used in deriving (13) all appear to depend on the same criteria. Their validity is favored by a large distance of the deuteron from the nucleus, a small nuclear charge and a low incident deuteron energy. Under these conditions, which imply a small electric field, the concept of the polarizability of the deuteron and a wave function of the form of (13) may be useful.

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Effect of the Nuclear Electric Field on the Elastic Scattering of Deuterons*

C. F. CLEMENT
Atomic Energy Research Establishment, Harwell, Berkshire, England
and
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
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The asymmetric action of the nuclear electric field on the deuteron is found to strongly affect the elastic scattering cross section on medium and heavy nuclei. Expressions for the scattering amplitude are found in the first and second Born approximations and in the adiabatic approximation. In the latter case the expression is evaluated and compared with experiment, giving rough agreement in magnitude, but not in shape, with deviations from Rutherford scattering found at low energies on medium and heavy nuclei.

The high-energy plane-wave limit of the second Born approximation amplitude is evaluated. It is peaked in the forward direction, is large in magnitude for heavy nuclei, and decreases with increase in energy. Consequences for deuteron optical models are discussed.

1. INTRODUCTION

THE Coulomb field of the nucleus acts on the proton in the deuteron and not on its center of mass. Thus the deuteron elastic scattering cross section is expected to differ from the Rutherford expression not only because of nuclear interactions. The separation of electric from nuclear effects can only be accomplished where one or the other is small. In this paper only the nuclear electric field is considered. The regions where its effect on elastic scattering might be expected to dominate are at incident deuteron energies below the Coulomb barrier, where nuclear effects are strongly inhibited, and for heavy nuclei. The electric field strength increases more rapidly with mass number than the nuclear field strength. The reason that the Coulomb perturbation is important for deuterons and not, say, for α particles is that the low binding energy of the deuteron enables it to be easily stretched and broken up by an asymmetrical force acting on it. There is definite experimental evidence[1] for large differences between the elastic scattering cross sections of deuterons and α particles on heavy nuclei. In the literature there have been two approaches to the problem and both are used here.

The most straightforward approach is the use of the first and second Born approximations. In this way effects arising from both real and virtual deuteron breakup can be included. Previous work by Nishida[2] has suggested that the large decrease from Rutherford scattering, observed at backward angles on heavy nuclei by Gove[3] and also by Rees and Sampson,[1] arises from the electric breakup of the deuteron. In his second paper he gives a qualitative classical theory which appears to show that dipole breakup alone is not responsible for the deviation. The quantum mechanical approach adopted here in Secs. 2 and 4 suffers from the difficulty of evaluating numerically the resulting second Born approximation amplitude. The evaluation of the high-energy limit is performed in Sec. 4 and the resulting amplitude is found to be large.

When proper spin wave functions of the deuteron are used, the first Born approximation also gives a contribution to the amplitude arising from the quadrupole moment of the deuteron. A tensor polarization of the deuteron is produced but the effect is very small.

The second approach arises from the use of the adiabatic approximation and corresponds to the physical picture that the modification in the elastic scattering arises mainly from the virtual breakup of the deuteron. This approach was adopted by Malenka

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[1] J. R. Rees and M. B. Sampson, Phys. Rev. **108**, 1289 (1957).

[2] Y. Nishida, Progr. Theoret. Phys. (Kyoto) **17**, 506 (1957); **19**, 389 (1958).

[3] H. E. Gove, Phys. Rev. **99**, 1353 (1955).

et al.[4] and Sawicki.[5] Malenka *et al.* gave a classical description of the process and Sawicki an approximate quantum mechanical description, both showing that the deviation from Rutherford scattering was of the order of a few percent in the backward direction. However, they did not choose the incident deuteron energy and target elements where the effect is most likely to be observed. These turn out to be energies not too far below the Coulomb barrier. In Sec. 3 an expression similar to that of Sawicki[5] is evaluated and compared to experiment. The comparison gives rough agreement in magnitude for incident deuteron energies below the Coulomb barrier on medium-A nuclei.

The relation between the adiabatic and second Born approximations is briefly discussed in Sec. 2. The results obtained appear to show that the Coulomb perturbation plays an important part in the elastic scattering of deuterons on heavy nuclei and on medium nuclei at low energies. Any deuteron optical model ought, therefore, to include its contribution. This subject is discussed in Sec. 5.

2. EXPRESSIONS FOR THE SCATTERING AMPLITUDE

In terms of relative and center-of-mass coordinates the Hamiltonian for a deuteron in the electric field of a nucleus of charge Z is

$$H=-\frac{\hbar^2}{M}\nabla_r^2-\frac{\hbar^2}{4M}\nabla_R^2+V(r)+\frac{Ze^2}{R}-P(\mathbf{r},\mathbf{R})=H_0-P(\mathbf{r},\mathbf{R}),$$

where

$$\mathbf{R}=(\mathbf{r}_n+\mathbf{r}_p)/2,\quad \mathbf{r}=\mathbf{r}_p-\mathbf{r}_n.$$

The nucleon-nucleon potential is $V(r)$ and the perturbing term which produces deviations from Rutherford scattering is

$$P(\mathbf{r},\mathbf{R})=Ze^2[1/R-1/|\mathbf{R}+\tfrac{1}{2}\mathbf{r}|].\tag{1}$$

We require the wave function of the deuteron to satisfy the equation

$$(H_0-E)\Psi(\mathbf{r},\mathbf{R})=P(\mathbf{r},\mathbf{R})\Psi(\mathbf{r},\mathbf{R}),\tag{2}$$

where

$$E=E_d+\epsilon_0.$$

The incident kinetic energy and binding energy of the deuteron are E_d, ϵ_0, respectively.

By standard methods the elastic scattering amplitude may be written as

$$F(\theta)=f_c(\theta)+f(\theta).\tag{3}$$

The Rutherford scattering amplitude, $f_c(\theta)$, and the correction to it, $f(\theta)$, are

$$f_c(\theta)=[\eta_d/2k_d\sin^2(\tfrac{1}{2}\theta)]\exp[-i\eta_d\ln\sin^2(\tfrac{1}{2}\theta)+i\pi+2i\sigma_0],$$

$$f(\theta)=\frac{1}{4\pi}\frac{4M}{\hbar^2}\int d^3\mathbf{r}d^3\mathbf{R}\,\psi_i^*(\eta_d,\mathbf{k}_f,\mathbf{R})\times\phi_0^*(\mathbf{r})P(\mathbf{r},\mathbf{R})\Psi(\mathbf{r},\mathbf{R}).\tag{4}$$

The initial wave number and Coulomb constant for the deuteron are k_d, η_d. The ground-state deuteron wave function is $\phi_0(r)$ and the center-of-mass wave function for the final state with ingoing spherical waves at infinity is

$$\psi_i(\eta_d,k_f,R)=\exp(-\tfrac{1}{2}\pi\eta_d)\Gamma(1-i\eta_d)\exp(i\mathbf{k}_f\cdot\mathbf{R})\times F[i\eta_d,1;-i(k_fR+\mathbf{k}_f\cdot\mathbf{R})].$$

Finally the elastic scattering cross section and its ratio to the Rutherford cross section are

$$d\sigma(\theta)=|f_c(\theta)+f(\theta)|^2,$$

$$d\sigma(\theta)/d\sigma_c(\theta)=|1+\Delta|^2,$$

where

$$\Delta=f(\theta)/f_c(\theta).\tag{5}$$

We will now discuss three possible forms for $\Psi(\mathbf{r},\mathbf{R})$ for substitution into (4) to determine $f(\theta)$.

