heat apparatus superconducting heater leads were not used; we estimate an uncertainty of about 1% in the value of ΔQ on these runs.

In the specific-heat runs at saturated vapor pressure the cavity was not filled completely, and the amount of gas which entered the cavity was accurately determined with a Toeppler pump which was previously calibrated. We estimate that the error in the amount of gas was less than 1%. On the high-pressure runs the cavity has to be completely filled; hence, in these runs it was important to know accurately the volume of the cavity. The cavity volume was obtained by measuring the weight of the cavity with and without acetone. Because of the O-ring flange incorporated in the cavity design, it was possible to accurately fill the cavity with no danger of air spaces. We estimate that the error in the determination of the cavity volume was 1%. The error in pressure on the elevated pressure runs was ± 0.25 atm.

ACKNOWLEDGMENTS

We wish to thank members of the low-temperature physics group at Yale who helped in a variety of ways in these experiments. We are particularly indebted to Bernard Bertman for his assistance in taking data and in the design of the equipment.

PHYSICAL REVIEW VOLUME 128, NUMBER 5 DECEMBER 1, 1962

Back Diffusion of Electrons in Nitrogen, Hydrogen, and Argon*

JOHN A. DAHLQUIST
Department of Physics, Washington University, St. Louis, Missouri
(Received June 25, 1962; revised manuscript received September 10, 1962)

The back diffusion of electrons to an emitting surface results in the decrease of i_0, the current leaving the cathode, to i, the current arriving at the anode in the presence of a gas. The ratio of i/i_0 is a function of the random thermal velocity and the drift velocity, and these velocities in turn are functions of E/p, the electric field strength-to-pressure ratio. An experiment was performed which controlled these velocities independently over wide ranges in the gases nitrogen, hydrogen, and argon. In accordance with theoretical prediction, the value of i/i_0 was found to decrease with increasing thermal velocity and to increase with increasing drift velocity. Typical measured values of i/i_0 for both thermal velocity and drift velocity appropriate to E/p of 2 V/cm per mm Hg were 0.06 in N_2, 0.075 in H_2, and 0.05 in Ar, indicating the low probability of escaping back diffusion at this value of E/p.

THE study of back diffusion of electrons to an emitting surface dates back to the early work of Thomson.[1] Thomson, by using a kinetic theory approximation, derived a formula for the ratio of the electron current with gas present to the current in vacuum:

$$\frac{i}{i_0}=\frac{1}{1+(c/4v)}, \tag{1}$$

where c is interpreted as the average speed of the electrons at the cathode, and v is the drift velocity appropriate to the electric field and pressure.

Varney[2] solved the one-dimensional diffusion equations, using standard techniques. Briefly, his procedure was to take the diffusion equation with a superimposed drift velocity, separate variables, solve the resulting ordinary differential equation, determine eigenvalues to satisfy initial conditions, and to set a sum of linear orthogonal solutions to satisfy the boundary conditions. In the semi-infinite case with zero concentration at $x=0$, the diffusion equation is solved by setting up an image solution. Integration over time of the flux crossing the plane $x=0$ results in the equation,

$$n_c=n_0e^{-\alpha v/D}, \tag{2}$$

where n_c is the number of particles out of an initial delta function of n_0 particles at $x=\alpha$ which return to the cathode.

Varney assumed that the injected electrons travel a free path in a normal manner before entering the diffusion process and so the delta function is replaced by a normal free-path distribution,

$$\phi(\alpha)d\alpha=(n_0/L)e^{-\alpha/L}d\alpha. \tag{3}$$

Integration then gives

$$n_c=\frac{n_0}{L}\int_0^X e^{-\alpha/L}e^{-\alpha v/D}d\alpha. \tag{4}$$

The symbol X, introduced as the upper limit of integration, takes on one of two values, whichever is less. Either it is the distance to the anode at which the electrons are again removed or it is the mean distance to the first inelastic impact the electrons may experience. In

* This research was supported in part by a grant from ARO(D), and from Research Corporation.

