$\Sigma^0-\Lambda^0$ Relative Parity from $\Lambda^0\rightarrow\Sigma^0$ Conversion Induced by the Coulomb Field of a Nucleus*

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Dreitlein and Primakoff have recently proposed the use of the $\Lambda^0\rightarrow\Sigma^0$ conversion induced by the Coulomb field of nuclei to determine the Σ^0 lifetime. It is shown here that the same experiment could also be used to measure the $\Sigma^0-\Lambda^0$ relative parity. Indeed, as a good approximation, the transversal polarizations (in the laboratory system) of the Λ^0 and Σ^0 particles are equal or opposite according as the $\Sigma^0-\Lambda^0$ relative parity is even or odd, respectively.

I. INTRODUCTION

A METHOD to determine the Σ^0 lifetime, $\tau(\Sigma^0)$, has recently been suggested by Dreitlein and Primakoff.[1] They show that the cross section for $\Lambda^0\rightarrow\Sigma^0$ conversion induced by the Coulomb field of a nucleus is proportional to $Z^2/\tau(\Sigma^0)$.[2] They also show that this type of conversion, in approximately forward directions with high-energy incident Λ^0 and high Z, dominates largely the $\Lambda^0\rightarrow\Sigma^0$ conversion induced by the strong coupling. The aim of this work is to discuss the information which might be obtained from this process (Fig. 1) by considering the polarization effects. Our conclusion is that such an experiment could also be used to measure the $\Sigma^0-\Lambda^0$ relative parity.

II. GENERAL ARGUMENTS

Our development is based on the following considerations:

(1) Let us consider the two-plane defined by the energy momenta of the Λ^0 and Σ^0 particles and the virtual photon γ. We choose a spacelike four-vector orthogonal to this plane as the quantization axis of the Λ^0 and Σ^0 spins. The virtual γ being a Coulomb photon (i.e., its electric vector and momentum are collinear), an extension of Bohr's theorem on intrinsic parities[3] to virtual particles allows us to write:

$$(-1)^{m_\Lambda-m_\Sigma}=\epsilon, \tag{1}$$

where m_Λ, m_Σ are the spin projections $\pm1/2$ of the Λ^0 and Σ^0 particles on the quantization axis; and $\epsilon(=\pm1)$ stands for the $\Sigma^0-\Lambda^0$ relative parity. The scattering matrix $S^{(\epsilon)}$ of the process Λ^0+N (nucleus) $\rightarrow\Sigma^0+N'$ (nucleus) is a two-by-two matrix in the polarization space. (We consider an unpolarized nuclei.) On account of the foregoing relation (1), one has the following explicit forms for $S^{(\epsilon)}$:

$$S^{(+)}=f^{(+)}\begin{pmatrix}S_0+S_3 & 0\\ 0 & S_0-S_3\end{pmatrix},\qquad S^{(-)}=f^{(-)}\begin{pmatrix}0 & S_1-iS_2\\ S_1+iS_2 & 0\end{pmatrix}. \tag{2a,b}$$

Here, $f^{(\epsilon)}$, S_0, S_1, S_2, and S_3 are dynamical terms to be determined on the basis of a phenomenological current for the vertex $\Lambda^0-\gamma-\Sigma^0$. Furthermore, since the $\Lambda^0\rightarrow\Sigma^0$ Coulomb conversion is only dominant for approximately forward directions, let us study this special case. For the strictly forward conversion, the process $\Lambda^0+N\rightarrow\Sigma^0+N'$ does not polarize, i.e., the polarization degree of the hyperon is conserved and the cross section is independent of the hyperon polarization. As we shall see later (Sec. IV), one has in that case $S_3=0$ or $S_1=0$. We conclude: For forward $\Lambda^0\rightarrow\Sigma^0$ Coulomb conversion the transversal polarizations in the laboratory system of the Λ^0 and Σ^0 particles are parallel or antiparallel, respectively, according to an even or odd $\Sigma^0-\Lambda^0$ relative parity; the longitudinal polarization (if any) being conserved in magnitude and sign.

(2) In a more general fashion, when dealing with non-strictly forward conversion, the nature of the vertex $\Lambda^0-\gamma-\Sigma^0$ [see Sec. III, (22a), (22b)] implies for the cross section of the process $\Lambda^0\rightarrow\Sigma^0+$virtual γ to be independent of the Λ^0 polarization and also the conservation of the polarization degree of the hyperon. (As a matter of fact, it turns out that S_1, S_2, S_3 are real quantities while S_0 is purely imaginary.) This result is comparable to the well-known theorem: In the Born approximation there is no polarization effect in the scattering of a spin-1/2 particle by an unpolarized target.

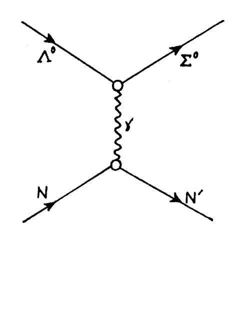

FIG. 1. Feynman graph for $\Lambda^0\rightarrow\Sigma^0$ conversion induced by a nuclear Coulomb field.

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[1] J. Dreitlein and H. Primakoff, Phys. Rev. **125**, 1671 (1962).

