Photoproduction of Pion Pairs near Threshold

P. CARRUTHERS*
Department of Physics, and Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York

AND

HOW-SEN WONG†
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

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The principal features of the reaction $\gamma+p \rightarrow p+\pi^++\pi^-$ in the photon lab energy range 300 to 800 MeV are shown to be accounted for quantitatively by the interaction current. The dominant process is the production of the 3–3 isobar along with an S-wave pion.

IN the study of the higher resonances occurring in pion-nucleon scattering, it has been customary to transcribe freely information from photoproduction ($\gamma+p \rightarrow N+\pi$) to the case of elastic scattering ($\pi+N \rightarrow \pi+N$). Despite the loss of Watson's theorem at such high energy, this practice has been reasonably successful when obvious differences between the two reactions are taken into account.[1] However, if one considers the production of pion pairs the correspondence between photon-induced and pion-induced reactions is by no means so complete. One of the most interesting (and hitherto unexplained) results obtained thus far is the behavior of the cross section for the reaction $\gamma+p \rightarrow p+\pi^++\pi^-$ in the energy range 400–800 MeV photon lab energy.[2] The cross section is observed to rise abruptly at a photon energy of about 500 MeV. At the energy of the second resonance ($E_\gamma=750$ MeV) the cross section is already decreasing. Such behavior is in distinct contrast to that of the reaction $\pi+N \rightarrow 2\pi+N$. (The shape of the inelastic pion-nucleon cross section mimics that of the elastic scattering.) The purpose of this paper is to point out that dominant features of the observed data may be explained by means of a very simple model in which the "interaction current" produces the 3-3 pion-nucleon isobar along with an S-wave recoil pion.

The proposed process is very similar to the electric dipole S-wave production of positive pions in the reaction $\gamma+p \rightarrow n+\pi^+$ near threshold. In the present case one produces the 3-3 isobar rather than a neutron. The abrupt rise in $\sigma(\gamma+p \rightarrow p+\pi^++\pi^-)$ is thus a consequence of the S-wave character of the extra pion. We note further that the threshold for producing a *real* 3-3 isobar and one pion is at 545-MeV photon energy, very near the observed jump in the cross section.

The low energy of the $\pi^+\pi^-p$ system near threshold suggests that one may safely utilize the approximations of the static theory. As a matter of fact, the process $\gamma+p \rightarrow N+2\pi$ was studied within this context several years ago by Cutkosky and Zachariasen.[3] In their approximation there are two contributions, due to the meson current and the interaction current. The meson current gives rise to two terms, which combine to give what has lately been called the Drell term[4] [Fig. 1(a)]. (So far as we know the static theory provides the only evidence that the meson current contribution is proportional to the 3-3 scattering amplitude when the exchanged meson is away from the pole.[5]) The other contribution, due to the interaction current, gives rise to the contribution shown in Fig. 1(b). The so-called nucleon current does not contribute in this approximation. As remarked by Cutkosky and Zachariasen, the interaction current dominates in the low-energy range. Near one BeV the meson current contribution is comparable to that of the interaction current. Although the static model becomes rather suspect at such energies. Itabashi found[6] that the Cal. Tech. data[7] for $\gamma+p \rightarrow p+\pi^++\pi^-$ at 1.2 BeV can be understood by means of the calculation of reference 3. However, the experiment of McLeod, Richert, and Silverman[8] shows that at this

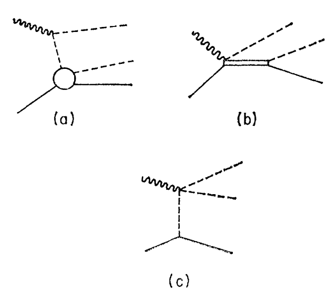

FIG. 1. The contributions of the meson current, the interaction current, and the one-pion exchange term are shown in Figs. (a), (b), (c), respectively.

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[3] R. E. Cutkosky and F. Zachariasen, Phys. Rev. **103**, 1108 (1956).
[4] S. D. Drell, Phys. Rev. Letters **5**, 278 (1960).
[5] The Feynman propagator for the exchanged pion corresponds, of course, to two time-ordered processes. One of these corresponds to the emission of two pions by the nucleon (one in the 3–3 state), the other to the argument given by Drell (see reference 4). The off-shell corrections given by the static theory are no doubt compensated for by the sacrifice of kinematical accuracy.
[6] K. Itabashi, Phys. Rev. **123**, 2157 (1961).
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energy there are also significant contributions from final states in which the ρ resonance is produced. The simplest process leading to the ρ is the one-pion exchange term of Fig. 1(c). Boccaletti and Selleri[9] recently found the processes of Figs. 1(a) and 1(c) to be negligible compared to the experimental value in the energy range of interest to us, 300–700 MeV.

