$Z_2^{(i)}$. In our model, there is no conserved current and we should not expect that a zero of $Z_2^{(i)}$ will occur to cancel the zero of Z.

VI. CONCLUSION

We have explored the possibility that the complex of fundamental interactions can be understood in terms of the stable self-generated solutions of the coupled Green's function equations of field theory. Since in this paper we have not tried to solve any integral equations, we have made no predictions which conclusively test the validity of this idea. Rather, we have consistently made the most naive approximations to the integral equations in order to reduce them to algebraic equations. Our purpose has been to show that there is the *possibility* of explaining fundamental interactions along these lines. There remains the more difficult practical problem of finding more reasonable approximations to the equations. There are also the basic problems dealing with stability criteria, the possible appearance of zero-mass particles,[14] and the existence of divergences, which must be understood before these ideas can become a complete theory.

ACKNOWLEDGMENT

We wish to thank Dr. S. Coleman for communicating to us some of his unpublished works with Dr. J. Goldstone.

[14] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

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Classical Radiation Recoil*

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The conditions under which a material system may recoil while emitting electromagnetic or gravitational radiation are investigated. The lowest order secular effects in the electromagnetic case arise from an interference of the electric dipole radiation with the electric quadrupole or magnetic dipole radiations. In the gravitational case, the lowest order terms involve the interference of the mass quadrupole radiation with the mass octopole or the flow quadrupole radiations. The investigation of the gravitational radiation recoil is carried out in complete analogy with the more elementary electromagnetic case, so that this paper should be accessible to physicists having no previous knowledge of general relativity theory.

1. INTRODUCTION AND NOTATIONS

It is well known that a material system can dissipate energy in the form of spherical waves radiated to infinity. In classical theory, these waves may be either electromagnetic[1] or gravitational,[2,3] and if they are emitted mostly in a preferred direction, then the emitting system will recoil in the opposite direction, like a rocket. However, while "photon rockets" have been largely publicized, the possibility of measuring gravitational radiation recoil is still far beyond our experimental techniques,[4] and even theoretical investigations have hitherto been restricted to a very special model.[5]

The main purpose of this paper is to present the general theory of gravitational radiation recoil. The calculations will be valid for any kind of motion (rotational, vibrational, or other) but with the condition that the material system remains localized within a finite volume. Only secular effects will be considered, i.e., those effects which do not average to zero over a long time interval.

In order to make these interesting questions accessible to the reader who is not a specialist of general relativity, we first present, in Sec. 2, the theory of electromagnetic radiation recoil, using the same tools as will later be needed in the gravitational case. It is found that the lowest order effects arise from the interference of the electric dipole radiation with either the electric quadrupole radiation or the magnetic dipole radiation. This could have been expected on general grounds, because the recoil force must be bilinear in the various multipoles (since the Poynting vector is), and the only way to construct a three-dimensional vector is to contract a 2^n pole with a 2^{n+1} pole. (Magnetic 2^n poles are homogeneous to electric 2^{n+1} poles.)

In gravitational theory, the analogs of the electric and magnetic 2^n poles are the mass and flow 2^n poles, which are defined in the same way, masses playing the role of charges. However, there can be neither mass nor

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[1] H. Lorentz, *The Theory of Electrons* (Dover Publications, New York, 1952), p. 254.
[2] A. Peres and N. Rosen, Ann. Phys. (New York) **10**, 94 (1960).
[3] A. Peres, Nuovo cimento **15**, 351 (1960).
[4] J. Weber, Phys. Rev. **117**, 306 (1959).
[5] W. B. Bonnor and M. Rotenberg, Proc. Roy. Soc. (London) **A265**, 109 (1961).

flow dipole radiations, because of the laws of conservation of linear and angular momenta.[6] One can therefore expect that the lowest order terms in the gravitational case will be due to the interference of the mass quadrupole radiation with either the mass octopole or the flow quadrupole radiations. The explicit calculations are carried out in Sec. 3, in a way which closely follows the pattern of the more elementary electromagnetic case. In particular, corresponding equations are labeled by the same numbers, with primes.

Finally, Sec. 4 is devoted to a brief review of our present status of knowledge of gravitational radiation. A possible program for future research is outlined.