I. The First Born Approximation

The first Born approximation arises from the replacement of $\Psi(\mathbf{r},\mathbf{R})$ in (4) by its unperturbed form $\psi_0(\eta_d,\mathbf{k}_d,\mathbf{R})\phi_0(r)$.

$$\psi_0(\eta_d,\mathbf{k}_d,\mathbf{R})=\exp(-\tfrac{1}{2}\pi\eta_d)\Gamma(1+i\eta_d)\exp(i\mathbf{k}_d\cdot\mathbf{R})\times F[-i\eta_d,1;i(k_dR-\mathbf{k}_d\cdot\mathbf{R})].\tag{6}$$

For $r<2R$, the perturbing term (1) may be expanded as

$$P(\mathbf{r},\mathbf{R})=\frac{Ze^2}{R}\sum_{n=1}^{\infty}\left(\frac{r}{2R}\right)^nP_n(\mathbf{r},\mathbf{R}).$$

The Legendre polynomial is a function of the angle between $\mathbf{r}$, $\mathbf{R}$. Thus the Born approximation contains integrals of the form

$$\int d^3\mathbf{r}\,\phi_0^*(\mathbf{r})r^nP_n(\mathbf{r},\mathbf{R})\phi_0(\mathbf{r}).$$

If the deuteron wave function is taken to be purely S state, these integrals all vanish. However, for a realistic deuteron wave function containing a D-state component there is a contribution from $n=2$ which is directly related to the deuteron's quadrupole moment. This term was investigated completely in the author's thesis.[6] It was found to have a negligible effect on the elastic scattering of deuterons at all values of E_d, Z. A tensor polarization of the deuteron is also produced. The terms expressing this polarization were found to be of the order of a few percent of the unpolarized

[4] B. J. Malenka, U. E. Kruse, and N. F. Ramsey, Phys. Rev. **91**, 1165 (1953).

[5] J. Sawicki, Acta Phys. Polon. **13**, 225 (1954).

[6] C. F. Clement, Ph.D. thesis, University of Cambridge, 1960 (unpublished).

term even in the most favorable cases, which are at low energies on light nuclei. There is thus no likelihood of their being detected experimentally.

The vanishing of the integral for $r<2R$ led French and Goldberger[7] to evaluate the integral for $r>2R$. Extremely small deviations from Rutherford scattering were obtained for large η_d. This integration is over a region where nuclear forces are strong and any contribution to the amplitude would be masked by nuclear effects. Since the first Born approximation gives such small deviations, we must use a higher order approximation for $\Psi(r,R)$.

II. The Second Born Approximation

Logically the next step is to consider the second Born approximation which requires a first-order solution of (2). A complete set of states for the internal motion of the deuteron are defined by

$$[-(\hbar^2/M)\nabla_r^2+V(\mathbf{r})]\phi_\lambda(\mathbf{r})=\epsilon_\lambda\phi_\lambda(\mathbf{r}),$$

where $\lambda\neq 0$ correspond to two-nucleon scattering states $\mathbf{k}$.

Then a complete set of states for the operator H_0-E are $\phi_\lambda(\mathbf{r})\psi_0(\eta_d',\mathbf{k}_d',\mathbf{R})$ where the ψ_0 are defined by (6). The outgoing-wave Green's function for the operator is then

$$G_0(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')=\sum_\lambda\int d^3\mathbf{k}_d'\,\phi_\lambda^*(\mathbf{r}')\psi_0^*(\eta_d',\mathbf{k}_d',\mathbf{R}')\times\phi_\lambda(\mathbf{r})\psi_0(\eta_d',\mathbf{k}_d',\mathbf{R})/(E-\epsilon_\lambda-E_d'+i\epsilon),$$

where the energy of the center-of-mass scattering state is

$$E_d'=\hbar^2k_d'^2/4M. \qquad (7)$$

Then using the first-order solution of (2) the second Born approximation scattering amplitude is

$$f(\theta)=-\frac{M}{\pi\hbar^2}\int d^3\mathbf{r}d^3\mathbf{R}d^3\mathbf{r}'d^3\mathbf{R}'\psi_i^*(\eta_d,\mathbf{k}_f,\mathbf{R})\phi_0^*(\mathbf{r})P(\mathbf{r},\mathbf{R})\,G_0(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')P(\mathbf{r}',\mathbf{R}')\phi_0(\mathbf{r}')\psi_0(\eta_d,\mathbf{k}_d,\mathbf{R}').$$

An attempt, which is described in Sec. 4, has been made to evaluate this integral. In order to reduce it to a manageable form several approximations must be made. The D-state component of the deuteron wave function is neglected and its spin dependence can then be omitted. The contribution of ϕ_0 in the Green's function is then negligible and the summation over λ is replaced by integration over the relative momentum $\mathbf{k}$. Since the exact form of the $\phi_\lambda(\mathbf{r})$ is not known, they are replaced by plane waves. That this approximation may be quite good is indicated by Ramsey *et al.*,[8] who used a free-particle two-body Green's function to evaluate the polarizability of the deuteron. Finally, the approximation is made of taking only the dipole term in the expansion of $P(\mathbf{r},\mathbf{R})$. That higher terms may be important is suggested by Nishida[2] but their inclusion would greatly increase the numerical work, and at low energies, where the Coulomb wave functions are small near the nucleus, the expansion is expected to converge rapidly.

After making the above approximations the correctly normalized scattering amplitude is

$$f(\theta)=-\frac{1}{\pi}\left(\frac{ZMe^2}{2\hbar^2}\right)^2\frac{1}{(2\pi)^6}\int d^3\mathbf{r}d^3\mathbf{R}d^3\mathbf{R}'d^3\mathbf{r}'d^3\mathbf{k}d^3\mathbf{k}_d'$$
$$\times\phi_0^*(\mathbf{r})\psi_i(\eta_d,\mathbf{k}_f,\mathbf{R})(r/R^2)\exp[i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')]P_1(\mathbf{r},\mathbf{R})$$
$$\times\psi_0(\eta_d',\mathbf{k}_d',\mathbf{R})\psi_0^*(\eta_d',\mathbf{k}_d',\mathbf{R}')(r'/R'^2)\phi_0(\mathbf{r}')P_1(\mathbf{r}',\mathbf{R}')$$
$$\times G_0(\eta_d,\mathbf{k}_d,\mathbf{R}')/(\tfrac{1}{4}k_d^2-\gamma^2-\tfrac{1}{4}k_d'^2-k^2+i\epsilon), \qquad (8)$$

where, in addition to (7), the energies are defined by

$$\epsilon_0=-\hbar^2\gamma^2/M,\quad \epsilon_k=\hbar^2k^2/M.$$

We finally turn to an approximation which leads to an expression needing considerably less numerical work.

