Dynamical Approach to the Selection Rule $|\Delta I|=\frac{1}{2}$*

E. R. McClIMENT AND K. NISHIJIMA
Department of Physics, University of Illinois, Urbana, Illinois
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By giving up the unrenormalizable Lagrangian theory, the problem of weak interactions was studied by means of the unsubtracted dispersion relations. In this formalism the selection rule $|\Delta I|=1/2$ in the nonleptonic decays of strange particles is shown to be a consequence of the charge independence of strong interactions. The unsubtracted dispersion relations further serve to determine the coupling constants in strong interactions as eigenvalues. Reasonable agreement with experiment was found for the nonleptonic hyperon decays for the even $\Sigma\Lambda$ parity.

I. INTRODUCTION

In the past several years much experimental evidence has been accumulated in favor of the selection rule $|\Delta I|=1/2$ in the nonleptonic decays of strange particles.[1,2] Since this is one of the few selection rules rather well established for weak interactions, we feel that there must be a deep reason why it is obeyed. If we take this point of view for granted, this selection rule must be a manifestation of the laws of nature underlying the yet unrevealed dynamical structure of weak interactions. This paper will be devoted to the study of the possibility of explaining or even deriving this selection rule starting from first principles alone. The first question immediately raised is what the first principles are, and it is one of the main subjects to answer this question.

First, it is clear that the Lagrangian formulation of weak interactions cannot give a solution to the above problem, since, in order to write down the interaction Lagrangian for the nonleptonic decays of strange particles, this selection rule must be built in from the beginning. In other words this rule must be assumed in the Lagrangian theory, and there is no possibility of explaining it. Thus, from our point of view, we must inevitably give up the Lagrangian formalism for weak interactions.

Although we have thus far denounced the Lagrangian approach, there are many reasons to believe that the Lagrangian approach is not very bad after all. We have no definite evidence against the Lagrangian theory, mainly because of the lack of our knowledge on the reliable solution of it for strong interactions. Also, the positive aspect of the Lagrangian theory should not be forgotten in connection with the brilliant successes in quantum electrodynamics. For this reason we shall take a rather conservative stand that strong interactions are described by renormalizable Lagrangian. There are people of the opinion that the difficulties associated with unrenormalizability occur only in the perturbation theory and that unrenormalizability does not give rise to serious troubles in better approximations. We think, however, that even in better approximations the difficulty would remain in one way or another. Thus, we are led to a very naive stand that strong interactions are represented by a renormalizable Lagrangian. This is perhaps the only way to keep the connection between quantum field theory and classical theory.

This naive point of view fails, however, in an effort to formulate the theory of weak interactions by means of renormalizable Lagrangians. It is extremely difficult or probably impossible to describe weak interactions in terms of renormalizable Lagrangians. For instance, Fermi interactions are unrenormalizable, and it is not possible to reproduce the V-A theory by combining renormalizable Yukawa-type interactions, so that we may conclude that weak interactions cannot be described by renormalizable Lagrangians. We interpret this fact as an indication that we should give up the Lagrangian approach for weak interactions; then the only method to describe weak interactions would be the dispersion approach. The application of the dispersion theory to weak interactions was initiated by Goldberger and Treiman, and they succeeded in calculating the π-μ decay.[3,4] Leaving details to later sections, we shall emphasize one particular aspect of this approach. They assumed an unsubtracted dispersion relation for the decay amplitude and succeeded in reproducing the experimental lifetime of the charged pion. In this paper we further extend this assumption to include all the decay amplitudes, namely, we assume unsubtracted dispersion relations for all the weak decay amplitudes. The physical significance of this assumption will be discussed at length in later sections, but we shall simply assume it here.

The formulation of weak interactions in terms of unsubtracted dispersion relations reminds us of the dispersion treatment of bound states proposed by Blankenbecler and Cook.[5] They assumed unsubtracted dispersion relations for vertex functions involving composite particles and showed how they lead to eigenvalue

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[1] F. S. Crawford, Jr., M. Cresti, R. L. Douglass, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **2**, 266 (1959).

[2] M. Schwarz, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, New York, 1960), p. 726.

[3] M. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

[4] M. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).

[5] R. Blankenbecler and L. F. Cook, Jr., Phys. Rev. **119**, 1745 (1960).

problems. It would not be unreasonable to assume that this formal analogy at the starting point might continue much further and that many aspects of the bound states are also shared by weak interactions. Take the deuteron, for instance: we can in principle explain why it is in the $^3S+^3D$ state but not in the ^1S state; determine the deuteron mass or determine the pion-nucleon coupling constant if the deuteron mass is given; and determine the D state to S state ratio. If the analogy really continues to be valid we would be able to explain why the nonleptonic decays of strange particles obey $|\Delta I|=1/2$ but not $|\Delta I|=3/2$, determine the coupling constants in strong interactions, and determine the branching ratios in decay processes. Assuming that this is really the case, we develop a theory of weak interactions and, in particular, the nonleptonic decays of hyperons will be studied in detail in connection with the selection rule $|\Delta I|=1/2$. In Sec. II we first translate the renormalizable Lagrangian theory of strong interactions into an appropriate dispersion language. Then in Sec. III we shall give a brief discussion of the problem of bound states, since the weak interactions, as formulated in this paper, bear strong resemblance to bound states. Then, based on the similarity to the bound-state problem, a theory of weak interactions is proposed in Sec. IV, and then the nonleptonic decays of hyperons will be discussed within the realm of our dispersion theory. Also the possibility of deriving the selection rule $|\Delta I|=1/2$ in the nonleptonic decays of strange particles will be shown in Sec. V. In the two Appendixes, we discuss the renormalization that we encountered in the nonleptonic decays of hyperons and the connection between conserved currents and subtractions.

II. DISPERSION FORMULATION OF RENORMALIZABLE LAGRANGIAN THEORY

We assumed that strong interactions are described by renormalizable Lagrangians, but we have to use dispersion relations rather than Lagrangians in the description of weak interactions. Therefore, in order to describe both strong and weak interactions coherently, it is desirable to give a dispersion description of strong interactions, and, in this section, we shall develop the dispersion formulation of strong interactions.

The S-matrix theory of strong interactions has been developed on the basis of two fundamental properties of the S matrix, namely, the unitarity and analyticity, but these are very general properties that should be satisfied by any theory and in order to specify a theory it is necessary to introduce subtractions in the dispersion relations and specify the values of subtraction constants. The introduction of subtractions, however, specifies the high-energy behaviors of scattering amplitudes to some extent, and we are yet very ignorant about them. There are still many obscure points in the dispersion theory.

Furthermore we do not have a complete set of dispersion relations that replaces the Lagrangian theory, and dispersion relations are known only for a relatively simple set of scattering amplitudes. It is possible, however, to write down a complete set of dispersion relations if we consider the Green's functions rather than the S-matrix elements. Green's functions satisfy both the parametric dispersion relations and the generalized unitarity condition as has been shown in a series of papers[6–9] by one of the present authors (KN), and this Green's function approach can be a possible substitute for the Lagrangian formulation. Since this formulation has been already discussed in detail, we shall not reproduce it here, but we shall base our arguments on this formulation.

In the dispersion theory one of the most important and difficult problems is the study of high-energy behavior of scattering or production amplitudes, and this problem is closely connected with the subtractions in dispersion relations. Since the solution of this problem is not available to us, we shall make a heuristic assumption concerning the numbers of subtractions needed in dispersion relations. At present, neither the solution of the Lagrangian theory nor that of the dispersion theory is known, except in the perturbation theory, so that they can be compared only in the perturbation theory.

Therefore it is perhaps reasonable to introduce the following postulate:

Postulate I. The subtractions in the parametric dispersion relations for Green's functions are fixed in such a way as to reproduce the solution of the renormalizable Lagrangian theory in perturbation theory.

