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Energy Distribution in Low-Energy Electron-Photon Showers in Lead Absorbers

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Detailed results for the energy distribution in electron-photon showers developing in lead absorbers are presented for the cases of primary electrons and photons of energy 50, 100, 200, 500, and 1000 MeV. These results show that Wilson's calculations on the same subject gave too low an energy degradation for electrons, thus making his electron numbers far too high.

The total track length of electrons of energy >10MeV for either electron or photon primaries is given by $T=0.032E_0$ radiation lengths with E_0 in MeV. It is also found that: (a) The average energy of the individual electrons lost because their energy is <10 MeV is 6.98 MeV; (b) the average energy of photons with $E<10$ MeV is 0.909 MeV; (c) the average energy of backscattered electrons with $E>10$ MeV is 14.0 MeV; (d) the average energy of backscattered photons with $E>10$ MeV is 13.1 MeV; (e) the average number of all photons produced per electron track length is 11.04.

I. INTRODUCTION

An extensive Monte Carlo program has recently been developed by Messel *et al.*[1] (hereafter referred to as MSVCB) for determining the radial and angular distribution of electrons and photons in electron-photon showers in various absorbers. In MSVCB results were presented for various electron radial and angular distribution functions for the cases of an electron primary energy of 1000 MeV and electron secondary energies than 10, 20, 50, 100, 200, and 500 MeV, with depths ranging from zero to 10 radiation lengths in lead and in emulsion absorbers. The authors took into account multiple scattering, collision losses, Compton effect, and Bethe-Heitler cross sections, accurate at low and high energies.

The above calculations for electron secondary energies greater than 10 MeV pointed to a considerable discrepancy between the only other comparable results put forward by Wilson[2] in 1952. In his paper, Wilson presented results of a Monte Carlo study of electron- and photon-initiated showers in lead for energies from 20 to 500 MeV. At that time, high-speed electronic digital computers were not generally available and Wilson used "a wheel of chance" for carrying out his Monte Carlo studies. His procedure was a simple graphical and mechanical one and necessitated making a number of approximations. One of these concerns the depth interval after which the wheel of chance is spun. Wilson divided the total depth of lead considered into intervals of 0.2 radiation length. He then followed electrons and photons through successive intervals, determining the fate of the particles in passing through the interval by the wheel of chance. Of course the distance that an electron or photon travels before an interaction depends upon its cross section which in turn depends critically upon the energy. It is thus quite possible that there is more than one interaction during the interval of 0.2 radiation length. In fact, the probability for the radiation of low-energy photons by electrons is almost unity for even much smaller intervals. It is thus to be expected that Wilson's results would give too small an energy degradation for the electrons into bremsstrahlung and consequently that his electron numbers would be too high. A further source of the same type of error in Wilson's results arises from his choice of 1 MeV for the lowest energy gamma ray emitted in the bremsstrahlung process. It is well known that the bremsstrahlung cross section becomes very large for the emission of low-energy quanta.

In view of the above we decided to modify the Monte Carlo program prepared by MSVCB in such a manner that we could display the energy degradation process in considerable detail for small showers developing in lead due to low-energy primary electrons and photons. Our method does not use the approximations made by Wilson and can be expected to give a more accurate picture of the showers' development.[3]

2. THE MONTE CARLO CALCULATIONS

The basic calculation was identical to that described in MSVCB and only differed in the information extracted. Though multiple scattering was included the lateral distribution of particles was ignored. For each depth interval we calculated the total energy and number of particles *lost* to the shower by each of the following ways: (a) electrons with $E<10$ MeV, (b) photons with $E<10$ MeV, (c) backscattered electrons with $E>10$ MeV, (d) backscattered photons with $E>10$ MeV.

The ionization loss and total length were calculated for all electrons until they were lost due to either (a) or

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[1] H. Messel, A. D. Smirnov, A. A. Varfolomeev, D. F. Crawford, and J. C. Butcher, J. Nuclear Phys. (to be published).

[2] R. R. Wilson, Phys. Rev. **86**, 261 (1952).

[3] *Note added in proof.* A private communication from Helen Thom points out an error in Wilson's paper. The curves labeled n_{st} are not the average number of electrons >8 MeV, but most likely (using his notation) $n_{st}+\langle\cos\theta\rangle_{av}n$ where $\theta=60°$. That is to say, these curves represent the total number of electrons which would be observed within 60° of the shower axis in a cloud chamber. Comparison by Miss Thom of our results with curves of secondary electrons $E>10$ MeV taken directly from Wilson's shower histories show good agreement.

