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Quantum Effects in Ultrasonic Attenuation in Metals in a Magnetic Field

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A study is carried out of quantum effects in the dependence of ultrasonic attenuation in metals on the intensity of an applied magnetic field. It is found that the coefficient of absorption of acoustic waves exhibits an oscillatory behavior as a function of the strength of the applied magnetic field. This paper limits its scope to the case in which $\omega\tau$, the product of the frequency of the sound wave and the relaxation time for the electrons, is much smaller than unity. The oscillations predicted in this work have an amplitude proportional to $(\hbar\omega_0/\zeta_0)^{3/2}$ for propagation at right angles to the magnetic field, and they are negligibly small for propagation in the direction of the magnetic field. The latter result is in contrast with the one described in the preceding paper (for $\omega\tau \gg 1$), where giant oscillations of the attenuation can occur.

I. INTRODUCTION

In the preceding paper[1] the authors have given a general analysis of the electrical conductivity tensor of a degenerate electron gas in the presence of a uniform magnetic field. The purpose of the present paper is to discuss those aspects of ultrasonic attenuation arising from electrons in metals in a magnetic field that have not been covered in previous semiclassical treatments.[2]

The notation used in this work is the same as that given in reference 1. The coefficient of absorption γ of sound waves can be obtained from Eq. (I-54) using the appropriate expressions for the tensors $\boldsymbol{\sigma}(\mathbf{q},\omega)$ and $\boldsymbol{\Gamma}$, when relaxation effects are neglected. However, the study given here differs from that in reference 1 in that we take into account the scattering of the electrons by thermal phonons or lattice imperfections by introducing a phenomenological relaxation time τ. This can be accomplished if we add a term $i\hbar(f-\bar{f}_0)/\tau$ to the left-hand side of the equation of motion [Eq. (I-7)] of the density matrix, where $\bar{f}_0$ is the local equilibrium distribution function referred to a system of coordinates that is moving with the positive ions rather than at rest in the laboratory system. The function $\bar{f}_0$ depends on the relative velocity of the electrons with respect to the positive ions and on the local value of the Fermi energy. It is possible to expand $\bar{f}_0$ about the true equilibrium value f_0; this procedure gives two correction terms. It turns out that the first correction is equivalent to adding a fictitious electric field of magnitude $m\mathbf{u}/e\tau$ to the true electric field. The second correction is a diffusion current that arises, in the case of longitudinal waves, from the changes in the local Fermi level that are caused by the successive expansions and compressions of the lattice as the sound wave propagates in the crystal. These effects have already been discussed in detail by Pippard,[3] so that further details are omitted here. There remains in the equation of motion of f the term $i\hbar(f-f_0)/\tau$, which can be formally taken into account in the expressions for the conductivity tensor by replacing ω by $\omega-i/\tau$. Once relaxation effects are included, the power absorbed per unit volume differs from that in Eq. (I-53) by an amount

$$-\tfrac{1}{2}\,\mathrm{Re}[(Nm/\Omega\tau)(\langle\mathbf{v}\rangle-\mathbf{u})\cdot\mathbf{u}^*],$$

which is returned coherently by the electrons to the acoustic wave. The quantity $\langle\mathbf{v}\rangle$ is the average local velocity of the conduction electrons. This is the so-called collision drag effect which has been extensively studied by Holstein,[4] and is particularly important at extremely high ultrasonic frequencies.

Because our interest is centered on quantum effects we shall consider the case in which the parameters $(\omega_0\tau)^{-2}$ and ξn_0 are much smaller than unity. However in the situations considered here these parameters may be comparable. In general, $\omega\tau \ll 1$ for ultrasonic frequencies ω of the order of several hundred megacycles per second or less and we shall restrict our considerations to this situation in the present work.

As in reference 1 the two geometrical arrangements in which acoustic waves propagate either at right angles to or parallel to the magnetic field $\mathbf{B}_0$ will be considered separately. In Sec. II we study propagation in a transverse magnetic field and in Sec. III we give some results of the analysis of ultrasonic attenuation in a longitudinal magnetic field. The direction of the applied magnetic field $\mathbf{B}_0$ is taken as the z axis of a Cartesian coordinate system. We can designate a given geometry by giving the directions of the wave vector $\mathbf{q}$ and the velocity field $\mathbf{u}$ of the ions in the form $(\hat{q},\hat{u})$. For example, the case of a transverse acoustic wave propagating in the y direction and polarized in the x direction is designated by the symbol (y,x), and the attenuation coefficient in this geometry is denoted by $\gamma(y,x)$.