This postulate enables us, following the prescriptions given in the previous papers, to fix the forms of the parametric dispersion relations completely, and we shall illustrate this statement by a simple example of the pion-nucleon interaction.[8]

Define a three-point function $\rho(x,y,z)$ by

$$\rho(xyz)=K_xD_y\bar{D}_z\langle 0|T[\varphi(x)\psi(y)\bar{\psi}(z)]|0\rangle, \quad (2.1)$$

where $K_x=\Box_x-\mu^2$ is the Klein-Gordon operator for the pion field; $D_y=\gamma\partial_y+m$ and $\bar{D}_z=\gamma^T\partial_z-m$ are the Dirac operators for the nucleon field. For simplicity we assume that the pion is neutral and expand ρ into a sum of all possible invariants:

$$\rho(xyz)=i\gamma_5\rho_a(xyz)+i\gamma_5\gamma_\mu\frac{\partial}{\partial x_\mu}\rho_b(xyz)+i\gamma_5\sigma_{\mu\nu}\frac{\partial^2}{\partial y_\mu\partial z_\nu}\rho_c(xyz). \quad (2.2)$$

Define the Fourier transform of $\rho_a(xyz)$ by

$$\rho_a(xyz)=\frac{-i}{(2\pi)^8}\int d^4p_1d^4p_2d^4p_3\delta(p_1+p_2+p_3)\times e^{i(p_1x+p_2y+p_3z)}\mathcal{G}_a(p_1p_2p_3); \quad (2.3)$$

[6] K. Nishijima, Phys. Rev. **119**, 485 (1960).
[7] K. Nishijima, Phys. Rev. **122**, 298 (1961).
[8] M. Muraskin and K. Nishijima, Phys. Rev. **122**, 331 (1961).
[9] K. Nishijima, Phys. Rev. **124**, 255 (1961).

then $\mathcal{G}_a$ is a function of scalar products of p's alone, i.e., p_1^2, p_2^2, and p_3^2. This $\mathcal{G}_a$ satisfies a once-subtracted parametric dispersion relation, and the subtraction constant is determined subject to the boundary condition

$$\mathcal{G}_a(p_1^2=-\mu^2, p_2^2=-m^2, p_3^2=-m^2)=g. \quad (2.4)$$

This subtraction introduces the interaction $ig\bar{\psi}\gamma_5\psi\varphi$ in the corresponding Lagrangian theory. A subtraction for $\mathcal{G}_b$ leads to the unrenormalizable pseudo-vector coupling in the corresponding Lagrangian theory so that $\mathcal{G}_b$ is assumed to satisfy the unsubtracted parametric dispersion relation. In this way, the subtractions in the dispersion relations are fixed with reference to the renormalizable Lagrangian.

III. DISPERSION APPROACH TO BOUND STATES

In the previous section we confined ourselves to perturbation theory, but clearly we need a better approximation method to deal with the problem of bound states. In reference 7 the definition of the composite particles distinct from elementary particles was given. Since the analogy between bound states and weak interactions motivated the present work, we shall briefly recapitulate the discussions on the bound states.

In the Lagrangian theory we have a clear-cut distinction between elementary and composite particles, since we introduce field operators only for those particles, either stable or unstable, that are supposed to be elementary. This distinction does not have a correspondence in dispersion theory, since we introduce field operators for all the stable particles no matter whether they are elementary or composite. In the S-matrix approach it is to some extent a matter of convenience to call a particle elementary or composite. We think, however, that it is still possible to make a distinction through subtractions in the parametric dispersion relations. In order to study this question, let us assume that the nucleon and pion are elementary and the deuteron is composite; then all the parameters related to the deuteron such as the rest mass, magnetic moment, etc., must be determined as functions of more fundamental parameters of the theory such as the nucleon mass, the pion mass, and the pion-nucleon coupling constant. This leads us to the following definition of the composite particles:

If no arbitrary parameters are introduced through the parametric dispersion relations for all the Green's functions involving a certain field operator φ_c for a stable particle c, we call c a composite particle.

As has been discussed before,[7] this definition does not necessarily exclude subtracted dispersion relations for Green's functions involving a composite particle. Indeed, if the subtraction constants are not arbitrary, they may satisfy subtracted dispersion relations. For example, the two-point Green's functions or the propagation functions satisfy twice-subtracted dispersion relations, but the subtraction constants are determined subject to the renormalization condition. Another example is given by the Green's functions corresponding to the electromagnetic vertex functions. In this case the subtraction constants are given by the universal charge, and we were led to the conclusion that the distinction between elementary and composite particles cannot be made by means of the universal electromagnetic interactions. A similar idea has been expressed by Feynman[10] in connection with the universal vector coupling in the Fermi interactions.

When no such universality is available, however, it is natural to assume that Green's functions involving composite particles satisfy unsubtracted dispersion relations. This certainly imposes a severe restriction on the high-energy behavior of composite particles, and there emerges a possibility of making the distinction through the measurements of high-energy cross sections.

Rigorous mathematical discussions on the Green's functions are rather difficult, since too many variables are involved, and we shall switch from Green's functions to S-matrix elements. The connection between these two approaches is not yet very clear; in particular we do not know whether an unsubtracted dispersion relation in one case necessarily requires an unsubtracted dispersion relation in the other. But, in order to make a heuristic argument on this subject, we will assume this correspondence whenever it is felt convenient.

First we would quote the work of Blankenbecler and Cook[5] who showed that the combination of an unsubtracted dispersion relation with unitarity leads to an eigenvalue problem thus enabling us to compute the binding energy of a bound state in principle. It is not clear in their approximation whether or not an unsubtracted dispersion relation always leads to an eigenvalue problem, but it perhaps does if an appropriate boundary condition is imposed upon the integral equations.

The second point we would like to emphasize is that the above-given definition of composite particles is consistent with our intuitive picture that particles with high spins would be composite. In order to prove that a particle is composite, we have to show the absence of subtractions in dispersion relations for all the Green's functions involving this particle, but, since it is rather hard, we shall give a plausible argument on the absence of subtractions in the dispersion relations for vertex functions and scattering amplitudes involving a high spin particle. We base our arguments on the unitarity condition and analyticity properties of such amplitudes. Lehmann, Symanzik, and Zimmermann[11] have shown that the pion-nucleon vertex function should be a decreasing function of the momentum transfer when nucleons are on the mass shell.

They started from the Källén-Lehmann integral

[10] R. P. Feynman, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 501.

[11] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo cimento **2**, 425 (1955).

representation of the two-point Green's function and assumed that the dispersion integral in this representation converges without subtraction. This already poses a severe restriction on the high-energy behavior of the weight function or the absorptive part of the propagation function. If we combine this restriction with the (generalized) unitarity condition for the propagation function we arrive at their result. In the application of the unitarity condition, the spins of particles in the intermediate states serve to determine the kinematical multiplicative factor, and the higher the spins, the more severe the restriction imposed on the high-energy behavior of the vertex functions as discussed in reference 7. The same argument appeared in the renormalization of Green's functions in perturbation theory.[9]

Froissart[12] studied the high-energy behavior of scattering amplitudes satisfying the Mandelstam representation and reached the conclusion that only a limited number of subtractions, and hence free parameters, can be introduced. In this case, too, the unitarity condition will give more and more severe restrictions on the high-energy behavior of the scattering amplitudes as the spins of particles in the intermediate states become higher and higher. Thus, vertex functions, as well as scattering amplitudes, involving high spin particles would satisfy unsubtracted dispersion relations. This argument renders support to the intuitive picture of composite particles that particles with high spins would inevitably be composite, and, at the same time, it suggests that Green's functions corresponding to unrenormalizable interactions will satisfy unsubtracted dispersion relations in the sense discussed in Sec. II.

IV. INTRODUCTION OF WEAK INTERACTIONS

As discussed in the previous section, it seems reasonable to assume unsubtracted dispersion relations for all the weak decay amplitudes, and we shall really assume it later. Instead of assuming the unsubtracted dispersion relations from the beginning, however, we shall develop our own arguments in quite a different way.

