Investigation of d-s Shell Nuclei with (d,He^3) Reactions*

BIBIJANA ČUJEC
University of Pittsburgh, Pittsburgh, Pennsylvania
(Received June 28, 1962)

The possibility of studying nuclear structure by (d,He^3) reactions with 15-MeV deuterons has been investigated and cross sections for transitions to low-lying states measured on a variety of nuclei (O^{16}, F^{19}, Al^{27}, P^{31}, S^{32}, Cl^{35}, Ca^{40}, Ni^{58}, Cu^{63}). Using distorted wave Born calculations, the spectroscopic factors have been extracted and some information obtained on the properties of the low-lying states. The ground states of O^{16}, P^{31}, S^{32}, Cl^{35}, and Ca^{40} are found to be essentially well described by the shell model. In the S^{32} ground state an admixture of about 13% of $s_{\frac{1}{2}}^2 d_{\frac{1}{2}}^2$ configuration has been established. The first excited state of Si^{30} ($E=2.24$ MeV, $J=2^+$) and the second excited state of P^{31} ($E=2.23$ MeV, $J=\frac{5}{2}^+$) appear as $d_{\frac{5}{2}}$ hole states. The ratio of spectroscopic factors shows that the Mg^{26} ground state ($J=0^+$) and excited state at 1.83 MeV ($J=2^+$) are rotational levels of a $K=0$ band. The probability of finding the deuteron ground state within the He^3 ground state was found to be ~ 0.7.

INTRODUCTION

STRIPPING reactions (d,p) and pickup reactions (p,d) and (d,t) have been widely and very successfully used in investigating nuclear structure.[1] It has been shown[2] that essentially the same information can be obtained by (d,t) reactions as by (p,d) reactions. In all of these widely investigated cases, a neutron is added to the target nucleus or picked up from it. The reactions (d,n) and (n,d), where a proton is transferred, have been studied in very few cases.[1] The reason is experimental trouble. On one hand the spectroscopy of neutrons is much more difficult than that of charged particles, and on the other hand, the generators for high-energy neutrons have much lower intensity. It is expected that the same information as is obtained by (n,d) reactions could be obtained by (d,He^3) reactions, provided that the outgoing He^3 particles are not too much affected by the Coulomb barrier or that this effect is appropriately taken into account. The aim of the present experiment was thus first to investigate the possibilities of studying nuclear structure by (d,He^3) reactions, initiated by 15-MeV deuterons which are available from the University of Pittsburgh cyclotron.

The fruitfulness of stripping and pickup reactions is due to the fact that the cross section can be written as

$$(d\sigma/d\Omega)(\theta)=A\sigma_{\text{calc}}(\theta)S, \qquad (1)$$

where A is a constant depending only on internal properties of incident and outgoing particles, $\sigma_{\text{calc}}(\theta)$ is the cross section which can be calculated using different approximations [Butler theory, distorted wave Born (DWB) approximation], and S is the spectroscopic factor or relative reduced width[1,3] which contains the characteristic information about nuclear states. By saying that it is possible to study nuclear structure by a certain reaction, we mean that it is possible to extract S from the measured $d\sigma/d\Omega$, which requires that $\sigma_{\text{calc}}(\theta)$ be given appropriately. In our case of (d,He^3) reactions we cannot expect that $\sigma_{\text{calc}}(\theta)$ will be given correctly by the simple Butler theory. However, the DWB approximation,[4] which takes into account the distortion of incident and outgoing waves due to both the nuclear and Coulomb potentials and also the Coulomb effect on the single-particle wave function of the picked-up proton, looks very promising.

The cross sections for a variety of nuclei between O^{16} and Zr^{90} have been measured, and corresponding DWB calculations have been performed using code SALLY.[4] The main investigation has, however, been limited to the d-s shell nuclei (O^{16} to Ca^{40}). The cases where the spectroscopic factor is well predicted by the shell model provide a check for the DWB calculation. Another check is provided by measuring the angular distribution for $S^{32}(d,He^3)P^{31}$ g.s. ($l=0$) and $Ca^{40}(d,He^3)$-K^{39} g.s. ($l=2$), and comparing with the calculations. The cross sections (absolute values) have been measured as accurately as possible in order to determine the constant A which depends on the interaction between the proton and deutron.

In the course of this experiment the measurements of (d,He^3) reactions of F^{19} and Al^{27}, initiated with 21.6-MeV deuterons, have been reported.[5]

MEASUREMENTS

As already mentioned, the reactions have been induced by 15-MeV deuterons. The targets used and reaction Q values are listed in Table I. The reaction products were analyzed by a magnetic spectrometer and detected in its focal plane by a CsI crystal, screened by a slit of 8-mm width and 25.5-mm height. The energies of different particles reaching the CsI crystal are related by $E_p=2E_d=3E_t=E_\alpha=\frac{3}{4}E_{He^3}$. In a typical case (Fig. 1) with $E_{He^3}=11.7$ MeV we have $E_p=E_\alpha=8.8$

* Supported by the National Science Foundation and the Office of Naval Research.

[1] M. H. Macfarlane and J. B. French, Revs. Modern Phys. **32**, 567 (1960). References to all stripping and pickup experiments performed to that time are quoted.

[2] A. I. Hamburger, Phys. Rev. **118**, 1271 (1960).

[3] J. B. French, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B.

[4] R. H. Bassel, R. M. Drisko, G. R. Satchler, Oak Ridge National Laboratory Report ORNL-3240 (unpublished).

[5] T. H. Braid and B. Zeidman, Bull. Am. Phys. Soc. **7**, 300 (1962).

TABLE I. Targets used and Q values.