III. The Adiabatic Approximation

The adiabatic approximation consists physically in regarding the period of the deuteron's internal motion as being fast compared to its motion through the Coulomb field. Thus the internal energy at a radius R is modified by an amount $\eta(R)$ which is related to the polarizability of the deuteron. In a previous paper[9] the latter quantity was calculated for a number of assumed deuteron wave functions, and a modified wave function, $\Psi(\mathbf{r},\mathbf{R})$, was suggested for a deuteron in the electric field of a nucleus. After substitution of such a wave function into (4) the modified scattering amplitude, in terms of the polarizability α, becomes

$$f(\theta)=\frac{M}{\pi\hbar^2}\frac{Z^2e^2\alpha}{2}\int_{R_0}d^3\mathbf{R}\,\psi_i^*(\eta_d,\mathbf{k}_f,\mathbf{R})\frac{1}{R^4}\psi_0(\eta_d,\mathbf{k}_d,\mathbf{R}). \qquad (9)$$

This expression was previously derived by Sawicki[5] using a different method. The integration over R must be cut off at the nuclear surface, and, for uniqueness, the expression should not be sensitive to R_0.

We may regard the adiabatic expression for $f(\theta)$ as being an approximation to the second Born approximation expression (8) in the following way. If the integration over k_d' in (8) is sharply peaked at a value equal to k_d such that the variation in the denominator can be neglected, we use

$$\int d^3\mathbf{k}_d'\,\psi_0^*(\eta_d',\mathbf{k}_d',\mathbf{R}')\psi_0(\eta_d',\mathbf{k}_d',\mathbf{R})=(2\pi)^3\delta(\mathbf{R}-\mathbf{R}').$$

[7] J. B. French and M. L. Goldberger, Phys. Rev. **87**, 899 (1952).
[8] N. F. Ramsey, B. J. Malenka, and U. E. Kruse, Phys. Rev. **91**, 1162 (1953).
[9] C. F. Clement, preceding paper [Phys. Rev. **128**, 2724 (1962)]

It follows that

$$f(\theta)\sim\frac{Z^2e^2}{2}\frac{M}{\pi\hbar^2}\left[\frac{Me^2}{2\hbar^2}\frac{1}{(2\pi)^3}\int d^3\mathbf{r}d^3\mathbf{r}'d^3\mathbf{k}\phi_0{}^*(\mathbf{r})r\right.$$
$$\left.\times\exp(i\mathbf{k}\cdot\mathbf{r}-i\mathbf{k}\cdot\mathbf{r}')r'P_1(\mathbf{r},\mathbf{R})P_1(\mathbf{r}',\mathbf{R}')\phi_0(\mathbf{r}')/(k^2+\gamma^2)\right]$$
$$\times\int d^3\mathbf{R}\,\psi_i{}^*(\eta_d,\mathbf{k}_f,\mathbf{R})\frac{1}{R^4}\psi_0(\eta_d,\mathbf{k}_d,\mathbf{R}).$$

The bracketed expression gives the polarizability of the deuteron in the method of Ramsey *et al.*,[8] and thus the whole expression reduces to (9).

Alternatively the validity of a polarized wave function necessary to produce (9) was examined in a previous paper.[9] It was found that such a wave function was best at low energies, for large value of R, and for relatively small values of Z. In Sec. 3 the expression is evaluated for medium and heavy nuclei. Even in the latter case (9) may be a good approximation at low energies, since the smallness of the center-of-mass wave functions for small R makes the greatest contribution arise from large values of R.

Finally we note that the adiabatic approximation neglects contributions to the amplitude arising from the real breakup of the deuterons. These appear in (8) from the pole in the denominator and their smallness is a necessary condition for the validity of the adiabatic approximation.

3. CROSS SECTION IN THE ADIABATIC APPROXIMATION

I. The Cross Section

In this section the expression (9) is evaluated numerically and the results are compared with experiment. A previous evaluation was made by Sawicki,[5] who replaced R^{-4} by $(R+R_0)^{-4}$, when the integral could be represented by a hypergeometric function. This treatment is, however, unsatisfactory for the following reason. For cases of interest when the incident energy is below the Coulomb barrier, the main contribution to the integral arises from values of R near R_c where

$$E_d=Ze^2/R_c.$$

For a typical case, $E_d=3.32$ MeV on Co^{59}, $R_c\sim 2R_0$. Thus the replacement results in the reduction of $f(\theta)$ by a factor of about $(2/3)^4$.

The method of evaluation used here is to expand the Coulomb wave functions in partial waves.

$$\psi_0(\eta_d,\mathbf{k}_d,\mathbf{R})=\sum_l[4\pi(2l+1)]^{1/2}i^l\exp(i\sigma_l)\times Y_l{}^0(\mathbf{R})F_l(k_dR)/k_dR. \quad (10)$$

$$\psi_i{}^*(\eta_d,\mathbf{k}_f,\mathbf{R})=\sum_{l'm'}4\pi(-1)^{m'}(-i)^{l'}\exp(i\sigma_{l'})\times Y_{l'}{}^{m'*}(\mathbf{R})Y_{l'}{}^{-m'*}(\mathbf{k}_f)\times F_{l'}(k_dR)/k_dR. \quad (11)$$

The spherical harmonics are referred to $\mathbf{k}_d$ as axis and F_l is the radial wave function which is regular at the origin.

The angular integrals in (9) immediately give $m'=0$, $l'=l$. We then define dimensionless radial integrals by

$$I_l(\rho_0,\eta_d)=\int_{\rho_0}^{\infty}F_l(\rho)\frac{1}{\rho^4}F_l(\rho)d\rho, \quad (12)$$

where $\rho_0=k_dR_0$.

On substitution of (9) into (5) the deviation from Rutherford scattering is specified by

$$\Delta(\theta)=-C\sin^2(\tfrac{1}{2}\theta)\exp[i\eta_d\ln\sin^2(\tfrac{1}{2}\theta)]\sum_l(2l+1)\times\exp[2i(\sigma_l-\sigma_0)]I_l(\rho_0,\eta_d)P_l(\cos\theta), \quad (13)$$

where

$$C=4Z^2e^2M\alpha k_d{}^2/\hbar^2\eta_d.$$

For a deuteron with laboratory energy E_d scattering on a nucleus mass number A the quantities in (13) were calculated from

$$k_d=A0.3104(E_d)^{1/2}/(A+2)\ \mathrm{F}^{-1},$$
$$M=AM_p/(A+2),\quad \eta_d=0.2239Z/(E_d)^{1/2}.$$

With α in units of 10^{-39} cm^3,

$$C=5.977(A/A+2)^3\alpha Z(E_d)^{3/2}\times 10^{-2}. \quad (14)$$

To find ρ_0 the minimum radius R_0 was taken as

$$R_0=1.3A^{1/3}+1.1\ \mathrm{F}.$$

For values of E_d below the Coulomb barrier, the dependence on ρ_0 was found to be slight. Since apart from C, Δ given by (13) is then a function of η_d only, the dependence of the deviation from Rutherford scattering for fixed η_d may be discussed. For small Δ

$$\sigma/\sigma_c\sim 1+2R\Delta.$$

Firstly, the deviation is linearly dependent on α. Secondly, for fixed η_d the ratio of the deviations on two elements 1 and 2 is roughly

$$\frac{\Delta_1}{\Delta_2}=\left(\frac{A_2}{A_2+2}\frac{A_1+2}{A_1}\right)^3\frac{Z_1{}^4}{Z_2{}^4}.$$

The incident energies on the two elements will, of course, be different. These variations provide a method of estimating the results to be expected from this theory for values of α, E_d, and Z other than those used here.