[1] J. J. Thomson, *Conduction of Electricity through Gases* (Cambridge University Press, New York, 1928), p. 466.

[2] R. N. Varney, Seventh Annual Gaseous Electronics Conference, New York University, October 14, 1954 (unpublished).

either case, once the electrons have passed to points where $\alpha > X$, they cannot return to the cathode.

Integration and substitution of $n = n_0 - n_c$, the number of electrons reaching the anode, finally gives

$$\frac{n}{n_0} = \frac{1+(D/vL)e^{-(X/L)(1+vL/D)}}{1+(D/vL)}. \tag{5}$$

Experimentally, the ratio of currents is measured and since $vL/D \ll 1$ the equation becomes

$$\frac{i}{i_0} = \frac{1+(D/vL)e^{-X/L}}{1+(D/vL)}, \tag{6}$$

where D is the diffusion constant, v is the drift velocity, L is the mean free path of the electrons, and X is the distance at which the electrons are removed from the back-diffusion process. Usually $X \gg L$ and so the equation becomes

$$\frac{i}{i_0} = \frac{1}{1+(D/vL)}. \tag{7}$$

For a given gas, the three parameters D, v, and L are all functions of E/p, the electric field-to-pressure ratio. In previous experiments there was no way of varying these parameters independently; in the present experiment this is done by using an experimental tube which has two drift regions.

The details of the tube are shown schematically in Fig. 1. An oxide-coated nickel filament serves as the source of electrons, and a field is applied between the filament and the grid. After numerous collisions with the gas molecules present, those electrons which have escaped back diffusion to the filament arrive at the grid with drift velocities and random thermal velocities appropriate to the field and the gas pressure. Most of the electrons arriving at the grid are collected by it, but a small percentage drift through the holes in the grid and find themselves in a new electric field. Again, after back-diffusion losses, the remaining electrons are collected by the plate.

The grid consists of two separate elements. The first is a Kovar tube, spotwelded to a flat nickel plate which has a $\frac{1}{4}$-in.-diam hole in the center. A mica disk, 0.10 mm thick and with a $\frac{1}{2}$-in.-diam hole in the center, separates the nickel plate from the second element of the grid.

The second element is a nickel mesh, tightly stretched and lightly spotwelded to a nickel ring. The ring is held to the nickel plate by stainless steel machine screws and is insulated from it by mica washers and the large mica disk. The mesh is sold commercially as "Lektromesh" and is made by the C. O. Jelliff Mfg. Company, Southport, Connecticut. It is 19 μ thick and has rather irregularly shaped holes whose centers are uniformly spaced on a square lattice, 63 μ on a side. The $\frac{1}{4}$-in. circle in the center contains between seven and eight thousand holes. The percentage of hole area to total area is 9.4%.

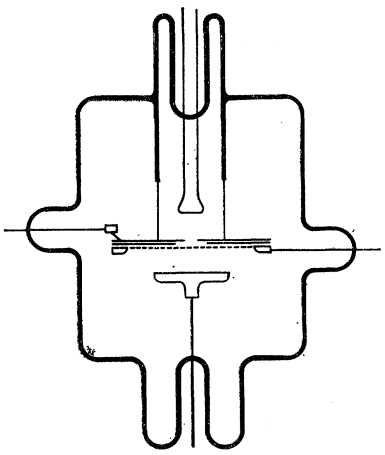

FIG. 1. Schematic diagram of the experimental tube. Reading from top to bottom, the parts shown are the filament, the nickel plate (attached to the Kovar tube), the mica insulator, the nickel mesh, and the collecting plate. The distance between the nickel plate and the mesh is exaggerated.

The collecting plate is a 3-cm solid nickel disk, placed 1 cm from the mesh.