Reaction	$Q_{g.s.}$	Target	Target thickness (mg/cm²)	Target thickness MeV (for He³)
$O^{16}(d,He^3)N^{15}$	−6.00	Ni oxidized[a]	1.12	0.20
$F^{19}(d,He^3)O^{18}$	−2.49	C_2F_4 on Au	1.4	0.25
$Al^{27}(d,He^3)Mg^{26}$	−2.75	Al	1.87	0.40
			0.23	0.05
$P^{31}(d,He^3)Si^{30}$	−1.85	P on Ag	4.5	0.87
$S^{32}(d,He^3)P^{31}$	−3.37	CdS on Ag	2.71	0.50
$Cl^{35}(d,He^3)S^{34}$	−0.87	$C_4H_5Cl_2$ with		
$Cl^{37}(d,He^3)S^{36}$	−2.87	evaporated Au	1.85	0.40
$Ca^{40}(d,He^3)K^{39}$	−2.83	Ca	1.95	0.38
$Ni^{58}(d,He^3)Co^{57}$	−2.41	Ni^{58}	2.33	0.27
$Cu^{63}(d,He^3)Ni^{62}$	−0.62	Cu^{63}	3.50	0.40
$Zr^{90}(d,He^3)Y^{89}$	−2.60	Zr	2.30	0.27

[a] For preparation see J. C. Armstrong and K. S. Quisenberry, Phys. Rev 122, 150 (1961).

MeV and E_d=4.4 MeV. In these energy regions, protons, alphas, and deuterons have continuous spectra. Therefore, it is not possible to detect groups of He^3 particles with the photographic plate method which has been widely and very successfully used[6] in this laboratory in investigating (d,p) and some (d,t) reactions. Here a more tedious method has to be used. With the slit before the crystal, only a small part of the spectrum is selected for detection. (With E_{He^3}=11.7 MeV, the energy interval corresponding to the slit of 8-mm width is 0.195 MeV.) The different particles are separated due to their different energy by a 256-channel pulse-height analyzer. The α particles appear below the protons since they produce in the CsI crystal a lower pulse than protons of the same energy. Moreover, before entering the crystal, the particles pass through a window of 1-mil Mylar (=3.5 mg/cm²) which reduces the energy of alphas more than that of protons. With He^3 particles of higher energy (Cu^{63} and Cl^{35} cases), an additional absorber of ½-mil aluminum has been placed on the crystal to provide enough separation between He^3 particles and protons. With this method He^3 particles to about 8 MeV ($Q\approx-6$ MeV) could be resolved; at lower energy they join to the deuteron peak.

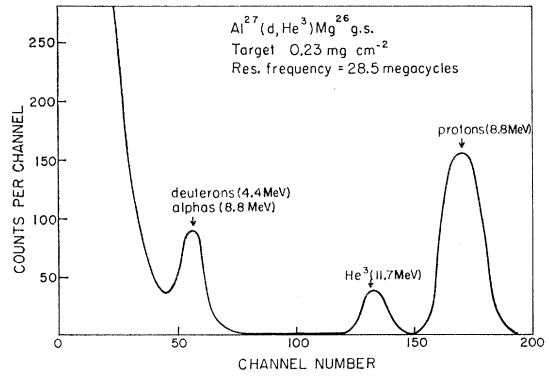

FIG. 1. Picture on the screen of the multichannel analyzer. Energies marked are determined by the magnetic field of the analyzing magnet and represent energies of particles before passing the detector window.

The field in the magnetic spectrometer—measured accurately by the proton resonance method—has been changed in such intervals (~200 kc) that the particle spectrum in the focal plane has been displaced for the width of the slit. To pass over one peak, usually, about five measurements had to be done; the number required obviously depended on the thickness of the target. With each measurement an irradiation between 50 and 200 μC—depending on the cross section of reaction studied—has been made and the spectrum given by 256-channel analyzer has been recorded. The pulses over about two half-widths of the He^3 peak (=20 channels) have been summed ($=N_i$). The sum $\sum_{i=1}^{n} N_i$ over individual measurements has been obtained, and the background, determined by measuring the spectrum above and below the He^3 peak, has been subtracted. For the solid angle the value determined by Hamburger[7] has been used. The absolute cross sections have an error of about 20%, due mainly to inaccuracy in the solid angle. The relative cross sections, referring to the same nucleus, are accurate within 10%.

RESULTS AND DISCUSSION

The results are presented in Table II. In the first and second column the target nuclei and the final states are listed, together with their isotopic spins and spins. In the third column the orbital angular momentum of the picked-up proton is given. In some cases its value follows already from conservation of angular momentum; in other cases it has been determined by measuring the cross section at different angles and comparing it with the calculated. For illustration, the angular distributions for l=0 and 2 are in Fig. 2. The comparisons between DWB curves and experimental results are about of this same quality for all angular distributions measured: $F^{19}(d,He^3)O^{18}$ g.s., $F^{19}(d,He^3)O^{18}$ 1.98 MeV, $P^{31}(d,He^3)Si^{30}$ g.s., $Al^{27}(d,He^3)Mg^{26}$ g.s., and $Al^{27}(d,He^3)Mg^{26}$ 1.83 MeV. The fourth column of Table II gives the experimental cross sections $\sigma_{exp}(\theta)$, measured at the angle θ (fifth column), where the angular distribution reaches its first maximum (with l=1, 2, and 3) or its second maximum (with l=0). The maximum has been predicted by DWB calculations and checked experimentally.

In columns 6, 7, and 8 the quantities related to DWB calculations are given. First, the orbit from which the proton is presumably picked up is specified. In some cases it is uniquely determined from conservation of angular momentum; in others the following assumptions have been made. In F^{19}, where a large mixing of $1d_{\frac{5}{2}}$ and $1d_{\frac{3}{2}}$ orbits occurs,[8] the l=2 proton is assumed to be

[6] B. L. Cohen, J. B. Mead, R. E. Price, K. S. Quisenberry, and C. Martz, Phys. Rev. 118, 499 (1960); B. L. Cohen and R. E. Price, *ibid.* 121, 1441 (1961).

[7] E. W. Hamburger, Ph.D. thesis, University of Pittsburgh, 1959 (unpublished).

[8] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) A229, 536 (1955).