The Coulomb phases in (13) were calculated using the recurrence relations for $\exp[2i(\sigma_l-\sigma_0)]$ which are

$$\exp[2i(\sigma_l-\sigma_0)]=A_l+iB_l,$$
$$A_l=\frac{l^2-\eta^2}{l^2+\eta^2}A_{l-1}-\frac{2l\eta}{l^2+\eta^2}B_{l-1},\quad A_0=1,$$
$$B_l=\frac{2l\eta}{l^2+\eta^2}A_{l-1}+\frac{l^2-\eta^2}{l^2+\eta^2}B_{l-1},\quad B_0=0.$$

They were checked by using the asymptotic expansion for $\sigma_l(\eta)$ for large l.

$$\sigma_l(\eta)=\beta(l+1)+\eta(\ln\gamma-1)+\frac{1}{\gamma}\left[-\frac{\sin\beta}{12}+\frac{\sin 3\beta}{360\gamma^2}-\frac{\sin 5\beta}{1260\gamma^4}+\cdots\right],$$

$$\gamma=[\eta^2+(l+1)^2]^{1/2},\quad \beta=\tan^{-1}[\eta/(l+1)].$$

The calculation of the radial integrals (12) remains to be considered.

II. Evaluation of the Radial Integrals

In the theory of Coulomb excitation[10] it is known that similar integrals to (12) are given accurately for large l or large η by the WKB approximation. We, therefore, use the approximation for large l and neglect the region inside the turning point.

$$I_l(\rho_0,\eta_d)=\int_{\rho_T}^{\infty}\frac{d\rho}{\rho^4}[f(\rho)]^{-1/2}\sin^2\left\{\frac{\pi}{4}+\int_{\rho_T}^{\rho}[f(\rho)]^{1/2}d\rho\right\},\quad(15)$$

where

$$f(\rho)=1-(2\eta_d/\rho)-(l(l+1)/\rho^2),$$

and ρ_T is the value of ρ for which $f(\rho)=0$.

The oscillating term in the integrand is neglected, a justifiable procedure for large η_d or l. Then the result of evaluating (15) is

$$I_l(\eta_d)=\frac{1}{2\eta_d^3}\left\{\frac{2(\epsilon^2+2)}{(\epsilon^2-1)^{5/2}}\arctan\left[\frac{(\epsilon^2-1)^{1/2}}{\epsilon+1+(2\epsilon+2\epsilon^2)^{1/2}}\right]-\frac{3}{2(\epsilon^2-1)^2}\right\},\quad(16)$$

where

$$\epsilon=[\eta_d^2+l(l+1)]^{1/2}/\eta_d.$$

The singularity in (16) at $\epsilon=1$ is only apparent and the integral reduces to its correct value at this point. For large $l(\gg\eta_d)I_l$ behaves as l^{-3} and the series (13) for Δ converges rapidly. The variation of Δ for large η_d may be discussed on the basis of the WKB approximation. In this case $A_l\sim-A_{l-1}$, $B_l\sim-B_{l-1}$ and, in the backward direction, $P_l(\cos\theta)\sim(-1)^l$ so that the phases for different l values in (13) are roughly coherent. Also ϵ in (16) is slowly varying as a function of η_d. Thus from (14) and (16) the variation of Δ with E_d, Z is

$$\Delta\sim(Z(E_d)^{3/2}/\eta_d^3)\sim(E_d^3/Z^2)\sim(Z^4/\eta_d^6).\quad(17)$$

The deviation from Rutherford scattering becomes smaller for large η_d because of the smaller penetration of the deuteron into the region inside the turning point. The expressions (17) afford an additional method of estimating deviations for large η_d from the deviations given subsequently.

For low values of l the integrals I_l were evaluated by using an auxiliary function $\phi_l(\eta_d,\rho)$ [see, for example, Froberg[11]]

$$F_l(\eta,\rho)=C_l(\eta)\rho^{l+1}\phi_l(\eta,\rho),$$

where

$$C_l^2(\eta)=\frac{2\pi\eta}{\exp(2\pi\eta)-1}\,\frac{1\times(1+\eta^2)\cdots(l^2+\eta^2)2^{2l}}{[(2l+1)!]^2}.$$

Then the integrals are

$$I_l(\rho_0,\eta_d)=C_l^2(\eta)\int_{\rho_0}^{\infty}\rho^{2l-2}\phi_l^2(\eta_d,\rho)d\rho.\quad(18)$$

The differential equation satisfied by ϕ_l is

$$\rho(d^2\phi_l/d\rho^2)+(2l+2)(d\phi_l/d\rho)+(\rho-2\eta)\phi_l=0.\quad(19)$$

Since the ϕ_l are normalized to unity at the origin, $I_0(\rho_0,\eta_d)$ is unbounded and behaves as ρ_0^{-1} as $\rho_0\to0$. The variation of the results with ρ_0 was investigated and the sensitivity was found to be small for incident energies below the Coulomb barrier. For example, the result of increasing R_0 by 1 F for Cu^{63} at $E_d=4.07$ was to reduce Δ by about 7% in the backward direction and by less at other angles. For all results quoted, the uncertainty in R_0 should not have produced an uncertainty in Δ of more than 10%.

For numerical purposes the integral (18) was evaluated up to a cutoff ρ_m. Initial values of ϕ_l were taken, by interpolation if necessary, from the NBS tables[12] and starting values of $\phi_l(\eta,\rho_0)$ were found from the relations

$$\phi_l'=\frac{\eta}{l+1}\phi_l-\frac{\rho}{2l+3}\left[1+\frac{\eta^2}{(l+1)^2}\right]\phi_{l+1}$$

$$=\frac{2l+1}{\rho}\left[\phi_{l-1}-\left(1+\frac{\eta\rho}{l(2l+1)}\right)\phi_l\right].$$

The differential equation (19) was then integrated out to ρ_m on a Mercury computing machine. Steps of 0.2 in ρ were found to give an accuracy of 1 part in 10^4 or better over an interval of 5 in ρ. Finally the integration in (18) was performed numerically by the method of Simpson's rule.

The remainder after integration to ρ_m may be written

$$I_R=\int_{\rho_m}^{\infty}[F_l(\rho)]^2\rho^{-4}d\rho.$$

This integral was estimated using the asymptotic power series for $F_l(\rho)$ given by Froberg.[11] After neg-

[10] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Modern Phys. **28**, 432 (1956).

[11] C. E. Froberg, Revs. Modern Phys. **27**, 399 (1955).

[12] *Tables of Coulomb Wave Functions*, National Bureau of Standard Applied Mathematics Series No. 17 (U. S. Government Printing Office, Washington, D. C., 1953), Vol. 1.

TABLE I. Comparison of the values of $I_5(\rho_0,\eta_d)$ calculated by direct integration, I_A, and by the WKB approximation, I_B.