[2] Results concerning this cross section have also been obtained by I. Ya Pomeranchuk and I. M. Shmushkevitch, Nuclear Phys. **23**, 452 (1951); N. S. C. Williams, Nuovo cimento **19**, 1278 (1961); B. Valuev, (to be published). For high Z its order of magnitude is millibarns.

[3] A. Bohr, Nuclear Phys. **10**, 486 (1959).

TABLE I. Complete summary of symbols.

Four-vectors		Unit four-vectors	
$\mathfrak{p}$	energy momentum of arbitrary particle	$\mathfrak{u}_\pi$	$\mathfrak{u}_\pi \equiv \mathfrak{p}_\pi / M_\pi$
$\mathfrak{p}_\pi$	energy momentum of π particle	$\mathfrak{d}$	$\mathfrak{s} = \eta \mathfrak{d}$
$\mathfrak{p}_\Lambda$	energy momentum of Λ particle	$\mathfrak{d}^{(\epsilon)}$	$\mathfrak{s}^{(\epsilon)} = \zeta^{(\epsilon)} \mathfrak{d}^{(\epsilon)}$
$\mathfrak{p}_\Sigma$	energy momentum of Σ particle	$\mathfrak{n}^{(0)}$	time component of a tetrad
$\mathfrak{p}_N$	energy momentum of nucleus	$\mathfrak{n}^{(i)}$	space components of a tetrad
$\mathfrak{q}$	energy momentum of virtual γ	$\mathfrak{n}$	orthogonal to the scattering plane
$\mathfrak{s}$	polarization of Λ^0 particle	$\mathfrak{u}$	$\mathfrak{u} \equiv \mathfrak{p}_\Lambda / M_\Lambda$
$\mathfrak{s}^{(\epsilon)}$	energy momentum of Σ^0 particle	$\mathfrak{u}'$	$\mathfrak{u}' \equiv \mathfrak{p}_\Sigma / M_\Sigma$
		$\mathfrak{d}$	polarization of virtual γ
Three-vectors (laboratory system)		**Stokes' vectors**	
$\mathbf{p}_\Lambda$	Λ^0 particle momentum	$\boldsymbol{\eta}$	Λ^0 particle
$\mathbf{p}_\Sigma$	Σ^0 particle momentum	$\boldsymbol{\zeta}^{(\epsilon)}$	Σ^0 particle
$\mathbf{n}$	$\mathbf{n} = (\mathbf{p}_\Lambda \times \mathbf{p}_\Sigma)/\lvert \mathbf{p}_\Lambda \times \mathbf{p}_\Sigma \rvert$		
$\mathbf{n}_1$	$\mathbf{n}_1 = (1/p_\Lambda)\mathbf{p}_\Lambda \times \mathbf{n}$		
$\mathbf{n}_2'$	$\mathbf{n}_2' = (1/p_\Sigma)\mathbf{p}_\Sigma \times \mathbf{n}$		
Parameters		**Scalars**	
α	Λ^0-decay parameter	E_Λ	Λ^0 energy in (lab system)
η	Polarization degree of Λ^0 particle	E_Σ	Σ^0 energy in (lab system)
$\zeta^{(\epsilon)}$	Polarization degree of Σ^0 particle	p_Λ	$p_\Lambda \equiv \lvert \mathbf{p}_\Lambda \rvert$ (lab system)
δ	$\delta \equiv 1 - p_\Sigma / p_\Lambda$	p_Σ	$p_\Sigma \equiv \lvert \mathbf{p}_\Sigma \rvert$ (lab system)
φ	angle between $\mathbf{u}$, $\mathbf{u}'$	M_Λ	Mass of Λ^0 particle
K	$K = 2M_{\Lambda^0} M_\pi [\triangle(M_\Lambda, M_p, M_\pi)]^{-1/2} = 1.397$; $\triangle(a,b,c) = (a+b+c)(a+b-c)(a-b+c)(a-b-c)$	M_Σ	Mass of Σ^0 particle

III. PHENOMENOLOGICAL DESCRIPTION[4]

We use the covariant formalism of polarization,[5,6] i.e., the polarization of a spin 1/2 particle is represented by a pseudo 4-vector orthogonal to its energy momentum. Let $\mathfrak{s} = \eta \mathfrak{d}$ be the polarization of the Λ^0 particle ($\mathfrak{d}^2 = -1$; $|\eta|$ is the degree of the Λ^0 polarization, $0 \leq |\eta|^2 \leq 1$). We assume that the Λ^0 beam is polarized. Usually this polarization will be transversal in the laboratory system. The Λ^0 decay is an analyzer of the Λ^0 polarization. Indeed the transition amplitude for $\Lambda^0 \to p + \pi^-$ is proportional [see Michel and Rouhaninejad,[7] formula (45)] to the function (see Table I for notation),

$$A = 1 - \alpha \eta K \mathfrak{u}_\pi \cdot \mathfrak{d}, \tag{3a}$$

where $\mathfrak{u}_\pi \equiv \mathfrak{p}_\pi / M_\pi$; K is a numerical constant; and α is the asymmetry parameter for the Λ^0-decay ($-1 \leq \alpha \leq 1$). In the Λ^0-rest system, the covariant expression (3a), becomes:

$$A = 1 - \alpha \eta \cos\phi, \tag{3b}$$

where ϕ is the angle between $\mathfrak{u}_\pi$ and $\mathfrak{d}$ in that system.