TABLE I. Results for a lead absorber with a primary 50-MeV electron (1000 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.178	5.728	0.011	0.000				0.913	0.457	1.370	
	1.220	5.198	0.254	0.000	3.906	10.579	0.494	29.879	9.539	39.419	0.78839
1.0	0.391	5.105	0.031	0.000				0.684	0.590	1.274	
	2.899	4.538	0.442	0.000	3.550	11.429	0.458	16.471	11.524	27.995	0.55992
1.5	0.457	3.713	0.052	0.000				0.357	0.611	0.968	
	3.388	3.207	0.753	0.000	2.536	9.885	0.331	6.719	11.382	18.101	0.36204
2.0	0.355	1.753	0.030	0.001				0.141	0.560	0.701	
	2.611	1.362	0.393	0.011	1.230	5.607	0.162	2.269	10.214	12.483	0.24966
2.5	0.218	0.646	0.014	0.000				0.048	0.495	0.543	
	1.596	0.565	0.170	0.000	0.444	2.776	0.059	0.736	8.976	9.713	0.19426
3.0	0.167	0.338	0.003	0.000				0.036	0.418	0.454	
	1.102	0.301	0.037	0.000	0.238	1.679	0.031	0.537	7.491	8.028	0.16056
4.0	0.228	0.424	0.007	0.001				0.019	0.308	0.327	
	1.498	0.349	0.082	0.011	0.279	2.220	0.037	0.343	5.464	5.807	0.11614
5.0	0.164	0.352	0.008	0.000				0.021	0.221	0.242	
	1.071	0.252	0.107	0.000	0.260	1.689	0.034	0.326	3.789	4.115	0.08231
6.0	0.103	0.235	0.003	0.000				0.012	0.170	0.182	
	0.686	0.173	0.036	0.000	0.149	1.044	0.020	0.189	2.882	3.071	0.06142
7.0	0.095	0.196	0.003	0.000				0.012	0.123	0.135	
	0.598	0.164	0.035	0.000	0.124	0.920	0.016	0.206	1.943	2.149	0.04299
8.0	0.078	0.150	0.001	0.000				0.004	0.087	0.091	
	0.492	0.133	0.010	0.000	0.107	0.742	0.014	0.049	1.358	1.406	0.02812
9.0	0.040	0.038	0.002	0.000				0.003	0.066	0.069	
	0.249	0.032	0.021	0.000	0.033	0.336	0.004	0.041	1.029	1.070	0.02141
10.0	0.025	0.043	0.001	0.000				0.006	0.050	0.056	
	0.166	0.015	0.010	0.000	0.039	0.230	0.005	0.079	0.760	0.840	0.01679
Totals	2.500	18.721	0.166	0.002							
	17.576	16.291	2.351	0.022	12.897	49.137	1.665				

(c) above.[4] The energy and number of electrons and photons left in the cascade ($E>10$ MeV) were tabulated as a check on the calculation. The showers were tracked in a lead absorber until they had penetrated 10 radiation lengths.

3. RESULTS

The results for a lead absorber with electron and photon primary energies of 50, 100, 200, 500, and 1000 MeV are presented in Tables (I–X). For the case of 1000-MeV primaries 500 cascades were analyzed, for 500-MeV primaries 750 cascades were analyzed, and for each of the remaining primary energies 1000 cascades were followed.

The tables give the following information:

Column 1 gives the depth measured in radiation lengths in lead (1 radiation length = 0.51 cm = 5.82 g/cm^2).

Column 2 gives the average number of electrons (top line) with energies less than 10 MeV which were lost to the shower in the depth interval considered. The bottom line gives the average total energy carried away by these electrons. For instance, in Table I we see that in the interval from 0 to 0.5 radiation length 0.178 electrons, each with energy less than 10 MeV, were lost to the shower and that these electrons carried away an average total energy of 1.22 MeV.

Column 3 gives the same information for photons of energy less than 10 MeV that column 2 does for electrons. The last two lines in the column, to the right of the line heading of "total," give the totals for the particles in that column. For instance in the case of column 3, Table I, one sees that 18.721 photons of energy less than 10 MeV and carrying a total of energy 16.291 MeV were lost to the cascade in the interval 0 to 10 radiation lengths.

Column 4 concerns backscattered electrons and gives the average number of electrons which are backscattered and lost to the shower in the depth interval considered while they have energies greater than 10 MeV. From Table I one sees that 0.011 electrons carrying a total energy of 0.254 MeV were backscattered and lost in the depth interval from 0 to 0.5 radiation length.

Column 5 gives the same information for the photons.

Column 6 shows the total energy lost by electrons of energy greater than 10 MeV in the form of ionization in the depth interval concerned.

Column 7 shows the total energy lost by the processes given in columns 2–5 for the depth interval considered. Thus, in Table I one sees that from 0 to 0.5 radiation length an energy of 10.579 MeV is lost from the cascade.

Column 8 the electron track length is given for the electrons while their energies are greater than 10 MeV in the depth interval concerned.

[4] To avoid the infrared catastrophe an artificial cutoff was introduced in the bremsstrahlung cross section whenever the energy ratio of the secondary photon to the primary electron was less than 2^{-16} (1.5×10^{-5}). For lower ratios the cross section was held constant, independent of secondary energy, at a value that gives the correct total cross section.

TABLE II. Results for a lead absorber with a primary 50-MeV photon (1000 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.108	0.884	0.014	0.001				0.284	0.836	1.120	
	0.739	0.796	0.249	0.014	0.621	2.420	0.080	7.151	40.427	47.578	0.95159
1.0	0.249	2.089	0.021	0.000				0.365	0.740	1.105	
	1.818	1.823	0.319	0.000	1.440	5.399	0.187	8.828	33.350	42.178	0.84359
1.5	0.301	2.283	0.018	0.001				0.337	0.685	1.022	
	2.242	2.004	0.261	0.020	1.642	6.169	0.214	7.712	28.302	36.014	0.72030
2.0	0.300	2.224	0.024	0.000				0.299	0.607	0.906	
	2.236	1.895	0.301	0.000	1.534	5.966	0.200	6.797	23.245	30.042	0.60086
2.5	0.243	1.798	0.020	0.000				0.249	0.549	0.798	
	1.763	1.357	0.262	0.000	1.247	4.629	0.162	5.671	19.733	25.403	0.50808
3.0	0.216	1.461	0.023	0.000				0.212	0.488	0.700	
	1.547	1.150	0.283	0.000	1.076	4.056	0.140	4.782	16.555	21.337	0.42675
4.0	0.358	2.207	0.022	0.001				0.142	0.394	0.536	
	2.647	1.998	0.273	0.011	1.533	6.461	0.200	3.011	11.872	14.883	0.29767
5.0	0.281	1.361	0.017	0.001				0.091	0.310	0.401	
	1.981	1.170	0.233	0.013	0.936	4.333	0.122	1.937	8.602	10.539	0.21079
6.0	0.236	0.967	0.008	0.000				0.053	0.238	0.291	
	1.694	0.790	0.117	0.000	0.662	3.263	0.086	1.155	6.111	7.265	0.14531
7.0	0.162	0.589	0.009	0.000				0.033	0.186	0.219	
	1.078	0.513	0.100	0.000	0.421	2.112	0.055	0.711	4.433	5.143	0.10287
8.0	0.104	0.384	0.004	0.000				0.024	0.138	0.162	
	0.692	0.341	0.047	0.000	0.250	1.331	0.033	0.572	3.239	3.811	0.07622
9.0	0.087	0.331	0.004	0.000				0.026	0.096	0.122	
	0.602	0.304	0.049	0.000	0.246	1.201	0.032	0.530	2.079	2.609	0.5218
10.0	0.066	0.240	0.003	0.000				0.013	0.074	0.087	
	0.460	0.170	0.038	0.000	0.163	0.831	0.021	0.280	1.479	1.777	0.03554
Totals	2.712	16.818	0.187	0.004							
	19.498	14.312	2.532	0.058	11.771	48.171	1.533				