TABLE II. Measured and calculated results.[a]

Target			Final state[b]				Experimental data			DWB calculations				Extraction of S and A				
	T	J		E (MeV)	T_0	J_0	l	σ_{exp} (mb/sr)	θ_{lab}	l_j	B_p (MeV)	σ_{calc} (mb/sr)	C^2	AS_{exp} $\approx S_{exp}$	$S/S_{g.s.}$	S_{theor}	$A=AS_{exp}/S_{theor}$
O^{16}	0	0^+	N^{15}	g.s.	$\frac{1}{2}$	$\frac{1}{2}^-$	1	2.26	15°	$1p_{1/2}$	11.49	1.17	$\frac{1}{3}$	3.87		4[d]	0.97[d]
F^{19}	$\frac{1}{2}$	$\frac{1}{2}^+$	O^{18}	g.s.	1	0^+	0	1.13	23°	$2s_{1/2}$	7.94	4.27	$\frac{2}{3}$	0.90	1	0.60[e]	0.67[e]
				1.98	1	2^+	2	0.41	20°	$1d_{5/2}+1d_{3/2}$	6.81	1.59	$\frac{2}{3}$	0.39	0.98		
				4.8			1	0.19	15°	$1p_{1/2}$	12.74	0.75	$\frac{2}{3}$	0.38	0.95		
Al^{27}	$\frac{1}{2}$	$\frac{5}{2}^+$	Mg^{26}	g.s.	1	0^+	2	0.50	20°	$1d_{5/2}$, $\Omega=\frac{5}{2}$	8.24	1.09	$\frac{2}{3}$	0.69	1	$\frac{1}{2}$[d]	1.38[d]
				1.83	1	2^+	2	0.67	20°	$1d_{5/2}$, $\Omega=\frac{5}{2}$	8.24	0.78	$\frac{2}{3}$	1.29	1.87		
				2.97	1	2^+	2	0.13	20°	$1d_{5/2}$, $\Omega=\frac{3}{2}$	9.93	0.44	$\frac{2}{3}$	0.44	0.64		
P^{31}	$\frac{1}{2}$	$\frac{1}{2}^+$	Si^{30}	g.s.	1	0^+	0	2.64	25°	$2s_{1/2}$	7.34	2.79	$\frac{2}{3}$	1.42	1	$\frac{3}{2}$,[d] 0.99[f]	0.95[d], 1.43[f]
				2.24	1	2^+	2	0.55	20°	$1d_{3/2}$	3.14	2.10	$\frac{2}{3}$	0.35	0.27		
										$1d_{5/2}$	7.84	0.67	$\frac{2}{3}$	1.23	0.87		
S^{32}	0	0^+	P^{31}	g.s.	$\frac{1}{2}$	$\frac{1}{2}^+$	0	2.25	25°	$2s_{1/2}$	8.86	1.09	$\frac{1}{2}$	4.14	1	4,[d] 2.77[g]	1.03,[d] 1.49[g]
				1.27	$\frac{1}{2}$	$\frac{3}{2}^+$	2	0.20	20°	$1d_{3/2}$	4.66	0.96	$\frac{1}{2}$	0.41	0.10		
				2.23	$\frac{1}{2}$	$\frac{5}{2}^+$	2	0.40	20°	$1d_{5/2}$	9.36	0.31	$\frac{1}{2}$	2.58	0.62		
Cl^{35}	$\frac{1}{2}$	$\frac{3}{2}^+$	S^{34}	g.s.	1	0^+	2	1.19	20°	$1d_{3/2}$	6.36	1.92	$\frac{2}{3}$	1.26		5/4[d]	1.01[d]
				2.13	1	2^+	2	<0.60	20°	$1d_{3/2}$	6.36	0.97	$\frac{2}{3}$	0.93[e]		$\frac{1}{4}$[d]	
Cl^{37}	$\frac{3}{2}$	$\frac{3}{2}^+$	S^{36}	g.s.	2	0^+	2	<1.85	20°	$1d_{3/2}$	8.36	0.55	$\frac{4}{5}$	4.21[e]		5/4[d]	
Ca^{40}	0	0^+	K^{39}	g.s.	$\frac{1}{2}$	$\frac{3}{2}^+$	2	1.60	20°	$1d_{3/2}$	8.32	0.42	$\frac{1}{2}$	7.64	1	8[d]	0.96[d]
				2.53	$\frac{1}{2}$		2	0.18	20°	$1d_{3/2}$	8.32	0.27	$\frac{1}{2}$	1.33	0.18		
Ni^{58}		0^+	Co^{57}	g.s.		$\frac{7}{2}^-$	3	0.23	18°	$1f_{7/2}$	7.90	0.06	···	3.83		8[d]	0.48[d]
Cu^{63}		$\frac{3}{2}^-$	Ni^{62}	g.s.		0^+	1	0.73	12°	$2p_{3/2}$	6.11	1.03	···	0.71		1[d]	0.71[d]

[a] $C^2=[C(T_0\frac{1}{2}T, M_{T_0}-\frac{1}{2}M_T)]^2$. S is given in terms of the isotopic spin formalism for nuclei of the d-s shell, but not for Ni^{58} and Cu^{63}. $AS_{exp}=\sigma_{exp}/C^2\sigma_{calc}$.
[b] *Nuclear Data Sheets*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C., 1960).
[c] Upper limit, determined by neglecting the other simultaneously recorded reaction.
[d] Calculated assuming pure j-j model (see text).
[e] Calculated in reference 1 using Redlich's wave functions.
[f] Transition $\alpha_1(S_{1/2}{}^2)_{1/2}\rightarrow(S_{1/2}{}^2)_0$ with $\alpha_1{}^2=0.66$.
[g] Calculated according to (10), with values quoted in the first line of Table III.