[4] See the Appendix for notation. The symbolism has been labeled in Table I.

[5] For the mathematical theory, see L. Michel and A. S. Wightman, Phys. Rev. **98**, 1190 (1955); L. Michel, Suppl. Nuovo cimento **14**, 95 (1959); A. S. Wightman, *Relations de Dispersion et Particules Elementaires* (Hermann et Cie, Paris, 1960), pp. 160–226.

[6] For the physical applications, see C. Bouchiat and L. Michel, Nuclear Phys. **5**, 416 (1958); J. Bernstein and L. Michel, Phys. Rev. **118**, 871 (1960); L. Michel and H. Rouhaninejad, *ibid.* **122**, 242 (1961); H. Rouhaninejad, PhD. thesis, Paris, 1961 (unpublished).

[7] See L. Michel and H. Rouhaninejad, reference 6.

The parameter α has recently been measured; however, in the experiment we study here, only $\alpha\eta$ has to be measured.

A right-handed orthonormal basis for the space time is a set of four-vectors such that

$$\mathfrak{n}^{(\alpha)} \cdot \mathfrak{n}^{(\beta)} = g^{\alpha\beta}, \qquad \frac{1}{4!} \epsilon^{\lambda\mu\nu\rho} n_\lambda^{(\alpha)} n_\mu^{(\beta)} n_\nu^{(\gamma)} n_\rho^{(\delta)} = -\epsilon^{\alpha\beta\gamma\delta}. \tag{4}$$

For a particle with energy momentum $\mathfrak{p}$ and mass M, let $\mathfrak{u}(\equiv \mathfrak{n}^{(0)}) = \mathfrak{p}/M$, $\mathfrak{n}^{(i)}$ be four four-vectors satisfying (4). We call them a "tetrad" for the given particle. In what follows, we choose the unit 4-vector $\mathfrak{n}$ orthogonal to the scattering three-plane of the process $\Lambda^0 + N \to \Sigma^0 + N'$ as the $\mathfrak{n}^{(3)}$ common component of all "tetrads" for the different particles:

$$n^\lambda = c \epsilon^{\lambda\mu\nu\rho} (\mathfrak{p}_\Lambda)_\mu (\mathfrak{p}_\Sigma)_\nu (\mathfrak{p}_N)_\rho, \tag{5}$$

where c is the normalization coefficient such that $n^\lambda n_\lambda = -1$.

Let $\mathfrak{u} = \mathfrak{p}_\Lambda / M_\Lambda$, $\mathfrak{n}^{(1)}$, $\mathfrak{n}^{(2)}$, $\mathfrak{n}(\equiv \mathfrak{n}^{(3)})$ stand for the Λ^0 tetrad. The $\mathfrak{n}^{(1)}$, $\mathfrak{n}^{(2)}$ four-vectors shall be chosen later [see Sec. IV, (26)]. The components η_i of the Λ^0 polarization in this tetrad are given by

$$\mathfrak{s} = \eta \mathfrak{d} = \boldsymbol{\eta} \mathbin{\blacksquare} \mathfrak{n} \equiv \textstyle\sum_i \eta_i \mathfrak{n}^{(i)}. \tag{6}$$

We use the symbolic scalar product $\blacksquare$ for summation over i. Usually the η_i are considered as the three components in a given tetrad of $\boldsymbol{\eta}$, the Stokes' vector of the particle. The density matrix for the polarization of the

Λ^0 particle is the two-by-two matrix:

$$\rho_\Lambda = \tfrac{1}{2}(1 - \mathfrak{S}\cdot\mathfrak{n}\,\bullet\,\boldsymbol{\tau}). \tag{7}$$

The density matrix for the Σ^0 particle $\rho_{\Sigma^0}{}^{(\epsilon)}$ is defined by

$$\rho_{\Sigma^0}{}^{(\epsilon)} = S^{(\epsilon)}\rho_\Lambda S^{(\epsilon)\dagger}, \tag{8}$$

where $S^{(\epsilon)}$ stands for the scattering matrix as given in (2a,b). We can write $\rho_{\Sigma^0}{}^{(\epsilon)}$ in the form

$$\rho_{\Sigma^0}{}^{(\epsilon)} = \tfrac{1}{2}(1 + \boldsymbol{\zeta}^{(\epsilon)}\bullet\boldsymbol{\tau}); \tag{9}$$

and, from (7), (8), and (9), one has for $\boldsymbol{\zeta}^{(\epsilon)}$

$$\boldsymbol{\zeta}^{(\epsilon)} = [2\,\mathrm{Tr}\rho_{\Sigma^0}{}^{(\epsilon)}]^{-1}\{\mathrm{Tr}(S^{(\epsilon)}S^{(\epsilon)\dagger}\boldsymbol{\tau}) + \sum_{j=1}^{3}\eta_j\,\mathrm{Tr}(S^{(\epsilon)}\tau_j S^{(\epsilon)\dagger}\boldsymbol{\tau})\}. \tag{10}$$