TABLE III. Results for a lead absorber with a primary 100-MeV electron (1000 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.133	5.894	0.003	0.000				1.019	0.816	1.835	
	0.891	6.062	0.039	0.000	4.147	11.138	0.511	64.273	24.595	88.868	0.88869
1.0	0.378	5.961	0.027	0.000				0.966	1.222	2.188	
	2.590	5.513	0.397	0.000	4.222	12.722	0.532	41.882	34.261	76.143	0.76144
1.5	0.565	5.864	0.036	0.001				0.846	1.361	2.207	
	3.897	5.385	0.530	0.021	4.068	13.900	0.521	27.040	35.201	62.242	0.62242
2.0	0.683	4.744	0.045	0.001				0.616	1.372	1.988	
	4.808	4.170	0.661	0.012	3.363	13.014	0.435	15.413	33.828	49.241	0.49241
2.5	0.654	3.401	0.040	0.000				0.393	1.260	1.653	
	4.715	2.968	0.491	0.000	2.424	10.598	0.316	9.078	29.562	38.640	0.38641
3.0	0.500	2.251	0.041	0.000				0.283	1.093	1.376	
	3.550	1.788	0.584	0.000	1.636	7.558	0.214	6.074	25.003	31.077	0.31077
4.0	0.764	2.762	0.049	0.000				0.134	0.816	0.950	
	5.355	2.326	0.733	0.000	1.922	10.336	0.252	2.743	18.000	20.743	0.20743
5.0	0.504	1.599	0.022	0.001				0.106	0.595	0.701	
	3.429	1.334	0.297	0.012	1.154	6.226	0.151	2.210	12.310	14.520	0.14520
6.0	0.420	1.161	0.022	0.000				0.073	0.410	0.483	
	2.873	0.937	0.297	0.000	0.835	4.942	0.110	1.334	8.242	9.576	0.09576
7.0	0.270	0.799	0.007	0.000				0.028	0.298	0.326	
	1.878	0.661	0.080	0.000	0.552	3.170	0.073	0.516	5.879	6.395	0.06395
8.0	0.144	0.365	0.005	0.000				0.020	0.233	0.253	
	0.973	0.285	0.057	0.000	0.272	1.587	0.036	0.343	4.465	4.807	0.04807
9.0	0.132	0.285	0.008	0.000				0.020	0.163	0.183	
	0.878	0.242	0.108	0.000	0.209	1.437	0.027	0.337	3.023	3.360	0.03360
10.0	0.094	0.241	0.006	0.000				0.019	0.117	0.136	
	0.612	0.168	0.086	0.000	0.159	1.025	0.021	0.322	2.013	2.335	0.02335
Totals	5.242	35.327	0.311	0.003							
	36.448	31.841	4.359	0.045	24.962	97.655	3.197				

TABLE IV. Results for a lead absorber with a primary 100-MeV photon (1000 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.094	1.178	0.004	0.000				0.390	0.883	1.273	
	0.624	1.095	0.053	0.000	0.848	2.621	0.107	17.954	79.424	97.378	0.97379
1.0	0.193	3.007	0.015	0.000				0.603	0.902	1.505	
	1.369	2.944	0.202	0.000	2.079	6.595	0.264	25.062	65.721	90.783	0.90784
1.5	0.336	3.886	0.018	0.000				0.674	0.995	1.669	
	2.368	3.533	0.221	0.000	2.772	8.893	0.354	24.492	57.386	81.879	0.81880
2.0	0.411	4.259	0.039	0.000				0.639	1.047	1.686	
	3.013	3.724	0.551	0.000	2.986	10.274	0.383	21.435	50.169	71.604	0.71604
2.5	0.476	3.843	0.041	0.000				0.537	1.061	1.598	
	3.539	3.521	0.557	0.000	2.676	10.293	0.345	17.259	44.059	61.318	0.61319
3.0	0.428	3.273	0.038	0.002				0.470	1.023	1.493	
	3.026	2.863	0.530	0.021	2.336	8.777	0.301	14.651	37.891	52.542	0.52542
4.0	0.786	5.460	0.057	0.002				0.345	0.921	1.266	
	5.566	4.505	0.822	0.025	3.787	14.706	0.490	10.080	27.752	37.832	0.37832
5.0	0.727	3.684	0.042	0.000				0.208	0.775	0.983	
	5.105	2.997	0.558	0.000	2.557	11.216	0.332	6.192	20.419	26.612	0.26612
6.0	0.521	2.312	0.034	0.000				0.134	0.606	0.740	
	3.605	1.954	0.533	0.000	1.664	7.756	0.216	3.643	15.191	18.834	0.18834
7.0	0.388	1.528	0.020	0.000				0.095	0.480	0.575	
	2.654	1.303	0.282	0.000	1.074	5.313	0.140	2.328	11.192	13.520	0.13520
8.0	0.319	1.026	0.020	0.000				0.069	0.363	0.432	
	2.126	0.889	0.270	0.000	0.740	4.025	0.096	1.682	7.812	9.494	0.09494
9.0	0.250	0.821	0.008	0.000				0.042	0.252	0.294	
	1.744	0.723	0.099	0.000	0.579	3.144	0.076	0.874	5.463	6.337	0.06338
10.0	0.164	0.388	0.007	0.000				0.022	0.185	0.207	
	1.087	0.370	0.103	0.000	0.270	1.830	0.035	0.550	3.957	4.507	0.04507
Totals	5.094	34.665	0.343	0.004							
	35.825	30.421	4.782	0.047	24.368	95.443	3.139				

