Although the experimental pion angular and momentum distributions have not as yet been accurately determined, the known results[2] agree qualitatively with the prediction of the static model, that the positive meson prefers to go at right angles to the photon beam, with the energy of the 3-3 resonance. Precise measurements of these quantities will be of great interest to test the details of the theory to a greater extent than is possible from total cross-section data. In particular, a more precise treatment of the Bose symmetry of the final pions may be required than that given by Cutkosky and Zachariasen.[3] These authors treated the mesons differently and reinstated symmetry by symmetrizing their results. If one treats the pions in a symmetrical way throughout then one obtains instead a set of coupled integral equations depending on two variables.[12]

(New York) **14**, 229 (1961)]. He finds a very large enhancement that could account for the remaining discrepancy with plausible parameters. Unfortunately, his calculations do not go beyond 550 MeV. It is important to ascertain whether the rescattering term gets small again at higher energy. We have also computed the *interference* term between Figs. 1(a) and 1(b). For an effective $\gamma\pi\pi\pi$ coupling $\Lambda=5$ [J. S. Ball, Phys. Rev. **124**, 2014 (1961) gives $|\Lambda|\leq 1.8$] the cross section is increased by about 4 μb at 500 MeV and 10 μb at 700 MeV.

We conclude that specifically electromagnetic processes (with no counterpart in the reaction $\pi+N\rightarrow 2\pi+N$) are responsible for the major features of the low-energy behavior of the reaction $\gamma+p\rightarrow p+\pi^{+}+\pi^{-}$. In contrast to the case of single pion production, the second resonance in pion-nucleon scattering appears to have little effect on the production of π^{+}, π^{-} pairs. Further work is required to evaluate and understand this situation. Presently, the authors are attempting to see to what extent available data on double-pion photoproduction can be understood on the basis of the processes of Fig. 1.

The authors are indebted to M. Simmons for his careful numerical work.

[12] P. Carruthers (unpublished).

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Arrival Times of Air Shower Particles at Large Distances from the Axis*

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A study has been made of the relative times of arrival of shower particles at large distances (200 to 1500 m) from the shower axis. Data were obtained at the MIT Volcano Ranch Station, using an array of 20 scintillation detectors, one of which was shielded part of the time. The shower size, direction, and core location were determined for each event. We describe the spatial distribution of shower particles at a given instant by means of three curved surfaces: the median surface for the penetrating particles (muons), the median surface for the electrons, and the extreme front. We find that the average median surface for the muons is approximately spherical, the center being located at an atmospheric depth of 320±70 g cm^{-2}, and that the average median surface for the electrons has a radius of curvature of about 1 km at a distance from the axis of 450 m. The electron radius of curvature increases at greater distances. Assuming that the extreme front is spherical, its average center must be located above 320 g cm^{-2}. We measured the radius for curvature of the extreme front for a small number of individual showers, but were not able to improve upon that limit.

I. INTRODUCTION

IN 1950–1951 several attempts were made to detect differences in arrival time between the particles which make up extensive air showers.[1–3] The first positive result was obtained by Jelley and Whitehouse[4] who measured the delays between successive pulses produced by air showers in a single scintillation counter. They found "delayed particles" associated with 0.85% of the showers, and measured the time distribution of the delayed particles. Their work was extended by Officer and Eccles.[5,6]

A different approach was introduced by Bassi, Clark, and Rossi.[7] They used an array of three scintillation counters. Relative time delays were measured for several configurations of the array, with counter separations up to 30 m. Using statistical methods of analysis they found that at a given instant most shower electrons lie in a flat disk of thickness between 1 and 2 m, and they found a lower limit of 1300 m for the radius of

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curvature of the shower front. By shielding one of the detectors they found that the penetrating particles lie in a slightly thicker disk (2–3 m) which lags the electrons by no more than 3 m. Finally, they showed that it was possible to estimate the direction of individual shower axes from measurements of arrival time at different points of a counter array, and they measured the zenith angle distribution of the directions of air showers. An experiment by Sugarman and DeBenedetti confirmed that the electron and muon disks are both very thin and that they practically coincide, near the shower axis.[8]

Many subsequent experiments have made use of arrival time measurements for the determination of air shower directions.[9–13] In some of these the detector separations have been much greater than 30 m, and the showers studied have been much larger than 10^5–10^6 particles, the size range to which the results of Bassi *et al.* apply. While there is no doubt that the timing technique is successful under those changed circumstances, there has been no report of measurements of the distribution in arrival times of shower particles at large distances from the axis, as such. The Cornell group,[10] using an array of diameter 900 m, has noted that arrival times do not usually conform to a plane, when such large distances are involved, but they have expressed this fact by attributing to each event a shower-front radius of curvature, which they find to vary over a wide range from shower to shower.

The present study was part of a program of measurements using a detector array of diameter 1770 m. Some of our results on the arrival time distribution have been mentioned in a preliminary publication.[14] Since the detector spacing (442 m) was so much greater than that used by Bassi *et al*, and because we intended to increase the spacing still further (and have done so, by a factor of two), it is clear that we needed the information for practical purposes. There are three particular points which the experimental design needs to take proper account of the distribution in arrival times:

(1) Choice of a characteristic feature of the detector pulse to use in finding shower directions. This could correspond to the mean arrival time, the time of arrival of the first particle, etc.