The polarization pseudo four-vector of the Σ^0 particle is

$$\mathfrak{S}^{(\epsilon)} = \boldsymbol{\zeta}^{(\epsilon)}\bullet\mathfrak{n}, \tag{11}$$

and the polarization degree is given by

$$\zeta^{(\epsilon)} = (-\mathfrak{S}^{(\epsilon)}\cdot\mathfrak{S}^{(\epsilon)})^{1/2} = (\boldsymbol{\zeta}^{(\epsilon)}\bullet\boldsymbol{\zeta}^{(\epsilon)})^{1/2}. \tag{12}$$

The scattering cross section has the form

$$\sigma^{(\epsilon)} = \sigma_0{}^{(\epsilon)}(1 + \boldsymbol{\eta}\bullet\boldsymbol{\lambda}^{(\epsilon)}), \tag{13}$$

where $\sigma_0{}^{(\epsilon)}$ means the cross section measured when polarizations are not observed and $\boldsymbol{\lambda}^{(\epsilon)}$ is given by

$$\boldsymbol{\lambda}^{(\epsilon)} = (\mathrm{Tr}S^{(\epsilon)}S^{(\epsilon)\dagger})^{-1}\,\mathrm{Tr}S^{(\epsilon)}\boldsymbol{\tau}S^{(\epsilon)\dagger}. \tag{14}$$

Taking into account the explicit forms (2a,b) one has for (10):

$$\zeta_1{}^{(\epsilon)} = \frac{\alpha^{(\epsilon)}\eta_1 - \epsilon\gamma^{(\epsilon)}\eta_2}{1+\beta^{(\epsilon)}\eta_3},\quad \zeta_2{}^{(\epsilon)} = \frac{\gamma^{(\epsilon)}\eta_1 + \epsilon\alpha^{(\epsilon)}\eta_2}{1+\beta^{(\epsilon)}\eta_3},\quad \zeta_3{}^{(\epsilon)} = \frac{\beta^{(\epsilon)} + \epsilon\eta_3}{1+\beta^{(\epsilon)}\eta_3}, \tag{15a,b,c}$$

where $\alpha^{(\epsilon)}$, $\beta^{(\epsilon)}$, $\gamma^{(\epsilon)}$ are six real numbers defined as follows:

$$\alpha^{(+)} = \frac{|S_0|^2 - |S_3|^2}{|S_0|^2 + |S_3|^2},\quad \beta^{(+)} = \frac{S_0S_3{}^* + S_0{}^*S_3}{|S_0|^2 + |S_3|^2},\quad \gamma^{(+)} = i\frac{S_0{}^*S_3 - S_0S_3{}^*}{|S_0|^2 + |S_3|^2}, \tag{16a,b,c}$$

$$\alpha^{(-)} = \frac{|S_1|^2 - |S_2|^2}{|S_1|^2 + |S_2|^2},\quad \beta^{(-)} = i\frac{S_1S_2{}^* - S_1{}^*S_2}{|S_1|^2 + |S_2|^2},\quad \gamma^{(-)} = \frac{S_1{}^*S_2 + S_1S_2{}^*}{|S_1|^2 + |S_2|^2}, \tag{17a,b,c}$$

which satisfy the relations

$$|\alpha^{(\epsilon)}|^2 + |\beta^{(\epsilon)}|^2 + |\gamma^{(\epsilon)}|^2 = 1. \tag{18}$$

Let us call $u(\mathfrak{p}, \pm\mathfrak{n})$ the Dirac amplitudes for a one-particle state of energy momentum $\mathfrak{p}$ and total polarization along $\pm\mathfrak{n}$. We describe the corresponding amplitudes for the Λ^0 particle by $u(\mathfrak{p}_\Lambda, l\mathfrak{n})$ and those for the Σ^0 particle by $u(\mathfrak{p}_\Sigma, s\mathfrak{n})$ where l, $s = \pm 1$. In an unpublished work, Michel has explicitly computed the two-by-two matrices $(T^A)_{sl}$:

$$(T^A)_{sl} = \langle\bar{u}(\mathfrak{p}_\Sigma, s\mathfrak{n})|\gamma^{(i,A)}|u(\mathfrak{p}_\Lambda, l\mathfrak{n})\rangle, \tag{19}$$

for the arbitrary 4×4 matrices $\gamma^\mu(\gamma^{(i,A)})$ when the Σ^0 tetrad is obtained from the Λ^0 tetrad by the Lorentz transformation which changes $\mathfrak{u} \equiv \mathfrak{p}_\Lambda/M_\Lambda$ into $\mathfrak{u}' \equiv \mathfrak{p}_\Sigma/M_\Sigma$ and leaves invariant all four-vectors orthogonal to both $\mathfrak{p}_\Lambda$ and $\mathfrak{p}_\Sigma$. The expressions for the T^A matrices corresponding to the 16 linearly independent matrices are given in Table II.