picked up from the barycenter of both levels. For $P^{31}(d,He^3)Si^{30}$ 2.24 MeV both possibilities of picking a $1d_{\frac{3}{2}}$ and $1d_{\frac{5}{2}}$ proton are considered individually. In the case of $Al^{27}(d,He^3)Mg^{26}$ the Nilsson orbits are specified. The binding energy B_p, listed in column 7, means the distance of the single-particle level (the orbit specified in column 6) from the top of the potential well. It is the most questionable parameter in the DWB calculations. It has been chosen in the following way: With all ground-state transitions the separation energy has been taken for B_p. Thus the position of one orbit in the well has been fixed. The spacings $1d_{\frac{5}{2}}-2s_{\frac{1}{2}}$ and $2s_{\frac{1}{2}}-1d_{\frac{3}{2}}$ have been assumed to be the same as for the corresponding single-particle levels in F^{17}: 0.50 and 4.20 MeV, respectively. (They are almost the same in O^{17}: 0.88 and 4.20 MeV.) In the case of F^{19}, the barycenter of both d levels is 1.125 MeV above the s level.[8] With Al^{27}, the Nilsson orbit $d_{\frac{5}{2}}$, $\Omega=\frac{3}{2}$ is assumed to be 1.69 MeV below the $\Omega=\frac{5}{2}$ orbit, corresponding to a deformation parameter $\eta=3$.[9]

With code SALLY the wave function of the picked-up proton has been calculated for the harmonic oscillator cutoff potential with appropriate radius, binding energy, and Coulomb barrier. The distorted incident and outgoing waves have been calculated using optical-model parameters obtained from elastic scattering of 15-MeV deuterons[10] and 25-MeV He^3 particles.[11] The integration has been performed over the whole region. The results give the cross section from 0° to 180°. The value at the first maximum (with $l=1$, 2, 3) or second maximum (with $l=0$) is listed in column 8 of Table II. In the neighborhood of this maximum,

$$\sigma_{calc}\approx\sigma_l(\theta)A^{-B_p}B^QC^{-Z}, \quad (2)$$

with

$$A=1.27, \quad B=1.20, \quad \text{and} \quad C=1.13.$$

This relation is valid in the region limited to d-s shell nuclei. When the Coulomb barrier for He^3 particles becomes appreciably higher than their energy, the cross section is depressed even more by Z than indicated with Eq. (2). This is well illustrated in comparing σ_{calc} for $_{29}Cu^{63}$ ($Q=-0.62$ MeV, $B_{Cb}=14.5$ MeV) and $_{40}Zr^{90}$ ($Q=-2.60$ MeV, $B_{Cb}=17.6$ MeV). σ_{calc} for Zr^{90} is lower by a factor 63 than that for Cu^{63} (reduced to the same Q value), while the relation (2) requires only a factor 4.4. Also, the characteristic angular distribution, which is still well pronounced with Cu^{63}, changes completely with Zr^{90}. It gets rather isotropic and even increasing towards higher angles (~50°). The predicted angular distribution for Cu^{63} has been experimentally confirmed, as σ_{exp} also seems to be in rather good agreement with $\sigma_{calc}S$. We did not succeed in observing the

[8] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).

[10] R. H. Drisko (private communication).

[11] G. W. Greenlees, J. S. Lilley, P. C. Rowe, and P. E. Hodgson, Nuclear Phys. **24**, 334 (1961).

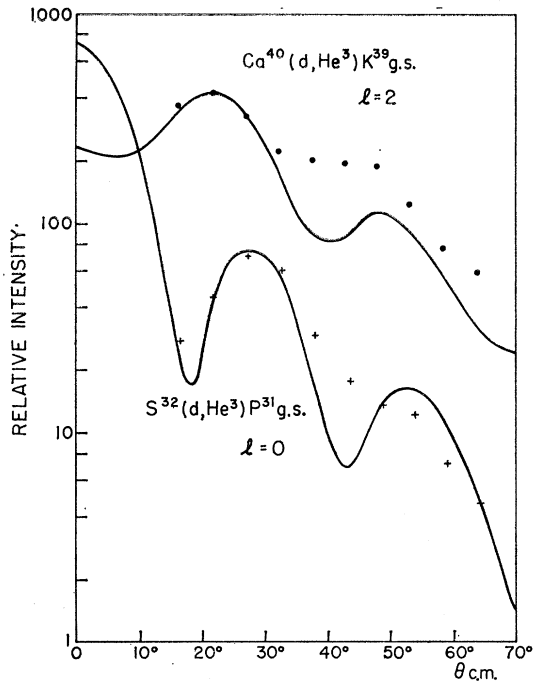

FIG. 2. Angular distributions for $S^{32}(d,He^3)P^{31}$ g.s. and $Ca^{40}(d,He^3)K^{39}$ g.s. The curves represent DWB calculations, the points experimental measurements. The curves are shifted arbitrarily along the ordinate.

reaction $Zr^{90}(d,He^3)Y^{89}$, although appreciable effort was spent. When the DWB calculations became available this became understandable, the predicted cross section being lower than experimental background.

In the last part of Table II the spectroscopic factors S_{exp} are extracted. Comparing them with S_{theor}, predicted by nuclear models, an attempt has been made to determine the constant A. Before going into details let us make some remarks about the isotopic spin formalism.

With d-s shell nuclei, where protons and neutrons fill the same orbits, the isotopic spin formalism has to be used. Here the spectroscopic factors related to the states of the same isotopic spin multiplet (nuclei with the same T but different M_T) are the same. Within this formalism, the neutrons and protons are treated as equivalent particles and it is always the total number of nucleons filling some orbit which has to be inserted in expressions[1] for S. Using this formalism, relation (1) has to be changed into

$$(d\sigma/d\Omega)(\theta)=A\sigma_{\text{calc}}(\theta)[C(T_0\tfrac{1}{2}T;M_{T_0}-\tfrac{1}{2}M_T)]^2S, \quad (3)$$

where $C(T_0\frac{1}{2}T;M_{T_0}-\frac{1}{2}M_T)$ is the Clebsch-Gordan coefficient coupling the isotopic spins of target, final, and transferred particles. From this expression values of S_{exp} or more accurately AS_{exp}, listed in column 10, have been extracted by inserting σ_{exp} for $d\sigma/d\Omega$.