The electrodes are enclosed in a Pyrex envelope and electrical connections are made by tungsten seals brought through the glass. The tube is connected to a standard glass vacuum system consisting of a mechanical pump, two single-stage mercury diffusion pumps, a mercury cutoff, a McLeod gauge, an ionization gauge, and a cold trap. After thorough baking in a degassing oven, a vacuum of 1×10^{-8} mm Hg is obtained. After activation of the filament a vacuum of 5×10^{-8} mm Hg is obtained with the filament on. The cold trap is cooled with liquid nitrogen at all times and all gases used must pass through the trap before reaching the experimental tube to avoid mercury contamination. The gases were obtained from Linde and Airco in 2-liter flasks and were listed as spectroscopically pure.

The value of i_0 is determined by taking the net electron current an infinitesimal distance before the grid and multiplying this current by the grid transmission factor. This net electron current is just the currents collected by the grid and the plate, so

$$i_0 = 0.094(I_g + I_p), \tag{8}$$

$$\frac{i}{i_0} = \frac{I_p}{0.094(I_g+I_p)}. \tag{9}$$

As shown in Fig. 1, the experimental tube breaks the electron drift space into two regions: in the first, which has a field strength-to-pressure ratio of $(E/p)_{12}$ the velocity of injection of the electrons into the second region is determined, as well as the magnitude of i_0. In the second region the diffusion takes place which results in the decrease of i_0 to i, the current collected by the plate. It is in the second region that the drift velocity is determined; therefore, the drift velocity is a function of $(E/p)_{34}$.

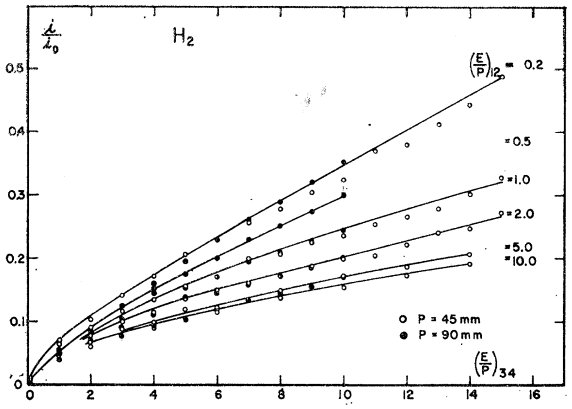

FIG. 2. Experimental data for hydrogen taken at pressures of 45 and 90 mm Hg. The values of i/i_0 for six values of $(E/p)_{12}$ are plotted against $(E/p)_{34}$.

The evaluation of D and L requires careful consideration. L enters from an integration of the diffusion equation and is the mean free path of the electrons in region two. However, the electrons have just entered region two from region one and still possess energies and velocities appropriate to region one. By the time they acquire the charactersitics of region two the electrons have traveled beyond the region of high concentration gradient (or else have diffused back to the cathode) and so do not participate in the back-diffusion process to any great extent. Thus, L should be written as a function of $(E/p)_{12}$.

By the same reasoning, D is evaluated in the region of greatest concentration gradient, and thus has the characteristics of region one, and not region two. The value of D is given as $D=(V^2/3\nu_m)_{av}$, where V is the velocity of the electrons and ν_m is the momentum transfer collision frequency. If the collision frequency is independent of electron velocity, a further simplification yields $D=cL/3$ and substitution into Eq. (3) gives

$$\frac{i}{i_0}=\frac{1}{1+(c/3v)}, \tag{10}$$

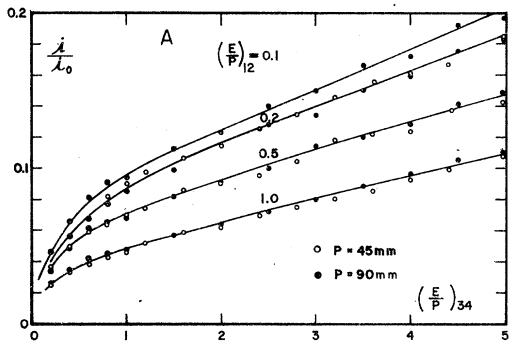

FIG. 3. Experimental data for argon taken at pressures of 45 and 90 mm Hg. The values of i/i_0 for four values of $(E/p)_{12}$ are plotted against $(E/p)_{34}$.