Element	E_d (MeV)	ρ_0	η_d	$100(I_B-I_A)/I_A$
C	1.8	1.6	1	−1.9
Al	2	2	2	−1.8
Co	4	3.6	3	−0.7
Co	2.28	2.8	4	−0.8
Pb	15.2	10	4.7	5.5

lecting the rapidly oscillating terms in the integrand an asymptotically convergent expression for I_R was found.

$$I_R=1/6\rho_m^3+\eta_d/16\rho_m^4+\cdots.$$

The cutoff was chosen at $\rho_m=20$ when, by taking the first term in I_R, the error is less than the second term, which is $4\eta_d 10^{-7}$.

In practice the direct integration method was used in evaluating $I_l(\rho_0,\eta_d)$ for $l\leq 5$, and the WKB method for $l\geq 5$. A comparison of the results for $l=5$ provides a check on both methods and is shown in Table I. It can be seen that the methods give consistent results and the WKB method should be accurate enough for $l>5$.

III. Comparison with Experiment

The calculation of Δ was programmed for a Mercury computer. The Legendre polynomials were calculated using recurrence relations, checked by direct computation, and the series (13) in l was summed to $l=40$. At this value convergence was assured for all cases considered. The value of the polarizability α was taken as 0.63 in units of 10^{-39} cm^3. This value was derived in a previous paper[9] from a Hulthén wave function of Hulthén and Sugawara[13] and is expected to be a good value for the deuteron.

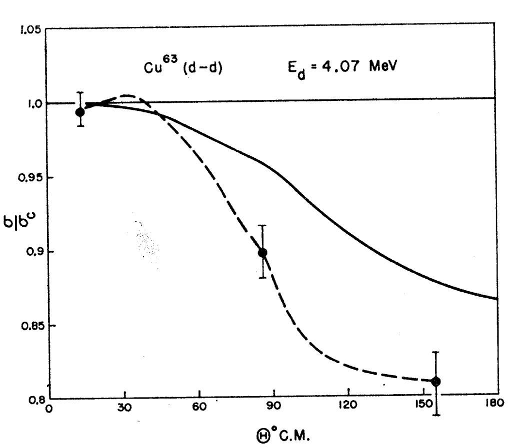

FIG. 1. Ratio of deuteron elastic scattering cross section to the Rutherford value for Cu^{63} with 4.07-MeV deuterons. The dashed curve represents the experiment of Slaus and Alford (see reference 14) and the solid curve is the theoretical prediction of the adiabatic approximation.

[13] L. Hulthén and M. Sugawara, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1959), Vol. 39, p. 1.

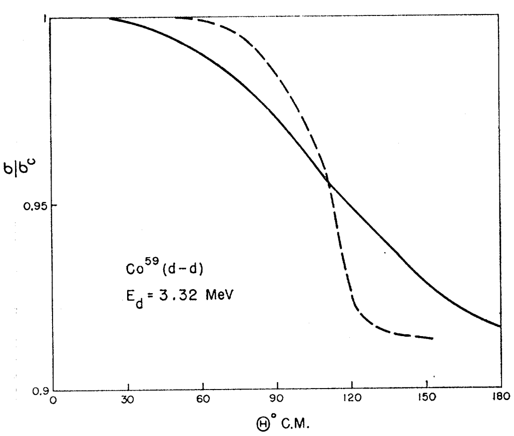

FIG. 2. Ratio of deuteron elastic scattering cross section to the Rutherford value for Co^{59} with 3.32-MeV deuterons. The dashed curve represents the experiment of Slaus and Alford (see reference 14) and the solid curve is the theoretical prediction of the adiabatic approximation.

Slaus and Alford[14] have reported a number of experiments on deuteron elastic scattering on elements ranging from Mg^{24} to Cu^{63} at laboratory energies of 3.32 and 4.07 MeV. On the lighter elements the energies are near that of the Coulomb barrier and the theory presented here is certainly inadequate because of nuclear effects. The latter should, however, be considerably reduced on the heavier elements where the incident energy is from 2 to 4 MeV below the Coulomb barrier. A comparison with theory is shown in two typical cases in Figs. 1 and 2. The experimental curves are somewhat rough but are adequate enough to give a comparison. The general features of the latter are that any agreement in magnitude of deviation is better at the lower energy and on the heavier elements. However, the shapes of the curves are different. The angles where the ratio σ/σ_c falls away from unity are comparable but the experimental curve exhibits a sharp drop whereas the theoretical curve is more slowly decreasing.

The adiabatic approximation is expected to be better for medium nuclei at low energies than for heavy nuclei.[9] Nevertheless, a comparison has also been made with experimental results on the latter. These have been reported by Gove[3] with $E_d=15.2$ MeV on Pb, Cindro and Wall[15] at 13.5 and 15 MeV on several nuclei, and Rees and Sampson[1] at 11 MeV on Ta, Au, Bi, and U. The latter show a considerable difference between d-d and comparable α-α deviations strongly suggesting an additional scattering mechanism in the deuteron case. In Figs. 3 and 4 are shown comparisons at the lower and higher energies. The rise in the backward direction of the theoretical curve in Fig. 4 arises from the large size of the adiabatic amplitude. Again agreement is considerably better at the lower energy.

[14] I. Slaus and W. Parker Alford, Phys. Rev. **114**, 1054 (1959).
[15] N. Cindro and N. S. Wall, Phys. Rev. **119**, 1340 (1960).

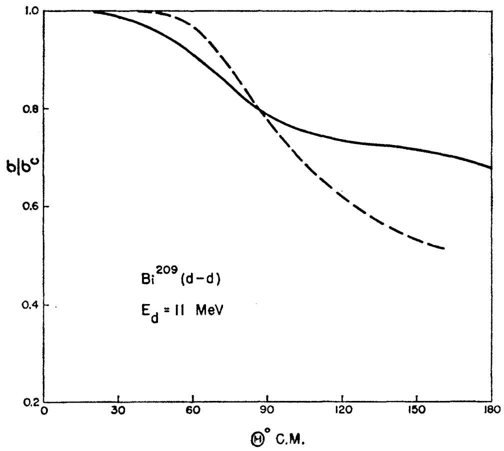

FIG. 3. Ratio of the deuteron elastic scattering cross section to the Rutherford value for Bi^{209} with 11-MeV deuterons. The dashed curve represents the experiment of Rees and Sampson (see reference 1) and the solid curve is the theoretical prediction of the adiabatic approximation.

The predicted variation of the deviation with E_d^3 for a given element was tested and was found to be approximately correct in the backward direction, as shown in Fig. 5.

In conclusion we may state that the comparison serves to show the large effect the electric field has on deuteron elastic scattering at low energies. At low enough energies the adiabatic approximation amplitude, which contains no arbitrary parameters, explains a large part of deviations from Rutherford scattering which have been observed. The second Born approximation, which is discussed next, can also include effects arising from the real breakup of the deuteron and should be a better approximation. A possible method of combining nuclear effects with the adiabatic approximation is discussed in Sec. 5.