Postulate I in Sec. II fixed the possible form of the fundamental set of equations in the dispersion approach, and, if we solve it in the perturbation theory, the only solution will be given by that of the renormalized Lagrangian theory for strong interactions. Various invariance requirements met by the Lagrangian theory of strong interactions will be shared by the solution of the dispersion theory. In order to include bound states, however, we have to solve the fundamental set of equations in better approximations than the perturbation theory. This gives rise to some difficulties in the dispersion theory due to the lack of uniqueness of the solution of the dispersion equations as shown by Castillejo, Dalitz, and Dyson.[13]

One possible way to overcome this difficulty and select a unique solution is to assume that the right solution is given by the renormalized Lagrangian theory provided that we can develop a better approximation method than the perturbation theory. Up to this point there is no essential benefit of formulating field theory in terms of dispersion relations, and we have not yet deviated from the conventional Lagrangian theory. A drastic change is introduced when we try to accommodate weak interactions into our formulation.

The Lagrangian theory is perhaps too restrictive to allow more than one solution, as we assumed above, but the dispersion theory is certainly more relaxed on this point. Therefore, we introduce our second postulate:

Postulate II. The fundamental set of equations in dispersion theory allows a one-parameter group of solutions,[14] and if we denote an arbitrary Green's function by $\mathcal{G}$,

(1) $\mathcal{G}$ can be expanded in powers of a small parameter λ, i.e.,

$$\mathcal{G}=\mathcal{G}_0+\lambda\mathcal{G}_1+\lambda^2\mathcal{G}_2+\cdots. \tag{4.1}$$

(2) $\mathcal{G}_0$ is given by the solution of the Lagrangian theory for strong interactions.

Let us assume that there are many invariance principles satisfied by the Lagrangian theory of strong interactions or equivalently by $\mathcal{G}_0$. These invariance principles are not necessarily satisfied by $\mathcal{G}_1$, $\mathcal{G}_2$, etc., and, if the introduction of a certain $\mathcal{G}$ violates an invariance principle, its lowest-order term $\mathcal{G}_0$ will vanish identically. In other words we are trying to destroy certain invariance principles by making use of the nonuniqueness of the solution of the dispersion theory.

The idea of finding noninvariant solutions of an invariant set of equations was first proposed by Heisenberg[15] and followed by Nambu and Jona-Lasinio.[16,17] Our approach is close to theirs in the basic ideas, but also differs from theirs in that violations are introduced only in weak interactions and also in that Lagrangian theory was given up to give ample room to accomodate noninvariant solutions.

The postulate I implies that weak-decay amplitudes, violating invariance principles and being unrenormalizable, should satisfy unsubtracted dispersion relations, or $\mathcal{G}_1$, $\mathcal{G}_2$, and further terms should satisfy unsubtracted dispersion relations.

First, we write down unsubtracted dispersion relations for $\mathcal{G}_1$ and then write down the unitarity condition in the first order of λ. Both dispersion relations and uni-

[12] M. Froissart, Phys. Rev. **123**, 1053 (1961).

[13] R. L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1956). This difficulty is related to the fact that the dispersion theory is incapable of accommodating unstable particles in the fundamental set of equations.

[14] This could be a several-parameter group of solutions as we shall see later, but this difference is irrelevant in the present discussion. Physically this parameter denotes the weak-coupling constant.

[15] W. Heisenberg, Revs. Modern Phys. **29**, 269 (1957).

[16] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[17] G. Jona-Lasinio and Y. Nambu, Phys. Rev. **124**, 246 (1961).

tarity condition are linear and homogeneous in $\mathcal{G}_1$. These equations have only the trivial solution, all $\mathcal{G}_1=0$, if solved in the series expansion in powers of the strong coupling constants. This feature bears a strong resemblance to the case of bound states, and we shall utilize this resemblance as a guide to formulate the theory of weak interactions.

If we solve the singular integral equations in an approximation better than the series expansion, however, there will be too many solutions in general, and we are forced to introduce further boundary conditions to select the physically acceptable solutions. For the time being, it is not clear what kind of boundary conditions we should employ, but, temporarily, we shall utilize the similarity of weak interactions to bound states. Take a simple Schrödinger equation for a radial wave function $u(r)$ and solve it for negative energies; then, as is well known, the function $u(r)$ increases exponentially for large values of r, but for special values of the parameter, i.e., the energy in this case, $u(r)$ falls off exponentially for large values of r, and the boundary condition in this case may be given by the requirement of rapid decrease of the amplitude or the wave function. We adopt this condition to our problem.

Postulate III. The first-order Green's functions $\mathcal{G}_1$ satisfy the following boundary conditions:

(1) $\mathcal{G}_1$ can survive only when the corresponding invariant[18] cannot exist in strong interactions because of its conflict with some of the invariance principles valid in strong interactions. In other words, $\mathcal{G}_1\neq 0$ only when $\mathcal{G}_0=0$.

(2) Among many other solutions of the singular integral equations for $\mathcal{G}_1$, the physically realizable solution is the one that falls off most rapidly at high energies, or for large values of invariant variables, e.g., the scaling parameter ξ.

(3) Time-reversal invariance is valid even in the presence of weak interactions.

Since the theory of weak interactions is formulated based on the formal analogy to bound states, weak interactions will share various important properties with bound states. One characteristic feature of weak interactions is that the weak decay amplitudes should satisfy unsubtracted dispersion relations. Goldberger and Treiman[3,4] assumed an unsubtracted dispersion relation for the π-μ decay amplitude and calculated the pion lifetime in accord with experiments. In the present approach, we generalized their assumption and assumed that all the weak decay amplitudes satisfy unsubtracted dispersion relations.

In the case of bound states Blankenbecler and Cook[5] suggested the possibility of deriving eigenvalue equations from unsubtracted dispersion relations. Although this has not yet been justified mathematically, this conclusion seems plausible on physical grounds, and, if the above analogy continues to be valid, the unsubtracted dispersion relations for the weak decay amplitudes will also lead to eigenvalue equations. We will discuss this point in the next section.

The structure of weak interactions as formulated here manifests itself in the way various invariance principles are violated. The conservation of such dichotomic variables as charge conjugation and parity is violated by the presence of weak decay amplitudes of the opposite transformation properties, and our formulation has very little to say about this in general. However, the isotopic spin does turn out to be violated in a particular way in our formalism.

The strong interactions are known to be approximately charge independent, and, for simplicity, let us assume that this is strictly true. First let us consider nonleptonic decays of a strange particle obeying $|\Delta S|=1$, and then expand the corresponding first-order Green's function $\mathcal{G}_a$ into isospin eigenfunctions, i.e.,

$$\mathcal{G}_a=\mathcal{G}_{1/2}+\mathcal{G}_{3/2}+\cdots, \tag{4.2}$$

where $\mathcal{G}_{1/2}$ transforms as a component of an isospin doublet $(I=1/2)$, $\mathcal{G}_{3/2}$ transforms as a component of an isospin quartet $(I=3/2)$, and so on. Corresponding to the strangeness change $|\Delta S|=1$, there occur only half-integral isospins in this expansion. Next, write down unitarity condition and unsubtracted dispersion relations for the Green's functions $\mathcal{G}_{1/2}$, $\mathcal{G}_{3/2}$, etc. Then since strong interactions are charge-independent and all the equations are linear in $\mathcal{G}_{1/2}$, $\mathcal{G}_{3/2}$, etc., the equations for $\mathcal{G}_{1/2}$'s, those for $\mathcal{G}_{3/2}$'s, etc., are completely decoupled from each other. Thus we can solve "1/2" equations, "3/2" equations, etc., separately. From the discussions given previously, each set leads to its own eigenvalue equations which serve to determine the strong coupling constants as eigenvalues. Generally, different sets of eigenvalue equations require different values for the same strong coupling constant, and they are incompatible with each other. Therefore only one set of equations will have a nontrivial solution other than zero. In order to answer the question of which set will have a nontrivial solution, we have to know precisely the strong interactions which enter as the kernels into the integral equations to determine the weak decay amplitudes, and, if we knew them, we could determine which one of the amplitudes in $\mathcal{G}_{1/2}$, $\mathcal{G}_{3/2}$, etc., would survive. Since such detailed knowledge is not available we will assume that $\mathcal{G}_{1/2}$ is the nonvanishing one; then this implies the selection rule $|\Delta I|=1/2$ in the nonleptonic decays of strange particles. The above arguments show that the selection rule $|\Delta I|=1/2$ is likely to be a consequence of the charge independence of strong interactions in the present scheme, and we realize that it is just in the realm of weak interactions, where an axiomatic dispersion approach reveals itself to be more capable of dealing with fundamental problems than the Lagrangian theory does.