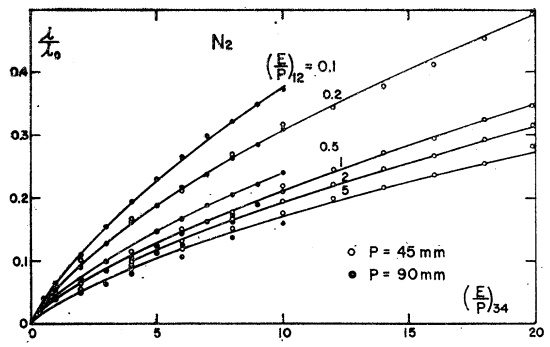

FIG. 4. Experimental data for nitrogen taken at pressures of 45 and 90 mm Hg. The values of i/i_0 for six values of $(E/p)_{12}$ are plotted against $(E/p)_{34}$.

where c is evaluated in region one and v in region two.

Since c and v are determined by varying E/p in regions one and two, respectively, Eq. (7) shows that an increase in $(E/p)_{12}$ decreases i/i_0 while an increase in $(E/p)_{34}$ increases i/i_0. Experimental results showing this dependence appear in Figs. 2, 3, and 4 for hydrogen, argon, and nitrogen, respectively. The data were taken at pressures of 45 and 90 mm Hg. The size of the tube prevented the use of pressures much below 45 mm, while the problems involved in building up high pressures and safely reducing them prevented the use of pressures much above 90 mm. A special mercury cutoff with a side tube for bleeding off gas was used at pressures up to 90 mm Hg.

The experimental data are compared with the predicted values of Eq. (7) in Figs. 5, 6, 7, and 8 over a wide

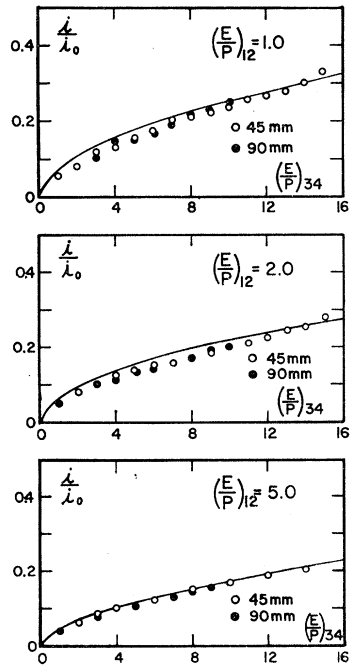

FIG. 5. Comparison of the experimental values of i/i_0 with theory for hydrogen. The experimental data are shown as circles.

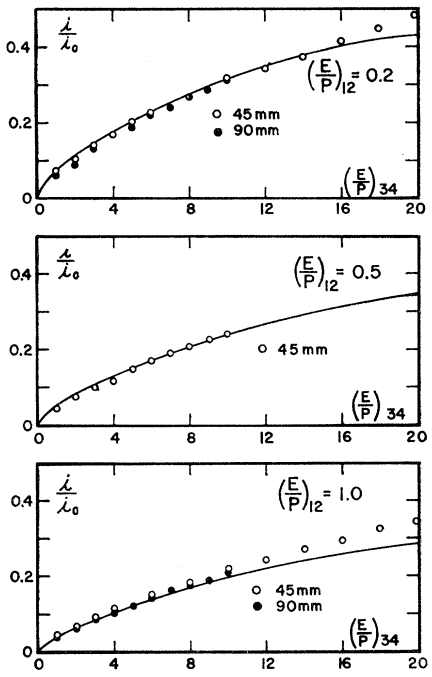

FIG. 6. Comparison of the experimental values of i/i_0 with theory for nitrogen. The experimental data are shown as circles.

range of $(E/p)_{12}$ and $(E/p)_{34}$ for the three gases. The agreement is seen to be good for hydrogen, excellent for nitrogen, and quite poor for argon. The values of v used in Eq. (7) were taken from Pack and Phelps[3] and Bradbury and Nielsen,[4] and the values of c from Townsend and Bailey.[5]