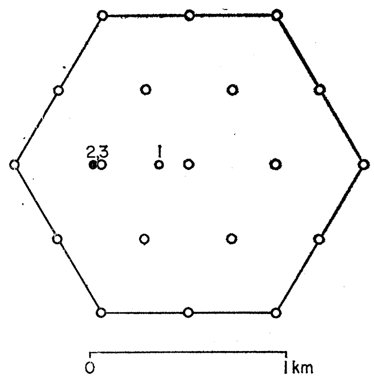

FIG. 1. Plan of the detector array during 1959–1960. Locations of the twentieth detector during runs 1, 2, and 3 are indicated.

(2) Choice of which pulses are to be used for finding the direction of a shower which has struck an array of detectors. (More generally, choice of the best weighting function for the various arrival time measurements.)

(3) Design of the equipment to register shower densities so as not to exclude any important fraction of the shower particles in case some were delayed with respect to others.

In addition, the arrival time distribution provides information on the structure of air showers. One can estimate the heights at which certain processes begin to occur and cease to occur.

II. METHOD

The MIT Volcano Ranch station is located near Albuquerque, New Mexico, at an elevation of 1800 m. An array of scintillation detectors is used to detect and measure air showers by the same general technique used in the earlier MIT Agassiz experiment. The present results are based on data obtained in 1959–1960 when the plan of the array was that shown in Fig. 1. The main array was made up of 19 detectors arranged in a pattern of triangles. We can divide the total period of operation (7 months) into three runs, according to the manner in which the 20th detector was employed. During the first run (3 months) it was located 150 m from the center, and its purpose was to give an unbiased measurement of the lateral distribution of particle density. During the second and third runs it was located adjacent to a normal detector, 442 m from the center. During the second run (1 week) it was unshielded. During the third (4 months) it was shielded by 10 cm of lead. The 20th detector and its associated electronic system were identical in construction to the other 19. The data from that detector were given identical treatment to data from the main detectors, with the following exceptions: (1) pulses were not used for detecting showers, (2) pulse times and amplitudes were not used for calculating shower direction, size, or core location.

Figure 2 shows the construction of a detector and of the shield used with the 20th detector during the third

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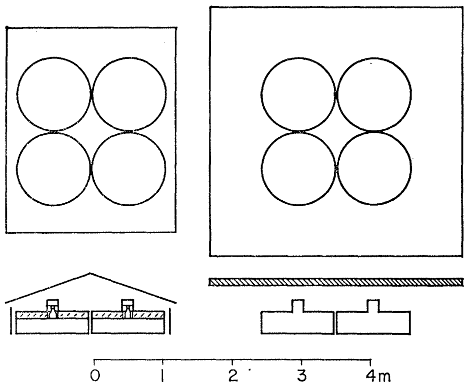

FIG. 2. Construction of a normal and of a shielded detector.

run. The area of each detector is 3.26 m^2. The four scintillator units that make up a detector operate in parallel.

Air shower events were selected and recorded by essentially the same means used in the Agassiz experiment. Signals from the various detectors were carried by coaxial cables to a central location where they were amplified and displayed on separate cathode-ray tubes. For the sake of symmetry, the coaxial cables were of equal length, regardless of the fact that some detectors were less distant than others. As in the Agassiz experiment, we employed Kraushaar's method to obtain four linear amplitude ranges, separated in time. The over-all delay of each signal, including the time required for transmission to the center and all incidental delays involved in the electronic system, was about 12 μ sec. These delays were adjusted to be equal, for the various channels, to within ± 0.01 μsec. Systematic checks during operation showed that the delays remained equal, within that tolerance. The detectors were located in a common horizontal plane, within ± 0.01 light μsec (rms deviation). The over-all rise time was about 0.25 μsec. The pulses were shaped by delay line clipping. The integration time was 1 μsec. The pulses produced by showers were photographed, and the photographs were projected for measurement. The time of each pulse was measured from a reference signal, common to all channels, to the time of first appearance of the pulse.

A preliminary study of the first showers that were recorded showed that the arrival times of large pulses correspond closely to a plane, whereas small pulses may have erratic delays, with respect to that plane, which are much larger than the errors of measurement. It was also noted that the small pulses frequently were "distorted"; that is, the pulse shapes showed erratic departures from the shape characteristic of a delta-function input. It seemed clear that, for distances as large as those which concerned us, the arrival time distribution of the individual shower particles is orders of magnitude broader than it is near the axis. Since the quantity we measure corresponds to the arrival time of the first particle, the measured times would fluctuate much less for many-particle pulses than for pulses corresponding to one or two particles.

As a result of our preliminary study, we adopted the following scheme for determining shower directions:

(1) In all cases the pulse times were used to determine a plane, which we call the "shower plane," and the direction of the shower was taken to be normal to this plane.