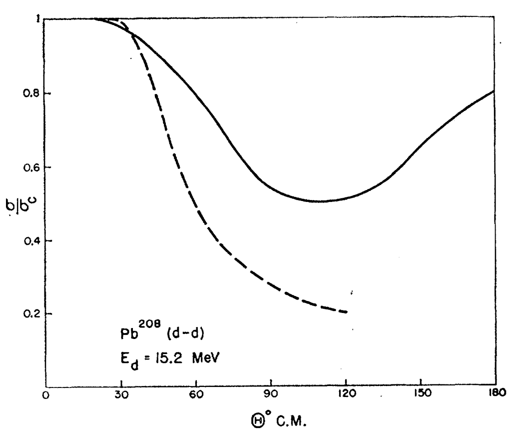

FIG. 4. Ratio of deuteron elastic scattering cross section to the Rutherford value for Pb^{208} with 15.2-MeV deuterons. The dashed curve represents the experiment of Gove (see reference 3) and the solid curve is the theoretical prediction of the adiabatic approximation.

4. CROSS SECTION IN THE SECOND BORN APPROXIMATION

I. Evaluation at Low Energies

In this section the evaluation of the second Born approximation expression (8) for the scattering amplitude is discussed. The full expression with Coulomb wave functions is reduced to a form for which numerical computation is feasible. An attempt at the latter is described in the author's thesis.[6] This was unsuccessful because of numerical difficulties, but the computation should be practical with a larger computing machine. The high-energy limit which is much easier to evaluate is discussed subsequently.

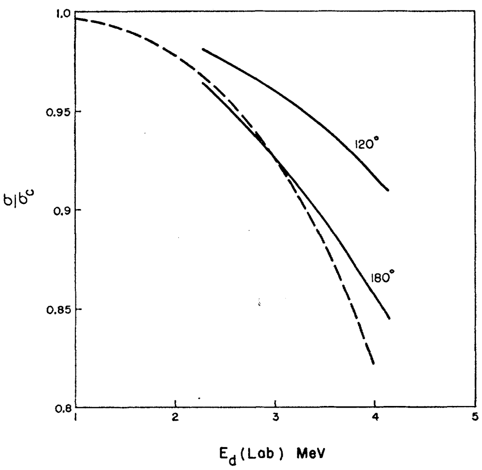

FIG. 5. Variation with energy of the deviation from Rutherford scattering of deuterons on Co^{59}. The solid curves are the calculated deviations at the angles shown and the dashed curve represents a deviation proportional to E_d^3 which is normalized at 3 MeV.

From (8) and (5) the evaluation is required of the quantity

$$\Delta(\theta)=-\frac{k_d}{\eta_d}\left(\frac{ZMe^2}{\hbar^2}\right)^2\frac{1}{(2\pi)^7}\sin^2(\theta/2)$$

$$\times\exp[i\eta_d\ln\sin^2(\theta/2)-2i\sigma_0]\int d^3\mathbf{r}d^3\mathbf{r}'d^3\mathbf{R}d^3\mathbf{R}'d^3\mathbf{k}$$

$$\times d^3\mathbf{k}_d'\phi_0^*(r)\psi_i^*(\eta_d,\mathbf{k}_f,\mathbf{R})(r/R^2)\exp(i\mathbf{k},\mathbf{r})P_1(\mathbf{r},\mathbf{R})$$

$$\times\psi_0(\eta_d',\mathbf{k}_d',\mathbf{R})\exp(-i\mathbf{k}\cdot\mathbf{r}')\psi_0^*(\eta_d',\mathbf{k}_d',\mathbf{R}')(r'/R'^2)$$

$$\times\phi_0(r')P_1(\mathbf{r}',\mathbf{R}')\psi_0(\eta_d,\mathbf{k}_d,\mathbf{R}')/(k'^2-k^2+i\epsilon),\quad(20)$$

where

$$k'^2=\tfrac{1}{4}k_d^2-\gamma^2-\tfrac{1}{4}k_d'^2.\quad(21)$$

The latter quantity is positive for a range of k_d' running from 0 to

$$K=(k_d^2-4\gamma^2)^{1/2}.$$

The procedure adopted consists in expanding the Coulomb wave functions, the exponentials, and the

Legendre polynomials in partial waves with all spherical harmonics referred to k_d as axis. The expansions for the Coulomb wave functions are given by (10) and (11), and the other cases are standard results. The deuteron wave function is taken as having a simple S-state form:

$$\phi_0(r)=[N_d/(2\pi)^{1/2}][\exp(-\gamma r)/r].$$

Integration over all the angular variables is then performed, and the resulting sum of Clebsch-Gordan coefficients over a third component of angular momentum leaves only radial integrals connecting partial waves with adjacent l values. These are the dipole integrals which appear in the theory of Coulomb excitation[10]:

$$M_{ll\pm1}^{-2}(k_d,k_d')=\frac{1}{k_dk_d'}\int_0^\infty dR\, F_l(k_dR)\frac{1}{R^2}F_{l\pm1}(k_d'R). \quad (21)$$

The integration over R, R' is extended over all space, certainly a valid approximation for large l and for low energies.

The integrals over r, r', k may be performed explicitly. They are of the form

$$I=\int drdr'dk\, r^2r'^2k^2\exp(-\gamma r-\gamma r')\times j_1(kr)j_1(kr')/(k'^2-k^2+i\epsilon).$$

We must consider two cases corresponding to k' given by (21) being real or imaginary. In both cases the integrals over r, r' are obtained from

$$\int r^2\exp(-\gamma r)j_1(kr)dr=\frac{2k}{(k^2+\gamma^2)^2}.$$

In the real case the resulting integral over k is obtained using contour integration. The result is

$$I=\frac{\pi}{8(k'^2+\gamma^2)^4}\left[-16ik'^3+\frac{1}{\gamma^3}(k'^6+9k'^4\gamma^2-9k'^2\gamma^4-\gamma^6)\right]. \quad (22)$$

The imaginary term corresponds to the residue at the pole. For the case with k' imaginary the result may be obtained by direct integration or by the replacement of k' by iq in (22).

$$I=-\tfrac{1}{8}\pi[(q^2+4q\gamma+\gamma^2)/\gamma^3(q+\gamma)^4].$$

Thus the final integrals over k_d' have the form

$$I_l^{\pm}=\int_0^K dk_d'\, k_d'^2\frac{1}{(k'^2+\gamma^2)^4}[-16ik'^3+(1/\gamma^3)(k'^6+9k'^4\gamma^2-9k'^2\gamma^4-\gamma^6][M_{ll\pm1}^{-2}(k_d,k_d')]^2-\int_K^\infty dk_d'\, k_d'^2\frac{(q^2+4q\gamma+\gamma^2)}{\gamma^3(q+\gamma)^4}[M_{ll\pm1}^{-2}(k_d,k_d')]^2. \quad (23)$$

When the incident energy, E_d, is less than the deuteron binding energy there are no intermediate real states allowed and the range of the second integral is from 0 to ∞.