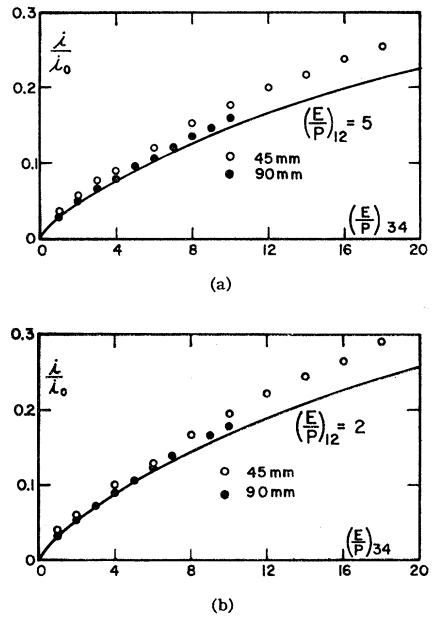

FIG. 7. Comparison of the experimental values of i/i_0 with theory for nitrogen. The experimental data are shown as circles.

[3] J. L. Pack and A. V. Phelps, Phys. Rev. **121**, 798 (1961).

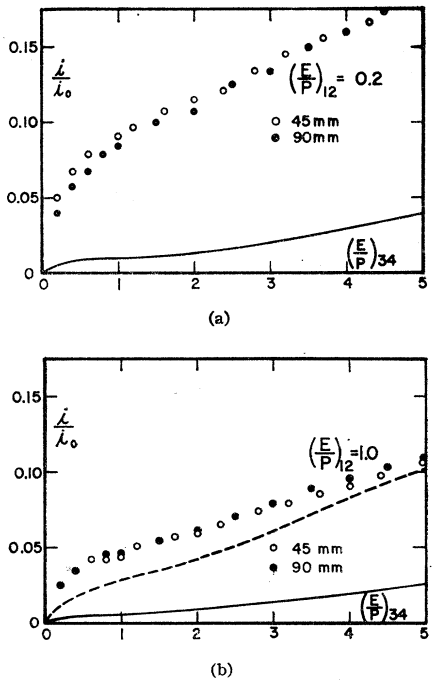

FIG. 8. Comparison of the experimental values of i/i_0 with theory for argon. The experimental data are shown as circles. The solid line is based on the unusually high values of c found by Townsend and Bailey, while the dashed line is based on the weighted average of D/vL.

Disagreement has also been reported by Theobald[6] between the Thomson theory and experimental results in argon. Theobald's experiment was done with photoelectrons in a tube consisting of two parallel plates. The disagreement in both experiments is in a large part explained by the extremely long Ramsauer free path of electrons in this gas. This fact invalidates the replacement of D by $cL/3$ where c and L are the average values of the speed and mean free path, averaged separately. Instead a weighted average of the form

$$\frac{D}{vL}=\frac{\int cLdN_c}{3v\int LdN_c}=\frac{\langle cL\rangle}{3v\langle L\rangle} \tag{11}$$

must be used, where dN_c is the number of electrons in the speed interval dc.

Unfortunately, the electron velocity distribution is not known over the range of E/p values used in the present experiment. Holstein[7] and Barbiere[8] have computed the energy distribution of electrons in gases by solving the Boltzmann transport equation, taking account of

[4] N. E. Bradbury and R. A. Nielsen, Phys. Rev. **49**, 388 (1936).
[5] R. H. Healey and J. W. Reed, *The Behavior of Slow Electrons in Gases* (Amalgamated Wireless Limited, Sydney, 1941), pp. 91–93.
[6] J. K. Theobald, J. Appl. Phys. **24**, 123 (1953).
[7] T. Holstein, Phys. Rev. **70**, 367 (1946).
[8] D. Barbiere, Phys. Rev. **84**, 653 (1951).

the variation of free paths with velocity and the possibility of inelastic collisions.

