions for pure $l=3$ transitions.

[17] G. Igo, W. Lorenz, and U. Schmidt-Rohr, Phys. Rev. **124**, 832 (1961).
[18] I. Slaus and W. P. Alford, Phys. Rev. **114**, 1054 (1959).
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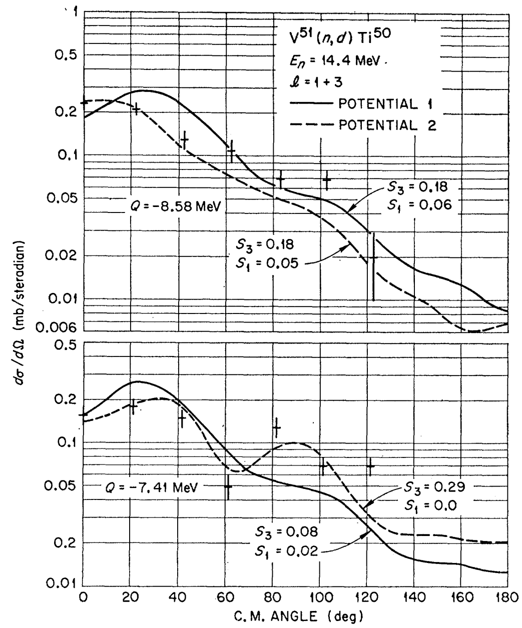

FIG. 7. DWBA predictions for the differential cross sections with small admixtures of $l=1$ when exciting the 2^+ and 4^+ states of Ti^{50}. The potential-2 curve for the $Q=-7.41$ MeV group is pure $l=3$, since any $l=1$ admixture impairs the agreement with experiment.

"measured" S are generally smaller than the shell-model predictions, although their ratios exhibit the same type of behavior, expecially for potential 2. The magnitudes may be low for several reasons. Firstly, the DWBA calculation is based upon a zero-range approximation for the neutron-proton interaction, and finite-range effects are expected to reduce the predicted cross sections and hence require larger S. In the plane-wave theory this effect would increase the S of Table I by roughly 30%. Also the magnitude of the predicted cross sections is quite sensitive to the radius chosen for the proton bound state (see below). Finally, the ambiguities in the deuteron optical potential lead to similar uncertainties in the cross sections; for example, potential 1 predicts cross sections roughly twice as great as those with potential 2. However, all of these uncertainties have far less effect on the relative cross sections. Of course, there are in addition over-all uncertainties of perhaps 20% or more in normalizing the theoretical curves to the experimental data.

Pickup transitions with $l=1$ to the 2^+ and 4^+ states in Ti^{50} are also allowed by angular momentum conservation, although the assumption of a pure f^n configuration in V^{51} would predict zero intensity for them. Figure 7 shows the effect of including some $l=1$ transition in these groups; it is assumed to involve pickup from the $2p$ orbit. The curves predicted by potential 1 improve the fit to the $Q=-8.58$ MeV group somewhat, but the improvement in fit to the $Q=-7.41$ MeV group is questionable. In neither case are the potential 2 fits any better. The spectroscopic factors used in drawing Fig. 7 are indicated there. The only conclusion one can draw is that $l=1$ admixtures are not demanded by the data, but that $2p$ spectroscopic factors of order 20 to 30% those for the $1f$ cannot be completely ruled out.

In order to study some other possible uncertainties in the analysis, further calculations were made. First, the effects of introducing spin-orbit coupling into the distorted-waves was investigated and found to be negligible. Next, variations in the proton bound-state wave function were tried. The effects of spin-orbit coupling are found to be negligible here also. However, the magnitude of the pickup cross section is much more sensitive to the radius chosen for the potential binding the proton, although the shape of the angular distribution is hardly changed. Radii of 1.2, 1.27, and $1.35A^{1/3}$ F were tried for the $Q=-7.41$-MeV group, and the $l=3$ peak cross sections predicted by potential 1 were in the ratios 0.7:1:1.5. The corresponding $1f$ wave function peaks at 3.2, 3.4, and 3.6 F, respectively. These radii roughly cover the range regarded as reasonable, so these results indicate the corresponding uncertainty in the absolute values of S extracted. A rather similar effect is found if the binding energy associated with the proton orbit is not equated to the separation energy $|Q|+2.23$ MeV, but given some effective value (such as might arise from rearrangement effects[20]). In particular, the shell-model interpretation would argue for the *same* effective binding energy for all four proton groups, since the same $1f$ proton state (in V^{51}) is involved in each. As expected, an increase in the binding energy leads to a decrease in the predicted cross section and vice versa. For potential 1, it is found the peak cross section of the $Q=-7.41$ MeV group changes by roughly 15% for each 1-MeV change in binding energy, but again the shape of the angular distribution is little changed.

IV. DISCUSSION

As noted in the preceding section, the experimental results are consistent with pure $l=3$ angular momentum transfer, although a small admixture of $l=1$ can also give reasonable fits with experiment when the 2^+ and 4^+ states of Ti^{50} are formed. Since V^{51} has $N=28$ and $Z=23$, and the ground state J^π is $7/2^-$, it is expected that the three extra protons would be mostly in the $f_{7/2}$ orbit. Hence an (n,d) reaction is most likely to occur through the pickup of a $f_{7/2}$ proton leading to the seniority $v=0$ ground and $v=2$ excited states in Ti^{50}. There is evidence from the j-j coupling calculations of Talmi and collaborators[21] that the states observed in Ti^{50} are states of the $(f_{7/2})^2$ configuration. It has been noted, for example, that level schemes very similar to that of Ti^{50} are found in ${}_{24}Cr_{28}{}^{52}$ which has two more protons, and also partly in ${}_{20}Ca_{22}{}^{42}$ and ${}_{26}Fe_{28}{}^{54}$. In

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[21] I. Talmi, in *Proceedings of the Rehovoth Conference on Nuclear Structure* (North-Holland Publishing Company, Amsterdam, 1958).

particular, Lawson and Uretsky,[22] making use of the experimentally observed energies of levels of $_{23}V_{28}{}^{51}$, calculated the energies of levels of the $(f_{7/2})^2$ configuration and obtained very good agreement with the observed levels in Ti^{50}.