[18] See Sec. II. The invariant corresponding to $\mathcal{G}_a$ is given by $i\gamma_5$.

The problem of deriving the selection rule $|\Delta I|=1/2$ will be discussed in more detail in the next section.

The scheme as described above should be able to be applied not only to nonleptonic decays, but also to leptonic decays as well. However, we encounter great complications in applying the unsubtracted dispersion relations to leptonic decays consistently. The reason for this difficulty is as follows: If we take, for instance, the decays

$$\pi\rightarrow\mu+\nu,\quad K\rightarrow\mu+\nu, \tag{4.3}$$

these processes require the study of the three-point Green's functions, or three-point vertex functions. It is customary, however to assume that μ and ν are emitted from the same point in the Feynman diagram, since they must both arise simultaneously from the weak interaction. This makes the three-point Green's functions behave as if they were two-point Green's functions as far as high-energy behavior is concerned. As we know, the high-energy behavior of two-point Green's functions are much worse than those of three-point Green's functions. Therefore, as long as we take the above customary approach, the introduction of subtractions seems to be indispensable in order to avoid divergences, but this does not conflict our basic philosophy of describing weak interactions in terms of unsubtracted dispersion relations. If we introduce the electromagnetic interactions through which leptons can interact with other particles strongly, the three-point functions behave really like three-point Green's functions and their high-energy behavior is improved. This problem of introducing electromagnetic interactions is particularly serious to π-μ and K-μ decays and also for the vector part of the Fermi interaction since they necessarily require subtractions if the electromagnetic interactions are neglected. In Appendix II an explanation will be given of why a conserved vector current requires a subtraction.

For the complication due to the electromagnetic interactions the derivation of eigenvalue equations for leptonic decays is extremely difficult[19] and we shall not attempt it here, but perhaps we may make speculations on this subject. If we take μ-e decay, then to a very good approximation we can neglect all the strong interactions except the electromagnetic interactions that are well known. For this process we write down two sets of equations corresponding to vector and axial vector couplings. If everything goes all right, we could determine two parameters as eigenvalues. There are two parameters involved in this problem, namely, the fine structure constant and the μ-meson to electron-mass ratio, and we might be able to determine them by solving eigenvalue equations. Unfortunately the equations involve infrared divergences, and we do not know how to overcome this difficulty within the framework of dispersion theory; we shall now stop our speculation.[20]

V. NONLEPTONIC DECAYS OF HYPERONS

In this section we discuss the vertex functions for nonleptonic decay of hyperons along the line described in the previous section. The decay of a hyperon Y (Λ or Σ) into a nucleon and a pion,

$$Y\rightarrow N+\pi,$$

is described by the amplitude,

$$\bar{u}_N(p)J_Y(s)=(2p_0q_0/m)^{\frac{1}{2}}\langle N\pi(-)|\bar{j}_Y(0)|0\rangle, \tag{5.1}$$

where $s=-P_Y^2$ is the square of the center-of-mass energy, p_0 is the final-state nucleon energy, q_0 is the final-state pion energy, and m is the nucleon mass. $\bar{j}_Y$ is defined by

$$\bar{j}_Y(x)=(-\gamma^T\cdot\partial+m_Y)\bar{\psi}_Y(x).$$

From the general invariance arguments, J may be expanded into a sum over various invariants as

$$J_Y(s)=F_1(s)+\frac{i\gamma\cdot P_Y}{\sqrt{s}}F_2(s)+\gamma_5\left(F_3(s)+\frac{i\gamma\cdot P_Y}{\sqrt{s}}F_4(s)\right). \tag{5.2}$$

The question of charge states will be dealt with later. The above rather arbitrary looking choice of invariants was made in order to simplify the decoupling of the integral equations which result from the dispersion relations. Application of the LSZ reduction formula to Eq. (5.1) yields, after neglecting an equal-time commutator,

$$J_Y(s)=i(2q_0)^{1/2}\int d^4x\, e^{-ipx}\langle\pi|\{j_N(x),\bar{j}_Y(0)\}\theta(x_0)|0\rangle, \tag{5.3}$$

where j_N is defined by

$$j_N(x)=(\gamma\cdot\partial+m)\psi_N(x).$$

Equation (5.3) is used to deduce the analytical properties of the amplitudes F_i. We shall write down the dispersion relations for the invariant functions F_i without proof:

$$F_i(s)=\frac{1}{\pi}\int_{-\infty}^{\infty}ds'\frac{\mathrm{Im}F_i(s')}{s'-s-i\epsilon}.\quad (i=1,2,3,4.) \tag{5.4}$$

In what follows we derive the integral equations for F_i in the one-pion approximation. If we had defined the functions F_i in (5.2) without the factor $\sqrt{s}$, these functions are analytic in the complex s plane cut along

[19] Consider the coupling between the π-μ decay and the axial vector part of the capture, then the derivation of the Goldberger-Treiman relation between the π-μ decay constant and the axial vector coupling constant from the Postulate III is relatively simple, but, in order to derive another linear relation between these constants, the introduction of electromagnetic interactions and other charge-dependent effects is indispensable.

[20] In addition to the problem of μ-e decay there seems to be a close connection between the fine structure constant and lepton masses. See Appendix II for details.

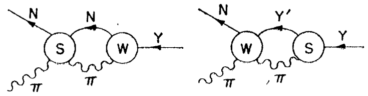

FIG. 1. Diagrams kept in the calculation of the absorptive part of the functions $F_i(s)$. S stands for strong interactions and W for weak interactions.

the positive real axis from the physical threshold $(m+\mu)^2$ to ∞, but because of the choice of the invariants in Eq. (5.2) we have introduced a left-hand cut from $-\infty$ to 0 in F_2 and F_4 due to the factor $\sqrt{s}$. Since, however, the branch point $s=0$ of the left-hand cut is far from the physical threshold $s=(m+\mu)^2$ we may neglect the left-hand cut in the one-pion approximation.

In order to calculate $\mathrm{Im}F_i(s)$, we make use of the absorptive part of J given by

$$A(s)=\tfrac{1}{2}(2q_0)^{1/2}\int d^4x\, e^{-ipx}\langle\pi|\{j_N(x),\bar{j}_Y(0)\}|0\rangle, \quad (5.5)$$

which we find from the time reversal invariance may be written

$$A(s)=\mathrm{Im}F_1(s)+\frac{i\gamma\cdot P_Y}{\sqrt{s}}\mathrm{Im}F_2(s)+\gamma_5\left(\mathrm{Im}F_3(s)+\frac{i\gamma\cdot P_Y}{\sqrt{s}}\mathrm{Im}F_4(s)\right). \quad (5.6)$$

In this calculation we shall treat both of the relative $\Sigma\Lambda$ parity cases, but calculations are explicitly shown only for the even parity case.

If we insert a complete set of intermediate states in Eq. (5.5) and use translational invariance, we obtain

$$A(s)=\tfrac{1}{2}(2q_0)^{1/2}(2\pi)^4\sum_n\langle\pi|j_N|n\rangle\times\langle n|\bar{j}_Y|0\rangle\delta(p_n-P_Y), \quad (5.7)$$

where p_n is the energy momentum of the intermediate state n. In what follows we employ the Goldberger-Treiman trick[3] and write the sum over states $|n\rangle$ as one-half the sum over "out" plus "in" states in order to insure reality of $A(s)$ in any approximation.

In Fig. 1 we indicate what terms will be kept in the calculation of $A(s)$.

Upon insertion of the contributions shown in Fig. 1 or in the one-pion approximation, we obtain

$$A(s)=\tfrac{1}{2}(2q_0)^{1/2}\frac{1}{(2\pi)^2}\Big[\int d^3p'd^3q'\,\langle\pi|j_N|\pi'N'\rangle\times\langle\pi'N'|\bar{j}_Y|0\rangle\delta(p'+q'-P_Y)+\int d^3p_{Y'}d^3q'\,\langle\pi|j_N|\pi'Y'\rangle\times\langle\pi'Y'|\bar{j}_Y|0\rangle\delta(p_Y'+q'-P_Y)\Big]. \quad (5.8)$$

In the first integral $\langle\pi|j_N|\pi'N'\rangle$ is proportional to the pion-nucleon scattering amplitude and $\langle\pi'N'|\bar{j}_Y|0\rangle$ is the original vertex; when the coefficient functions are inserted into the dispersion relations, we therefore get integral equations for $F_i(s)$.