(2) The electronic triggering requirement insured that there were at least three pulses per shower, each of which corresponded to at least ten particles. In some cases (the smallest showers we recorded) all of the remaining pulses were smaller than three particles. In such cases the shower plane was calculated from the times of the three largest pulses.

(3) If there were four to seven pulses larger than three particles, the shower plane was fitted to their times by the method of least squares.

(4) If there were more than seven pulses larger than three particles, the times of the seven largest pulses were used.

(5) For a few of the largest showers all or nearly all of the detectors were struck by three or more particles. In some such cases we redetermined the shower plane using all of these pulses, as a special test.

In effect, we weighted the pulse times according to the numbers of particles that produced them. The weight was set equal to one if the number of particles was three or more, and zero if the number was one or two. The additional requirement that no more than seven times be used for routine calculations had the effect of eliminating data from distances larger than at most 800 m, regardless of pulse size.

The calculations of shower direction (zenith angle θ, azimuth angle φ), shower size (N), and core location were performed by an electronic computer. The computer also calculated, for each shower, the following quantities of interest for this paper:

(1) $D = t_{\text{obs}} - t_{\text{calc}}$, the apparent delay in μsec of each observed pulse time with respect to the shower plane,

(2) R, the perpendicular distance in meters of each detector from the shower axis,

(3) $\sigma = [(q-3)^{-1} \sum_q D^2]^{1/2}$, the standard deviation of the q delay times used in finding the shower plane.

III. ERRORS

We begin by defining two characteristics of an air shower, the "extreme front" and the "tangent plane", which we can determine, approximately, from our measurements. By "extreme front" we mean a surface drawn at a given instant through the particles which, at that instant, have traveled furthest in the forward direction, at various distances from the axis. If we assume that secondaries from the first interaction (or they, together with *their* secondaries) can travel in nearly straight lines with nearly the speed of light, then the extreme front will be nearly spherical with a radius of curvature equal to the distance of this front from the location of the first interaction. By "tangent plane" we mean the plane tangent to the extreme front and perpendicular to the shower axis.[15]

For the purposes of our study we need a reference time with which to compare individual observed pulse times. We use the quantity t_{calc}, which is an approximation to the time t at which the tangent plane passed through the point in question. We need to estimate the error of t_{calc} with respect to t. That error will consist of three parts, corresponding to (1) a systematic displacement of the shower plane with respect to the tangent plane near the shower axis, (2) a random displacement near the axis, and (3) a random displacement, resulting from errors in shower direction, which increases with distance from the axis. We will estimate each of the three.

The deviation of an individual time measurement from t will be partly instrumental and partly the result of fluctuations in the shower. To estimate the instrumental error we selected all events in the second and third runs in which the pulses from the twentieth detector and the adjacent normal detector corresponded to 10 particles or more, and compared the observed times for the two detectors. The 171 time differences, $D_{20}-D_{adj}$, have a standard deviation of 0.086 ± 0.007 μsec. Most of the measurements were at distances of about 200 m from the shower axis. Because the distances were small and the pulses were large, we consider that fluctuations were negligible and the dispersion was almost entirely instrumental. We obtain the value 0.061 μsec for the instrumental standard deviation of a single time measurement.

For the same 171 events we determined the standard deviation of D_{20}, the difference between the observed time for the twentieth detector and the time corresponding to the shower plane. We obtained the value 0.081 ± 0.007 μsec. From that result and the value 0.061 μsec for the instrumental error of the observed times we conclude that the standard deviation of the random error of t_{calc} with respect to t near the axis is about 0.053 μsec.

The mean values $\langle D_{20}\rangle_{av}$ and $\langle D_{adj}\rangle_{av}$ were -0.032 ±0.007 and -0.049 ± 0.007 μsec, respectively. The difference between the two mean values is hardly significant, and is in the range expected for *systematic* instrumental differences between detectors. The fact that both of the mean values are about -0.04 μsec, rather than zero, is probably significant. Many of the times used for determining shower planes correspond to pulses produced by 3 to 10 particles, i.e., to smaller pulses than those used in finding $\langle D_{20}\rangle_{av}$ and $\langle D_{adj}\rangle_{av}$. We interpret the value -0.04 μsec to mean that on the average our shower planes lag behind the tangent planes by about 0.04 μsec, which corresponds to 12 m.

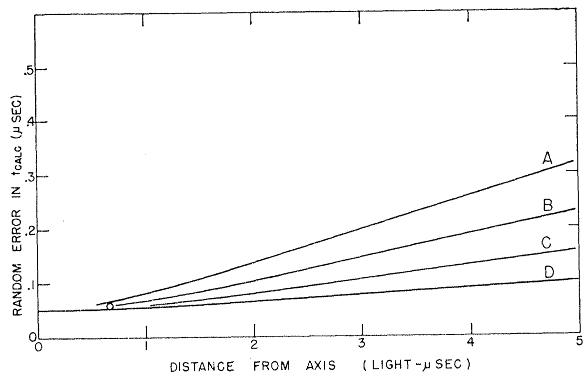

FIG. 3. Expected random error in t_{calc} with respect to t. Curves A, B, and C apply to showers whose directions were determined by the routine method, using no more than seven pulse times. Curve A holds for $N=1$ to 2×10^7; curve B, for $N=2$ to 5×10^7; and curve C, for $N>5\times10^7$ particles. Curve D applies to the events of Table III.