From the results of Cutkosky and Zachariasen, we find the total cross section for the production of a doubly charged isobar and a recoil π^-:

$$\sigma_{+-}=\frac{\alpha}{\pi}\frac{1}{k(1+k/E_k)}\times\int_1^{\omega_{\max}} d\omega_+\, F^2(k_-)\sigma_{33}(\omega_+)(k_-/k_+), \quad (1)$$

where $k_\pm$, $\omega_\pm$ are the momenta and energies for the mesons of indicated charge, $\alpha=e^2/4\pi$ is 1/137, k is the c.m. photon energy, E_k is the c.m. energy of the initial proton, σ_{33} is the total 3-3 pion-nucleon cross section, and $F(k_-)$ is the factor (nearly unity)

$$F(q)=1-\left(\frac{\omega_q}{2k}-\frac{1}{4kq}\ln\frac{\omega_q+q}{\omega_q-q}\right). \quad (2)$$

The small second term in Eq. (2) arises from the S-wave projection of the meson current term. To compute the energy available to the recoil meson we take

$$W=M+\omega_++\omega_-, \quad (3)$$

where W is the total c.m. energy (including nucleon motion) $W=k+(k^2+M^2)^{1/2}$. M is the nucleon mass. Taking the pion mass to be unity, the maximum energy of the positive meson is $\omega_{\max}=W-M-1$. [σ_{+-} is measured in units of $(\hbar/\mu c)^2=20$ mb.] We have used the phase space appropriate for a static nucleon but retained the correct photon flux factor, in view of the numerical importance of the latter correction. Finally, in order to account for the case in which the π^- comes off in the 3-3 state we multiply Eq. (1) by 10/9 to obtain the total cross section.[3] This factor is included in the computed curves.

The presence of the sharply peaked function $\sigma_{33}(\omega)$ under the integral in Eq. (1) makes it clear that σ_{+-} is large only for $\omega_{\max}\gtrsim\omega_r$, the 3-3 resonance energy. For orientation and a quick estimate of the size of σ_{+-} let us use a delta-function approximation for the 3-3 resonance. Using the effective range formula of Chew and Low[10] one finds for a narrow width Γ:

$$\sigma_{33}(\omega)\approx(32\pi^2f^2/3)k\delta(\omega-\omega_r), \quad (4)$$

where $f^2=0.08$. Thus for $W>M+\omega_r+1$, Eq. (1) becomes

$$\sigma_{+-}\approx\frac{32\pi\alpha f^2}{3}\frac{k_-F^2(k_-)}{k(1+k/E_k)}. \quad (5)$$

[9] D. Boccaletti and F. Selleri, Nuovo cimento **12**, 1099 (1961).
[10] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

For the values $F\approx1$, $k_-\approx1$, $k\approx3$ we find $\sigma_{+-}\approx86$ μb, which is about the maximum value observed. Figure 2(a) shows 10/9 the right-hand side of Eq. (5) as a function of laboratory photon energy. The threshold behavior is, of course, distorted by the approximation of Eq. (4).

The total cross section obtained using the experimental value of σ_{33} (we used the Breit-Wigner form with parameters given by Gell-Mann and Watson[11]) is shown in Fig. 2(b). Although the computed value is slightly low near the peak region the combination of experimental uncertainity with the proverbial (and well deserved) difficulty in the calculation of production amplitudes make the resemblance between theory and experiment quite satisfactory. The excellent agreement near threshold, and the sudden rise (at the correct energy) to a value close to the experimental value are especially satisfying. It should be noted that it is not quite correct to compare the delta-function approximation, Eq. (4), with the more exact calculation, because the area under the experimental curve σ_{33} is rather less than the "area" associated with Eq. (4). If we adjust the strength of the delta function in (4) so that the areas coincide we obtain the curve Fig. 2(c) as the proper prediction for a very narrow 3-3 resonance. We doubt whether any conclusions can be drawn from the better agreement obtained with a narrow resonance for 500 MeV $<E_\gamma<$700 MeV. It appears more likely that any deficiency in cross section in this interval is due to other neglected processes leading to isobar formation.[12]

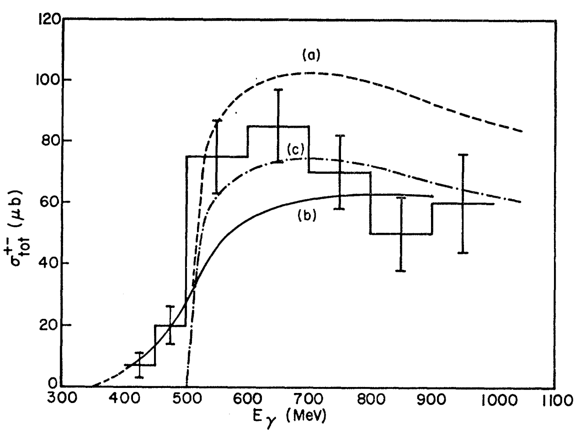

FIG. 2. The histogram is the cross section for the reaction $\gamma+p\rightarrow p+\pi^++\pi^-$ vs laboratory photon energy as measured by Chasan *et al.* (reference 2). Curves (a), (b), (c) result from various theoretical approximations. Curve (a) is derived from the delta-function approximation leading to Eq. (5). Curve (b) is obtained by numerical integration of Eq. (3) using the experimental 3–3 resonance cross section. Curve (c) is the delta-function approximation adjusted to give the same area for the 3–3 resonance as the experimental value.