TABLE V. Results for a lead absorber with a primary 200-MeV electron (1000 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	total $E>10$	Fraction left
0.5	0.124	5.721	0.001	0.000				1.133	1.148	2.281	
	0.779	6.405	0.017	0.000	4.360	11.561	0.522	133.744	54.702	188.446	0.94223
1.0	0.394	7.092	0.017	0.000				1.298	1.887	3.185	
	2.669	7.105	0.264	0.000	5.093	15.131	0.629	92.081	81.233	173.314	0.86657
1.5	0.705	8.132	0.037	0.003				1.341	2.301	3.642	
	5.025	7.547	0.596	0.034	5.799	19.001	0.732	66.934	87.386	154.320	0.77160
2.0	0.883	8.397	0.060	0.000				1.260	2.492	3.752	
	6.287	7.653	0.881	0.000	5.896	20.717	0.753	48.465	85.114	133.579	0.66790
2.5	1.009	7.672	0.064	0.002				1.119	2.469	3.588	
	7.256	6.807	0.917	0.031	5.395	20.407	0.695	35.879	77.297	113.176	0.56588
3.0	1.021	6.707	0.070	0.001				0.886	2.348	3.234	
	7.246	5.766	0.984	0.012	4.645	18.652	0.601	24.989	69.533	94.522	0.47261
4.0	1.694	9.146	0.124	0.000				0.550	1.969	2.519	
	12.124	7.807	1.689	0.000	6.481	28.101	0.841	14.293	52.102	66.395	0.33198
5.0	1.366	5.992	0.081	0.000				0.341	1.551	1.892	
	9.487	5.014	1.128	0.000	4.201	19.830	0.547	8.827	37.727	46.553	0.23277
6.0	0.982	3.940	0.056	0.000				0.227	1.203	1.430	
	6.829	3.556	0.713	0.000	2.847	13.945	0.371	4.996	27.592	32.588	0.16294
7.0	0.765	2.466	0.039	0.000				0.137	0.916	1.053	
	5.289	2.167	0.530	0.000	1.678	9.663	0.220	3.209	19.712	22.920	0.11460
8.0	0.547	1.587	0.021	0.000				0.104	0.681	0.785	
	3.608	1.459	0.250	0.000	1.111	6.428	0.145	2.381	14.109	16.490	0.08245
9.0	0.384	1.186	0.014	0.000				0.080	0.507	0.587	
	2.642	0.952	0.171	0.000	0.812	4.578	0.106	1.657	10.252	11.910	0.05955
10.0	0.271	0.790	0.013	0.000				0.048	0.398	0.446	
	1.821	0.686	0.179	0.000	0.515	3.201	0.068	0.932	7.775	8.706	0.04353
Totals	10.146	68.828	0.597	0.006							
	71.061	62.924	8.320	0.077	48.834	191.215	6.231				

TABLE VI. Results for a lead absorber with a primary 200-MeV photon (1000 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.056	1.319	0.002	0.000				0.470	0.947	1.417	
	0.327	1.191	0.032	0.000	1.022	2.572	0.126	40.107	157.320	197.427	0.98714
1.0	0.181	3.702	0.018	0.000				0.788	1.132	1.920	
	1.214	3.785	0.307	0.000	2.692	7.998	0.335	58.506	130.922	189.428	0.94714
1.5	0.345	5.314	0.022	0.000				0.958	1.430	2.388	
	2.511	5.068	0.300	0.000	3.707	11.586	0.465	61.383	116.463	177.846	0.88923
2.0	0.503	6.103	0.034	0.001				1.054	1.740	2.794	
	3.587	5.656	0.452	0.011	4.385	14.091	0.554	56.956	106.795	163.751	0.81876
2.5	0.618	6.823	0.053	0.000				1.085	1.900	2.985	
	4.355	6.375	0.746	0.000	4.783	16.259	0.608	50.608	96.888	147.496	0.73748
3.0	0.769	6.823	0.062	0.000				1.014	2.002	3.016	
	5.510	6.308	0.907	0.000	4.772	17.496	0.609	45.246	84.757	130.003	0.65002
4.0	1.618	11.787	0.112	0.002				0.774	1.997	2.771	
	11.483	10.280	1.544	0.024	8.401	31.733	1.078	28.101	70.155	98.256	0.49128
5.0	1.420	8.697	0.098	0.001				0.588	1.794	2.382	
	9.844	7.600	1.347	0.014	6.147	24.952	0.793	19.952	53.315	73.267	0.36634
6.0	1.343	6.511	0.085	0.001				0.412	1.444	1.856	
	9.338	5.603	1.125	0.010	4.594	20.671	0.595	12.620	39.933	52.553	0.26277
7.0	0.997	4.600	0.051	0.001				0.259	1.170	1.429	
	6.995	3.963	0.675	0.012	3.250	14.895	0.422	7.260	30.384	37.644	0.18822
8.0	0.761	3.035	0.038	0.000				0.179	0.926	1.105	
	5.252	2.538	0.510	0.000	2.147	10.448	0.279	5.099	22.074	27.173	0.13586
9.0	0.551	1.977	0.026	0.000				0.113	0.725	0.838	
	3.798	1.750	0.321	0.000	1.386	7.255	0.131	2.973	16.921	19.894	0.09947
10.0	0.412	1.410	0.022	0.001				0.081	0.577	0.658	
	2.738	1.098	0.277	0.014	0.985	5.111	0.128	2.520	12.260	14.780	0.07390
Totals	9.575	68.101	0.623	0.007							
	66.953	61.215	8.544	0.084	48.271	185.067	6.173				