A few remarks are necessary about keeping only the contributions of Fig. 1. We have first of all neglected K-meson couplings since baryon conservation implies that the smallest mass state containing K mesons is the $N\bar{K}$ state which is far above the threshold $s=(m+\mu)^2$ for the rescattering term. We also omitted single-baryon intermediate states since they do not contribute to F_i when the initial hyperon is on the mass shell as we shall discuss in the Appendix I.[21]

In this calculation we neglect the $\Sigma\Lambda$ mass difference and treat the inhomogeneous Born term [Fig. 1(b)] by perturbation theory, since its contribution begins at $s=(M+\mu)^2$, where M is the averaged hyperon mass which is above the threshold by at least one-pion mass. This is about the same position as the neglected $N2\pi$ intermediate state would begin to contribute, anyway, which is further reason for not attempting a better means of calculating the Born term than perturbation theory. The diagrams used in the calculation of the inhomogeneous Born term are shown in Fig. 2. The perturbation calculation of the Born term is supposed to give an overestimate, since we approximate the unknown hyperon decay vertices by constant values, while the true vertex functions are assumed to fall off at high energies.

In writing down the integral equations for F_i we have to know the pion-nucleon scattering amplitude $\langle\pi|j_N|\pi'N'\rangle$, but, since the precise form of this scattering amplitude is not known at high energies, we decided to use the form given by perturbation theory. This makes the result of the present calculation unreliable quantitatively, but we hope that it will be reliable qualitatively.

In the inhomogeneous Born term, we dropped the single-baryon intermediate states, which means that we are concerned with the amputated vertex functions F_i but not with three-point Green's functions F_i including the vacuum-polarization-type corrections. In order to write down the integral equations for the amputated vertex functions consistently we have to be careful in dropping contributions involving bubbles in the diagram, and in the present calculation we kept only the

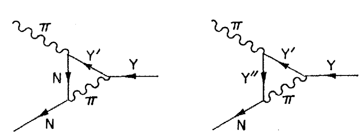

FIG. 2. Diagrams used to calculate the inhomogeneous Born term in Eq. (5.8).

[21] The inclusion of the single baryon states certainly changes the functions $F_i(s)$ but it does not change the values of F's on the mass shell.

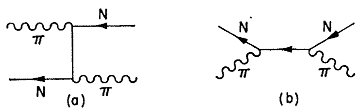

FIG. 3. Feynman diagrams for pion-nucleon scattering in the lowest order. The first graph (a) was kept and the second one (b) was dropped in the calculation.

diagram in Fig. 3(a) but dropped Fig. 3(b) for the pion-nucleon scattering, since the latter always gives rise to bubbles when connected to the hyperon decay vertices.

Before proceeding, we must take up the question of charge states. For this purpose we note that the amplitude J, in general, may be written for Λ decay

$$J_\alpha^{(\Lambda)}(s)=J^{(0)}(s)\tau_\alpha(1-\tau_3)/2, \qquad (5.9)$$

for decay of Λ into a nucleon and a pion in charge state α. For Σ decay we may write

$$J_{\alpha\beta}^{(\Sigma)}(s)=(J^{(+)}(s)\delta_{\alpha\beta}+J^{(-)}(s)\tfrac{1}{2}[\tau_\alpha,\tau_\beta]) \times(1-\tau_3)/2, \qquad (5.10)$$

for decay of a Σ in charge state β into a nucleon and a pion in charge state α. If we define amplitudes $J^{(1)}$ and $J^{(3)}$ proportional to the amplitudes for decay of a Σ into pure $I=1/2$ and $I=3/2$ states, respectively, then

$$J^{(1)}=J^{(+)}+2J^{(-)},$$

and

$$J^{(3)}=J^{(+)}-J^{(-)}. \qquad (5.11)$$

The integral equations for these amplitudes are uncoupled since the pion-nucleon scattering is diagonal in the isospin. Here we are concerned with decay amplitudes obeying $|\Delta I|=1/2$, which as has been discussed in the previous section are decoupled from all others not obeying this rule.

In writing down the inhomogeneous Born term represented by Fig. 2, we assumed the following *effective* Lagrangians[22]:

$$\mathcal{L}_\Sigma=\bar{\psi}_N\frac{(1-\tau_3)}{2}(a_\Sigma+b_\Sigma\gamma_5)\boldsymbol{\psi}_\Sigma\cdot\boldsymbol{\varphi}+\bar{\psi}_N\boldsymbol{\tau}\frac{(1-\tau_3)}{2} \times(a_\Sigma'+b_\Sigma'\gamma_5)\cdot\boldsymbol{\psi}_\Sigma\times\boldsymbol{\varphi}+\text{Herm. conj.}, \qquad (5.12)$$

$$\mathcal{L}_\Lambda=\bar{\psi}_N\boldsymbol{\tau}\frac{(1-\tau_3)}{2}(a_\Lambda+b_\Lambda\gamma_5)\psi_\Lambda\boldsymbol{\varphi}+\text{Herm. conj.},$$

where the boldface symbols represent vectors in charge space. This does not mean that we assumed the existence of Lagrangians describing weak-decay interactions, but rather the a's and b's are the values of vertex functions on the mass shell. These Lagrangians are consistent with $|\Delta I|=1/2$ rule.

[22] See, for instance, H. Umezawa, Suppl. Progr. Theoret. Phys. (Kyoto) 7, 67 (1959).

With these preliminaries we can manipulate Eq. (5.8) and get

$$\mathrm{Im}F_i^{(\kappa)}(s)=C_\kappa\sum_{j=1}^{4}K_{ij}(s)\,\mathrm{Re}F_j^{(\kappa)}(s)\theta(s-(m+\mu)^2) +G_i^{(\kappa)}(s)\theta(s-(M+\mu)^2), \quad (\kappa=0,1,3) \qquad (5.13)$$

with

$$K_{ij}(s)=\frac{G^2}{16\pi}\frac{q}{W}\begin{bmatrix} M(s) & s^{1/2}N(s) & 0 & 0\\ s^{1/2}N(s) & M(s) & 0 & 0\\ 0 & 0 & M(s) & -s^{1/2}N(s)\\ 0 & 0 & -s^{1/2}N(s) & M(s)\end{bmatrix}, \qquad (5.14)$$

where $C_\kappa=-1$ for $\kappa=0$, 1, and $+2$ for $\kappa=3$. G is the pion-nucleon coupling constant, and q/W is the center-of-mass momentum over total energy. The form of $K_{ij}(s)$ indicates the obvious fact that there is no coupling between parity conserving and nonconserving amplitudes, and the functions $M(s)$ and $N(s)$ involve kinematical factors and the S- and P-wave pion-nucleon scattering amplitudes. The important point is that they depend only on the strong interactions. The $G_i^{(\kappa)}(s)$ in Eq. (5.13) denote the contributions from the inhomogeneous Born term and are linear in the a's and b's.