We wish next to estimate the error in our determinations of shower direction. The uncertainty in direction will depend on shower size since the amount of useful time data increases with increasing size. The random instrumental errors discussed above are partly responsible for the errors in direction but fluctuations in arrival time are also significant.

To choose the most unfavorable case in which it was possible to make an estimate, we selected all showers in which the number of times used for finding the direction was four, one more than the minimum number. In 80% of the 153 cases, the smallest of the four pulses corresponded to 3 to 6 particles; that is, belonged to the class for which arrival time fluctuations were expected to be especially noticeable. We computed the over-all standard deviation

$$[153^{-1}\sum D^2]^{1/2},$$

and obtained the value 0.110 ± 0.009 μsec. The size of the showers was about 2×10^7 particles.

To make a similar test for larger showers, we chose cases in which seven times had been used to find the shower direction, and all of the pulses corresponded to six or more particles. We had recorded about 500 showers of this type, so we used only the first 100 and the last 100. The over-all standard deviation was 0.111 ± 0.008 μsec. The size of the showers was about 10^8 particles.

Knowing the standard deviation of the individual time measurements used for finding the shower direction, we can calculate the errors in shower direction, for various cases. We find that for the smallest showers, where only three times were used, the root-mean-square error angle is 7.5°. For showers large enough so that seven times were used, we find that the error is

[15] The definitions are necessarily imprecise for an individual shower because the number of particles is finite. However, we believe that the characteristics are sufficiently well defined to be useful for showers as large as those which concern us. Rigorous definitions can be given as limits for an ensemble of showers produced by primaries with the same energy and direction which begin to develop at the same depth in the atmosphere.

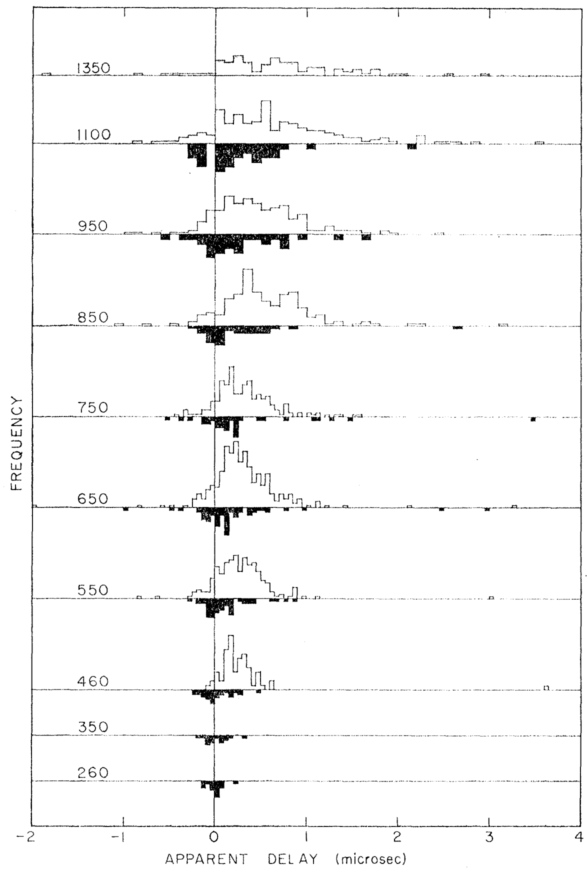

FIG. 4. Single-particle delay histograms. Distance in meters from the shower axis is given at the left. The filled-in histograms are for muons. They are shown inverted beneath the corresponding histograms for electrons plus muons, obtained using unshielded detectors There were no data for muons at 1350 m, and there were no .data from unshielded detectors at 260 m or 350 m. In each case the normalization of the upper histogram is arbitrary. If there is a corresponding muon histogram it has been normalized so that there is approximate agreement with the proportion of muons as given by an independent measurement [See Eq. (1)].

3.5°. For the largest showers, we find that the directions obtained by making use of all of the arrival time information have an expected error of about 2°.

The values given above apply to vertical showers. For moderately inclined showers an approximate correction is given by multiplying those values by secθ, where θ is the zenith angle. However, we are going to show that arrival time fluctuations are smaller for showers which have large inclinations, so that the errors in direction may be nearly independent of zenith angle. It may also be noted that for most purposes errors in azimuth are unimportant. The expected errors in zenith angle are, of course, less than the values given above, in most cases.

Having estimated the errors in direction and the random error in t_{calc} near the axis, we can make a superposition and find the standard random error in t_{calc} with respect to t for all distances. Figure 3 shows the result for several classes of showers, from the smallest to the largest that we may deal with.

TABLE I. Single-particle delays, in μsec, with respect to the tangent plane, for various distances from the shower axis.