Assuming a pure j-j model without configuration mixing, it is very easy to obtain spectroscopic factors S_{theor} for the ground-state transitions. In the case of O^{16}, P^{31}, S^{32}, Cl^{37}, and Ca^{40} the transitions[12] $(p_{\frac{3}{2}}^4)_{00}\rightarrow(p_{\frac{1}{2}}^3)_{\frac{1}{2}\frac{1}{2}}$, $(s_{\frac{1}{2}}^3)_{\frac{1}{2}\frac{1}{2}}\rightarrow(s_{\frac{1}{2}}^2)_{10}$, $(s_{\frac{1}{2}}^4)_{00}\rightarrow(s_{\frac{1}{2}}^3)_{\frac{1}{2}\frac{1}{2}}$, $(d_{\frac{5}{2}}^5)_{1\frac{3}{2}}\rightarrow(d_{\frac{5}{2}}^4)_{20}$, and $(d_{\frac{3}{2}}^8)_{00}\rightarrow(d_{\frac{3}{2}}^7)_{\frac{1}{2}\frac{3}{2}}$ lead to unique final states[13]; the corresponding S_{theor}, determined by the third sum rule,[1] are 4, 3/2, 4, 5/4, and 8, respectively. In the case of Al^{27} and Cl^{35} the ground-state transitions $(d_{\frac{5}{2}}^{11})_{\frac{1}{2}\frac{5}{2}}\rightarrow(d_{\frac{5}{2}}^{10})_{10}$ and $(d_{\frac{3}{2}}^3)_{\frac{1}{2}\frac{3}{2}}\rightarrow(d_{\frac{3}{2}}^2)_{10}$, respectively, lead to final states which are unique only when fixing both isotopic spin and seniority. Thus, in determining S_{theor} the fifth sum rule[1] has to be used, giving $S_{\text{theor}}=1/2$ and 5/4, respectively. In the case of Ni^{58} and Cu^{63}, S_{theor} has been determined by means of simple relations given by French,[14] taking into account just one kind of nucleon and neglecting isotopic spin. The thus determined S_{theor} are listed in column 12 with a superscript d. They represent actually the upper limits, since S get smaller when configuration mixing occurs.

As mentioned already in the Introduction, A is dependent only on the internal properties of the incident and outgoing particles, that is, on their spins and on the interaction between the transferred particle and the lightest particle [proton and deuteron in the (d,He^3) case]. For (d,He^3) it is given by[4]

$$A=\tfrac{4}{3}a^2D_0^2(1.018\times10^4)^{-1},$$

where a^2 is the probability of finding deuteron ground state within He^3 ground state, the D_0^2 is the strength of the interaction between proton and deuteron in the zero-range approximation. In (d,p) reactions, where D_0^2 refers to the interaction between proton and neutron, $D_0^2(1.018\times10^4)^{-1}$ is unity in the "zero-range approximation," and 1.5 when the Hulthén function is used to describe the deuteron. Assuming that the interaction between proton and deuteron is the same as that between neutron and proton, we have

$$A=\tfrac{4}{3}a^2$$

in the zero-range approximation, and

$$A=2a^2$$

describing proton+deuteron with the Hulthén function.

The experimental values for A are listed in the last column of Table II. Values with superscript d represent lower limits, since they have been obtained with the upper limits for S_{theor}. The mean value (omitting the Ni^{58} and Cu^{63} cases) is $A_{\min}=1.05$, which gives $a_{\min}^2=0.53$ (with the Hulthén function). The best values are those obtained with P^{31} and S^{32} using the appropriate wave functions in calculating S_{theor}. From the mean value $A=1.46$ we get $a^2=0.73$ (with the Hulthén function). These values have to be considered only as rough estimates since they include the inaccuracies in σ_{exp} ($\sim$20%), in σ_{calc}, and in the assumption $V_{pd}=V_{np}$.

[12] The subscripts outside the brackets refer to isotopic spin T and spin J of the nucleus.

[13] B. H. Flowers, Proc. Roy. Soc. (London) **A212**, 248 (1952).

[14] Reference 3, Eq. (32); or reference 1, Eq. (III–150).

However, they show that a^2 is rather close to the maximum value 0.75 allowed by the spin functions.[15] This result disagrees with the result obtained from (d,t) reactions by means of Butler theory, where the corresponding probability of finding the deuteron ground state within the triton ground state was found[16] to be an order of magnitude smaller.

Discussion of Individual Reactions

$$F^{19}(d,He^3)O^{18}$$

Wave functions for $A=18$ and $A=19$ nuclei have been calculated by Elliott and Flowers[8] and Redlich,[17] with very similar results, considering interactions between nucleons outside the O^{16} core. From Redlich's wave function there[1] is found for the ground state transition the spectroscopic factor $S_{\text{g.s.}}=0.60$. For the excited state the theory[18] predicts the ratio $S_{2^+}/S_{\text{g.s.}}=0.87$. Experimentally, $S_{\text{g.s.}}$ is smaller by a factor 1.5 to 2.1, which shows a lower occupation of the $2s_{\frac{1}{2}}$ orbit. A similar, although less obvious, result was obtained previously[19] with $F^{19}(n,d)O^{18}$ g.s., where the reduced width extracted by means of a Butler calculation is $S\theta_0^2=0.012$. The comparison with S_{theor} is restricted by the uncertainly known $\theta_0^2(2s)$; however, the result seems[1] to be smaller than expected. The experimental ratio $S_{2^+}/S_{\text{g.s.}}=0.98$ is rather close to the theoretical prediction.

The excited state observed at 4.8 MeV may be identical with a known state at 5.01 MeV. It is reached by picking up an $l=1$ proton and is one of many possible states formed by coupling one hole in the p shell with the three outer nucleons.

$$Al^{27}(d,He^3)Mg^{26}$$

If Al^{27} were a pure $d_{\frac{5}{2}}^{-1}$ state, and the Mg^{26} ground and first excited states were $d_{\frac{5}{2}}^{-2}$ states with $J=0$ and 2, the ratio $S_2/S_{\text{g.s.}}$ would be 5. According to the rotational model, however, the ratio of spectroscopic factors for transitions to two levels of the same rotational band is[1,20]

$$\frac{S(J_0')}{S(J_0)}=\frac{2J_0'+1}{2J_0+1}\frac{[C(J_0'jJ;K_0\Omega K)]^2}{[C(J_0jJ;K_0\Omega K)]^2}, \tag{4}$$

which in our case gives $S_2/S_{\text{g.s.}}=1.78$, in rather good agreement with experimental value 1.87. That the Mg^{26} ground and first excited states could be rotational levels was suggested[1] already by analyzing the reaction $Mg^{25}(d,p)Mg^{26}$, where the experimentally rather roughly determined $S_2/S_{\text{g.s.}}\approx 0.7$ was considered to be not too far from the theoretically predicted value 0.4. An analogous situation has been established[21] for Mg^{24} by studying the reaction $Mg^{25}(d,t)Mg^{24}$, for which the ratio $S_2/S_{\text{g.s.}}$ has been found to be 2.0 in rather good agreement with the predicted one (1.78).