The insertion of Eq. (5.13) into the dispersion relations (5.4) yields a set of coupled integral equations of the singular Omnès type.

$$\mathrm{Re}F_i^{(\kappa)}(s)=\frac{P}{\pi}\sum_i\int_{(m+\mu)^2}^{\infty}ds' \times\frac{C_\kappa K_{ij}(s')\,\mathrm{Re}F_j(s')+G_i^{(\kappa)}(s')\theta(s'-(M+\mu)^2)}{s'-s}. \qquad (5.15)$$

The matrix in Eq. (5.14) which we call $L_{ij}(s)$ is diagonalized by

$$U=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & -1 & 1\end{bmatrix}, \qquad (5.16)$$

so that

$$UK(s)U^{-1}=\frac{G^2}{16\pi}\frac{q}{W}UL(s)U^{-1}=\frac{G^2}{16\pi}\frac{q}{W}D(s) \qquad (5.17)$$

is a diagonal matrix. After first uncoupling the Eqs. (5.15) by the diagonalization procedure we can write down the solution in terms of linear combinations of the F_i.[23,24]

$$\sum_j U_{ij}\,\mathrm{Re}F_j^{(\kappa)}(s)=\sum_i \frac{C_\kappa U_{ij}G_j^{(\kappa)}(s)D_i(s)\theta(s-(M+\mu)^2)}{1+C_\kappa^2D_i^2(s)}+\frac{e^{\varphi_i(s)}}{[1+C_\kappa^2D_i^2(s)]^{1/2}}\frac{P}{\pi}\times\int_{(M+\mu)^2}^{\infty}ds'\frac{e^{-\varphi_i(s')}\sum_j U_{ij}G_j^{(\kappa)}(s')}{(s'-s)[1+C_\kappa^2D_i^2(s')]^{1/2}} \quad (5.18)$$

with

$$\varphi_i(s)=\frac{P}{\pi}\int_{(m+\mu)^2}^{\infty}\frac{\tan^{-1}C_\kappa D_i(s')ds'}{s'-s}. \quad (5.19)$$

Since we are assuming unsubtracted dispersion relations for F_i, the high-energy behavior of the solution is important and we approximate $D_j(s)$ and $G_j^{(\kappa)}(s)$ by their asymptotic values which are constants. Then

$$\varphi_i(s)-\varphi_i(s')\simeq\frac{1}{\pi}\tan^{-1}\left(\frac{C_\kappa G^2}{16\pi}\right)\ln\left[\frac{s-(m+\mu)^2}{s'-(m+\mu)^2}\right], \quad (5.20)$$

and, thus, we get

$$\sum_j U_{ij}\,\mathrm{Re}F_j^{(\kappa)}(s)\simeq\sum_i \frac{C_\kappa U_{ij}G_j^{(\kappa)}(s)D_i(s)}{1+C_\kappa^2D_i^2(s)}\theta(s-(M+\mu)^2)+\frac{\sum_j U_{ij}G_j^{(\kappa)}(\infty)}{[1+C_\kappa^2D_i^2(s)]^{\frac{1}{2}}}\frac{P}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{ds'}{s'-s}\times\left[\frac{s-(m+\mu)^2}{s'-(m+\mu)^2}\right]^{(1/\pi)\tan^{-1}(C_\kappa G^2/16\pi)}. \quad (5.21)$$

In order for F_j to satisfy unsubtracted dispersion relations in a more accurate calculation, we must have $D_i(\infty)=0$ and we dropped it. This calculation makes sense if in all cases

$$(1/\pi)\tan^{-1}C_\kappa G^2/16\pi>0. \quad (5.22)$$

It is apparent from Eq. (5.21) that this makes the integral converge but also leads to the divergence of $F_i(s)$ at infinity. However, this was not unexpected since in the inhomogeneous Born term we approximated decay amplitudes of hyperons by constants, and we believe that actual vertex functions are well behaved at infinity.[25] Since the arc tangent is a many-valued function, we can choose an appropriate branch for each case to satisfy (5.22).

When we put s on the hyperon mass shell, we get

$$J(M^2)=F_1(M^2)-F_2(M^2)+\gamma_5[F_3(M^2)-F_4(M^2)], \quad (5.23)$$

which just involves two of the linear combinations $U_{ij}F_j(M^2)$ in Eq. (5.21), and they can be expressed in terms of the effective coupling constants a's and b's in (5.12). Both a's and b's are slightly complex, but, because of the small phase shifts in pion-nucleon scattering at the energy corresponding to the hyperon decays, we may regard them as real. Then for $s=M^2$ the right-hand side of Eq. (5.21) can be expressed in terms of the strong-coupling constants G, $g_{\Sigma\Lambda\pi}$, and $g_{\Sigma\Sigma\pi}$, and the weak-decay amplitudes on the mass shell which are just the a's and b's. After carrying out the integral in Eq. (5.21), we find for the even $\Sigma\Lambda$ parity case

$$\begin{aligned}a_\Lambda&=0.23(g_{\Sigma\Lambda\pi}/16\pi)\times[G(a_\Sigma-a_{\Sigma}')+2g_{\Sigma\Sigma\pi}a_{\Sigma}'-g_{\Sigma\Lambda\pi}a_\Lambda],\\ a_\Sigma-a_{\Sigma}'&=0.25\{(g_{\Sigma\Lambda\pi}/16\pi)[2Ga_\Lambda-g_{\Sigma\Lambda\pi}(a_\Sigma+a_{\Sigma}')]+(g_{\Sigma\Sigma\pi}/16\pi)[G(a_\Sigma-a_{\Sigma}')+g_{\Sigma\Sigma\pi}(2a_\Sigma+a_{\Sigma}')-g_{\Sigma\Lambda\pi}a_\Lambda]\},\\ a_\Sigma+2a_{\Sigma}'&=0.23\{(g_{\Sigma\Lambda\pi}/16\pi)[-Ga_\Lambda+g_{\Sigma\Lambda\pi}(2a_{\Sigma}'-a_\Sigma)]+(g_{\Sigma\Sigma\pi}/16\pi)[-2Ga_\Sigma-2g_{\Sigma\Sigma\pi}(a_{\Sigma}'-a_\Sigma)+2g_{\Sigma\Lambda\pi}a_\Lambda]\}.\end{aligned} \quad (5.24a)$$

$$\begin{aligned}b_\Lambda&=0.35(g_{\Sigma\Lambda\pi}/16\pi)\times[G(2b_{\Sigma}'-b_\Sigma)+2g_{\Sigma\Sigma\pi}b_{\Sigma}'-g_{\Sigma\Lambda\pi}b_\Lambda],\\ b_\Sigma-b_{\Sigma}'&=0.65\{(g_{\Sigma\Lambda\pi}/16\pi)[-2Gb_\Lambda-g_{\Sigma\Lambda\pi}(b_\Sigma+b_{\Sigma}')]+(g_{\Sigma\Sigma\pi}/16\pi)[G(b_\Sigma-3b_{\Sigma}')+g_{\Sigma\Sigma\pi}(2b_\Sigma+b_{\Sigma}')-g_{\Sigma\Lambda\pi}b_\Lambda]\},\\ b_\Sigma+2b_{\Sigma}'&=0.35\{(g_{\Sigma\Lambda\pi}/16\pi)[Gb_\Lambda+g_{\Sigma\Lambda\pi}(2b_{\Sigma}'-b_\Sigma)]+(g_{\Sigma\Sigma\pi}/16\pi)[2Gb_\Sigma+2g_{\Sigma\Sigma\pi}(b_\Sigma-b_{\Sigma}')+2g_{\Sigma\Lambda\pi}b_\Lambda]\}.\end{aligned} \quad (5.24b)$$

For the odd-parity case we obtain a similar set of equations. These equations may be written in the form

$$A\begin{pmatrix}a_\Lambda\\a_\Sigma\\a_{\Sigma}'\end{pmatrix}=0,\quad B\begin{pmatrix}b_\Lambda\\b_\Sigma\\b_{\Sigma}'\end{pmatrix}=0, \quad (5.25)$$

where A and B are 3 by 3 matrices whose matrix elements are functions of G, $g_{\Sigma\Lambda\pi}$, $g_{\Sigma\Sigma\pi}$, and the masses M, m, μ. We can find nonvanishing solutions of (5.25) if and only if

$$\det A=\det B=0. \quad (5.26)$$

TABLE I. The numerical values of the coupling constants $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$.

$\Sigma\Lambda$ relative parity	Even	Odd
$g_{\Sigma\Lambda\pi}^2/4\pi$	7.1	3.8
$g_{\Sigma\Sigma\pi}^2/4\pi$	1.2	0.25

[23] N. I. Muskhelishvili, *Singular Integral Equations* (Noordhoff, Groningen, Holland, 1953).

[24] R. Omnès, Nuovo cimento 8, 316 (1958).

[25] The solution of the singular integral Eq. (5.15) is by no means unique, but it is impossible to improve the convergence of $F_i(s)$ at infinity. In other words, the solution (5.18) is the right one that falls off most rapidly at infinity in accord with the boundary condition given in Postulate III.