Distance (m)	Shielded detector Mean	Shielded detector Median	Unshielded detector $\theta<45°$ Mean	Unshielded detector $\theta<45°$ Median	Unshielded detector $\theta>45°$ Mean
260	0.03 ± 0.02	0.04 ± 0.03	...	...	..
350	0.04 ± 0.03	0.03 ± 0.04	...	...	0.16 ± 0.09
460	0.05 ± 0.04	0.02 ± 0.05	0.27 ± 0.02	0.26 ± 0.03	0.19 ± 0.05
550	0.14 ± 0.03	0.07 ± 0.04	0.30 ± 0.02	0.29 ± 0.03	0.18 ± 0.05
650	0.14 ± 0.03	0.13 ± 0.04	0.34 ± 0.02	0.31 ± 0.03	0.28 ± 0.07
750	0.26 ± 0.10	0.14 ± 0.10	0.36 ± 0.02	0.32 ± 0.03	0.26 ± 0.08
850	0.20 ± 0.04	0.10 ± 0.08	0.56 ± 0.04	0.50 ± 0.05	0.32 ± 0.08
950	0.30 ± 0.08	0.18 ± 0.10	0.51 ± 0.04	0.45 ± 0.05	...
1100	0.38 ± 0.07	0.20 ± 0.10	0.70 ± 0.05	0.58 ± 0.06	0.38 ± 0.07
1350	...	...	0.69 ± 0.08	0.64 ± 0.10	...

IV. RESULTS

The large pulses were used for finding the shower plane, so as to minimize the effect of fluctuations. To investigate the fluctuations themselves we made use of the smallest pulses, corresponding to one particle. Measurements of $t_{\text{obs}}-t_{\text{calc}}$ were sorted according to distance from the shower axis. For each distance interval we plotted the frequency distribution of the apparent delays. We then found the mean and the median of each distribution. This was done separately for pulses from the normal, unshielded detectors and the shielded detector. For the unshielded detectors the data were also separated into two classes according to the zenith angle of the showers. Two of the three sets of frequency distributions are shown in Fig. 4. The mean and median values are given in Table I. Values in the table have been corrected for the systematic displacement between shower plane and tangent plane.

Figure 4 and Table I summarize most of our direct experimental results. We will give a few more details on how the data were selected. Then we will present some quantities derived from Fig. 4 and Table I, and we will give an interpretation of our results. Finally, we will discuss our evidence in relation to other investigations of extensive air showers.

In obtaining the results for penetrating particles the only requirement was that the observed pulse height should correspond to less than 1.5 particles per detector area. All of the showers recorded in the third run were included, without regard to shower size or zenith angle. In obtaining the results for shower particles, penetrating or not, there was an additional requirement. Not all of the showers were used, but only those which had been scanned "with special care." Also, as mentioned above, the showers were separated into two classes according to the zenith angle.

We made the requirement of special care in scanning because we wished to find delay distributions for *single particles*. For the detectors that make up the main array, the average deflection produced on one of the cathode-ray tubes by a single particle is only one millimeter. This is well above the noise level, and such pulses can be counted with nearly 100% efficiency if the scanning is done with care and with that intention. Otherwise the efficiency might be only 50%, and the apparent "single-particle" pulses might be heavily contaminated with pulses produced by two or more particles. Several sections of record, containing about 400 showers in all, were scanned with special care in this regard, and 1500 "single-particle" pulses were observed. For the shielded detector we used greater amplification. The deflection produced by an average single particle was 3.5 mm, so no special pains were needed to insure 100% scanning efficiency. Of course, some of the "single-particle" pulses were undoubtedly produced by two particles, which were not resolved. However, studies of the equipment and of the pulse-height distributions for shower particles and for cosmic-ray muons make us confident that the amount of contamination is of the order of 10 or 20%, so that we make no serious error in calling our results "arrival time distributions for single particles." The inclusion of two- or three-particle pulses will, of course, cause the arrival time distributions to be less broad than for a pure sample of single particles.[16] We should note that, because single particles produce such small deflections, the instrumental error in measuring their times is probably about 0.10 μsec, or twice as great as for large pulses.

The data from unshielded detectors come from showers which are nearly all in the size range $10^7 < N < 10^8$. The median size is about 2×10^7 particles. The data from the shielded detector apply to showers which are about twice as large, on the average. We consider it quite unlikely, *a priori*, that the phenomena under study depend significantly on shower size, over any range as small as a few decades. We have made a number of small-scale tests, one of which will be mentioned later, which support this view, but we have not made a detailed search for possible size-dependence.

TABLE II. Median delay of electrons at various distances from the shower axis.

Distance (m)	Delay (μ sec)
460	0.32±0.04
550	0.37±0.04
650	0.36±0.04
750	0.42±0.04
850	0.71±0.06
950	0.53±0.06
1100	0.85±0.08

Returning to Fig. 4 and Table I, the first striking feature is that penetrating particles have considerably smaller delays, on the average, than shower particles as a whole. Considering the range of distances with which we deal, it is safe to assume that virtually all of the penetrating particles are muons, so we will say that the shielded data refer to muons. The unshielded data refer to a mixture of muons and electrons. We have measured the proportion of muons to shower particles, a quantity which we call k. Our preliminary result for vertical showers can be represented by the following empirical formula:

$$k = 0.061(R/100)^{0.74}, \qquad 80 < R < 1000 \qquad (1)$$

where R is distance in meters from the shower axis.[17] If one normalizes the unshielded distributions (for showers with $\theta < 45)^\circ$ to unity and the shielded distributions to $k(R)$, one can obtain by subtraction the arrival time distributions for *electrons*. We have done this, and we give in Table II the median delay for individual electrons as a function of distance from the shower axis.