II. ATTENUATION OF ACOUSTIC WAVES IN A TRANSVERSE MAGNETIC FIELD

The purpose of this section is to study quantum effects in the absorption of acoustic waves traveling in a direction perpendicular to the applied magnetic field.

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[1] J. J. Quinn and S. Rodriguez, preceding paper [Phys. Rev. 128, 2487 (1962)]. Hereafter, when referring to equations in this reference we shall designate them by the roman numeral I followed by the number of the equation. For example, Eq. (54) in the preceding paper is referred to as Eq. (I-54).

[2] M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. 117, 937 (1960).

[3] A. B. Pippard, Phil. Mag. 46, 1104 (1955).

[4] T. D. Holstein, Phys. Rev. 113, 479 (1959).

We take the direction of propagation along the y-axis. Assuming $(\omega_0\tau)^{-2}$ and ξn_0 to be much smaller than unity, we obtain the following expressions for the components of the electrical conductivity tensor:

$$\sigma_{xx}=\frac{\sigma_0}{1+i\omega\tau}\left[4\xi\frac{W}{\hbar\omega_0}+\left(\frac{1+i\omega\tau}{\omega_0\tau}\right)^2\right], \tag{1}$$

$$\sigma_{yy}=\frac{\sigma_0}{(\omega_0\tau)^2}(1+i\omega\tau)\left[1-\tfrac{3}{2}\xi\frac{W}{\hbar\omega_0}-\left(\frac{1+i\omega\tau}{\omega_0\tau}\right)^2\right], \tag{2}$$

$$\sigma_{xy}=-\sigma_{yx}=-\frac{\sigma_0}{\omega_0\tau}\left[1-3\xi\frac{W}{\hbar\omega_0}-\left(\frac{1+i\omega\tau}{\omega_0\tau}\right)^2\right], \tag{3}$$

and

$$\sigma_{zz}=\frac{\sigma_0}{1+i\omega\tau}\left[1-4\xi\frac{W_z}{\hbar\omega_0}\right]. \tag{4}$$

These relations have been obtained using the expansion of the Laguerre polynomials,[5] and retaining only terms to first order in the parameters $(\omega_0\tau)^{-2}$ and ξn_0. The symbols W and W_z which have been introduced, represent the average energy per particle for the motion in the x-y plane and in the z direction, respectively:

$$W=N^{-1}\sum_{nk_yk_z}(n+\tfrac{1}{2})\hbar\omega_0 f_0(E_{nk_z}), \tag{5}$$

and

$$W_z=N^{-1}\sum_{nk_yk_z}(\hbar^2k_z^2/2m)f_0(E_{nk_z}). \tag{6}$$

The quantity σ_0 is the ordinary dc electrical conductivity $\sigma_0=\omega_p^2\tau/4\pi$.

With the modifications considered in the introduction, namely the phonon drag and diffusion effects, the power absorbed per unit volume is

$$Q=\tfrac{1}{2}\,\mathrm{Re}[(N/\Omega)e\mathbf{u}^*\cdot\boldsymbol{\varepsilon}-(m/e\tau)\mathbf{u}^*\cdot\mathbf{j}^{(1)}], \tag{7}$$

where $\boldsymbol{\varepsilon}=\mathbf{E}+m\mathbf{u}/e\tau$. There exist two relations connecting $\mathbf{j}^{(1)}$ and $\mathbf{E}$. The first, which is a consequence of Maxwell's equations, relates the total current density, $\mathbf{J}=\mathbf{j}^{(1)}-(N/\Omega)e\mathbf{u}$, to the electric field, and is given by (I-58). For propagation in the y direction, $\mathbf{\Gamma}$ is diagonal, and its components are

$$\Gamma_{xx}=\Gamma_{zz}=i\beta\sigma_0 \tag{8}$$

and

$$\Gamma_{yy}=-i\omega/4\pi. \tag{9}$$

In Eq. (8) $\beta=c^2q^2/4\pi\omega\sigma_0$, and we have neglected the ratio s^2/c^2 as compared to unity. The second relation between $\mathbf{j}^{(1)}$ and $\mathbf{E}$ is obtained from the solution of the equation of motion of the density matrix as done in reference 1, taking into account the effects which are due to the finite relaxation time. This equation can be expressed in the form

