# Electric Polarizability of the Deuteron*

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The polarizability of the deuteron is calculated for several assumed deuteron wave functions, and the values obtained are compared with previous results. The best value is $0.63\times10^{-39}$ cm$^3$. The method used gives an approximate wave function of the deuteron in the nuclear electric field. The approximations of the method are examined and are found to be best for low-energy deuterons incident on relatively light nuclei.

## 1. INTRODUCTION

In some physical situations it is legitimate to a first approximation to regard an electric field acting on a deuteron as being constant. It is thus of interest to find a modified ground-state wave function and the modified energy of the deuteron in such a field. The modified energy is specified by the polarizability of the deuteron which has been calculated in various ways by a number of authors.[1–3] In Secs. 2 and 3 modified wave functions and their corresponding polarizabilities are calculated by a direct method for several assumed deuteron wave functions, and the results are compared with those previously obtained.

Since the deuteron has no bound excited states, a wave function representing a bound deuteron in a constant electric field can only be approximate. However, such an approximation may be partially valid for a deuteron in the nuclear electric field. This situation is discussed in Sec. 4. Finally, in Sec. 5 the validity of the approximations used is discussed.

## 2. MODIFIED DEUTERON WAVE FUNCTIONS

The Hamiltonian for a deuteron in a constant electric field $\mathbf{E}$ is

$$H=-(\hbar^2/2M)(\tfrac{1}{2}\nabla_R^2+2\nabla_r^2)+V(r)-e\mathbf{E}\cdot\mathbf{r}_p,$$

where $V(r)$ is the two-nucleon potential, and the relative and center-of-mass coordinates are taken as

$$\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n,\quad \mathbf{R}=\tfrac{1}{2}(\mathbf{r}_p+\mathbf{r}_n).$$

On separating in these coordinates the relative wave equation becomes

$$[-(\hbar^2/2M)\nabla_r^2+V(r)-\tfrac{1}{2}e\mathbf{E}\cdot\mathbf{r}]\psi(r)=[\epsilon+\eta(E)]\psi(r),\quad (1)$$

where $\eta(E)$ is the change in the binding energy $\epsilon$ for the ground state.

The unperturbed deuteron satisfies the equation

$$[-(\hbar^2/2M)\nabla_r^2+V(r)]\psi_0(r)=\epsilon\psi_0(r).\quad (2)$$

Assuming that a form for $\psi(r)$ is known, $\eta$ may be found by multiplying (1) on the left by $\psi_0^*(r)$, integrating over $\mathbf{r}$ and using (2).

$$\eta(E)=-\tfrac{1}{2}e\int\psi_0(r)\mathbf{E}\cdot\mathbf{r}\psi(r)d^3\mathbf{r}\Big/\int\psi_0^*(r)\psi(r)d^3\mathbf{r}\quad (3)$$

We attempt to solve Eq. (1) assuming a form for $\psi_0(r)$.

### Gaussian Wave Function

$$\psi_0(r)=\exp(-\beta^2r^2)$$

This wave function is unrealistic for a deuteron and is introduced for comparison and because (1) may be solved exactly. The normalized solution is

$$\psi(r)=(2\beta^2/\pi)^{3/4}\exp(-\delta E/\sqrt{2}\beta-\beta^2r^2-\delta\mathbf{E}\cdot\mathbf{r}),$$

where

$$\delta=eM/8\hbar^2\beta^2,$$
$$\eta(E)=-(e^2M/64\hbar^2\beta^4)E^2.$$

### Wilson Wave Function

$$\psi_0(r)=(\gamma/2\pi)^{1/2}[\exp(-\gamma r)/r]$$

For this wave function an exact solution has not been found, but we can find a solution valid for sufficiently small $E$ and $r$.

We first write

$$\psi(r)=\psi_0(r)[1+\psi_1(r)].$$

After the use of (1) and (2) the equation for $\psi_1(r)$ becomes

$$2(\hbar^2/M)(1/r^2+\gamma/r)\mathbf{r}\cdot\nabla\psi_1-(\hbar^2/M)\nabla^2\psi_1=-[\tfrac{1}{2}e\mathbf{E}\cdot\mathbf{r}+\eta(E)][1+\psi_1(r)].$$

This equation may be satisfied to the first order in $E$ by choosing

$$\psi_1(r)=\delta r\mathbf{E}\cdot\mathbf{r},\quad \delta=eM/8\hbar^2\gamma.\quad (4)$$

The energy change $\eta(E)$ is obtained from (3) correct

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[1] N. F. Ramsey, B. J. Malenka, and U. E. Kruse, Phys. Rev. **91**, 1162 (1953).
[2] J. Sawicki, Acta Phys. Polon. **13**, 225 (1954).
[3] B. W. Downs, Phys. Rev. **98**, 194 (1955).