After the substitutions for the expansions in (20) have been made, the expression for Δ in terms of the integrals becomes

$$\Delta(\theta)=C\sin^2(\theta/2)\exp[i\eta_d\ln\sin^2(\theta/2)]\times\sum_l[(l+1)I_l^{+}+lI_l^{-}]\exp[2i(\sigma_l-\sigma_0]P_l(\cos\theta), \quad (24)$$

where

$$C=(2/3\pi)(k_dZ^2/\eta_d)(Me^2/\hbar^2)^2N_d^2.$$

The principal difficulty in the evaluation of the integrals I_l lies in the calculation of a sufficiently large number of dipole matrix elements given by (21). For large η_d or l they can be calculated fairly accurately by the WKB approximation.[10] However, unless the integrals I_l are found accurately the resulting value of $\Delta(\theta)$ is unreliable because of the cancellations which occur because of the phase changes in the sum over l. From an attempted calculation of (24) the following qualitative conclusions have been drawn. There is a sharp peaking in contributions to I_l from intermediate states with $k_d'\sim k_d$, a necessary condition for the validity of the adiabatic approximation. The imaginary part of I_l is considerably smaller than the real part. This result implies that any observed deviations from Rutherford scattering do not mainly arise from the *real* electric breakup of the deuteron, contrary to the suggestion of Nishida.[2]

II. The High-Energy Limit

At high incident energies a substantial part of the elastic deuteron cross section may arise from the Coulomb field asymmetry. The dipole approximation is not expected to be as good as at low energies, but should give some idea of the amplitude.

The high-energy limit of the previous theory is obtained by taking the limit as E_d becomes large, keeping Z fixed. Then the Coulomb wave functions become plane waves and

$$\eta_d\to0,\quad \eta_d'\to0,\quad \sigma_l(\eta_d)\to0.$$

A region of finite η_d' should be retained in intermediate states, but will give a small contribution in the integration over k_d' in (23). The only major change in the previous formula is the replacement of the Coulomb dipole integrals by integrals of the form

$$M_{ll\pm1}(k_d,k_d')=\int_0^\infty j_l(k_dR)j_{l\pm1}(k_d'R)dR.$$

Using the formulas of Watson[16] the integrals appear

[16] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, New York, 1944).

as simple series which are, for $k_d > k_d'$,

$$M_{l\,l+1}=\frac{1}{k_d}\left(\frac{k_d'}{k_d}\right)^{l+1}\sum_{n=0}^{\infty}a_{nl}\left(\frac{k_d'}{k_d}\right)^{2n},$$

$$M_{l\,l-1}=\frac{1}{k_d}\left(\frac{k_d'}{k_d}\right)^{l-1}\sum_{n=0}^{\infty}b_{nl}\left(\frac{k_d'}{k_d}\right)^{2n},$$

where

$$a_{nl}=\frac{1}{4}\frac{(n+l)\cdots(n+1)}{(n+l+\frac{3}{2})\cdots(n+\frac{1}{2})}, \qquad l\geq 1,$$

$$a_{n0}=\frac{1}{(2n+3)(2n+1)},$$

$$b_{nl}=-\frac{1}{4}\frac{(n+l-1)\cdots(n+1)}{(n+l-\frac{1}{2})\cdots(n-\frac{1}{2})}, \qquad l\geq 2,$$

$$b_{n1}=-\frac{1}{(2n+1)(2n-1)}.$$

For $l=0$ we may also write

$$M_{01}=\frac{1}{2k_d}+\frac{k_d^2-k_d'^2}{4k_dk_d'^2}\ln\left(\frac{k_d-k_d'}{k_d+k_d'}\right).$$

For $k_d'>k_d$ the integrals are obtainable from the above expressions by interchanging k_d and k_d' and using the series with the appropriate l values. In the limit of k_d equal to k_d' we finally have

$$M_{l\,l+1}=M_{l+1\,l}=1/2k_d(l+1).$$

The recurrence relations satisfied by the integrals are

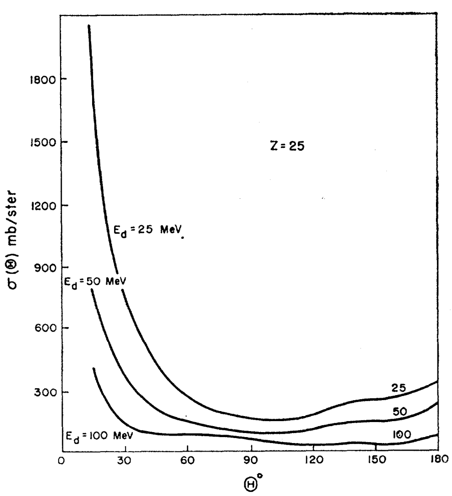

FIG. 6. Contribution of the electric field perturbation to the elastic deuteron cross section in the high-energy limit of the second Born approximation. The curves are for $Z=25$ at the energies indicated.

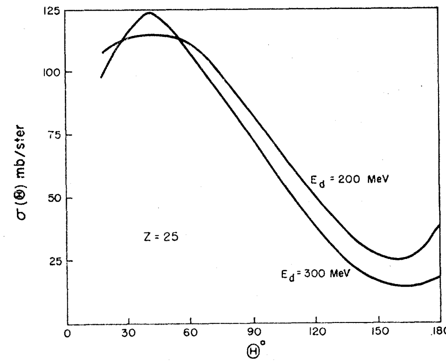

FIG. 7. Contribution of the electric field perturbation to the elastic deuteron cross section in the high-energy limit of the second Born approximation. The curves are for $Z=25$ at the energies indicated.

$$M_{l+1\,l}=[1/k_d(2l+1)](2lk_d'M_{l\,l-1}-k_d'M_{l\,l+1}),$$
$$M_{l+1\,l+2}=[1/2(l+2)k_d'][(2l+3)k_dM_{l\,l+1}-k_d'M_{l+1\,l}].$$

By using the above relations and series the matrix elements were calculated up to $l=45$.

We no longer wish to find the ratio to the Coulomb amplitude so that from (24) and (5) the partial scattering amplitude is

$$f(\theta)=C\sum_l[(l+1)I_l^+ + lI_l^-]P_l(\cos\theta).$$

The integrals I_l^+ are given by (23). When all the wave numbers appearing in the $I_l^\pm$ are taken in units of 10^{12} cm^{-1} the value of the constant is

$$C=(1/3\pi)(Me^2/\hbar 2)^2N_d^2Z^2\times 10^{-44}$$
$$=4.23Z^2\times 10^{-14}\text{ cm}. \qquad (25)$$

The value for N_d^2 is the same as that used in Sec. 3.

The total partial cross section arising from $f(\theta)$ is

$$\sigma=4\pi C^2\sum_l[1/(2l+1)]|(l+1)I_l^+ + lI_l^-|^2.$$

To calculate the amplitude and partial cross section a program was written for a Mercury computer. For integration over k_d' the lower limit was somewhat arbitrarily taken as 10 (in units of 10^{12} cm^{-1}). This corresponds to $\eta_d'\sim 0.07Z$ in intermediate states, certainly too small a cutoff for large Z. The upper limit was taken at a value of k_d' for which k_d'/k_d equaled 10^{-1}. The integration was performed numerically by Simpson's rule, the number of points being increased until no further change was observed in the results. The other quantities were calculated as in Sec. 3, the summation over l being taken up to $l=36$, certainly a sufficiently large value.