TABLE VII. Results for a lead absorber with a primary 500-MeV electron (750 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.141	5.719	0.000	0.000				1.289	1.681	2.971	
	0.866	6.631	0.000	0.000	4.764	12.260	0.554	332.066	155.672	487.738	0.97548
1.0	0.460	8.744	0.020	0.000				1.777	2.853	4.631	
	3.071	9.147	0.325	0.000	6.567	19.110	0.796	257.837	210.789	468.626	0.93726
1.5	0.897	11.709	0.047	0.000				2.125	3.869	5.995	
	6.204	11.378	0.637	0.000	8.492	26.710	1.052	202.705	239.189	441.895	0.88379
2.0	1.268	13.684	0.079	0.000				2.392	4.655	7.047	
	8.911	13.109	1.141	0.000	9.981	33.142	1.255	161.966	246.785	408.751	0.81751
2.5	1.660	15.136	0.105	0.003				2.525	5.080	7.605	
	11.679	14.146	1.549	0.028	10.919	38.322	1.385	133.143	237.280	370.423	0.74085
3.0	1.933	15.585	0.115	0.000				2.469	5.216	7.685	
	13.579	14.167	1.639	0.000	11.186	40.572	1.428	112.142	217.662	329.804	0.65961
4.0	4.224	29.221	0.249	0.000				1.985	5.015	7.000	
	29.708	25.928	3.632	0.000	20.770	80.038	2.669	74.906	174.802	249.708	0.49942
5.0	3.816	21.540	0.263	0.000				1.381	4.416	5.797	
	27.202	19.160	3.847	0.000	15.416	65.625	1.989	50.732	133.271	184.002	0.36801
6.0	3.055	16.131	0.181	0.001				1.069	3.711	4.780	
	21.374	13.847	2.576	0.016	11.405	49.219	1.475	33.680	101.064	134.745	0.26949
7.0	2.616	11.792	0.160	0.001				0.699	2.968	3.667	
	17.944	10.098	2.386	0.021	8.305	38.753	1.079	20.706	75.248	95.954	0.19191
8.0	1.981	8.083	0.105	0.001				0.471	2.331	2.801	
	13.664	7.170	1.467	0.013	5.708	28.023	0.742	12.705	55.192	67.897	0.13579
9.0	1.493	5.457	0.071	0.001				0.304	1.812	2.116	
	9.950	4.821	0.926	0.017	3.795	19.508	0.495	7.540	40.806	48.346	0.09669
10.0	1.135	3.461	0.056	0.001				0.205	1.372	1.577	
	7.633	2.856	0.718	0.021	2.483	13.711	0.324	5.009	29.610	34.618	0.06924
Totals	24.681	166.263	1.451	0.009							
	171.785	152.458	20.844	0.116	119.791	464.994	15.242				

TABLE VIII. Results for a lead absorber with a primary 500-MeV photon (750 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.037	1.556	0.001	0.000				0.551	1.093	1.644	
	0.244	1.681	0.017	0.000	1.157	3.099	0.138	106.279	390.620	496.899	0.99380
1.0	0.197	4.751	0.009	0.000				1.093	1.624	2.717	
	1.242	4.843	0.153	0.000	3.475	9.712	0.422	159.617	327.569	487.186	0.97438
1.5	0.508	7.613	0.027	0.001				1.473	2.415	3.888	
	3.493	7.682	0.392	0.015	5.598	17.180	0.689	164.348	305.657	470.004	0.94001
2.0	0.804	9.981	0.043	0.000				1.888	3.099	4.987	
	5.528	9.325	0.592	0.000	7.313	22.758	0.912	169.746	277.498	447.244	0.89449
2.5	1.108	11.783	0.081	0.000				2.057	3.801	5.859	
	7.877	11.035	1.153	0.000	8.568	28.633	1.077	157.833	260.762	418.595	0.83719
3.0	1.415	13.379	0.097	0.001				2.177	4.303	6.480	
	9.947	12.346	1.410	0.019	9.560	33.283	1.210	141.310	243.989	385.299	0.77060
4.0	3.531	27.681	0.220	0.001				2.037	4.683	6.720	
	24.956	25.040	3.111	0.030	19.608	72.745	2.501	103.014	209.506	312.520	0.62504
5.0	3.421	24.160	0.229	0.001				1.776	4.579	6.355	
	24.191	20.898	3.124	0.014	17.221	65.448	2.208	75.987	171.033	247.020	0.49404
6.0	3.511	20.531	0.220	0.001				1.423	4.135	5.557	
	24.620	17.614	3.020	0.016	14.702	59.971	1.893	54.509	132.505	187.014	0.37403
7.0	2.972	15.517	0.185	0.001				1.024	3.567	4.591	
	20.782	13.459	2.555	0.016	11.083	47.895	1.433	32.609	106.457	139.066	0.27813
8.0	2.421	11.408	0.140	0.000				0.749	2.963	3.712	
	16.949	9.826	1.924	0.000	8.143	36.842	1.055	23.393	78.770	102.163	0.20433
9.0	1.879	8.248	0.092	0.000				0.501	2.417	2.919	
	13.045	7.032	1.134	0.000	5.708	26.918	0.740	14.903	60.332	75.235	0.15047
10.0	1.493	5.743	0.065	0.000				0.341	1.909	2.251	
	10.162	4.854	0.897	0.000	4.043	19.956	0.526	9.164	46.111	55.275	0.11055
Totals	23.299	162.351	1.411	0.008							
	163.035	145.636	19.482	0.110	116.178	444.441	14.803				