TABLE II. Σ-decay constants.

Relative ΣΛ parity	Even Sol. 1	Even Sol. 2	Odd Sol. 1	Odd Sol. 2	Exp.
$\tau(\Sigma^-)$	0.8×10^{-10}	2.3×10^{-10}	0.7×10^{-12}	4×10^{-12}	1.6×10^{-10}
$\tau(\Sigma^+)$	0.4×10^{-10}	1.3×10^{-10}	0.7×10^{-12}	5×10^{-12}	0.8×10^{-10}
$(\Sigma^+\rightarrow p+\pi^0)/(\Sigma^+\rightarrow n+\pi^+)$	3	3	10	10	~1
$\alpha(\Sigma^+\rightarrow p+\pi^0)$	0.33	0.98	0.004	0.026	0.78[a]

[a] See reference 29.

These two equations are sufficient to determine the two unknown parameters $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$. Thus Eqs. (5.25) may be looked upon as eigenvalue equations to determine the two unknown strong-coupling constants $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$, while G was assumed to be known, i.e., $G^2/4\pi=15$. The simultaneous equations (5.26) were solved numerically, and the solutions are tabulated in Table I.

In both cases we find a rather interesting common result

$$g_{\Sigma\Lambda\pi}{}^2/4\pi\gg g_{\Sigma\Sigma\pi}{}^2/4\pi, \tag{5.27}$$

which is in contradiction with global symmetry but consistent with the R invariance recently proposed by Sakurai.[26]

Once the eigenvalues are known, we may proceed to solve Eq. (5.25) for the eigenvectors or the ratios of Σ to Λ decay constants. Since the equations are linear in the decay constants we cannot fix the normalization, and this is generally the case as required by the existence of one parameter group of solutions in Postulate II. At this point the analogy between bound states and weak interactions breaks down, since the problem of bound states is essentially a nonlinear problem and the normalization of the solution is uniquely determined by the nonlinear dynamical equations.

In the present case we determine the normalization of the decay constants with reference to the experimental values, namely, the lifetime $\tau(\Lambda)$ and asymmetry parameter $\alpha(\Lambda)$. Before doing this, however, there is one quantity which may be determined independently of the Λ data, the sign of the ratio $\alpha(\Sigma^+\rightarrow p+\pi^0)/\alpha(\Lambda)$. We find that, irrespective of the Σ-Λ relative parity assignment, this ratio is negative, in accord with experiment.[27–29]

All of the decay amplitudes are of the form $a+b\gamma_5$ and, when they are expressed in the two-component form, both the lifetime and asymmetry parameter are symmetric in the S- and P-wave amplitudes. Therefore, comparison with experimental values gives generally two solutions for a_Λ and b_Λ corresponding to S-dominant and P-dominant solutions.

Solution 1:

$$a_\Lambda=\pm9.2\times10^{-8},\quad b_\Lambda=\pm4.6\times10^{-6}.$$
(S-dominant solution)

Solution 2: (5.28)

$$a_\Lambda=\pm2.4\times10^{-7},\quad b_\Lambda=\pm1.8\times10^{-6}.$$
(P-dominant solution)

We used the experimental lifetime $\tau(\Lambda)=2.5\times10^{-10}$ sec and $\alpha(\Lambda)=-0.67$ [29] to determine a_Λ and b_Λ. Once a_Λ and b_Λ are determined we can compute a_Σ, b_Σ and a_Σ', b_Σ', and, hence, all the parameters related to the Σ decay.

For the even relative parity case, the agreement with experiment is not bad in view of the roughness of our calculations, whereas for odd relative parity there is strong disagreement. On the basis of this calculation then, we obtain evidence in favor of the even ΣΛ relative parity assignment. This is in accord with the recent experimental assignment of the even relative ΣΛ parity by the Berkeley group.[30] However, it must be stressed that, due to the drastic approximations involved, we consider the agreement with experiment as rather fortuitous and should not be taken too seriously.

In the present calculations we found

$$\alpha(\Sigma^+\rightarrow n+\pi^+)\approx\alpha(\Sigma^-\rightarrow n+\pi^-)\approx\alpha(\Sigma^+\rightarrow p+\pi^0)$$

for both solutions. Experimentally, the first two are nearly zero and thus in conflict with experiment. This seems to be a manifestation of the roughness of our approximations.

In connection with the inequality (5.27) we shall make a brief remark. In the absence of the K-meson coupling the Ξ decay amplitudes are decoupled from the ΣΛ decays, but the structure of the equations for Ξ decay amplitudes would be almost the same as that for the decays treated here provided that strong interactions are symmetric between nuclear and cascade particle and their mass difference does not play an important role.

Indeed the equations for the Ξ decay problem are obtained by replacing the nucleon wave function by the

[26] J. J. Sakurai, Phys. Rev. Letters **7**, 426 (1961).
[27] R. W. Birge and W. B. Fowler, Phys. Rev. Letters **5**, 254 (1960).
[28] J. Leitner, L. Gray, E. Harth, S. Lichtman, J. Westgard, M. Block, B. Bruckner, A. Engler, R. Gessaroli, A. Kovacs, T. Kikuchi, C. Meltzer, H. O. Cohn, W. Bugg, A. Pevsner, P. Schlein, M. Meer, N. T. Grinellini, L. Lenainara, L. Monari, and G. Puppi, Phys. Rev. Letters **7**, 264 (1961).
[29] E. F. Beall, B. Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters **7**, 285 (1961); **8**, 75 (1962).
[30] R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters **8**, 175 (1962).

cascade wave function in our calculations. Therefore if R invariance holds we will find almost the same values for $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$ in this problem. If the K-meson coupling is introduced, these two problems are no longer separate, but the new coupled set of equations will give again almost the same values for $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$ provided that the K-meson coupling is relatively weak because of the higher threshold energy. Then the decay amplitudes will still keep the approximate R invariance, and, hence, we will get $\alpha(\Xi)\simeq-\alpha(\Lambda)$ as predicted by Sakurai.[26] This is also consistent with experiment.[31]

Our final remark is concerned with the selection rule $|\Delta I|=1/2$. We assumed the existence of the decay amplitudes obeying $|\Delta I|=1/2$ and determined the coupling constants $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$ as eigenvalues. Then from the arguments given in Sec. IV amplitudes not obeying this rule will identically vanish at least in the present approximation. In other words, the presence of the amplitudes obeying $|\Delta I|=1/2$ excludes other amplitudes not obeying this rule, and this is a consequence of charge independence of strong interactions.

APPENDIX I. RENORMALIZATION IN HYPERON DECAYS

The problem that we encountered in hyperon decays is how to renormalize the cross self-energy represented by the Feynman diagram in Fig. 4. We dropped the contribution of this diagram for the reason that it would vanish when the hyperon Y is on the mass shell. We shall explain this statement in what follows.

Without loss of generality we assume that all the particles in this problem are scalar, and we use the notation:

$$\varphi_N(x)=\varphi_1(x),\quad \varphi_Y(x)=\varphi_2(x);\quad m_N=m_1,\quad m_Y=m_2. \tag{I1}$$

The two-point Green's functions are defined by

$$\Delta'_{Fij}(x-y)=\langle 0|T[\varphi_i(x),\varphi_j^\dagger(y)]|0\rangle, \tag{I2}$$

and follow the Källén-Lehmann representation.

We consider this problem in the first order of weak interactions and treat hyperons as if they were stable since their instability is introduced in second order. If weak interactions are introduced, there are no quantum numbers with which to distinguish between a nucleon and a hyperon of the same charge and the propagation functions obey the Källén-Lehmann representation:

$$\Delta'_{Fij}(x)=\sum_{k=1,2}C_{ij}(k)\Delta_F(x,m_k^2)+\int d\kappa^2\,\rho_{ij}(\kappa^2)\Delta_F(x,\kappa^2), \tag{I3}$$

where both $C_{ij}(k)$ and $\rho_{ij}(\kappa^2)$ are 2-by-2 Hermitian matrices. We are particularly interested in the matrices $C_{ij}(1)$ and $C_{ij}(2)$ that are given by

$$C_{ij}(k)\propto\langle 0|\varphi_i(0)|N_k\rangle\langle N_k|\varphi_j^\dagger(0)|0\rangle =\langle 0|\varphi_i(0)|N_k\rangle\langle 0|\varphi_j(0)|N_k\rangle^*, \tag{I4}$$

where N_k denotes a one-nucleon state for $k=1$ or a one-hyperon state for $k=2$.