Since, according to (25), the cross section is proportional to Z^4 it was calculated for $Z=25$ only and for various values of the incident energy. The angular distributions for $E_d=25$, 50, 100 MeV are shown in Fig. 6 and those for $E_d=200$, 300 MeV in Fig. 7. In addition the scattering amplitudes at 25, 50 MeV are given in Fig. 8. All the curves were calculated down to

$\theta=15°$. Below this value Coulomb scattering will generally predominate. For the limits of k_d' taken the imaginary part of the amplitude retained the behavior shown in Fig. 8 at all energies. On the other hand, the real part was strongly peaked forward only at the lowest energies. As a function of energy the partial total cross section is shown in Fig. 9.

The results are not realistic for several reasons. Firstly, no nuclear effects are included. Secondly, the Z^4 dependence of the cross section is incorrect. Even for η_d, η_d' small the matrix elements are considerably overestimated by the approximation. The cutoffs for integration over k_d' are arbitrary and the contribution of the region of finite η_d' is too large. Altogether the cross sections given might be taken as upper limits. Nevertheless we may draw the following conclusions. It is clear that the electric field perturbation produces a considerable effect on the elastic scattering of deuterons on medium and heavy nuclei. The effect seems to lie mainly in the forward direction, particularly with the imaginary part of the amplitude which arises from the real breakup of deuterons. At low energies the contribution will increase with energy so that in Fig. 9 the curve should be peaked at some value. On heavy nuclei the calculated effect is so large that the use of the second Born approximation is suspect.

5. DEUTERON OPTICAL MODELS

For the elastic scattering of nucleons optical models have proved a success in that they account for certain gross features of the scattering process, in particular variation of the cross section with mass number A and

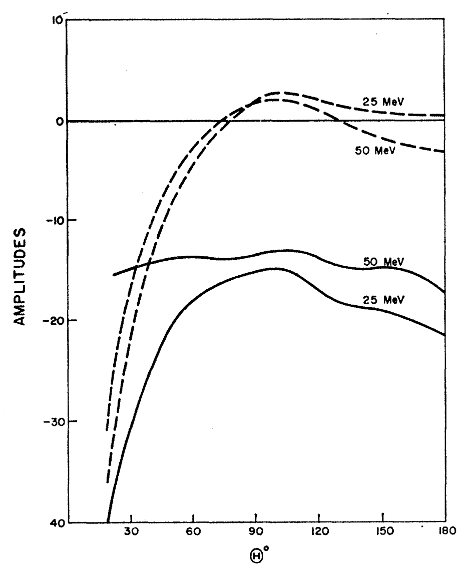

FIG. 8. Contribution of the electric field perturbation to the deuteron elastic scattering amplitudes in the high-energy limit of the second Born approximation. The real parts are given by the solid curves and the imaginary parts by the dashed curves. To obtain the amplitudes in units of 10^{-13} cm, the values indicated must be multiplied by $4.23\times10^{-3}Z^2$.

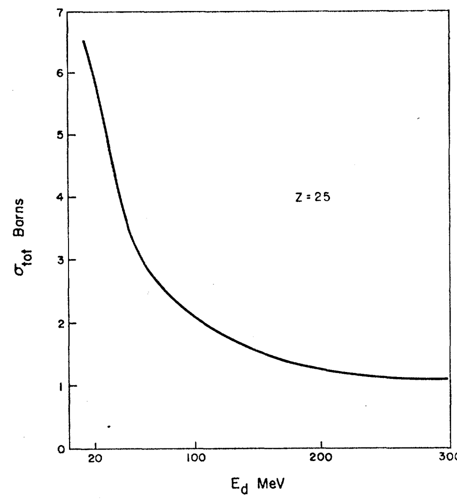

FIG. 9. Contribution of the electric field perturbation to the total elastic deuteron cross section in the high-energy limit of the second Born approximation, showing the variation of cross section with energy for $Z=25$.

with energy. Optical models have also been applied to the elastic scattering of deuterons by authors including Slaus and Alford[14] and Hodgson *et al.*[17] They have also been used in the study of stripping reactions by the distorted-wave method.[18] We wish to point out here that the use of nuclear optical models for deuterons is a suspect and in many cases not a useful procedure. If the electric field perturbation on the deuteron is as large for medium and heavy nuclei, as has been suggested in the rest of the paper, the use of a nuclear optical model obscures the fact that a large part of deuteron elastic scattering takes place outside the nuclear surface. The parameters obtained from fitting angular distributions are then somewhat meaningless.

The procedure of using such an optical model to describe the deuteron wave function in a distorted wave calculation is even more suspect. Oppenheimer and Phillips[19] found originally that a strong deuteron polarization was necessary to explain the differences in magnitude between d-p and d-n reactions on heavy nuclei. A center-of-mass optical model is incapable of providing this polarization. From the results of Sec. 4 we believe that, even at energies above the Coulomb barrier, electric effects on the deuteron are comparable to nuclear effects on heavy nuclei. Thus, only on light nuclei or on medium A nuclei at high energies would deuteron elastic scattering arise only from nuclear forces.

On light or medium nuclei at low energies one might be able to regard the asymmetric electric potential $P(\mathbf{r},\mathbf{R})$ as a perturbation on the motion of a deuteron described by a center of mass potential $V(R)$. The

[17] P. E. Hodgson, J. Agiular, A. García, and J. B. A. England, Nuclear Phys. **22**, 138 (1961).
[18] W. Tobocman, Phys. Rev. **115**, 98 (1959).
[19] J. R. Oppenheimer and M. Phillips, Phys. Rev. **48**, 500 (1935).

adiabatic approximation ought still to be approximately valid at sufficiently low energies and would give for the scattering amplitude, corresponding to (3), the expression

$$f(\theta)=f_0(\theta)+\frac{M}{\pi\hbar^2}\frac{Z^2e^2\alpha}{2}\int_{R_0}^{\infty}d^3\mathbf{R}\,\psi^{(-)*}(\mathbf{R})(1/R^4)\psi^{(+)}(\mathbf{R}).$$

The wave functions $\psi^{(+)}$, $\psi^{(-)}$ have boundary conditions of outgoing and incoming waves, respectively, at infinity and satisfy the equation

$$\left[-\frac{\hbar^2}{4M}\nabla_R{}^2+\frac{Ze^2}{R}+V(R)\right]\psi(R)=E_d\psi(R).$$

The scattering amplitude arising from $\psi^{(+)}(R)$ is $f_0(\theta)$ which includes the Rutherford scattering contribution.

One would be able to evaluate the integral as in Sec. 4, the only difference being that the radial integrals would have the form

$$I_l(\rho_0,\eta_d)=\int_0^{\infty}\frac{1}{\rho^4}\frac{d\rho}{4}\{F_l(\rho)+iG_l(\rho)+\exp(2i\delta_l)[F_l(\rho)-iG_l(\rho)]\}^2.$$

Here δ_l is the complex phase shift arising from $V(R)$. For high partial waves the δ_l would be negligible and the WKB approximation could be used for the integrals. The electric forces will always dominate in the high partial waves because of their effectively larger radius of interaction.

The meaning of a potential $V(R)$ obtained by fitting experimental angular distributions with an expression of the form would still be obscure. The modification of the deuteron wave function outside the nucleus by electric forces would affect the deuteron's interaction with the nucleus. However, $V(R)$ could at least be parametrized within the nuclear volume.

To conclude, we have shown that the asymmetry of the Coulomb field's action on the deuteron gives a considerable contribution to the elastic scattering of deuterons on medium and heavy nuclei.

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