TABLE IX. Results for a lead absorber with primary 1000-MeV electron (500 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.116	5.358	0.002	0.000				1.328	2.102	3.430	
	0.679	6.784	0.026	0.000	4.881	12.370	0.554	640.400	347.229	987.629	0.98763
1.0	0.514	9.516	0.004	0.000				2.186	3.724	5.910	
	3.253	10.396	0.058	0.000	7.512	21.219	0.896	516.463	449.944	966.408	0.96641
1.5	1.024	15.184	0.054	0.000				3.100	5.332	8.432	
	6.945	14.863	0.958	0.000	11.245	34.011	1.377	438.090	494.301	932.391	0.93239
2.0	1.768	20.388	0.106	0.002				3.822	6.816	10.638	
	12.467	20.060	1.547	0.027	14.841	48.943	1.847	375.062	508.322	883.384	0.88339
2.5	2.380	25.030	0.126	0.002				4.304	8.162	12.466	
	16.512	24.177	1.738	0.023	18.020	60.470	2.267	318.568	504.338	822.906	0.82291
3.0	3.018	26.992	0.182	0.000				4.540	8.916	13.456	
	21.378	24.240	2.479	0.000	19.566	67.663	2.478	294.337	460.823	755.160	0.75516
4.0	7.190	55.230	0.480	0.000				4.126	9.856	13.982	
	51.383	49.722	6.517	0.000	39.387	147.009	5.025	201.071	406.894	607.965	0.60797
5.0	7.468	49.232	0.466	0.006				3.392	9.478	12.870	
	51.938	44.182	6.674	0.082	34.571	137.447	4.437	134.900	335.522	470.422	0.47042
6.0	7.036	39.252	0.404	0.006				2.636	8.234	10.870	
	49.464	34.573	5.733	0.084	27.958	117.813	3.606	91.948	260.536	352.484	0.35248
7.0	5.954	29.604	0.308	0.006				1.882	6.904	8.786	
	41.285	25.469	4.297	0.070	20.682	91.803	2.676	63.296	197.318	260.614	0.26061
8.0	4.546	21.804	0.262	0.000				1.418	5.688	7.106	
	31.342	18.735	3.689	0.000	15.485	69.251	2.007	43.534	147.766	191.301	0.19130
9.0	3.800	15.584	0.194	0.000				0.946	4.506	5.452	
	26.021	13.659	2.772	0.000	10.648	53.100	1.384	26.766	111.398	138.164	0.13816
10.0	2.842	10.842	0.144	0.002				0.684	3.478	4.162	
	19.254	9.052	1.872	0.022	7.781	37.981	1.012	18.462	81.710	100.172	0.10017
Totals	47.658	324.016	2.732	0.024							
	331.920	295.914	38.360	0.308	232.577	899.079	29.568				

TABLE X. Results for a lead absorber with a primary 1000-MeV photon (500 cascades analyzed).

	Energy loss							Energy left in cascade			
Depth	Electrons $E<10$	Photons $E<10$	Electrons $N<0$	Photons $N<0$	Ionization	Total	Electron track length	Electrons $E>10$	Photons $E>10$	Total $E>10$	Fraction left
0.5	0.026	1.702	0.002	0.000				0.614	1.218	1.832	
	0.145	1.983	0.044	0.000	1.453	3.625	0.169	223.876	772.497	996.373	0.99637
1.0	0.234	5.234	0.010	0.000				1.286	2.146	3.432	
	1.524	5.697	0.152	0.000	4.099	11.472	0.489	319.237	665.666	984.903	0.98490
1.5	0.548	9.052	0.024	0.000				1.908	3.306	5.214	
	3.864	9.631	0.379	0.000	6.765	20.638	0.822	347.267	617.012	964.279	0.96428
2.0	1.058	13.412	0.050	0.002				2.774	4.542	7.316	
	7.271	14.025	0.775	0.024	10.039	32.134	1.238	358.111	574.063	932.173	0.93217
2.5	1.590	18.372	0.086	0.000				3.450	5.714	9.164	
	11.175	17.416	1.251	0.000	13.502	43.344	1.686	354.458	534.368	888.826	0.88883
3.0	2.232	21.668	0.110	0.000				3.790	6.926	10.716	
	15.803	20.309	1.735	0.000	15.965	53.813	2.009	323.545	511.400	834.945	0.83495
4.0	5.878	48.108	0.298	0.000				3.866	8.312	12.178	
	41.338	44.450	4.150	0.000	34.409	124.346	4.362	247.199	463.330	710.528	0.71053
5.0	6.498	47.728	0.378	0.002				3.588	3.702	12.290	
	45.392	43.248	5.321	0.031	34.075	128.067	4.347	210.473	371.818	582.291	0.58229
6.0	6.398	42.920	0.376	0.002				3.178	8.480	11.658	
	44.817	38.266	5.069	0.029	30.923	119.103	3.961	146.162	316.904	463.067	0.46307
7.0	6.156	37.052	0.386	0.006				2.470	7.708	10.178	
	43.147	32.602	5.378	0.065	26.183	107.374	3.367	103.700	251.891	355.592	0.35559
8.0	5.252	28.330	0.352	0.004				1.840	6.824	8.664	
	36.462	24.737	4.847	0.052	20.022	86.121	2.583	68.793	200.619	269.412	0.26941
9.0	4.330	21.080	0.240	0.004				1.378	5.744	7.122	
	29.943	18.188	3.397	0.072	14.681	66.281	1.900	43.114	159.926	203.040	0.20304
10.0	3.790	15.646	0.188	0.000				0.990	4.626	5.616	
	26.282	13.353	2.574	0.000	11.002	53.211	1.428	31.143	118.619	149.763	0.14976
Totals	43.992	310.304	2.500	0.020							
	307.165	283.906	35.070	0.273	223.117	849.531	28.362				

Column 9 gives at the depth shown the average number of electrons with energies greater than 10 MeV (top line) and the energy carried by these electrons.

Column 10 gives the same type of information for the photons as column 9 does for electrons.