Throughout this paper, we use natural units $c=G=4\pi\epsilon_0=1$. Greek indices run from 0 to 3, Latin ones from 1 to 3. The Minkowski tensor, diag$(+1,\ -1,\ -1,\ -1)$, is denoted by $g_{\alpha\beta}$ (this is *not* the usual convention in general relativity) and serves to raise and lower indices. An index placed after an already defined symbol means partial differentiation. However, time derivatives are also denoted by left superscripts, e.g., ${}^7T^{\alpha\beta}\equiv\partial^7T^{\alpha\beta}/\partial t^7$.

Finally, we note that as a consequence of our assumption that the system is localized within a finite volume, it follows that one can take effectively, *in the approximation required to compute the secular effects of radiation*, that $\int F_\alpha dV=0$, where F is any function of the field sources.[2] For $\alpha=1,2,3$, this equation is a trivial consequence of the Gauss theorem. For $\alpha=0$, it can be written as $d(\int FdV)/dt=0$, and means that the average value of the time derivative over a long time interval vanishes in the required approximation. Throughout this paper, we shall make an extensive use of this property, in the form

$$\int AB_\alpha dV=-\int A_\alpha BdV, \tag{1}$$

A and B being any two functions of the material sources.

2. ELECTROMAGNETIC RADIATION RECOIL

We consider a system of charges characterized by a current density J^α which satisfies the continuity equation

$$J^\alpha{}_\alpha=0. \tag{2}$$

The electromagnetic potential A^α in the Lorentz gauge

$$A^\alpha{}_\alpha=0, \tag{3}$$

satisfies the field equations

$$\Box A^\alpha=4\pi J^\alpha. \tag{4}$$

The electromagnetic field is

$$F_{\alpha\beta}=A_{\beta\alpha}-A_{\alpha\beta}, \tag{5}$$

and the 4-force on the material system is given by

$$K_\alpha=\int F_{\alpha\beta}J^\beta dV. \tag{6}$$

The zero component of K_α represents the total flow of energy, and is well known.[1] The three spatial components represent the net thrust on the material system due to the reaction of the outgoing radiation.

From (1) and (2) it follows that

$$\int A_{\alpha\beta}J^\beta dV=-\int A_\alpha J^\beta{}_\beta dV=0, \tag{7}$$

whence

$$K_m=\int J^\alpha A_{\alpha m}dV. \tag{8}$$

To evaluate (8), we solve (4) for A^α, taking the retarded potential solution

$$A_{\rm ret}{}^\alpha=\tfrac{1}{2}(A_{\rm ret}{}^\alpha-A_{\rm adv}{}^\alpha)+\tfrac{1}{2}(A_{\rm ret}{}^\alpha+A_{\rm adv}{}^\alpha). \tag{9}$$

The last parenthesis, however, is time symmetric, and thus cannot contribute to any secular radiation effect. Thus, for our purpose, we can effectively take that

$$A^\alpha=\tfrac{1}{2}(A_{\rm ret}{}^\alpha-A_{\rm adv}{}^\alpha), \tag{10}$$

$$=\frac{1}{2}\int[J^\alpha(t-R)-J^\alpha(t+R)]dV/R, \tag{11}$$

where $R=|\mathbf{x}-\mathbf{X}|$, $\mathbf{x}$ being the argument of A^α and $\mathbf{X}$ the variable of integration. We now expand (11) about t, by means of the Taylor theorem. (This may impose some restrictions on the time derivatives of J^α, namely, that the velocities and accelerations of the charges are not too large.) Assuming that such an expansion is valid, we obtain

$$A_\alpha=-\int[{}^1J_\alpha+({}^3J_\alpha R^2/3!)+({}^5J_\alpha R^4/5!)+\cdots]dV, \tag{12}$$

whence

$$A_{\alpha m}=-\int[(2\,{}^3J_\alpha/3!)+(4\,{}^5J_\alpha R^2/5!)+\cdots]R^m dV. \tag{13}$$

Before we substitute this result into (8), we note that for any function Z

$$\int {}^nJ_0ZdV=-\int {}^{n-1}J_kZ_kdV, \tag{14}$$

in virtue of (1) and (2). In particular

$$A_{0m}=-\int\{[2\,{}^2J_m/3!]+[4\,{}^4J_k(\partial R^2R^m/\partial R^k)/5!]\cdots\}dV. \tag{15}$$

[6] J. Boardman and P. G. Bergmann, Phys. Rev. **115**, 1318 (1959).