We expect that the proportion of muons should be greater for inclined than for vertical showers. Comparisons of particle densities registered by our shielded and unshielded detectors agree qualitatively with this expectation, but the data are scanty. As yet we can give no formula analogous to Eq. (1) which would apply to showers with large zenith angles. It may be interesting to note in this connection that the unshielded delay distributions for showers with $\theta > 45^\circ$ are intermediate between those for vertical showers (unshielded) and those for muons. The comparison suggests that for distances greater than about 700 m the inclined showers consist mainly of muons. At smaller distances an increasing admixture of electrons is indicated. The decrease in arrival time fluctuations for very inclined showers tends to offset the reduction in baseline which would otherwise lead to reduced accuracy in measurements of the shower direction for such events.

[16] It is a simple matter to synthesize multiple-particle arrival time distributions when the parent single-particle distribution is known, but to accomplish the reverse is not easy. The detectors used by Bassi *et al.* (see reference 7) did not have adequate pulse-height resolution to permit separating single and multiple-particle pulses on that basis. Those authors illustrated the manner in which multiple-particle contamination might influence their results, but they did not give a numerical estimate of the net uncertainty. Inspection of their experimental arrangement suggests that one should interpret their claim, "our method of selecting showers insures that the average number of traversals is close to one," to mean that the average number was two or three, in which case the reported dispersions would be significantly smaller than the dispersions for single electrons. Sugarman and DeBenedetti (see reference 8) took the useful precaution of masking down the area of one of their detectors so that, during their Run III, 80% of the pulses were expected to correspond to single electrons. They found that the standard deviation of the pulse time distribution was appreciably greater than for a previous run in which only one third of the pulses were produced by single electrons. For our detectors, vertical cosmic-ray muons give a pulse-height distribution which has a relative width at half-maximum equal to 0.5, so pulse-height discrimination should be effective in distinguishing between single- and multiple-particle pulses. For a test, we compared the observed number of particles to the expected number, which is given by the computer. For 90% of the "single-particle" pulses, the expected number of particles was less than one. It was practically never greater than two. For most of the distributions of Fig. 4 the contamination must be nil. For the distributions nearest the shower axis it could be as great as 20%.

[17] J. Linsley, L. Scarsi, and B. Rossi, J. Phys. Soc. Japan **17**, Suppl. A-III, 91 (1962).

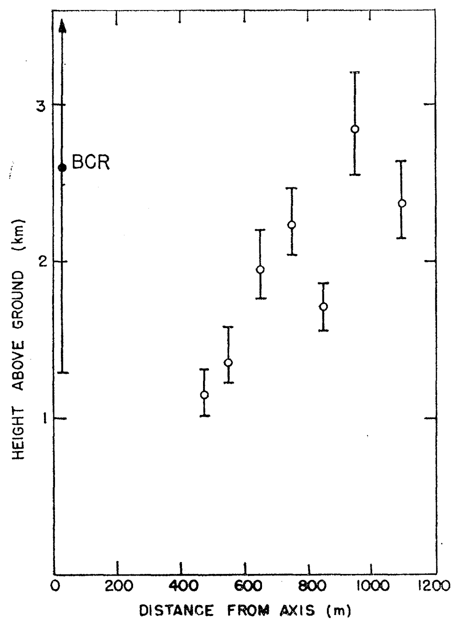

FIG. 5. Location of the median source for electrons observed at various distances from the shower axis. The point marked BCR represents the sea-level measurement of Bassi, *et al.* (see reference 7).

The remainder of this section will be concerned with the median delays for muons and electrons, separately. We have chosen to deal with median rather than average delays because the median, as a statistic, has simpler transformation properties. It can be seen in Table I that the quantitative differences between medians and averages would be insignificant for most purposes, except possibly for the muons.

Previous investigators[7,10] have described departures from the "disk" of Bassi *et al.* by ascribing a radius of curvature to the shower front. That mode of description assumes that one is dealing, in effect, with a point source. In fact, we know that shower particles originate from something more like a line source, and the model of a line source accounts in a natural way for the observation that, *in individual events*, particles arrive at a given (large) distance from the axis with widely varying time delays. Nevertheless, "radius of curvature" can be useful if, for example, we associate it with the median delay, and if we allow it to vary with distance from the shower axis to the point of observation. In that case it serves to locate the median source from which particles observed at that distance were emitted. We calculated the radius of curvature corresponding to median delays at various distances, for electrons and for muons. The results for electrons are shown in Fig. 5. Beyond 500 m there appears to be a linear increase in source height with distance. The rate of increase would correspond to a characteristic angle of emission of about 23°. We also show the result of Bassi *et al.* for essentially zero distance from the axis. The simplest hypothesis by which their result and ours could be reconciled is that the height of the median source is approximately constant and equal to 1 km for the electrons that are observed at distances 0 to about 500 m from the axis. Do the two regions correspond to two different mechanisms for transporting the electrons to points off the axis? Also, it is not clear how the results might vary with elevation. To be noncommittal we have expressed distances in meters. Possibly, if Molière units were used, the results would be valid for elevations other than that of Volcano Ranch.