The sum of the spectrographic factors, $\sum_J S$, should give the average number of nucleons in the $f_{7/2}$ orbit in the V^{51} ground state.[4,5] In Table I we see that this sum falls short of 3 for both potential 1 and 2. However, for the reasons discussed above, the relative values are more reliable than the absolute values and are compared with the theory in Table I also. Within the uncertainties of both the experiment and its analysis, the relative S values are consistent with the shell-model predictions for a pure $(f_{7/2})^3$ configuration.

There is evidence for seniority mixing in the titanium isotopes from the $Ti^{50}(d,t)$ reaction.[23] Although neutron pickup from the $N=28$ shell is occurring, two strong $l=3$ groups are seen, indicating that the Ti^{49} ground state has seniority $v=1$ with only 60% probability (if only $f_{7/2}$ configurations are assumed). Similar results have been seen in the $Fe^{54}(He^3,\alpha)$ reaction,[24] which also involves pickup from the $N=28$ shell. States of nondefinite seniority for the $1f_{7/2}$ shell have already been applied to the problem.[25] However, for nuclei such as V^{51} and Ti^{50} which both have a closed neutron shell, the technique used in this reference does not lead to seniority mixing if only pure $f_{7/2}$ configurations are considered.[25] Hence any departure of the spectroscopic factors from the seniority predictions must be due, on this model, to configuration mixing. However, as already indicated, more precise experimental data and a better understanding of the DWBA theory are required before these questions can be answered in detail.

The possibility of small $l=1$ contributions to the proton group exciting the 2^+ and 4^+ levels in Ti^{50} would require some $2p_{3/2}$ or $2p_{1/2}$ proton admixture into the ground state of V^{51}. Neutron pickup with $l=1$ has been observed in the $Ti^{50}(d,t)$ reaction,[23] even though $N=28$ here, and it has been estimated that the average number of $2p$ neutrons in the Ti^{50} ground state is 0.4 ± 0.2. A similar admixture of $2p$ protons in the $Z=28$ shell has been observed in the $Ni^{58}(d,He^3)$ reaction.[26] Both these examples involve even targets; however, the reaction $Ca^{42}(d,p)$ leads to Ca^{43}, the neutron analog of V^{51}. In particular, the angular distribution for the $\frac{3}{2}^-$ level at 0.593 MeV also shows $l=1$ capture,[27] although on the simplest model this level would be interpreted as a state of the $(1f_{7/2})^3$ configuration and no stripping would be allowed. The spectrographic factor S_1 is estimated[4] to be about 0.05, as compared to $S_3=0.79$ for the $l=3$ transition to the ground state. The amount of $2p$ admixture is thus of the same order of magnitude as is compatible with the present data.

If the V^{51} and Ti^{50} states are almost pure $(1f_{7/2})$, the lowest order contributions to an $l=1$ transition will arise from admixtures to the V^{51} ground state of the type $(f_{7/2}{}^2)_J p_{1/2}$ and $(f_{7/2}{}^2)_J p_{3/2}$, which then overlap with the predominantly $(f_{7/2})^2$ configuration of the corresponding state with spin J in Ti^{50}. If the amplitudes of these admixtures are written $\beta(J_j)$ (with $j=\frac{1}{2}$ or $\frac{3}{2}$), the spectroscopic factor for $l=1$ becomes $S_1(J)\approx\sum_j|\beta(J_j)|^2$. A rough estimate of the $\beta(J_j)$ may be obtained using perturbation theory with a δ-function internucleon force, and assuming a knowledge of the energy separation Δ_j of the $(f_{7/2})^3$ and $(f_{7/2})^2 p_j$ configurations, and of ϵ_s, the singlet strength times radial overlap. The result is then[28]

$$\beta(4\tfrac{1}{2})=-0.23\epsilon_s/\Delta_{1/2},$$
$$\beta(4\tfrac{3}{2})=0.19\epsilon_s/\Delta_{3/2},$$
$$\beta(2\tfrac{3}{2})=0.37\epsilon_s/\Delta_{3/2}.$$

($j=\frac{1}{2}$ is not allowed when $J=2$). A reasonable estimate for ϵ_s/Δ_j is roughly $\frac{1}{3}$, giving then $S_1(4)\approx0.01$ and $S_1(2)\approx0.015$. These crude estimates are of the same order of magnitude as the S_1 allowed by the data (Fig. 7).

The most significant fact concerning the $l=1$ transitions is that very small amounts of p-state impurity in the V^{51} wave function will produce large effects on the deuteron angular distribution because $l=1$ transitions are dynamically favored over those with $l=3$. Thus we can conclude that the percentage of admixture of p orbitals in the V^{51} wave function is small, if present at all. It is interesting to note that the pairing model calculations of Kisslinger and Sorensen[5] for the V^{51} ground state also lead to an almost pure $(f_{7/2})^3$ configuration.

ACKNOWLEDGMENTS

We would like to express our thanks to Dr. J. B. French, Dr. A. Zucker, Dr. G. Alaga, and Dr. M. Paić for helpful discussions, and to Dr. R. H. Bassel and Dr. R. M. Drisko for assistance with the calculations.

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[28] We are indebted to J. Ginnochio for assisting us with these calculations.