TABLE I. Polarizability of the deuteron. The values of $\alpha$ are in units of $10^{-39}$ cm$^3$.

| Reference | Method used | Deuteron wave function | Polarizability $\alpha$ |
|---|---|---|---|
| Ramsey *et al.*[a] | Green's function | Hulthèn | 0.56 |
| Sawicki[b] | Kirkwood's variational method | Hulthèn | 0.32 |
| | | Wilson | 0.21 |
| Downs[c] | Variational method with odd-parity state interaction $V$ | Hulthèn with $V$ Serber force | 0.56 |
| | | $V$ scalar square well | 0.58 |
| | Present method | Gaussian | 0.075 |
| | | Wilson | 0.445 |
| | | Hulthèn (as above) | 0.42 |
| | | Hulthèn (a) | 0.622 |
| | | Hulthèn (b) | 0.629 |

[a] See reference 1. [b] See reference 2. [c] See reference 3.

to order $E^2$.

$$\eta(E)=-(e^2M/64\hbar^2\gamma^4)E^2.$$

Since the polarizability is only well defined to order $E^2$, this method should give it its correct value subject to the other approximations to be discussed. Neglecting the normalization factor of order $E^2$, the normalized solution of (1) to order $E$ is

$$\psi(r)=(\gamma/2\pi)^{1/2}[\exp(-\gamma r)/r][1+\delta r\mathbf{E}\cdot\mathbf{r}].$$

### Hulthèn Wave Function

$$\psi_0(r)=r^{-1}[\exp(-\gamma r)-\exp(-\Gamma r)]$$

A similar method to that employed in the previous case results in the wave function

$$\psi(r)=\frac{1}{(4\pi)^{1/2}}\left(\frac{1}{2\gamma}+\frac{1}{2\Gamma}-\frac{2}{\gamma+\Gamma}\right)^{-1/2}\frac{1}{r}[(1+\delta_1\mathbf{E}\cdot\mathbf{r})\times\exp(-\gamma r)-(1+\delta_2\mathbf{E}\cdot\mathbf{r})\exp(-\Gamma r)],$$
$$\delta_1=eM/8\hbar^2\gamma,\quad \delta_2=eM/8\hbar^2\Gamma. \tag{5}$$

However, in the derivation of (5) in addition to neglecting second order terms in $E$ a term

$$(\delta_1-\delta_2)r\mathbf{E}\cdot\mathbf{r}[-(\hbar^2/2M)\nabla^2+V-\epsilon]\frac{1}{r}\exp(-\gamma r) \tag{6}$$

has been omitted. This term is small for large $r$ since $r^{-1}\exp(-\gamma r)$ is a good approximation to the unperturbed wave function. The energy change derived from (5) is

$$\eta(E)=-\frac{Me^2}{4\hbar^2}\left(\frac{1}{2\gamma}+\frac{1}{2\Gamma}-\frac{2}{\gamma+\Gamma}\right)^{-1}\left[\frac{1}{\gamma}\frac{1}{(2\gamma)^4}+\frac{1}{\Gamma}\frac{1}{(2\Gamma)^4}-\left(\frac{1}{\Gamma}+\frac{1}{\gamma}\right)\frac{1}{(\gamma+\Gamma)^2}\right]E^2. \tag{7}$$

Since $\Gamma\sim 7\gamma$, the parts of $\eta(E)$ arising from the terms containing $\delta_2$ are very small compared to the first term and for practical purposes can be omitted. This corresponds to the neglect of (6) and shows that the change of energy and thus the polarizability are determined by the asymptotic form of the unperturbed wave function. However, the amount of wave function in the asymptotic part, corresponding to the normalization factor in (5), is important in the determination of $\eta(E)$. This suggests the use of a wave function with the correct asymptotic normalization $N_H$.

$$\psi_0(r)=[N_H/(4\pi)^{1/2}](1/r)[\exp(-\gamma r)-\exp(-\Gamma r)].$$

A wave function of this form is quoted by Hulthèn and Sugawara[4] for the $S$-state part of the deuteron wave function with the important parameters $\gamma$, $N_H$ determined from two-nucleon data. The best forms for $\eta(E)$ and $\psi(r)$ are then obtained by the replacement of $[1/2+1/2\Gamma-2/(\Gamma+\gamma)]^{-1}$ by $N_H^2$ in (5) and (7).