Column 11 shows the totals for columns 9 and 10. Thus one sees from Table I that at a depth of 0.5 radiation length there is a total of 1.370 electrons and photons, each particle with an energy greater than 10 MeV, and carrying a total of 39.419 MeV.

Column 12 gives the fraction of the primary energy still carried by the electrons and photons which have energies greater than 10 MeV at the depth shown.

The last figure in column 7 together with the primary energy times the last figure in column 12 should add up to the primary energy, and where this does not occur exactly it is due to rounding off errors in the computation.

Some of the interesting statistics obtained from the results in Tables I to X are summarized in Table XI. The ratios shown have been calculated for each of our five energies 50, 100, 200, 500, and 1000 MeV and for electron and photon primaries.

Column 2 shows the ratio of total electron track length for electrons >10 MeV divided by the primary energy. One sees in Tables I to X that in each case at a

TABLE XI. Statistics obtained from Tables I–X.

		Percentage energy loss to							
Primary energy (MeV)	(Track length)/E_0	Electrons $E<10$ MeV	Photons $E<10$ MeV	Electrons $n<0$	Photons $n<0$	Ionization	Energy decay a	b	Penetration
				Primary electron					
50	0.03359	36.28	32.80	4.89	0.04	25.99	0.43±0.02	0.331±0.007	2.31
100	0.03237	38.21	31.93	4.61	0.05	25.20	0.87±0.03	0.363±0.004	5.96
200	0.03199	38.67	31.71	4.64	0.04	24.94	1.26±0.02	0.339±0.003	9.52
500	0.03188	39.19	31.09	4.89	0.05	24.78	1.93±0.03	0.332±0.002	13.76
1000	0.03155	40.37	30.44	4.81	0.02	24.37	2.35±0.03	0.315±0.002	17.33
				Primary photon					
50	0.03184	40.71	29.64	5.09	0.12	24.45	1.21±0.02	0.351±0.003	5.13
100	0.03261	38.16	31.29	5.23	0.05	25.27	1.59±0.03	0.357±0.003	7.75
200	0.03229	38.11	31.82	4.90	0.05	25.13	1.78±0.02	0.320±0.002	11.16
500	0.03191	40.23	30.07	4.94	0.02	24.74	2.10±0.05	0.292±0.004	15.94
1000	0.03144	41.28	29.78	4.72	0.03	24.19	2.54±0.06	0.282±0.003	19.64

depth of 10 radiation lengths a varying fraction of the primary energy is still being carried by electrons and photons of energy >10 MeV. Thus, to obtain the ratios given, one must carry out a simple extrapolation to the point where the shower particles all drop below 10 MeV. Let T be the total electron track length corresponding to an energy loss E (column 7 of previous tables), with E_0 the primary energy. Then we calculated the coefficients A and B in

$$T/E=A+BE,$$

giving each point a weighting factor $1/(1-E/E_0)$. The extrapolated value is obtained by setting $E=E_0$.

Though this track length ratio shows a slow variation with primary energy, the total electron track length (for $E>10$ MeV) of a shower for either electron or photon primary may be expressed with considerable accuracy by

$$T=0.032E_0 \text{ radiation length}$$

or

$$T=0.0163E_0 \text{ cm},$$

with E_0 in MeV.

In columns 3 to 7 we give the percentage of primary energy lost by each of the five processes mentioned in Sec. 2. The values shown are extrapolated in the same way as the track length ratio. With the exception of 50-MeV primary photon the percentages generally show a slow but systematic variation with primary energy.

Columns 8 and 9 show the results of fitting an exponential decay to the total energy in the tail of the cascade,

$$E/E_0=ae^{-bt},$$

where t is measured in radiation lengths from the origin. The 50, 100, 200, and 500-MeV showers were fitted from $t=4$ to $t=10$ radiation lengths while the 1000-MeV showers were fitted from $t=5$ radiation lengths.

The coefficients a show a simple logarithmic dependence on primary energy which is best illustrated by considering the penetration depth of a shower. This is the depth at which the expected total remaining energy is 10 MeV. The values for this penetration depth in radiation lengths are tabulated in the end column. The errors shown for a and b are the least-square fitting errors.

With the exception of the 50-MeV primary photon shower these penetration depths can be expressed as follows:

(a) Primary electron,

$$t_p=5.0 \log_e(E_0/30.5) \text{ radiation lengths.}$$

(b) Primary photon,

$$t_p=5.2 \log_e(E_0/22.7) \text{ radiation lengths.}$$

In all the shower statistics calculated the 50-MeV primary photon shower has slightly anomalous results. This is because the photon mainly produces low-energy electrons which do not produce photons >10 MeV. This leaves the shower deficient in low-energy photons. To a much lesser extent this applies to the 100-MeV photon showers.

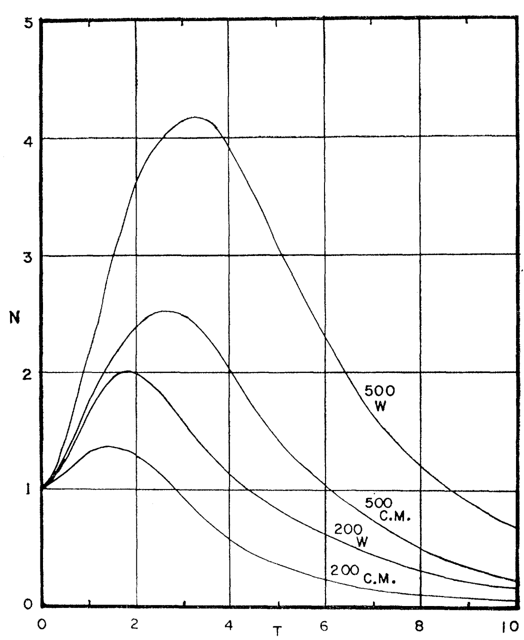

FIG. 1. The expected number of electrons with energy $\geqslant 10$ MeV in a shower plotted against depth T, in radiation lengths, for an incident electron in lead absorber. The energy is marked on the curves in MeV. Results given by Wilson (indicated by W) for electrons with energy $\geqslant 8$ MeV are shown for comparison.