$$\boldsymbol{\varepsilon}=\mathbf{R}\cdot\mathbf{j}^{(1)}, \tag{10}$$

where the nonvanishing components of $\mathbf{R}$ are

$$R_{xx}=\sigma_{yy}(\sigma_{xx}\sigma_{yy}+\sigma_{xy}^2)^{-1}, \tag{11}$$

$$R_{xy}=-R_{yx}=-\sigma_{xy}(\sigma_{xx}\sigma_{yy}+\sigma_{xy}^2)^{-1}, \tag{12}$$

$$R_{yy}=\frac{\sigma_{xx}}{\sigma_{xx}\sigma_{yy}+\sigma_{xy}^2}-\frac{4\pi i}{3}\left(\frac{qv_0}{\omega_p}\right)^2\frac{1}{\omega(1+i\omega\tau)}, \tag{13}$$

$$R_{zz}=\sigma_{zz}^{-1}. \tag{14}$$

After some straightforward but tedious manipulations one obtains the following expressions for the attenuation coefficients

$$\gamma(y,x)=\frac{zm}{M\tau s_t}\,\mathrm{Re}\left[\frac{(1-i\beta)(\sigma_0R_{xx}-1)}{1-i\beta\sigma_0R_{xx}}\right], \tag{15}$$

$$\gamma(y,y)=\frac{zm}{M\tau s_l}\,\mathrm{Re}\left[\frac{\sigma_0\sigma_{xx}}{\sigma_{xx}\sigma_{yy}+\sigma_{xy}^2}-1-\frac{1}{3}\frac{(qv_0\tau)^2}{1+\omega^2\tau^2}-\frac{i\beta(\sigma_0R_{xy})^2}{1-i\beta\sigma_0R_{xx}}\right], \tag{16}$$

$$\gamma(y,z)=\frac{zm}{M\tau s_t}\,\mathrm{Re}\left[\frac{(1-i\beta)(\sigma_0R_{zz}-1)}{1-i\beta\sigma_0R_{zz}}\right]. \tag{17}$$

In these equations s_l and s_t are the velocities of longitudinal and transverse sound waves, respectively. Using the relation $\omega\tau\ll1$ together with the assumption $\beta\ll1$, which is satisfied under a wide range of experimental conditions, we find

$$\gamma(y,x)=(zm\xi/2M\tau s_t)(W/\hbar\omega_0), \tag{18}$$

$$\gamma(y,y)=\frac{zm}{M\tau s_l}\left[\frac{4\xi(\omega_0\tau)^2}{1+\omega^2\tau^2}\frac{W}{\hbar\omega_0}-\frac{1}{3}\frac{(qv_0\tau)^2}{1+\omega^2\tau^2}+(\beta\omega_0\tau)^2\left(1-\frac{3W}{2\hbar\omega_0}\right)\right], \tag{19}$$

$$\gamma(y,z)=(4zm\xi/M\tau s_t)(W_z/\hbar\omega_0). \tag{20}$$

The quantities W and W_z are evaluated using a method which has been described in detail by Wilson.[6] They exhibit, as functions of B_0, an oscillatory character whose origin is the same as that of the de Haas–van Alphen oscillations of the magnetic susceptibility of metals. The formulas for W and W_z are

$$W=(2\zeta_0/5)\left[1+\frac{5\pi}{4}\left(\frac{\hbar\omega_0}{2\zeta_0}\right)^{1/2}\frac{kT}{\zeta_0}\times\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{1/2}}\frac{\sin[(2\pi n\zeta_0/\hbar\omega_0)-\pi/4]}{\sinh[2n\pi^2kT/\hbar\omega_0]}+\frac{5}{6}\left(\frac{\hbar\omega_0}{2\zeta_0}\right)^2\right], \tag{21}$$

[5] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 2, p. 188.