The first term gives no contribution to the recoil, as may be seen with the help of (1), (8), and (14). We finally obtain, from (8), (13), and (14)

$$K_m = -\int\int \{J^0(\mathbf{x})[4\,{}^4J_k(\mathbf{X})(\partial R^2R^m/\partial R^k)/5!] + J^k(\mathbf{x})[2\,{}^3J_k(\mathbf{X})R^m/3!]\}d^3\mathbf{x}d^3\mathbf{X}, \quad (16)$$

where we have omitted the contribution from higher order terms (hitherto denoted by three points). By repeated use of (1) and (14), this can be brought to the form

$$K^m = (2/15)\,{}^1D^k(4E^{km} - E^{mk} - \delta^{km}E^{nn}), \quad (17)$$

where ${}^1D^k = \int {}^1J^0x^kdV = \int J^kdV$ and $E^{mn} = \int {}^3J^mx^ndV$. We now introduce the magnetic dipole moment

$$M^{mn} = \int (J^mx^n - J^nx^m)dV, \quad (18)$$

and the electric quadrupole moment

$$Q^{mn} = \int J^0(x^mx^n - \tfrac{1}{3}\delta^{mn}x^kx^k)dV, \quad (19)$$

from which we obtain, with the help of (14)

$${}^1Q^{mn} = \int (J^mx^n + J^nx^m - (2/3)\delta^{mn}J^kx^k)dV. \quad (20)$$

With these notations, (17) can finally be written

$$K^m = -{}^2D^k[(1/5)\,{}^3Q^{km} + (1/3)\,{}^2M^{km}]. \quad (21)$$

It is interesting to compare this formula with the one for the rate of *energy* loss, which is given by the *squares* of ${}^2D^k$, ${}^3Q^{km}$, and ${}^2M^{km}$ (no interference).[7]

3. GRAVITATIONAL RADIATION RECOIL

We now consider a material system characterized by a stress-energy tensor $T^{\alpha\beta}$ which satisfies the conservation law

$$T^{\alpha\beta}{}_{\beta} = 0. \quad (2')$$

The gravitational potential $V^{\alpha\beta}$ is a symmetric tensor which satisfies the De Donder gauge

$$V^{\alpha\beta}{}_{\beta} = 0, \quad (3')$$

and the Einstein field equations

$$\Box V^{\alpha\beta} = 16\pi T^{\alpha\beta}. \quad (4')$$

These equations actually are nonlinear, because the stress-energy tensor $T^{\alpha\beta}$ must take into account contributions from all fields, *including* the gravitational field. We shall write

$$T^{\alpha\beta} = {}_mT^{\alpha\beta} + {}_gT^{\alpha\beta}, \quad (22)$$

where the subscripts m and g stand for "matter" and "gravitational field," respectively. ${}_gT^{\alpha\beta}$ is a complicated function of the $V^{\alpha\beta}$ (and only of them). It contains terms which are quadratic in the first derivatives of $V^{\alpha\beta}$ and also some second derivatives of $V^{\alpha\beta}$, the coefficients themselves being algebraic functions of $V^{\alpha\beta}$.[8,9] Fortunately, the exact form of ${}_gT^{\alpha\beta}$ is not required for our purpose.

We now introduce the following notations[10]:

$$\mathfrak{g}^{\alpha\beta} = g^{\alpha\beta} + V^{\alpha\beta}, \quad \mathfrak{g}_{\alpha\beta}\mathfrak{g}^{\beta\gamma} = \delta_\alpha{}^\gamma, \quad (23)$$

and

$$F_{\beta\gamma}{}^\alpha = \tfrac{1}{2}[\mathfrak{g}_{\pi\beta}V^{\pi\alpha}{}_\gamma + \mathfrak{g}_{\pi\gamma}V^{\pi\alpha}{}_\beta + \mathfrak{g}^{\alpha\pi}(\tfrac{1}{2}\mathfrak{g}_{\beta\gamma}\mathfrak{g}_{\mu\nu} - \mathfrak{g}_{\beta\mu}\mathfrak{g}_{\gamma\nu})V^{\mu\nu}{}_\pi]. \quad (5')$$

$F_{\beta\gamma}{}^\alpha$ is the *gravitational field* and has 40 independent components, instead of only 6 for the electromagnetic field. Incidentally, let us note that the mathematical meaning of the equivalence principle simply is that in the Einstein equations (4′), $g^{\alpha\beta}$ and $V^{\alpha\beta}$ never appear separately, but only in the combination $\mathfrak{g}^{\alpha\beta}$.[11,12] We shall, however, not make use of this important property in the present work, but only of the fact that ${}_gT^{\alpha\beta}$ satisfies

$$16\pi\,{}_gT^{\alpha\beta}{}_\beta \equiv (16\pi\,{}_gT^{\beta\gamma} - \Box V^{\beta\gamma})F_{\beta\gamma}{}^\alpha. \quad (24)$$