The transition matrix for the process described by the graph of Fig. 1 is given by[8]

$$S^{(+)} \equiv \langle\Sigma^0|j^{\mu(+)}(0)|\Lambda^0\rangle a_\mu = \rho\bar{u}(\mathfrak{p}_\Sigma, s\mathfrak{n})\{F_1{}^{(+)}(\mathfrak{q}^2)i\gamma^\mu a_\mu + iF_2{}^{(+)}(\mathfrak{q}^2)\sigma^{\mu\nu}q_\nu a_\mu\}u(\mathfrak{p}_\Lambda, l\mathfrak{n}), \tag{22a}$$

$$S^{(-)} \equiv \langle\Sigma^0|j^{\mu(-)}(0)|\Lambda^0\rangle a_\mu = \rho\bar{u}(\mathfrak{p}_\Sigma, s\mathfrak{n})\{F_1{}^{(-)}(\mathfrak{q}^2)\gamma^5\gamma^\mu a_\mu + iF_2{}^{(-)}(\mathfrak{q}^2)\gamma^5\sigma^{\mu\nu}q_\nu a_\mu\}u(\mathfrak{p}_\Lambda, l\mathfrak{n}), \tag{22b}$$

where $F_1{}^{(\epsilon)}(\mathfrak{q}^2)$ [$F_2{}^{(\epsilon)}(\mathfrak{q}^2)$] are the electric [magnetic] form factors of the vertex $\Lambda^0-\gamma-\Sigma^0$ depending on the square of the momentum transfer ($\mathfrak{q} = \mathfrak{p}_\Lambda - \mathfrak{p}_\Sigma$); ρ stands for the electromagnetic form factor of the nucleus. We include in ρ the Z dependence and the normalization factors as well. The four-vector $\mathfrak{a}$ describes the polarization of the virtual photon; it is such that

$$\mathfrak{a}\cdot\mathfrak{q} = 0,\quad \mathfrak{a}\cdot\mathfrak{a} = 1. \tag{23a,b}$$

In the following we shall only consider the magnetic $F_2{}^{(\epsilon)}(\mathfrak{q}^2)$ form factor term, because the contribution of the electric $F_1{}^{(\epsilon)}(\mathfrak{q}^2)$ form factor is negligible for the

TABLE II. Explicit expressions for the T^A matrices [see formula (19)] corresponding to the 16 linearly independent $\gamma^{(i,A)}$ matrices.[a]

$\gamma^{(i,A)}$	T^A
1	$W/2$
$i\gamma^\mu$	$(1/W)(u'^\mu + u^\mu + i\epsilon^{\mu\nu\rho\sigma}\boldsymbol{\tau}\bullet\mathbf{n}_\nu u_\rho u_\sigma')$
$\sigma^{\mu\nu}$	$(1/2W)[\epsilon^{\mu\nu\rho\sigma}\boldsymbol{\tau}\bullet\mathbf{n}_\rho(\mathfrak{u}'+\mathfrak{u})_\sigma - i(\mathfrak{u}\wedge\mathfrak{u}')^{\mu\nu}]$
$\gamma^5\gamma^\mu$	$(W/2)\boldsymbol{\tau}\bullet\mathbf{n}^\mu - 1/W\,\boldsymbol{\tau}\bullet\mathbf{n}^\nu u_\nu' u^\mu$
γ^5	$(-i/W)\boldsymbol{\tau}\bullet\mathfrak{n}\cdot\mathfrak{u}'$
$\gamma^5\sigma^{\mu\nu}$	$(1/2W)\{[(\mathfrak{u}'+\mathfrak{u})\wedge\boldsymbol{\tau}\bullet\mathfrak{n}]^{\mu\nu} + i\epsilon^{\mu\nu\rho\sigma}u_\rho' u_\sigma\}$

[a] Here $\mathfrak{p}_\Sigma = M_\Sigma\mathfrak{u}'$, $\mathfrak{p}_\Lambda = M_\Lambda\mathfrak{u}$, and $W = [2(1+\mathfrak{u}\cdot\mathfrak{u}')]^{1/2}$. The symbol $\wedge$ means external product, i.e., $(\mathfrak{u}\wedge\mathfrak{u}')^{\mu\nu} = u^\mu u'^\nu - u^\nu u'^\mu$.

[8] See references 1 and 2 for a discussion on the vertex function of the process involved. See also G. Feldman and T. Fulton, Nuclear Phys. 8, 106 (1958); J. Dreitlein and B. Lee, Phys. Rev. 124, 1274 (1961).

approximately forward $\Lambda^0 \to \Sigma^0$ Coulomb conversion. However we can prove that the general statements given in Sec. II are still valid if the electric form factor is not neglected. Indeed, the $\Lambda^0-\gamma-\Sigma^0$ vertex function considered as a complex function of the squared momentum transfer ($t \equiv \mathfrak{q}^2$) has no singularities on the negative real axis of the complex t plane. This negative real axis is the physical region for the $\Lambda^0 \to \Sigma^0$ Coulomb conversion. Time-reversal invariance requires that there is no complex phase between the electric and magnetic form factors. It suffices to apply the formulas of Table II to (22a,b) to obtain the general results of Sec. II.

We finally recall how the Σ^0 polarization can be analyzed. Let us call $\mathfrak{n}''^{(\alpha)}$ the tetrad for the Λ^0 particle of the decay process $\Sigma^0 \to \Lambda^0 + \gamma$. The corresponding correlation function [see reference 7, formula (64)] is

$$H = 1 - \alpha\zeta^{(\epsilon)} \cos\theta_1 \cos\theta_2, \qquad (24)$$

where $\cos\theta_1 = -\mathfrak{d}^{(\epsilon)}\cdot\mathfrak{n}$ (see Table I); and $\cos\theta_2 = -K\mathfrak{u}_\pi \cdot \mathfrak{n}''^{(\epsilon)}$. Here the quantity to be measured is $\alpha\zeta^{(\epsilon)}$, α being the same parameter as in (3a,b) and $\zeta^{(\epsilon)}$ the polarization degree of the Σ^0 particle (12).