A numerical integration of the Barbiere distribution of electrons in argon for $E/p=1$ was performed, using the free path data of Ramsauer and Kollath.[9] The result was $\langle cL\rangle/\langle L\rangle=4.4\times10^7$ cm/sec, a drastically smaller value than that given by Townsend and Bailey. A high probable error, possibly as high as 10%, should be assigned to this value because of the uncertainty in reading the Ramsauer and Barbiere values.

The resulting curve based on the above value is plotted as a dashed line in Fig. 8. The agreement between experiment and the modified Varney theory is seen to be much better, with close agreement at high $(E/p)_{34}$. It is believed that the disagreement at low $(E/p)_{34}$ can be explained by the use of Eq. (6). At low E/p values an appreciable number of electrons have free paths comparable with the 1-cm drift space used, even at pressures as high as 90 mm Hg.

The use of Eq. (6) is also called for at high values of E/p where inelastic collisions occur. In this case X is the mean distance to the first inelastic collision, and this value can be used to calculate an inelastic collision probability. Experimental evidence as well as Barbiere's theoretical calculations indicate that very small inelastic collision probabilities greatly alter the energy distribution of electrons in argon.

Actually a weighted average in computing $D/(vL)$ should be used for H_2 and N_2, but since the free path is relatively constant this is not necessary. It is only in argon, where the free path behaves almost like a delta function, that the weighted average is necessary.

In conclusion, the methods of back diffusion are capable of giving quantitative results in molecular gases, and can conceivably be used in place of the cross-diffusion techniques to give values of c and other parameters. Confirmation of the Theobald results under an extended range of all variables and the successful analysis of the discrepancies with the simple theory in the case of the rare gases give the present results considerable value for applied purposes. For such use Figs. 2, 3, and 4 are especially significant.

The author wishes to express his gratitude to Professor R. N. Varney, who proposed and directed this work, for his assistance and advice during the course of the work.

[9] S. C. Brown, *Basic Data of Plasma Physics* (John Wiley & Sons, Inc., New York, 1959), p. 19.

PHYSICAL REVIEW VOLUME 128, NUMBER 5 DECEMBER 1, 1962

Osmotic Pressure of He^3 in Liquid He^4, with Proposals for a Refrigerator to Work below 1°K

H. LONDON AND G. R. CLARKE
Atomic Energy Research Establishment, Harwell, England

AND

ERIC MENDOZA
Physical Laboratories, University of Manchester, Manchester, England

(Received September 21, 1961; revised manuscript received May 14, 1962)

An experimental study has been carried out of the osmotic pressure of solutions of the isotope He^3 in liquid He^4 at low temperatures, between 0.8 and 1.2°K. A superleak, a tube packed with a fine powder, acted as a semipermeable membrane which allowed only the superfluid He^4 to pass. The conclusion from these experiments was that the measured osmotic pressures were in reasonable agreement with values expected from the thermodynamic relations with other equilibrium properties of the mixtures, notably their vapor pressures. Thermodynamic equilibrium therefore seemed to have been attained under the conditions of the experiments. The second half of this paper concerns a study of the cooling which must take place during the adiabatic dilution of He^3 by He^4. If the dilution is carried out at low temperatures where the solutions separate into two phases, the absorption of heat is estimated to be usefully large. After dilution the solution can be distilled, condensed and recirculated so as to make a continuously acting refrigerator. It should be possible to operate at temperatures of 0.1°K or below.

1. INTRODUCTION

THE object of this paper is to explore the feasibility of a new kind of refrigerator for operation below 1°K. The proposed method uses as the working substance a solution of He^3 in He^4; the cooling is produced by a process of adiabatic dilution and is followed by a distillation in order to reconcentrate the diluted solution and then recirculate it. Thus, the evaporation of the He^3 takes place in two stages—the dilution of the He^3 by He^4 followed by the evaporation of the dilute liquid. This, it is calculated, should allow very low temperature (below 0.1°K) to be reached.

At low temperatures where the entropy of liquid He^4 is very small, a solution of He^3 in He^4 can profitably be