The spectroscopic factor for rotational levels is given by[1,20]

$$S=\rho^2\frac{2J_0+1}{2J+1}[C(J_0jJ;K_0\Omega K)]^2a_{nlj}^2(|\Omega|) \tag{5}$$

($\rho^2=2$ when either K or $K_0=0$, and $\rho^2=1$ otherwise). For $Al^{27}(d,He^3)Mg^{26}$ g.s. ($K_0=0$, $\Omega=\frac{5}{2}$, $j=\frac{5}{2}$), the value is $S=\frac{1}{3}$, assuming $a_{nlj}^2=1$. Although not explicitly mentioned, it seems that (5) gives the spectroscopic factor without the isotopic spin formalism. Thus, using our notation, it is actually $C^2S=\frac{1}{3}$ or $S=\frac{1}{2}$.

The transition to the next 2^+ level at 2.97 MeV could be imagined as picking up a proton from the next, completely filled $\Omega=\frac{3}{2}$ orbit. Coupling $\Omega=\frac{5}{2}$ of the upper Nilsson orbit with $\Omega=\frac{3}{2}$ of the lower, we have $K_0=1$ or 4. It is reasonable to expect the $K_0=1$ band to be lower than the $K_0=4$ band. For transitions to this level ($J_0=2$, $K_0=1$), according to relation (5), the ratio $S/S_{\text{g.s.}}=1.07$. In this case we expect to find a level with $J_0=1$ of the band $K_0=1$ below 2.97 MeV. A level at 1.33 MeV has been recently[22] found with the $Si^{29}(n,\alpha)$-Mg^{26} reaction. We tried to detect this level with the $Al^{27}(d,He^3)$ reaction, however without success. The reason may be too small a cross section. The theoretically predicted cross section amounts to only 0.06 mb/sr which is actually of the same order as background. Another possibility is that the level 2^+ at 2.97 MeV is the lowest level of the band $K_0=2$. The predicted ratio for this case, $S/S_{\text{g.s.}}=0.53$, is in rather good agreement with the experimental value 0.64.

$$P^{31}(d,He^3)Si^{30}$$

Let us adopt for the P^{31} ground state the wave function proposed by Macfarlane and French[1]:

$$\psi(P^{31}\text{ g.s.})_{\frac{1}{2}\frac{1}{2}}=\alpha_1(s_{\frac{1}{2}}^3)_{\frac{1}{2}\frac{1}{2}}+\alpha_2(s_{\frac{1}{2}}^2)_{01}d_{\frac{3}{2}}+\alpha_3s_{\frac{1}{2}}(d_{\frac{3}{2}}^2)_{10}+\alpha_4s_{\frac{1}{2}}(d_{\frac{3}{2}}^2)_{01}, \tag{6}$$

with

$$\alpha_1^2=0.66,\quad \alpha_2^2=\alpha_3^2=0.04,\quad \alpha_4^2=0.26, \tag{7}$$

extracted from measurements[23] of $P^{31}(d,p)P^{32}$. Assuming that the Si^{30} ground state is mainly $(s_{\frac{1}{2}}^2)_{10}$, the spectroscopic factor for ground-state transition is $S_{\text{g.s.}}=\frac{3}{2}\alpha_1^2=0.99$.

The only component of the Si^{30} 2.24-MeV state with $T=1$, $J=2$, which can be reached by picking a proton from wave function (6), is $(s_{\frac{1}{2}}d_{\frac{3}{2}})_{12}$. Assuming that this component is largely predominating we get[24] $S_2=\frac{9}{16}\alpha_4^2$. The ratio of both spectroscopic factors is thus $S_2/S_{\text{g.s.}}=3\alpha_4^2/8\alpha_1^2=0.148$, using values (7). In the present

[15] A. Werner, Nuclear Phys. **1**, 9 (1956).
[16] M. El Nadi and L. Abou Hadid, Nuclear Phys. **8**, 51 (1958).
[17] M. G. Redlich, Phys. Rev. **98**, 199 (1955); **110**, 468 (1958).
[18] J. P. Elliott (private communication, quoted in reference 19).
[19] F. L. Ribe, Phys. Rev. **106**, 767 (1957).
[20] G. R. Satchler, Ann. Phys. (New York) **3**, 275 (1958).
[21] E. W. Hamburger and A. G. Blair, Phys. Rev. **119**, 777 (1960).
[22] G. Dearnaley and A. T. G. Ferguson, *Proceedings of the Rutherford Jubilee Conference, Manchester, 1961* (Academic Press Inc., New York, 1961).
[23] P. B. Daitch and J. B. French, Phys. Rev. **87**, 500 (1952).
[24] Reference 1, Eqs. (III-68) and (III-83).