(This holds identically in the $V^{\alpha\beta}$.) It then follows from (2′) and (4′) that

$${}_mT^{\alpha\beta}{}_\beta = {}_mT^{\beta\gamma}F_{\beta\gamma}{}^\alpha. \quad (25)$$

The left-hand side is the divergence of the matter part of the stress-energy tensor, and is therefore equal to the force density acting on matter from all the fields *not* included in ${}_mT^{\alpha\beta}$. It is just the gravitational force density. Thus, the total gravitational force acting on our material system is

$$K^\alpha = \int F_{\beta\gamma}{}^\alpha\,{}_mT^{\beta\gamma}dV. \quad (6')$$

The zero component of K^α represents the total flow of energy, and is well known.[2,3] The three spatial components represent the net thrust on the material system due to the reaction of the outgoing radiation.

Up to this point, our treatment has been rigorous, and

[7] L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1951), p. 206.

[8] A. Papapetrou, Proc. Roy. Irish Acad. **A52**, 11 (1948).

[9] A. Peres, Nuovo cimento **11**, 617 (1959).

[10] For the reader who is familiar with general relativity and may be scandalized by the approach of this paper, let us add that our $\mathfrak{g}^{\alpha\beta}$ is the usual $\mathfrak{g}^{\alpha\beta}$ of general relativity, and that our ${}_mT^{\alpha\beta}$ is what is usually called $-\mathrm{Det}(g_{\gamma\delta})T^{\alpha\beta}$. It is a second-rank tensor density, and our $F_{\beta\gamma}{}^\alpha$ are linear combinations of the Christoffel symbols. Finally,

$$16\pi\,{}_gT^{\alpha\beta} \equiv \Box V^{\alpha\beta} - \mathrm{Det}(g_{\gamma\delta})(R^{\alpha\beta} - \tfrac{1}{2}Rg^{\alpha\beta}).$$

[11] R. H. Dicke, Phys. Rev. Letters **7**, 359 (1961).

[12] W. H. Thirring, Ann. Phys. (New York) **16**, 96 (1961).

no approximations were made. We shall now suppose that the $V^{\alpha\beta}$ are small, compared to unity (at the surface of the Sun, the largest one, V^{00}, is of the order of 10^{-6}) so that we can safely replace, in (5′), $\mathfrak{g}^{\alpha\beta}$ and $\mathfrak{g}_{\alpha\beta}$ by $g^{\alpha\beta}$ and $g_{\alpha\beta}$. We can thus write

$$F_{\beta\gamma}{}^{\alpha}=\tfrac{1}{2}(V^{\alpha}{}_{\beta\gamma}+V^{\alpha}{}_{\gamma\beta}+\tfrac{1}{2}g_{\beta\gamma}V_{\pi}{}^{\pi\alpha}-V_{\beta\gamma}{}^{\alpha}). \quad (5'')$$

For the same reason, we can replace ${}_{m}T^{\alpha\beta}$ in the right-hand side of (6′) by $T^{\alpha\beta}$.

From (1) and (2′) it thus follows that

$$\int V^{\alpha}{}_{\beta\gamma}T^{\beta\gamma}dV=-\int V^{\alpha}{}_{\beta}\,T^{\beta\gamma}{}_{\gamma}dV=0, \quad (7')$$

whence

$$K^{m}=\frac{1}{4}\int T^{\alpha\beta}(2V_{\alpha\beta m}-g_{\alpha\beta}V^{\gamma}{}_{\gamma m})dV, \quad (8')$$

$$=\frac{1}{4}\int[T^{00}(V^{00}{}_{m}+V^{kk}{}_{m})-4T^{0k}V^{0k}{}_{m}+2T^{pq}V^{pq}{}_{m}+T^{pp}(V^{00}{}_{m}-V^{kk}{}_{m})]dV. \quad (8'')$$

To evaluate (8″) we solve (4′) for $V^{\alpha\beta}$, taking the retarded potential solution

$$V_{\text{ret}}{}^{\alpha\beta}=\tfrac{1}{2}(V_{\text{ret}}{}^{\alpha\beta}-V_{\text{adv}}{}^{\alpha\beta})+\tfrac{1}{2}(V_{\text{ret}}{}^{\alpha\beta}+V_{\text{adv}}{}^{\alpha\beta}). \quad (9')$$