[11] M. Gell-Mann and K. M. Watson, Ann. Rev. Nuclear Sci. **4**, 219 (1954).
[12] *Note added in proof.* M. Monda (to be published) has carried out a calculation similar to ours. He has also evaluated the rescattering of one of the final pions in Fig. 1(a) using methods similar to those developed by one of us [P. Carruthers, Ann. Phys.

Although the experimental pion angular and momentum distributions have not as yet been accurately determined, the known results[2] agree qualitatively with the prediction of the static model, that the positive meson prefers to go at right angles to the photon beam, with the energy of the 3-3 resonance. Precise measurements of these quantities will be of great interest to test the details of the theory to a greater extent than is possible from total cross-section data. In particular, a more precise treatment of the Bose symmetry of the final pions may be required than that given by Cutkosky and Zachariasen.[3] These authors treated the mesons differently and reinstated symmetry by symmetrizing their results. If one treats the pions in a symmetrical way throughout then one obtains instead a set of coupled integral equations depending on two variables.[12]

We conclude that specifically electromagnetic processes (with no counterpart in the reaction $\pi+N\rightarrow 2\pi+N$) are responsible for the major features of the low-energy behavior of the reaction $\gamma+p\rightarrow p+\pi^{+}+\pi^{-}$. In contrast to the case of single pion production, the second resonance in pion-nucleon scattering appears to have little effect on the production of π^{+}, π^{-} pairs. Further work is required to evaluate and understand this situation. Presently, the authors are attempting to see to what extent available data on double-pion photoproduction can be understood on the basis of the processes of Fig. 1.

The authors are indebted to M. Simmons for his careful numerical work.

(New York) **14**, 229 (1961)]. He finds a very large enhancement that could account for the remaining discrepancy with plausible parameters. Unfortunately, his calculations do not go beyond 550 MeV. It is important to ascertain whether the rescattering term gets small again at higher energy. We have also computed the *interference* term between Figs. 1(a) and 1(b). For an effective $\gamma\pi\pi\pi$ coupling $\Lambda=5$ [J. S. Ball, Phys. Rev. **124**, 2014 (1961) gives $|\Lambda|\leq 1.8$] the cross section is increased by about 4 μb at 500 MeV and 10 μb at 700 MeV.

[12] P. Carruthers (unpublished).

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Arrival Times of Air Shower Particles at Large Distances from the Axis*

JOHN LINSLEY AND LIVIO SCARSI†
Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts
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A study has been made of the relative times of arrival of shower particles at large distances (200 to 1500 m) from the shower axis. Data were obtained at the MIT Volcano Ranch Station, using an array of 20 scintillation detectors, one of which was shielded part of the time. The shower size, direction, and core location were determined for each event. We describe the spatial distribution of shower particles at a given instant by means of three curved surfaces: the median surface for the penetrating particles (muons), the median surface for the electrons, and the extreme front. We find that the average median surface for the muons is approximately spherical, the center being located at an atmospheric depth of 320 ± 70 g cm^{-2}, and that the average median surface for the electrons has a radius of curvature of about 1 km at a distance from the axis of 450 m. The electron radius of curvature increases at greater distances. Assuming that the extreme front is spherical, its average center must be located above 320 g cm^{-2}. We measured the radius for curvature of the extreme front for a small number of individual showers, but were not able to improve upon that limit.

I. INTRODUCTION

IN 1950–1951 several attempts were made to detect differences in arrival time between the particles which make up extensive air showers.[1–3] The first positive result was obtained by Jelley and Whitehouse[4] who measured the delays between successive pulses produced by air showers in a single scintillation counter. They found "delayed particles" associated with 0.85% of the showers, and measured the time distribution of the delayed particles. Their work was extended by Officer and Eccles.[5,6]

A different approach was introduced by Bassi, Clark, and Rossi.[7] They used an array of three scintillation counters. Relative time delays were measured for several configurations of the array, with counter separations up to 30 m. Using statistical methods of analysis they found that at a given instant most shower electrons lie in a flat disk of thickness between 1 and 2 m, and they found a lower limit of 1300 m for the radius of

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† Present address: Istituto Nazionale di Fisica Nucleare, Milan, Italy.
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