## 3. POLARIZABILITY

For an adiabatically applied uniform field $E$ the polarizability is defined as

$$\alpha=-2\eta(E)/E^2,$$

where $\eta$ is of the order of $E^2$.

From the forms for $\eta(E)$ found in Sec. 2, the results are

$$\text{(i) Gaussian}\quad \alpha=Me^2/32\hbar^2\beta^4,$$
$$\text{(ii) Wilson}\quad \alpha=Me^2/32\hbar^2\gamma^4, \tag{8}$$
$$\text{(iii) Hulthèn}\quad \alpha=\frac{Me^2}{4\hbar^2}N_H^2\left[\frac{1}{\gamma}\frac{1}{(2\gamma)^4}+\frac{1}{\Gamma}\frac{1}{(2\Gamma)^4}-\left(\frac{1}{\Gamma}+\frac{1}{\gamma}\right)\frac{1}{(\Gamma+\gamma)^4}\right]. \tag{9}$$

Numerical values obtained from these expressions are compared with those of other authors in Table I. For the purposes of comparison results are first given using

[4] L. Hulthèn and M. Sugawara, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1959), Vol. 39, p. 1.

the same parameters.

Wilson $\gamma^{-1}=4.5\text{F}$.

Hulthèn $\gamma=0.17R_H^{-1}$, $\Gamma=1.19R_H^{-1}$, $R_H=0.675\text{F}$.

For the Gaussian wave function $\beta$ was chosen so that $\langle r\rangle$ is the same as that for the Hulthèn wave function.

Values of $\alpha$ are also given for the parameters of Hulthèn and Sugawara.[4] Since these parameters determine mainly the asymptotic form of the deuteron wave function, they are probably the best quoted. The changes with different deuteron $D$-state probabilities are negligible and only the case of $P_D=4\%$ is used. The terms containing $\Gamma$ in (9) are of the order of $10^{-4}$ times the first term. There remain two cases (a) and (b) corresponding to different effective ranges for the triplet even central force. In both $\gamma^{-1}=4.316\text{F}$.

Case (a) $\rho=1.704\text{F}$, $N_H^2=3.305\gamma$,

Case (b) $\rho=1.734\text{F}$, $N_H^2=3.343\gamma$.

In addition we can compare the analytic expressions for $\alpha$ with those of Sawicki.[2]

Wilson $\alpha=Me^2/72\hbar^2\gamma^4$,

Hulthèn $$\alpha=\frac{2}{7}\frac{Me^2}{\hbar^2}\left[\frac{1}{4\gamma^3}-\frac{4}{(\gamma+\Gamma)^3}+\frac{1}{4\Gamma^3}\right]^2\frac{\gamma^2\Gamma^2(\gamma+\Gamma)^2}{(\Gamma-\gamma)^4}.$$

The difference with (8) and (9) suggests that Sawicki's method gives too small a value for $\eta(E)$.

Ramsey *et al.*[1] also found the polarizability arising from the coupling of the $S$ and $D$ states in the deuteron. Since it is a factor of 10 smaller than the pure $S$-state value, it could be neglected in any application. We have also neglected the fact that $V(r)$ is state dependent. The results of Downs,[3] who considered this dependence while using the same formula for $\alpha$ as Ramsey *et al.*,[1] appear to show that it has little effect on $\alpha$.

It is evident from the dependence of $\alpha$ on $\gamma$, $N_H$ that its value is sensitive to the form of the tail of the deuteron wave function. No practical method exists at present for the measurement of $\alpha$ except possibly the elastic scattering of deuterons at low energies. This method is discussed in a subsequent paper.

### 4. DEUTERON IN THE NUCLEAR ELECTRIC FIELD

The deuteron may be polarized while being scattered by a nucleus, a situation which led Oppenheimer and Phillips[5] to explain the preponderance of $d$-$p$ over $d$-$n$ reactions at low incident deuteron energies on heavy elements. We shall briefly discuss to what extent the approximations used in this paper give a polarized deuteron wave function.