The last parenthesis, however, is time symmetric, and thus cannot contribute to any secular effects. Thus, for our purpose, we can effectively take that

$$V^{\alpha\beta}=\tfrac{1}{2}(V_{\text{ret}}{}^{\alpha\beta}-V_{\text{adv}}{}^{\alpha\beta}), \quad (10')$$

$$=2\int[T^{\alpha\beta}(t-R)-T^{\alpha\beta}(t+R)]dV/R, \quad (11')$$

where $R=|\mathbf{x}-\mathbf{X}|$, $\mathbf{x}$ being the argument of $V^{\alpha\beta}$ and $\mathbf{X}$ the variable of integration. We now expand (11′) about t, by means of the Taylor theorem. (This may impose some restrictions on the time derivatives of $T^{\alpha\beta}$, namely, that the velocities and accelerations of the masses are not too large.) Assuming that such an expansion is valid, we obtain

$$V^{\alpha\beta}=-4\int[{}^{1}T^{\alpha\beta}+({}^{3}T^{\alpha\beta}R^{2}/3!)+({}^{5}T^{\alpha\beta}R^{4}/5!)+({}^{7}T^{\alpha\beta}R^{6}/7!)+\cdots]dV, \quad (12')$$

whence

$$V^{\alpha\beta}{}_{m}=-4\int[2\,{}^{3}T^{\alpha\beta}/3!)+(4\,{}^{5}T^{\alpha\beta}R^{2}/5!)+(6\,{}^{7}T^{\alpha\beta}R^{4}/7!)+\cdots]R^{m}dV. \quad (13')$$

Before we substitute this result into (8″), we note that for any function Z

$$\int {}^{n}T^{00}ZdV=\int {}^{n-2}T^{pq}Z_{pq}dV, \quad (14')$$

and

$$\int {}^{n}T^{0k}ZdV=\int {}^{n-1}T^{km}Z_{m}dV, \quad (14'')$$

in virtue of (1) and (2′). In particular

$$V^{00}{}_{m}=-4\int[(4\,{}^{3}T^{pq}/5!)(R^{2}R^{m})_{pq}+(6\,{}^{5}T^{pq}/7!)(R^{4}R^{m})_{pq}+\cdots]dV, \quad (15')$$

and

$$V^{0k}{}_{m}=4\int[(2\,{}^{2}T^{km}/3!)+(4\,{}^{4}T^{kn}/5!)(\partial R^{2}R^{m}/\partial R^{n})+\cdots]dV. \quad (15'')$$

We thus obtain, from (8″), (13′) (15′), and (15″)

$$\begin{aligned}K^{m}=-\int\int\{&T^{00}[(4\,{}^{3}T^{pq}/5!)(R^{2}R^{m})_{pq}\\&+(6\,{}^{5}T^{pq}/7!)(R^{4}R^{m})_{pq}+\cdots+(2\,{}^{3}T^{kk}R^{m}/3!)\\&+(4\,{}^{5}T^{kk}R^{2}R^{m}/5!)+\cdots]+4T^{0k}[(2\,{}^{2}T^{km}/3!)\\&+(4\,{}^{4}T^{kn}/5!)(\partial R^{2}R^{m}/\partial R^{n})+\cdots]\\&+T^{kk}[(4\,{}^{3}T^{pq}/5!)(R^{2}R^{m})_{pq}+\cdots\\&-(2\,{}^{3}T^{pp}R^{m}/3!)-\cdots]\\&+2T^{pq}[(2\,{}^{3}T^{pq}R^{m}/3!)+\cdots]\}d^{3}xd^{3}X. \quad (16')\end{aligned}$$

Further use of (1), (14′), and (14″) shows that the last terms in the first, second, and third row vanish, and it remains

$$\begin{aligned}K^{m}=-\int\int &T^{rs}(\mathbf{x})^{3}T^{uv}(\mathbf{X})\{[6(R^{4}R^{m})_{rsuv}/7!]\\&+[4\delta_{uv}(R^{2}R^{m})_{rs}/5!]-[16\delta_{ru}(R^{2}R^{m})_{sv}/5!]\\&+[4\delta_{ru}\delta_{sv}R^{m}/3!]+[4\delta_{rs}(R^{2}R^{m})_{uv}/5!]\\&-[2\delta_{rs}\delta_{uv}R^{m}/3!]\}d^{3}xd^{3}X, \quad (16'')\end{aligned}$$