IV. EXPLICIT CALCULATIONS AND RESULTS

Since only magnetic form factors are considered, we have for the right-hand side terms in (22a,b) (see Table II):

$$S^{(+)} = f^{(+)} a_\mu q_\nu \{\epsilon^{\mu\nu\rho\sigma} \boldsymbol{\tau} \cdot n_\rho (\mathfrak{u}' + \mathfrak{u})_\sigma - i(\mathfrak{u} \wedge \mathfrak{u}')^{\mu\nu} \tau_0\}, \quad (25a)$$

$$S^{(-)} = f^{(-)} a_\mu q_\nu \{[(\mathfrak{u}' + \mathfrak{u}) \wedge \boldsymbol{\tau} \cdot \mathfrak{n}]^{\mu\nu} + i\epsilon^{\mu\nu\rho\sigma} u_\rho' u_\sigma \tau_0\}, \quad (25b)$$

with

$$f^{(\epsilon)} = (i/2W)\rho F_2^{(\epsilon)}(\mathfrak{q}^2); \quad W = [2(1 + \mathfrak{u}\cdot\mathfrak{u}')]^{1/2}.$$

In the laboratory system we choose the Λ^0 tetrad as follows: $\mathfrak{n}^{(0)} \equiv \mathfrak{u}$; $\mathfrak{n}^{(3)} \equiv \mathfrak{n}$, as was defined in (5) (unit four-vector orthogonal to the scattering three-plane); and $\mathfrak{n}^{(1)}$, $\mathfrak{n}^{(2)}$ are such that

$$\mathfrak{n}^{(1)} = (0, \mathbf{n}_1), \quad \mathfrak{n}^{(2)} = [p_\Lambda / M_\Lambda, (E_\Lambda \mathbf{p}_\Lambda)/(M_\Lambda p_\Lambda)], \quad (26a,b)$$

where $p_\Lambda \equiv |\mathbf{p}_\Lambda|$ and $\mathbf{n}_1 = (1/p_\Lambda)\mathbf{p}_\Lambda \times \mathbf{n}$. The transversal and longitudinal polarizations (in the lab system) of the Λ^0 particle are then given by

$$\eta_t = -\mathfrak{s}\cdot\mathfrak{n}(\equiv \eta_3), \quad \eta_{t'} = -\mathfrak{s}\cdot\mathfrak{n}^{(1)}(\equiv \eta_1),$$
$$\eta_l = -\mathfrak{s}\cdot\mathfrak{n}^{(2)}(\equiv \eta_2), \quad (27a,b,c)$$

η_t means transversal polarization collinear to $\mathbf{n}$ (unit vector orthogonal to the scattering plane in the lab system); $\eta_{t'}$ means transversal polarization collinear to $\mathbf{n}_1$ [(26a)]; η_l is the longitudinal polarization. We neglect the kinetic energy of the recoiling nucleus ($E_\Lambda = E_\Sigma$), and we define

$$\delta \equiv 1 - p_\Sigma / p_\Lambda \approx (M_\Sigma^2 - M_\Lambda^2)/2p_\Lambda^2,$$
$$2(1-\delta)\sin^2\varphi/2 \approx \varphi^2/2, \quad (28)$$

where $\varphi \equiv \cos^{-1}(\mathbf{u}\cdot\mathbf{u}')$, i.e., the lab angle between $\mathbf{p}_\Lambda$ and $\mathbf{p}_\Sigma$. δ is a small quantity for sufficiently large p_Λ; for $p_\Lambda = 4M_\Lambda = 4.4$ BeV/c, $\delta \simeq 0.0043$. On account of these approximations (28), we finally obtain

$$S_0 = -iE_\Lambda (p_\Lambda/M_\Lambda)^2 (1 + M_\Lambda/M_\Sigma)$$
$$\times [(1 - M_\Lambda/M_\Sigma)\delta + (1 + M_\Lambda/M_\Sigma)\varphi^2/2], \quad (29a)$$

$$S_3 = p_\Lambda (1 + M_\Lambda/M_\Sigma) p_\Lambda / p_\Sigma (1-\delta)\varphi, \quad (29b)$$

$$S_1 = -E_\Lambda (p_\Lambda/M_\Lambda)(1 + M_\Lambda/M_\Sigma)(1-\delta)\varphi, \quad (30a)$$

$$S_2 = p_\Lambda [(1 + M_\Lambda/M_\Sigma)(\delta + \varphi^2/2)$$
$$- (p_\Lambda^2 / M_\Lambda M_\Sigma)(\delta - \varphi^2/2)]. \quad (30b)$$

Therefore, one has $\beta^{(\epsilon)} = 0$ [see (16b), (17b)], and from (13) and (14) $\sigma = \sigma_0$; i.e., the cross section is independent of the Λ^0 polarization. The $\boldsymbol{\zeta}^{(\epsilon)}$ components [see (15a,b,c)] become

$$\zeta_1^{(\epsilon)} = \alpha^{(\epsilon)}\eta_{t'} - \epsilon\gamma^{(\epsilon)}\eta_l, \quad \zeta_2^{(\epsilon)} = \gamma^{(\epsilon)}\eta_{t'} + \epsilon\alpha^{(\epsilon)}\eta_l,$$
$$\zeta_3^{(\epsilon)} = \epsilon\eta_t, \quad (31a,b,c)$$

and $\boldsymbol{\zeta}^{(\epsilon)2} = \boldsymbol{\zeta}^{(\epsilon)} \cdot \boldsymbol{\zeta}^{(\epsilon)} = \eta^2$; i.e., the polarization degree of the hyperon is conserved. These are the quantitative expressions of the statements given in Sec. II.2.