[6] A. H. Wilson, *The Theory of Metals* (Cambridge University Press, New York, 1958), 2nd ed., pp. 160–168.

and

$$W_z=\frac{1}{5}\zeta_0\left[1-\frac{5\pi}{2}\left(\frac{\hbar\omega_0}{2\zeta_0}\right)^{1/2}\frac{kT}{\zeta_0}\times\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{1/2}}\frac{\sin[(2\pi n\zeta_0/\hbar\omega_0)-\pi/4]}{\sinh[2n\pi^2kT/\hbar\omega_0]}-\frac{5}{12}\left(\frac{\hbar\omega_0}{2\zeta_0}\right)^2\right]. \quad (22)$$

III. ATTENUATION OF ACOUSTIC WAVES IN A LONGITUDINAL MAGNETIC FIELD

With the approximation $\omega\tau\ll1$, considered in this paper, the coefficient of ultrasonic attenuation in the case of propagation parallel to the applied magnetic field turns out to be the same as in the semiclassical limit to a high degree of accuracy. In fact, up to and including terms of the order of $(\hbar\omega_0/\zeta_0)^2$ we find no quantum effects similar to those discussed in Sec. II. This result differs radically from that found in the case in which $\omega\tau\gg1$. For example, in the latter situation, the coefficient of attenuation for $\omega_0\gg\omega$ experiences giant oscillations which have been described in detail elsewhere.[7] When $\omega\tau\approx1$, the expressions for the attenuation coefficients are unwieldy and we have not been able to obtain results of the simplicity of those exhibited in Eqs. (18)–(20). The giant oscillations predicted by Gurevich *et al.*[7] have been observed by Korolyuk and Prushchak[8] in a Zn sample where the ratio of the resistivity at liquid helium temperature to that at room temperature is 3×10^{-5} and at an ultrasonic frequency of 200 Mc/sec.

To summarize, there exists two types of oscillatory quantum effects in ultrasonic attenuation in metals as a function of an applied magnetic field. The first, which is described in Sec. II, is an oscillation in γ of small amplitude [proportional to $(\hbar\omega_0/2\zeta_0)^{3/2}$] superimposed on the ordinary semiclassical coefficient of attenuation. The second appears in the form of giant oscillations when the acoustic wave propagates in the direction of the magnetic field. The first type of oscillations become observable when $\omega_0\tau$ and ω_0/qv_0 are much larger than unity, while the second type is observable only if $\omega_0\gg\omega>\tau^{-1}$.

[7] V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Soviet Phys.—JETP 13, 552 (1961).
[8] A. P. Korolyuk and T. A. Prushchak, Soviet Phys.—JETP 14, 1201 (1962).

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Simple Model for Nucleation around Dislocations*

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The effect of the mechanical stresses at a dislocation, via magnetostriction, on the theoretical nucleation field for magnetization reversal is approached by assuming a cylindrical region in which the magnetocrystalline anisotropy vanishes. The complete spectrum of eigenvalues is studied for this model, and it is found that the buckling mode yields the lowest eigenvalues for hard materials with reasonable size for the defective region. The turnover to the curling mode is at a radius of about 300 Å for MnBi and 550 Å for $BaFe_{12}O_{19}$. To obtain the observed value of nucleation field in $BaFe_{12}O_{19}$, the radius of the cylinder with $K=0$, should be about 350 Å, which seems plausible for a dislocation.

1. INTRODUCTION

It has been suggested by Rathenau *et al.* that domain walls might start to nucleate at regions where the magnetocrystalline anisotropy constant is low, because of some structural imperfections.[1] This possibility has been studied for simple one-dimensional models,[2] giving rather encouraging results for hard materials.[3] These are extended here to three dimensions, giving a fuller description of the calculations mentioned in a recent review.[3]

Since measurements for hard materials are usually carried out on crystals in the form of thin plates, the calculation reported here assumes the material to be an infinite slab of width $2a$, perpendicular to the z axis, which is assumed to be an easy axis, and at which direction the external magnetic field is applied. The imperfection is assumed to be a region in the form of a cylinder of radius R, perpendicular to the slab, in which the magnetocrystalline anisotropy coefficient $\bar{K}$ vanishes. In cylindrical coordinates one has, thus, putting the z axis along the center line of the cylinder,

* This work will be included in a thesis by C. Abraham to be submitted to the Hebrew University, Jerusalem, in partial fulfilment of the requirements for a degree of Doctor of Philosophy.
[1] G. W. Rathenau, J. Smit, and A. L. Stuyts, Z. Physik 133, 250 (1952).
[2] A. Aharoni, Suppl. J. Appl. Phys. 30, 70S (1959); Phys. Rev. 119, 127 (1960); C. Abraham and A. Aharoni, *ibid.* 120, 1576 (1960).
[3] Amikam Aharoni, Revs. Modern Phys. 34, 227 (1962).