The results for muons are shown in Fig. 6. In this case it seemed appropriate to express heights in units of g cm^{-2}, measured from the top of the atmosphere. The average inclination of the showers (about 25°) was taken into account. Although there is an indication that muons observed farther from the axis tend to originate at greater heights, all the measurements are consistent with a rather well-defined source region with the median source at an atmospheric depth of 320 ± 70 g cm^{-2}. Thus, half of the muons appear to originate in the first few generations of the nucleonic cascade.

The preceding results were obtained by averaging together data from large numbers of showers. In that way we could measure median and average delays with better precision than we could measure individual pulse times, provided that the errors in individual measurements are symmetrical, which we believe to be the case. However, we would also like to measure the delays, with respect to a plane, of the very first particles to arrive at large distances; that is, we would like to measure the curvature of the extreme front. To do this using the type of data shown in Fig. 4 would require much larger samples and much more detailed knowledge of the instrumental resolution. We can say, of course,

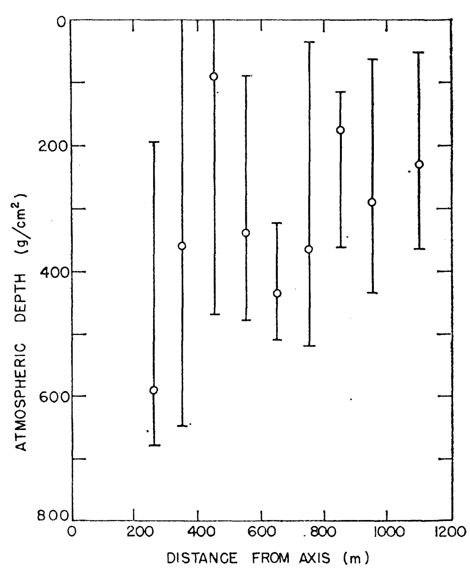

FIG. 6. Location of the median source for muons observed at various distances from the shower axis.

TABLE III. Source locations corresponding to the extreme front, for individual events.

Serial number	Size	Zenith angle	Source location (g/cm^2) Most probable[a]	Limit[b]
29283	2.0×10^9	48°	0	160
23020	1.2	20°	90	370
27288	1.1	38°	150	450
24518	1.0	29°	330	540
25479	1.0	15°	210	500
27237	0.8	28°	490	640

[a] In some cases the most probable value is rather well defined. In others it is not.
[b] Locations at a greater depth in the atmosphere are ruled out with 90% confidence. In none of the cases could a source location at zero depth be ruled out with 90% confidence.

that the source of the particles in the extreme front must be located at an atmospheric depth less than 320 g/cm^2, the value that applies to the *median* source for the muons.

To see if we could do better than that, we selected the most favorable cases at our disposal and found, for those individual showers, source locations corresponding to the earliest detected particles. The showers had sizes ranging from 8×10^8 to 2×10^9 particles and were the six largest which struck inside the array boundary during the period to which this paper refers. For each event we fitted the pulse times to a sphere, by a least-squares method, and computed the location of the center of the sphere. We also found 90% confidence limits for each source location measurement. The results are shown in Table III.

The average source for the six events is located about 220 g/cm^2 below the top of the atmosphere. Although, in view of the large errors, the six measurements are consistent with a "unique" source position (one that is fairly well localized with respect to the position of the first interaction), the best values vary over a wide range. If one were to accept these differences at face value, one might infer that there are large intrinsic fluctuations in the nuclear processes involved. The differences are too large to represent, directly, fluctuations in the height of the first interaction. However, fluctuations in the height of the first interaction could have an indirect influence on the early stages of muon development because of the density effect. If, by chance, the first interaction takes place very high in the atmosphere, pion decay is favored over interaction, so development of the muons might begin almost immediately. If the first interaction takes place at a lower level, development of the muons might not begin, effectively, until a still lower level, because most of the pions produced in the first few generations of the nuclear cascade might interact, rather than decay. By a similar argument, the height of origin of the first detectable muons might be greater, on the average, for inclined than for vertical showers. Also, the onset of muon production may be less well defined for the largest showers than for those which are produced by somewhat less energetic primary particles.

We emphasize that the source whose location one measures by this last technique is the source of the first *detectable* signal that a shower has begun to develop. Thus, until the desired limit has been reached, the result one obtains will depend on properties of the detecting system; in particular, on the area of the detectors. In the six cases of Table III we doubt that the limit was reached. For that reason, in addition to the largeness of the errors, we caution against interpreting Table III as evidence for or against (1) large intrinsic fluctuations, or (2) density effects such as those mentioned. The results prove that most showers of this size begin to develop rather high in the atmosphere, rarely lower than about 5 km, measured from sea level. They do not really add anything to the conclusions drawn from averaging together data on many showers, except that they apply to showers which are much larger.