In the case of strictly forward $\Lambda^0 \to \Sigma^0$ Coulomb conversion $\varphi = 0$, and $\gamma^{(\epsilon)} = \beta^{(\epsilon)} = 0$; $\alpha^{(\epsilon)} = \epsilon$. The longitudinal and transversal components (in the lab system) of the Σ^0 polarization are then given by

$$\zeta_t^{(\epsilon)} = \epsilon\eta_t, \quad \zeta_{t'}^{(\epsilon)} = \epsilon\eta_{t'}, \quad \zeta_l^{(\epsilon)} = \eta_l; \quad (32a,b,c)$$

the transverse components conserve or reverse their sign according to even or odd $\Sigma^0 - \Lambda^0$ relative parity, respectively, while the longitudinal polarization conserves its magnitude and sign. This was stated in Sec. II.1.

To define the transversal and longitudinal components, in the laboratory system, of the Σ^0 polarization when dealing with nonstrictly forward $\Lambda^0 \to \Sigma^0$ conversion, we introduce a new Σ^0 tetrad as follows:

$$\mathfrak{n}'^{(0)} = \mathfrak{p}_\Sigma / M_\Sigma, \quad \mathfrak{n}'^{(1)} = (0, \mathbf{n}_2'),$$
$$\mathfrak{n}'^{(2)} = [p_\Sigma / M_\Sigma, (E_\Sigma \mathbf{p}_\Sigma)/(M_\Sigma p_\Sigma)], \quad \mathfrak{n}'^{(3)} = \mathfrak{n}, \quad (33)$$

with $\mathbf{n}_2' = (\mathbf{p}_\Sigma \times \mathbf{n})/p_\Sigma$. The transversal and longitudinal components of the Σ^0 polarization are then,

$$\zeta_t^{(\epsilon)} = -\mathfrak{s}^{(\epsilon)}\cdot\mathfrak{n}'^{(3)}, \quad \zeta_{t'}^{(\epsilon)} = -\mathfrak{s}^{(\epsilon)}\cdot\mathfrak{n}'^{(1)}, \quad \zeta_l^{(\epsilon)} = -\mathfrak{s}^{(\epsilon)}\cdot\mathfrak{n}'^{(2)},$$

with

$$\mathfrak{s}^{(\epsilon)} = \boldsymbol{\zeta}^{(\epsilon)} \cdot \mathfrak{n} = \zeta_1^{(\epsilon)}\mathfrak{n}^{(1)} + \zeta_2^{(\epsilon)}\mathfrak{n}^{(2)} + \zeta_3^{(\epsilon)}\mathfrak{n}^{(3)}.$$

The $\boldsymbol{\zeta}^{(\epsilon)}$ components were defined in (31a,b,c): The explicit results for $\zeta_t^{(\epsilon)}$, $\zeta_{t'}^{(\epsilon)}$, and $\zeta_l^{(\epsilon)}$ are

$$\zeta_t^{(\epsilon)} = \epsilon\eta_t, \quad (34a)$$

$$\zeta_{t'}^{(\epsilon)} \approx \eta_{t'}\left[\alpha^{(\epsilon)}\left(1 - \frac{\varphi^2}{2}\right) - \gamma^{(\epsilon)}\frac{E_\Lambda}{M_\Lambda}\varphi\right]$$
$$- \epsilon\eta_l\left[\gamma^{(\epsilon)}\left(1 - \frac{\varphi^2}{2}\right) + \alpha^{(\epsilon)}\frac{E_\Lambda}{M_\Lambda}\varphi\right], \quad (34b)$$

$$\zeta_l{}^{(\epsilon)} \approx \eta_{t'}\left\{\alpha^{(\epsilon)}\frac{E_\Sigma}{M_\Sigma}\varphi-\gamma^{(\epsilon)}\frac{1}{M_\Lambda M_\Sigma}\times\left[p_\Lambda{}^2\left(\frac{\varphi^2}{2}-\delta\right)+M_\Lambda{}^2\left(1-\frac{\varphi^2}{2}\right)\right]\right\}-\epsilon\eta_l\left\{\gamma^{(\epsilon)}\frac{E_\Sigma}{M_\Sigma}\varphi+\alpha^{(\epsilon)}\frac{1}{M_\Lambda M_\Sigma}\times\left[p_\Lambda{}^2\left(\frac{\varphi^2}{2}-\delta\right)+M_\Lambda{}^2\left(1-\frac{\varphi^2}{2}\right)\right]\right\}. \quad (34c)$$

From (34), one reads: The orthogonal components to the scattering plane of the Λ^0 and Σ^0 polarizations are equal or opposite according to even or odd $\Sigma^0-\Lambda^0$ relative parity, respectively. We remark that this holds in both four and three dimensions, the scattering plane in the latter case being defined in the laboratory system.