In terms of relative and center-of-mass coordinates and neglecting nuclear interactions the wave function of the deuteron must satisfy the equation

$$\left[-\frac{\hbar^2}{4M}\nabla_R^2-\frac{\hbar^2}{M}\nabla_r^2+\frac{Ze^2}{|\mathbf{R}+\frac{1}{2}\mathbf{r}|}+V(r)\right]\psi(\mathbf{R},\mathbf{r})=(\epsilon+E)\psi(\mathbf{R},\mathbf{r}),\quad(10)$$

where $E$ is the incident deuteron kinetic energy.

The unperturbed solution is found by letting the Coulomb field act on the center of mass of the deuteron.

$$\psi_0(\mathbf{R},\mathbf{r})=\phi_0(\mathbf{R})\psi_0(\mathbf{r}),$$

where $\phi_0(\mathbf{R})$ is the appropriate Coulomb wave function

$$\phi_0(\mathbf{R})=\exp(-\tfrac{1}{2}\pi\eta_d)\Gamma(1+i\eta_d)\times\exp(i\mathbf{k}\cdot\mathbf{R})F[-i\eta_d,1;i(kR-\mathbf{k}\cdot\mathbf{R})].$$

The deuteron wave number is $k$ and

$$\eta_d=Ze^2M_d/\hbar k.$$

Except for the region where $\mathbf{r}_p\cdot\mathbf{r}_n<0$, which is expected to be unimportant at low energies as the deuteron is stretched across the nucleus, the perturbation may be written.

$$\frac{1}{|\mathbf{R}+\frac{1}{2}\mathbf{r}|}-\frac{1}{R}=-\frac{1}{R}\sum_n\left(\frac{r}{2R}\right)^nP_n(\mathbf{r},\mathbf{R}).$$

The Legendre polynomial is a function of the angle between $\mathbf{r}$ and $\mathbf{R}$.

In the adiabatic approximation and taking only the dipole term in the expansion the next order solution of (10) is obtained by solving the equations

$$\left[-\frac{\hbar^2}{M}\nabla_r^2+V(r)-\tfrac{1}{2}eZe\frac{\mathbf{R}\cdot\mathbf{r}}{R^3}\right]\phi_1(\mathbf{r},\mathbf{R})=[\epsilon+\eta(R)]\phi_1(\mathbf{r},\mathbf{R}),\quad(11)$$

$$[-(\hbar^2/M)\nabla_R^2+(Ze^2/R)]\phi_1(\mathbf{R})=[E-\eta(R)]\phi_1(\mathbf{R}),$$
$$\psi_1(\mathbf{r},\mathbf{R})=\phi_1(\mathbf{R},\mathbf{r})\phi_1(\mathbf{R}).\quad(12)$$

Equation (11) corresponds to (1) if the constant electric field is identified as

$$\mathbf{E}=Ze\mathbf{R}/R^3.$$

Approximate solutions of (11) may then be found by the procedure adopted in Sec. 2.

To the lowest order the solution of Eq. (12) is still $\phi_0(R)$ since $\eta(R)$ is of order $R^{-4}$. The solution of (12) to the next order gives the change in the elastic scattering of deuterons in the adiabatic approximation. This has been considered by Sawicki[2] and Malenka *et al.*,[6] and a more complete discussion will be given in a future paper. If we take the unperturbed deuteron

[5] J. R. Oppenheimer and M. Phillips, Phys. Rev. **48**, 500 (1935).

[6] B. J. Malenka, U. E. Kruse, and N. F. Ramsey, Phys. Rev. **91**, 1165 (1953).

wave function as having the Wilson form, an approximate solution of (10) is

$$\psi_1(\mathbf{r},\mathbf{R})=N_H\frac{\exp(-\gamma r)}{r}\left[1+\delta r\frac{Ze\mathbf{R}\cdot\mathbf{r}}{R^3}\right]\phi_0(R), \quad (13)$$

where $\delta$ is given by (4).