TABLE III. Values of the parameters used.[a]

Assumptions	β/β_1 or β_4/β_1	$(\beta_2^2+\beta_3^2+\beta_4^2)/\beta_1^2$	β_1^2	$\beta_2^2+\beta_3^2+\beta_4^2$
$\alpha_1^2=0.66,\ \alpha_2=\alpha_3=\alpha_4=\alpha$	0.23	0.16	0.86	0.14
$\beta_2=\beta_3=\beta_4=\beta,\ \gamma_2=\gamma_3=\gamma_4$	−0.19	0.12	0.89	0.11
$\alpha_1^2=0.66,\ \alpha_2^2=\alpha_3^2=0.04,\ \alpha_4^2=0.26$	0.24	0.17	0.86	0.14
$\beta_2=\beta_3=\beta_4=\beta,\ \gamma_2=\gamma_3=\gamma_4$	−0.19	0.11	0.90	0.10
$\alpha_1^2=0.66,\ \alpha_2=\alpha_3=0,\ \alpha_4^2=0.34$	0.45	0.21	0.83	0.17
$\beta_2=\beta_3=0,\ \gamma_2=\gamma_3=0$	−0.32	0.10	0.91	0.09

[a] The calculations have been performed taking for $(\alpha^2/\alpha_1^2)^{1/2}$ and $(\alpha_4^2/\alpha_1^2)^{1/2}$, the positive value of the roots; the result is, however, not very sensitive to this assumption.

experiment $S_2/S_{\text{g.s.}}=0.27$, which is higher by a factor 1.8 and requires $\alpha_4^2/\alpha_1^2=0.72$ or, with α_2 and α_3 unchanged, $\alpha_1^2=0.53$ and $\alpha_4^2=0.39$, which seems very unlikely. It is, however, also possible to reach a 2^+ state by picking up a proton from closed $d_{\frac{5}{2}}$ orbit. The sum over all states formed by coupling one $d_{\frac{5}{2}}$ hole with the three outer nucleons is 12. With regard to this sum, the experimental $S_2=1.23$ appears rather low. However, comparing this S with that for transition $S^{32}(d,\text{He})P^{31}$ 2.23 MeV, which is evidently reached by picking up a $d_{\frac{5}{2}}$ proton (see following paragraph), we may conclude that the Si^{30} first excited state is one of the states formed by coupling one $d_{\frac{5}{2}}$ hole with the three outer nucleons.

$S^{32}(d,He^3)P^{31}$

The assumption made already with P^{31}—that the $d_{\frac{5}{2}}$ orbit is completely filled—has been experimentally confirmed for S^{32} by the fact that no $l=2$ transition to $\frac{5}{2}^+$ levels have been observed[1] in the $S^{32}(d,p)S^{33}$ reaction. Since two-particle excitations are lower in energy than one particle, we write the S^{32} ground-state wave function:

$$\psi(S^{32}\text{ g.s.})_{00}=\beta_1(s_{\frac{1}{2}}^4)_{00}+\beta_2(d_{\frac{5}{2}}^2)_{00}+\beta_3(s_{\frac{1}{2}}^2)_{10}(d_{\frac{5}{2}}^2)_{10}+\beta_4(s_{\frac{1}{2}}^2)_{01}(d_{\frac{5}{2}}^2)_{01}, \quad (8)$$

where we expect β_1 to be appreciably larger than other β's. Adopting (6) for the P^{31} ground state wave function and taking for the P^{31} 1.27-MeV state ($T=\frac{1}{2}$, $J=\frac{3}{2}^+$) all possible $(s_{\frac{1}{2}}^2)d_{\frac{3}{2}}$ configurations:

$$\psi(P^{31}\ 1.27\text{ MeV})_{\frac{1}{2}\frac{3}{2}}=\gamma_2(s_{\frac{1}{2}}^2)_{00}d_{\frac{3}{2}}+\gamma_3(s_{\frac{1}{2}}^2)_{10}d_{\frac{3}{2}}+\gamma_4(s_{\frac{1}{2}}^2)_{01}d_{\frac{3}{2}}, \quad (9)$$

we get[1] for the spectroscopic factors

$$S_{\text{g.s.}}(s_{\frac{1}{2}})=4\alpha_1^2\beta_1^2\left(1+\frac{1}{\sqrt{6}}\frac{\alpha_3}{\alpha_1}\frac{\beta_3}{\beta_1}+\frac{1}{\sqrt{2}}\frac{\alpha_4}{\alpha_1}\frac{\beta_4}{\beta_1}\right)^2, \quad (10)$$

$$S_{\frac{3}{2}}(d_{\frac{3}{2}})=2(\beta_2\gamma_2+\beta_3\gamma_3+\beta_4\gamma_4)^2. \quad (11)$$

From the experimentally determined ratio $S_{\frac{3}{2}}/S_{\text{g.s.}}=0.10$ we get—making different assumptions regarding the α's, β's, and γ's, specified in Table III—for the ratio $(\beta_2^2+\beta_3^2+\beta_4^2)/\beta_1^2$ values between 0.10 and 0.21, which gives an admixture of 9 to 17% of $s_{\frac{1}{2}}^2d_{\frac{3}{2}}^2$ configurations.

It is impossible to reach a $\frac{5}{2}^+$ state by removing a nucleon from the S^{32} g.s. wave function (8). Therefore, we have to interpret the transition $S^{32}(d,\text{He}^3)P^{31}$ 2.23 MeV ($T=\frac{1}{2}$, $J=\frac{5}{2}^+$) as picking up a proton from the closed $d_{\frac{5}{2}}$ orbit. The sum rule predicts $S=12$. Experimentally $S_{\frac{5}{2}}$ is about five times less.

$Cl^{35}(d,He^3)S^{34}$ and $Cl^{37}(d,He^3)S^{36}$

The only direct comparison with experiment is possible for the $Cl^{35}(d,\text{He}^3)S^{34}$ g.s. transition. We have $S_{\text{exp}}=1.26$ in good agreement with $S_{\text{theor}}=5/4$, showing that the Cl^{35} ground state is a rather pure $d_{\frac{3}{2}}$ state. The reactions $Cl^{35}(d,\text{He}^3)S^{34}$ 2.13 MeV and $Cl^{37}(d,\text{He}^3)S^{36}$ g.s. have been recorded together. We can therefore estimate only the upper limits for S_{exp} by taking into account just one transition and neglecting the other. We can also calculate the total cross section by using the theoretically predicted spectroscopic factors. What we get is smaller by a factor 2 than the experimental value. This suggests that S^{34} 2.13 MeV may have a configuration different from $d_{\frac{3}{2}}^2$. Some attempt has been made to detect in the angular distribution the participation of an $s_{\frac{1}{2}}$ picked-up proton, but without success.