Let us consider the case when the Λ^0 polarization is purely transversal (in the lab system). Then we have

$$\eta_t=\eta\sin\theta,\quad \eta_l=0,\quad \eta_{t'}=\eta\cos\theta, \quad (35a,b,c)$$

where θ is the angle between the Λ^0 polarization and $\mathbf{n}$ in the laboratory system. The transversal and logitudinal components of the Σ^0 polarization become

$$\zeta_t{}^{(\epsilon)}=\epsilon\eta\cos\theta, \quad (36a)$$

$$\zeta_{t'}{}^{(\epsilon)}\approx\eta\sin\theta[\alpha^{(\epsilon)}(1-\varphi^2/2)-\gamma^{(\epsilon)}(E_\Lambda/M_\Lambda)\varphi], \quad (36b)$$

$$\zeta_l{}^{(\epsilon)}\approx\eta\sin\theta\{\alpha^{(\epsilon)}(E_\Sigma/M_\Sigma)\varphi-\gamma^{(\epsilon)}(M_\Lambda M_\Sigma)^{-1}\times[p_\Lambda{}^2(\varphi^2/2-\delta)+M_\Lambda{}^2(1-\varphi^2/2)]\}. \quad (36c)$$

Remarks:

(a) Strictly forward $\Lambda^0\to\Sigma^0$ conversion for purely transversal Λ^0 polarization:

$$\zeta_l{}^{(\epsilon)}=0,\quad \zeta_t{}^{(\epsilon)}=\epsilon\eta\cos\theta,\quad \zeta_{t'}{}^{(\epsilon)}=\epsilon\eta\sin\theta. \quad (37a,b,c)$$

(b) The Λ^0 polarization is collinear to $\mathbf{n}$ (unit vector orthogonal to the scattering plane). Then

$$\zeta_t{}^{(\epsilon)}=\epsilon\eta,\quad \zeta_{t'}{}^{(\epsilon)}=\zeta_l{}^{(\epsilon)}=0. \quad (38a,b,c)$$

(c) The purely transversal Λ^0 polarization is in the scattering plane. Then

$$\zeta_t{}^{(\epsilon)}=0,$$
$$\zeta_{t'}{}^{(\epsilon)}=\eta[\alpha^{(\epsilon)}(1-\varphi^2/2)-\gamma^{(\epsilon)}(E_\Lambda/M_\Lambda)\varphi], \quad (39a,b,c)$$
$$\zeta_l{}^{(\epsilon)}=\eta\{\alpha^{(\epsilon)}(E_\Sigma/M_\Sigma)\varphi-\gamma^{(\epsilon)}(M_\Lambda M_\Sigma)^{-1}\times[p_\Lambda{}^2(\varphi^2/2-\delta)+M_\Lambda{}^2(\delta-\varphi^2/2)]\}.$$

(d) Finally, we notice that from a formal point of view there is much analogy between the $\Lambda^0\to\Sigma^0$ Coulomb conversion we have considered and the OPEC (one-pion-exchange contribution) to the process $\Lambda^0+N\to\Sigma^0+N$. Since a π meson is a pseudoscalar particle, the application of Bohr's theorem to the vertex $\Lambda^0-\pi-\Sigma^0$ allows us to predict the following result: The components of the Λ^0 and Σ^0 polarization on any space-like four-vector orthogonal to the two-plane defined by the vertex $\Lambda^0-\pi-\Sigma^0$ are parallel or antiparallel according to odd or even $\Sigma^0-\Lambda^0$ relative parity, respectively (i.e., the opposite to the Coulomb photon exchange case).

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APPENDIX NOTATION

We use natural units: $\hbar=c=1$. The fundamental metric tensor is taken as $g^{00}=-g^{11}=-g^{22}=-g^{33}=1$; $g^{\mu\nu}=0$ if $\mu\neq\nu$. a^μ are the contravariant components of the 4-vector $\mathfrak{a}$. We use the scalar product notation: $\mathfrak{a}\cdot\mathfrak{b}=a^\mu b_\mu=a_\mu b^\mu=a^\mu b^\nu g_{\mu\nu}$. Latin indices run from 1 to 3; Greek indices run from 0 to 3. The symbol * means complex conjugate; † means Hermitian conjugate; $\boldsymbol{\tau}\cdot\mathbf{a}$ is a short hand for $\sum_i \tau^{(i)}a^{(i)}$; $\boldsymbol{\tau}$ are the usual Pauli matrices (with τ_3 diagonal); τ_0 is the 2-by-2 unit matrix; $\epsilon^{\lambda\mu\nu\rho}$ is the totally antisymmetric tensor of rank 4 ($\epsilon^{0123}=1$). Dirac's γ^μ matrices are defined by $\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=-2g^{\mu\nu}$ and we take $\sigma^{\mu\nu}=(2i)^{-1}[\gamma^\mu,\gamma^\nu]_-$. The adjoint spinor of u is $\bar{u}=u^\dagger\mathbf{A}$, where $\mathbf{A}$ is a matrix such that $(\mathbf{A}i\gamma^\mu)^\dagger\equiv\mathbf{A}i\gamma^\mu$.