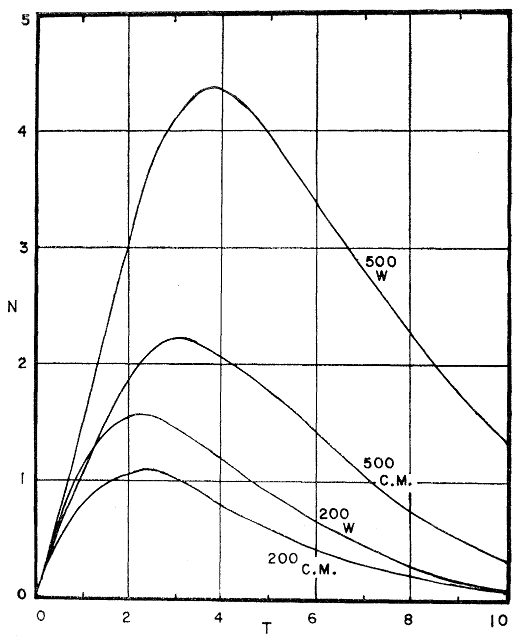

FIG. 2. The expected number of electrons with energy $\geqslant 10$ MeV in a shower plotted against depth T, in radiation lengths, for an incident photon in lead absorber. The energy is marked on the curves in MeV. Results given by Wilson (indicated by W) for electrons with energy $\geqslant 8$ MeV are shown for comparison.

Some other statistics which appear to be nearly independent of primary energy are:

(1) The average energy of the individual electrons lost because their energy is <10 MeV is 6.98 MeV.

(2) The average energy of photons with $E<10$ MeV is 0.909 MeV.

(3) The average energy of backscattered electrons with $E>10$ MeV is 14.0 MeV.

(4) The average energy of backscattered photons with $E>10$ MeV is 13.1 MeV.

(5) The average number of all photons produced per electron track length is 11.04.

These shower parameters are not only nearly independent of the energy but also of the nature of the primary particle. If one considers the development of showers from the point of equivalent remaining energy then once past the shower maximum the showers look essentially the same and are nearly independent of the nature of the primary particle and the magnitude of its primary energy. It should be remembered that the results presented are only averages of the showers considered. An individual cascade can exhibit wide variations.

A comparison between Wilson's results and those given in the present work is shown in Figs. 1 and 2, where we have plotted the number of electrons with energy >10 MeV due to primary electrons of energy 500 and 200 MeV (Fig. 1) against depth in the lead absorber measured in radiation lengths. Figure 2 shows the same results for primary photons. On the same figures we have plotted Wilson's results for the number of electrons with energy >8 MeV. The discrepancy is very large amounting to almost 100% at the maximum in the case of primary photon results. The discrepancy cannot be accounted for by the fact that in our case we are considering electrons with energy >10 MeV whereas Wilson considers particles with $E>8$ MeV. As we have already pointed out, the average energy of electrons with $E<10$ MeV is 6.98 MeV. Apparently Wilson's method of calculation gave too low an energy degradation for the electrons, thus accounting for the differences above.

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μ-Mesonic Atom Studies of Ti, Fe, Cu, Zn, Tl, Pb, and Bi†*

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This paper presents the results of a precision re-evaluation of the μ-mesonic x-ray transition energies for Tl, Pb, Bi, Ti, Fe, Cu, and Zn. The $2p-1s$ and $3d-2p$ transition regions were studied for Tl, Pb, and Bi, and the $2p$ fine structure splitting was determined with good precision from both sets of measurements. The Tl, Pb, and Bi transition energies were determined to about 0.2%, while the $2p$ state fine structure splitting was determined to within 2 to 3%. The results are in good agreement with the theoretically predicted values of Ford and Wills, based on the charge distributions implied from the high-energy electron scattering experiments. An anomaly in the observed intensity ratio of the $2p_{1/2}-1s$ to $2p_{3/2}-1s$ transitions for Bi and Tl is discussed. It is believed to be due to an excitation of the first excited state of the initial nucleus. The observed $2p-1s$ transition energies for Ti, Fe, Cu, and Zn are given, with experimental uncertainties of 0.2 to 0.3%. The comparison with the theoretically predicted values is much improved relative to the earlier experimental results.

I. INTRODUCTION

SINCE the original studies of the $2p-1s$ μ-mesonic x rays by Fitch and Rainwater,[1] we have at various times attempted to improve the precision of these results with respect to the $2p-1s$ transition energy for elements near $Z=82$. The earlier $2p-1s$ spectra for Tl, Pb, and Bi were of such a shape that a reasonable interpretation required an assumption of approximately the expected 190-keV fine structure splitting of the $2p_{3/2}$, $2p_{1/2}$ levels. However, the experimental curves had limited statistical accuracy, and the energy calibration using the 4.43-MeV γ ray from a Pu-Be source required a considerable energy and shape extrapolation, which limited the absolute accuracy of the energy evaluations.

In this paper we report new results for the $2p-1s$, and also for the $3d-2p$ x rays from Tl, Pb, and Bi, which we believe to be an order of magnitude superior to the earlier results. A much better absolute evaluation of the x-ray energies was obtained by use of a fast neutron irradiated circulating water target which yielded 6.134±0.006-MeV γ rays, associated with a transition from a well-studied excited state of O^{16}. These x-ray energies are found to be in excellent agreement

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* The adjective "mesonic" is retained here, even though the muon has lately been excluded from the meson family, since no satisfactory new label has been invented to cope with the situation. The term "μ-leptonic" or "muonic" are possible.
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1 V. L. Fitch and J. Rainwater, Phys. Rev. 92, 789 (1953).