The probability of the deuteron being polarized is obtained by integrating $\phi_1(r,R)$ over $r$.

$$P=(1/128)(Me^2/\hbar^2)^2Z^2/\gamma^6R^4.$$

We take $\gamma=0.2315\mathrm{F}^{-1}$ and with $R$ in fermis.

$$P=(0.0612/R^4)Z^2.$$

Evidently this result has some validity only when $R$ is outside the nuclear surface and $Z$ is sufficiently small so that $P\ll 1$.

The validity of the approximations used and of a wave function such as (13) will now be discussed.

## 5. VALIDITY OF THE APPROXIMATIONS

Strictly for a deuteron the polarizability is not a well-defined concept. This is because it is not sufficiently strongly bound, and the potential $-\frac{1}{2}e\mathbf{E}\cdot\mathbf{r}$ in (1) becomes numerically greater than $\epsilon$ for large enough $r$, implying the possibility of a free neutron and proton outside some radius. Thus a static deuteron would decay with time and in solving (1) we would be faced with the problem of satisfactory boundary conditions at infinity. We consider the physical situation for a deuteron in the nuclear electric field. For an incident energy less than the binding energy of the deuteron there can be no outgoing waves in the deuteron's internal motion. Thus a wave function of the form (13) does satisfy the appropriate boundary conditions as $r$ tends to infinity. For greater incident energies the component of the wave function corresponding to real deuteron breakup is omitted. However, (13) may still represent a fair approximation to the component corresponding to virtual breakup.

Apart from examining whether the wave function satisfies the appropriate boundary conditions we also substitute it in the differential equation (10) and examine the terms neglected. After the basic approximation is made of taking only the dipole term in the expansion for $r<2R$, the equation becomes

$$(H-\epsilon-E)\psi_1(\mathbf{r},\mathbf{R})=-N_H[\exp(-\gamma r)/r]$$
$$\times\{(\delta Z^2e^3r(\mathbf{r}\cdot\mathbf{R})^2/2R^6)\phi_0(R)+(Ze\delta\hbar^2/2MR^3)$$
$$\times[\mathbf{r}-3\mathbf{R}(\mathbf{r}\cdot\mathbf{R})/R^2]\cdot\nabla_R\phi_0(R)\}. \quad (14)$$

The first term is the one neglected in the approximate method of solving Eq. (1). Its ratio to the term neglected by taking the unperturbed solution of Eq. (10) is

$$\Delta=\delta r(Ze\mathbf{R}\cdot\mathbf{r}/R^3)=\delta r(\mathbf{E}\cdot\mathbf{r}).$$

When $\Delta$ is of the order of unity the approximation is certainly unsatisfactory. In the worst case when the neutron is on the nuclear surface and the deuteron is in a straight line position with the center of the nucleus, we obtain

$$R=R_0+\tfrac{1}{2}r,\quad R_0=r_0A^{1/3},$$
$$\Delta\sim 0.019Zr^2/(r_0A^{1/3}+\tfrac{1}{2}r)^2,$$

where $r_0$, $r$ are in fermis.

TABLE II. Breakdown of the perturbation approximation. The value of $r$ for which $\Delta\sim 1$ is $r_c$, which is given in fermis.

| Element | $A$ | $Z$ | $r_c$ |
|---|---|---|---|
| Up to Mg | | 12 | none |
| Mn | 55 | 25 | 24 |
| Sn | 116 | 50 | 11.8 |
| Pb | 208 | 82 | 10 |

When $r_0=1.3$ the values of $r$ for which $\Delta\sim 1$ are shown for some typical elements in Table II. It can be seen that the approximation is good enough to describe almost all of the tail of the wave function for light elements, whereas for heavy elements it breaks down outside a certain radius $r_c$. For larger values of $R$ when the deuteron is further from the nucleus the approximation is better. Thus for incident deuteron energies below the Coulomb barrier, where the center-of-mass wave function $\phi_0(R)$ falls off exponentially near the nucleus, the first term in (14) is small over the important regions of configuration space.