V. DISCUSSION: THE PRESENT RESULTS IN RELATION TO PREVIOUS INVESTIGATIONS

Going first to the study by Jelley and Whitehouse[4] and later work on similar lines by Officer and Eccles,[5,6] it would appear that their "delayed particles" do not represent a distinct component of air showers, but were associated with the relatively few large showers which struck far from their equipment, in competition with many small showers which struck nearby.

The results of the Cornell group[10] represent measurements of the same type as those we give in Table III. It appears to us that the radii of curvature which one attempts to measure in this way are so large that substantial improvements in technique will be needed before one can obtain physically significant results. Until one has achieved adequate resolution, one's measurements will represent a limit which is essentially instrumental, and may differ from one experiment to another. We believe that this accounts for the difference between the Cornell and MIT results.

The rather large spread in arrival time of shower particles at distances greater than a Molière unit from the axis needs to be given consideration when one measures particle densities by means of proportional detectors, such as scintillation counters. In the case of the MIT Agassiz experiment,[9] the potential difficulties were not foreseen, and the electronic integration time was made quite small, about 0.1 μsec. In general, the pulse produced by a group of particles whose arrivals are distributed over a time interval Δt will be appreciably smaller than if they had arrived simultaneously, unless Δt is much smaller than the integration time employed. For small to moderate distances electrons predominate in air showers. In the present experiment, we find that the median radius of curvature for such distances is about 1 km, and that the spread in arrival time is about equal to the median delay. The Agassiz experiment was carried out at sea level. Let us assume that the electron radius of curvature scales with elevation in the same manner as the Moliere unit. Then at a distance from the axis equal to one Molière unit we expect a time spread of about 0.01 μsec, which is much smaller than 0.1 μsec. However, the lateral distribution measurements extended to about 400 m from the axis, or 5 Molière units. At the latter distance we expect the time spread to be about 0.3 μsec. The pulses observed at such great distances were small, and there is a tendency for fluctuations from the average distribution in arrival time to be beneficial. However, there is no doubt that for distances greater than $r\sim 1$ shower densities were underestimated, in that experiment, by an amount which increased with distance. We would expect errors of a few percent at $r=2$, 20 or 30% at $r=3$, and a factor of 2 at 4 or 5 Molière units. We must expect that the lateral distributions which were derived from the density measurements are too steep at large distances. Disagreement of the type expected is found, in fact, between the lateral distribution measured in the present experiment (integration time 1.0 μsec) and the one measured at Agassiz.[17]

When one goes to greater elevations, the integration time must be increased in order to avoid errors of this type. The assumptions used in analyzing the sea-level experiment imply that the spread in arrival time (in μsec) at a given distance from the axis (in Molière units) should increase with elevation in the same manner as the Molière unit (in meters). In the Bolivian-MIT El Alto experiment the integration time used at an elevation of 630 g cm^{-2} was the same as that used in the Agassiz experiment, and one would expect an increased distortion of the lateral distribution at large distances. In fact, the preliminary measured lateral distribution agrees with NKG(1.0) rather than NKG(1.3), which fitted the Agassiz measurements.[11,18,19]

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[19] *Note added in proof.* By NKG(s) we mean the Greisen approximation to the lateral distribution function calculated by Nishimura and Kamata, for age parameter s.

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$Al^{27}(p,3pn)Na^{24}/C^{12}(p,pn)C^{11}$ Cross-Section Ratio in the GeV Region*

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The $Al^{27}(p,3pn)Na^{24}/C^{12}(p,pn)C^{11}$ cross-section ratio has been measured at nine energies between 0.4 and 17 GeV. Corrections have been applied for the loss of C^{11} by diffusion from the thin plastic foils which were used, and the effects of this loss on existing data are discussed. From the measured ratios and published absolute cross sections for the $C^{12}(p,pn)C^{11}$ reaction, an excitation function for the standard monitor reaction, $Al^{27}(p,3pn)Na^{24}$, was obtained. The cross section at 3 GeV was found to be 9.1±0.5 mb, lower than the previously accepted value of 10.5 mb. Cross sections relative to the $Al^{27}(p,3pn)Na^{24}$ cross section were also obtained for the production of Na^{22}, F^{18}, N^{13}, C^{11}, and Be^7 from aluminum and for Be^7 from carbon in this energy region. These cross sections are essentially independent of energy between 6 and 28 GeV. An exception is Be^7 production from aluminum, which increases slightly between 3 and 28 GeV.

INTRODUCTION

THE first attempts[1,2] to obtain absolute activation cross sections in the GeV region utilized an indirect method to obtain the $Al^{27}(p,3pn)Na^{24}$ cross section. It was found to be 8.8 mb at 2.2 GeV and 8.1 mb at 3 GeV, indicating decreases from the value of 10.8 mb at 0.45 GeV[3] which had been used for normalization. When direct measurements of the absolute[4] $\sigma_C(C^{11})$ became available at 2 and 3 GeV [5] and at 3, 4.5, and 6

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[4] We will use subsequently the notation $\sigma_X(Y)$ to denote the cross section for producing Y from target X.

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