$Ca^{40}(d,He^3)K^{39}$

In addition to the ground-state transition also a weak transition to the K^{39} 2.53-MeV level has been detected. By comparing the cross sections at 20° and 25°, $l=2$ seems more likely than $l=0$. This suggests the K^{39} 2.53-MeV level to be a $d_{\frac{3}{2}}^{-1}$ level with an excited core.

CONCLUSIONS

(1) (d,He^3) reactions used in connection with DWB calculations are well suited for studying nuclear structure provided that the energy of the He^3 particles is not appreciably lower than the Coulomb barrier, in which case the cross section falls rapidly and also the characteristic angular distribution disappears.

(2) In S^{32}, the ground state is an admixture of about 13% of $s_{\frac{1}{2}}^2d_{\frac{3}{2}}^2$ configurations to the predominating $s_{\frac{1}{2}}^4$ one.

(3) It is striking that the ratios $AS_{\text{exp}}/S_{\text{theor}}$ (last column of Table II), with S_{theor} determined by assuming just the pure j-j model (just the principal configuration), are almost the same for O^{16}, P^{31}, S^{32}, Cl^{35}, and Ca^{40}.

Allowing for S^{32} and P^{31} a contribution of about 20% of other configurations, we have to expect the same also for the closed shell nuclei O^{16} and Ca^{40}. The observed $l=2$ transition leading to the K^{39} 2.53-MeV state is an additional evidence that the Ca^{40} ground state is not a pure closed shell state.

(4) The first excited state of Si^{30} ($J=2^+$, $E=2.24$ MeV) as well as the second excited state of P^{31} ($J=\frac{5}{2}^+$, $E=2.23$ MeV) are $d_{\frac{1}{2}}$ hole states.

(5) It is somewhat surprising and not well understood that in the cases $F^{19}(d,He^3)O^{18}$ 4.8 MeV, $P^{31}(d,He^3)Si^{30}$ 2.24 MeV, and $S^{32}(d,He^3)P^{31}$ 2.23 MeV, where the proton is picked up from the lower, completely filled orbit, the spectroscopic factors S_{exp} are so low. In the first two cases they contribute only about one-tenth and in the last case about one-fifth to the total sum.

(6) The Mg^{26} ground state ($J=0$) and excited state at 1.83 MeV ($J=2$) are rotational levels of the $K=0$ band. The nature of the state at 2.97 MeV remains unclear.

(7) The probability of finding the deuteron ground state within the He^3 ground state was found to be ~ 0.7.

ACKNOWLEDGMENTS

Sincere thanks are due to Dr. B. L. Cohen for suggesting the experiment and his stimulating interest, to Dr. N. Austern, Dr. H. McManus, and Dr. R. M. Drisko for helpful discussions, to J. E. Hay for introducing the measuring technique, to G. Fodor for preparing the targets, to J. DeFrancesco and other cyclotron staff for their help in performing the measurements.

PHYSICAL REVIEW VOLUME 128, NUMBER 5 DECEMBER 1, 1962

Angular Distributions for the $F^{19}(d,n)Ne^{20}$ Reaction*

ROGER W. FINLAY† AND GEORGE E. OWEN
Department of Physics, Johns Hopkins University, Baltimore, Maryland

(Received July 13, 1962)

Angular distributions of neutrons corresponding to transitions to the ground and first (1.63 MeV) excited states in the reaction $F^{19}(d,n)Ne^{20}$ have been measured at incident deuteron energies of 1.0 and 1.3 MeV. Neutrons were detected using a single stilbene crystal spectrometer. Pulse shape discrimination in stilbene was employed to eliminate background radiation. The angular distributions of the ground-state neutrons were consistent with the predictions of nuclear stripping theory when exchange terms were incorporated in the analysis.

1. INTRODUCTION

THE reaction $F^{19}(d,n)Ne^{20}$ has been studied previously at deuteron bombarding energies of 9.06, 3.57 and 2.17 MeV.[1–3] Analyses of the angular distributions were in each case attempted in terms of nuclear stripping theory. Calvert *et al.*[1] obtained satisfactory fits to the angular distributions corresponding to transitions to the ground state of Ne^{20} in terms of the Butler[4] theory by choosing the value $1_p=0$ for the angular momentum with which the proton is captured. At lower energies[2,3] the data were represented better by choosing $1_p=2$.

Benenson *et al.*[2] have suggested that a study of the energy dependence of the ground-state neutron angular distribution would be desirable to find the crossover from the $1_p=0$ and $1_p=2$ distribution. The purpose of this paper is to report the preliminary results of this study.

In addition to the angular distributions of the ground state and first excited state neutrons, the neutron yields were taken for deuteron energies between 0.65 and 1.3 MeV.

2. EXPERIMENTAL ARRANGEMENT

Two types of targets were used in this experiment. CaF_2 targets were prepared by vacuum evaporation onto a thin nickel backing. The thickness of the evaporated film could be obtained by carefully weighing the target backing before and after evaporation. A typical result of this weighing might be (104 ± 5) μg. The area and density of the films were known so the thickness could be calculated. A crude check on this method was made by examining the width of the resonance peak of the $F^{19}(p,\alpha,\gamma)O^{16}$ reaction. Using the values of the stopping cross section of protons in CaF_2 tabulated by Bader *et al.*,[5] the results of the two thickness determinations were compared.

* Supported in part by the U. S. Atomic Energy Commission.
† Present address: University of Ohio, Athens, Ohio.
[1] J. M. Calvert, A. A. Jaffe, and E. E. Maslin, Proc. Phys. Soc. (London) **A68**, 1017 (1955).
[2] R. E. Benenson, H. Y. Chen, and J. J. Lidofsky, Phys. Rev. **122**, 874 (1961).
[3] S. Morita and K. Takeshita, J. Phys. Soc. Japan, **13**, 1241 (1958).
[4] S. T. Butler, Phys. Rev. **80**, 1095 (1950); Proc. Roy. Soc. (London) **A208**, 559 (1951); Phys. Rev. **88**, 685 (1952).
[5] M. Bader, R. E. Pixley, F. S. Mozer, and W. Whaling, Phys. Rev. **103**, 32 (1956).