where we have omitted the contributions from higher order terms (hitherto denoted by three points). A lengthy but straightforward calculation then leads to

$$K^{m}=(8/105)A^{rs}(6\delta^{mr}B^{skk}+6B^{msr}-11B^{rsm}), \quad (17')$$

where

$$A^{rs}=\int({}^{1}T^{rs}-\tfrac{1}{3}\delta^{rs}\,{}^{1}T^{kk})dV, \quad (26)$$

and

$$B^{rsm}=\int({}^2T^{rs}-\tfrac{1}{3}\delta^{rs}\,{}^2T^{kk})x^m dV. \tag{27}$$

We now introduce the mass quadrupole moment

$$Q^{mn}=\int T^{00}(x^m x^n-\tfrac{1}{3}\delta^{mn}x^k x^k)dV, \tag{19$'$}$$

the mass octopole moment

$$H^{mns}=\int T^{00}[x^m x^n x^s-\tfrac{1}{5}x^k x^k(\delta^{mn}x^s+\delta^{ns}x^m+\delta^{sm}x^n)]dV, \tag{19$''$}$$

and the flow quadrupole moment

$$N^{mns}=\int[(T^{0s}x^m-T^{0m}x^s)x^n+(T^{0s}x^n-T^{0n}x^s)x^m$$
$$-\tfrac{1}{2}x^k x^k(2T^{0s}\delta^{mn}-T^{0m}\delta^{ns}-T^{0n}\delta^{ms})$$
$$+\tfrac{1}{2}T^{0k}x^k(2x^s\delta^{mn}-x^m\delta^{ns}-x^n\delta^{ms})]dV. \tag{18$'$}$$

(Notice that $Q^{mm}=H^{mms}=N^{mms}=N^{mss}=0$.) With the help of (14′) and (14″) we then obtain

$${}^3Q^{mn}=2A^{mn}, \tag{20$'$}$$

$${}^4H^{mns}=2[B^{mns}+B^{nsm}+B^{smn}-\tfrac{2}{5}(\delta^{mn}B^{skk}+\delta^{ns}B^{mkk}+\delta^{sm}B^{nkk})], \tag{20$''$}$$

$${}^3N^{mns}=B^{smn}+B^{snm}-2B^{mns}+\tfrac{1}{2}(\delta^{sm}B^{nkk}+\delta^{sn}B^{mkk}-2\delta^{mn}B^{skk}), \tag{20$''$}$$

and (17′) can finally be written as

$$K^m={}^3Q^{rs}[(8/45)\,{}^3N^{rsm}-(2/63)\,{}^4H^{rsm}]. \tag{21$'$}$$

On the other hand, the rate of loss of *energy*[2,3] would be given by the *squares* of ${}^3Q^{rs}$, ${}^3N^{rsm}$, and ${}^4H^{rsm}$.

It is interesting to note that the final result can be expressed in terms of T^{00} (the mass density) and T^{0k} (the momentum density) only [see Eqs. (18′), (19′), and (19″)], without explicit reference to the internal stresses T^{mn}.

4. SUMMARY AND OUTLOOK

Gravitational radiation, which only a few years ago was a rather mysterious phenomenon, is by now fairly well understood. In the present paper, the author has endeavored to show that it can be investigated by exactly the same techniques as electromagnetic radiation. While the present method can easily be generalized to higher approximations (i.e., higher multipole moments), its domain of applicability is limited to the vicinity of the material system emitting the radiation. Roughly speaking, the series (12) and (12′) will converge rapidly only in a domain much smaller than the wavelength.

Quite a different problem is the investigation of the asymptotic properties of gravitational radiation at large distances from its sources. This question has recently been clarified by Sachs[13] and is also fairly well understood by now. The remaining problem which has not yet been solved is to establish the correspondence between the asymptotic properties of gravitational radiation and the structure of its sources. This can perhaps be done by a WKB technique, as recently used by Suna[14] in the case of electromagnetic radiation. The main difficulty is that the "gauge" used by Sachs[13] is quite different from (3′). (It is, in spherical polar coordinates, $V^{00}=-1$, $V^{0r}=r^2\sin\theta$, $V^{0\theta}=V^{0\phi}=0$.) We may hope that this last problem will be solved in the not too distant future. *Note added in proof.* This question has recently been clarified by T. A. Morgan and A. Peres (to be published).

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[13] R. Sachs (to be published).
[14] A. Suna (to be published).