If the adiabatic approximation is valid, the neglect of the second term in (14) is justified and conversely. The derivative of the Coulomb wave function is

$$\nabla_R\phi_0(R)=\exp(-\tfrac{1}{2}\pi\eta_d)\Gamma(1+i\eta_d)\exp(i\mathbf{k}\cdot\mathbf{R})$$
$$\times\{i\mathbf{k}F[-i\eta_d,1;i(kR-\mathbf{k}\cdot\mathbf{R})]$$
$$+k\eta(\mathbf{R}/R-\mathbf{k}/k)F[-i\eta_d+1,2;i(kR-\mathbf{k}\cdot\mathbf{R})]\}. \quad (15)$$

From an examination of the second term in comparison to the original perturbation we may draw the following conclusions. Firstly, the term neglected is of order $R^{-3}$ so that the approximation is valid for sufficiently large $R$. Secondly, as a function of $E$, $Z$ the second term becomes relatively smaller with decrease in magnitude. The second term in (15) has an extra factor $Z$ so that for large $Z$ the adiabatic approximation may break down in a region near the nucleus. Also, the first term in (15) contains $\sqrt{E}$ so that the adiabatic approximation breaks down for high energies.

We finally consider whether a wave function $\psi_1(\mathbf{r},\mathbf{R})$ such as (13) should be normalized for integration over $r$. The normalization factor would be proportional to $R^{-4}$ and its neglect is consistent with the other approximations made.

In conclusion we may state that the approximations used in deriving (13) all appear to depend on the same criteria. Their validity is favored by a large distance of the deuteron from the nucleus, a small nuclear charge and a low incident deuteron energy. Under these conditions, which imply a small electric field, the concept of the polarizability of the deuteron and a wave function of the form of (13) may be useful.

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# Effect of the Nuclear Electric Field on the Elastic Scattering of Deuterons*

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The asymmetric action of the nuclear electric field on the deuteron is found to strongly affect the elastic scattering cross section on medium and heavy nuclei. Expressions for the scattering amplitude are found in the first and second Born approximations and in the adiabatic approximation. In the latter case the expression is evaluated and compared with experiment, giving rough agreement in magnitude, but not in shape, with deviations from Rutherford scattering found at low energies on medium and heavy nuclei. The high-energy plane-wave limit of the second Born approximation amplitude is evaluated. It is peaked in the forward direction, is large in magnitude for heavy nuclei, and decreases with increase in energy. Consequences for deuteron optical models are discussed.

## 1. INTRODUCTION

THE Coulomb field of the nucleus acts on the proton in the deuteron and not on its center of mass. Thus the deuteron elastic scattering cross section is expected to differ from the Rutherford expression not only because of nuclear interactions. The separation of electric from nuclear effects can only be accomplished where one or the other is small. In this paper only the nuclear electric field is considered. The regions where its effect on elastic scattering might be expected to dominate are at incident deuteron energies below the Coulomb barrier, where nuclear effects are strongly inhibited, and for heavy nuclei. The electric field strength increases more rapidly with mass number than the nuclear field strength. The reason that the Coulomb perturbation is important for deuterons and not, say, for $\alpha$ particles is that the low binding energy of the deuteron enables it to be easily stretched and broken up by an asymmetrical force acting on it. There is definite experimental evidence[1] for large differences between the elastic scattering cross sections of deuterons and $\alpha$ particles on heavy nuclei. In the literature there have been two approaches to the problem and both are used here.

The most straightforward approach is the use of the first and second Born approximations. In this way effects arising from both real and virtual deuteron breakup can be included. Previous work by Nishida[2] has suggested that the large decrease from Rutherford scattering, observed at backward angles on heavy nuclei by Gove[3] and also by Rees and Sampson,[1] arises from the electric breakup of the deuteron. In his second paper he gives a qualitative classical theory which appears to show that dipole breakup alone is not responsible for the deviation. The quantum mechanical approach adopted here in Secs. 2 and 4 suffers from the difficulty of evaluating numerically the resulting second Born approximation amplitude. The evaluation of the high-energy limit is performed in Sec. 4 and the resulting amplitude is found to be large.

When proper spin wave functions of the deuteron are used, the first Born approximation also gives a contribution to the amplitude arising from the quadrupole moment of the deuteron. A tensor polarization of the deuteron is produced but the effect is very small.

The second approach arises from the use of the adiabatic approximation and corresponds to the physical picture that the modification in the elastic scattering arises mainly from the virtual breakup of the deuteron. This approach was adopted by Malenka

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[1] J. R. Rees and M. B. Sampson, Phys. Rev. **108**, 1289 (1957).
[2] Y. Nishida, Progr. Theoret. Phys. (Kyoto) **17**, 506 (1957); **19**, 389 (1958).
[3] H. E. Gove, Phys. Rev. **99**, 1353 (1955).