If the field operators are properly normalized or renormalized to satisfy the asymptotic condition, we find

$$(2P_{N0})^{\frac{1}{2}}\langle 0|\varphi_1(0)|N_1\rangle=(2P_{Y0})^{\frac{1}{2}}\langle 0|\varphi_2(0)|N_2\rangle=1, \tag{I5}$$

and, consequently,

$$C_{11}(1)=C_{22}(2)=1. \tag{I6}$$

This condition does not determine the matrices $C_{ij}(1)$ and $C_{ij}(2)$ uniquely, since the asymptotic condition does not give further conditions to determine the nondiagonal elements. In the presence of quantum numbers to distinguish between a nucleon and a hyperon there were no nondiagonal elements, and now we have no conditions for determining them. This corresponds to the fact that in field theory there is no one-to-one correspondence between field operators and particles,[32] e.g., if we take appropriate linear combinations of φ_1 and φ_2 in such a way as to satisfy

$$\varphi_1'=a_{11}\varphi_1+a_{12}\varphi_2,\quad \varphi_2'=a_{21}\varphi_1+a_{22}\varphi_2, \tag{I7}$$

$$\langle 0|\varphi_1'|N_1\rangle=\langle 0|\varphi_1|N_1\rangle,\quad \langle 0|\varphi_2'|N_2\rangle=\langle 0|\varphi_2|N_2\rangle, \tag{I8}$$

then the primed operators are as useful as the unprimed ones in the dispersion theory and the coefficients a_{ij} are quite arbitrary except for (I8). All the physical results should be independent of the choice of a's, however. For this reason we will employ such a gauge that makes the calculations simplest, i.e., we choose a special gauge by imposing additional conditions

$$\langle 0|\varphi_1(0)|N_2\rangle=\langle 0|\varphi_2(0)|N_1\rangle=0. \tag{I9}$$

The choice of such a gauge is indispensable in the perturbation theory in order to renormalize self-energies corresponding to the diagram in Fig. 4.

Otherwise we would have to introduce gauge-dependent divergent constants into the calculations since the above gauge is the only one that is free from the cross self-energy type divergences.

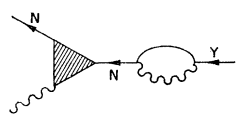

FIG. 4. A cross self-energy-type Feynman diagram.

[31] W. B. Fowler, R. W. Birge, P. Eberhard, R. Ely, M. L. Good, W. M. Powell, and H. K. Ticho, Phys. Rev. Letters 6, 134 (1961).

[32] See, for instance, K. Nishijima, Phys. Rev. 111, 995 (1958).

In the above gauge we find

$$\Delta'_{F\,ij}(x)=\delta_{ij}\Delta_F(x,m_i^2)+\int d\kappa^2\,\rho_{ij}(\kappa^2)\Delta_F(x,\kappa^2), \quad \text{(I10)}$$

and the structure of the pole term is completely determined.[33]

It is clear that the contribution of the first Feynman diagram vanishes when the hyperon is on the mass shell since,

$$\langle 0|\varphi_Y(0)|N\rangle=0. \quad \text{(I11)}$$

Several authors kept the Feynman diagram in Fig. 4 on the basis of the argument that the pole term dominates in the neighborhood of the pole, but the result is gauge-dependent unless the vertex corrections of the same order are properly included. In our special gauge we can discard the contributions from the cross self-energy-type diagrams when the hyperon is on the mass shell and we kept only the vertex-type diagrams in Sec. V.

APPENDIX II. CONSERVED VECTOR CURRENTS AND SUBTRACTIONS

In the text we discussed the necessity of subtractions for such leptonic decay amplitudes as

$$\pi\to\mu+\nu,\quad K\to\mu+\nu,$$

when the electromagnetic interactions are neglected. The reason was that the three-point vertex functions behave at high energies as badly as two-point functions do because of the currently accepted structure of weak interactions that the lepton pair is emitted from the same point in the Feynman diagram.

The conserved vector part of the β-decay amplitude is given by

$$\langle p|\mathfrak{J}_\mu^\dagger(0)|n\rangle\bar{u}_e\gamma_\mu(1+\gamma_5)u_\nu, \quad \text{(II1)}$$

except for a multiplicative constant. We shall study the dispersion relation for the nuclear vertex function here in the approximation of neglecting all the charge-dependent effects.

We introduce the isotopic spin current density $\boldsymbol{\mathfrak{J}}_\mu$, which is conserved, i.e.,

$$\partial\boldsymbol{\mathfrak{J}}_\mu/\partial x_\mu=0. \quad \text{(II2)}$$

The boldface denotes a vector in charge space and the isospin I is defined by

$$\mathbf{I}=\int d^3x\,\boldsymbol{\mathfrak{J}}_0(x). \quad \text{(II3)}$$

The current density $\mathfrak{J}_\mu^\dagger$ in (II1) is given by

$$\mathfrak{J}_\mu=\mathfrak{J}_{\mu1}-i\mathfrak{J}_{\mu2},\qquad \mathfrak{J}_\mu^\dagger=\mathfrak{J}_{\mu1}+i\mathfrak{J}_{\mu2}. \quad \text{(II4)}$$

If $\varphi_a(x)$ denotes the field operator of a charge multiplet with an isospin I, then $\boldsymbol{\mathfrak{J}}_0$ satisfies the commutation relation

$$[\varphi_a(x),\boldsymbol{\mathfrak{J}}_0(y)]=\delta^3(\mathbf{x}-\mathbf{y})\boldsymbol{\omega}_{ab}\varphi_b(y)\quad\text{for}\quad x_0=y_0, \quad \text{(II5)}$$

where the matrices ω_1, ω_2, and ω_3 are the $(2I+1)$-dimensional irreducible representation of I_1, I_2 and I_3. The subscripts a and b run from $-I$ to $+I$. Taking the Hermitian conjugate of (II5), we get

$$[\varphi_a^\dagger(x),\boldsymbol{\mathfrak{J}}_0(y)]=-\delta^3(\mathbf{x}-\mathbf{y})\boldsymbol{\omega}_{ab}^*\varphi_b^\dagger(y)\quad\text{for}\quad x_0=y_0. \quad \text{(II6)}$$

By making use of the technique developed in reference (6) we can easily show

$$\begin{aligned}(\partial/\partial x_\mu)T[\boldsymbol{\mathfrak{J}}_\mu(x)\varphi_a(y)\varphi_b(z)\cdots]&=-\delta(x-y)\boldsymbol{\omega}_{aa'}T[\varphi_{a'}(y)\varphi_b(z)\cdots]\\&\quad-\delta(x-z)\boldsymbol{\omega}_{bb'}T[\varphi_a(y)\varphi_{b'}(z)\cdots]-\cdots.\end{aligned} \quad \text{(II7)}$$

As a special case of (II7) we get

$$(\partial/\partial x_\mu)\langle 0|T[\boldsymbol{\mathfrak{J}}_\mu(x)\varphi_a(y)\varphi_b^\dagger(z)]|0\rangle=-\boldsymbol{\omega}_{ab}[\delta(x-y)-\delta(x-z)]\Delta_F'(y-z), \quad \text{(II8)}$$

where

$$\langle 0|T[\varphi_a(y)\varphi_b^\dagger(z)]|0\rangle=\delta_{ab}\Delta_F'(y-z).$$

Now define the ρ function by

$$\boldsymbol{\varrho}_\mu(xyz)_{ab}=(-i)K_yK_z\langle 0|T[\boldsymbol{\mathfrak{J}}_\mu(x)\varphi_a(y)\varphi_b^\dagger(z)]|0\rangle, \quad \text{(II9)}$$

then ρ satisfies the equation

$$\begin{aligned}(\partial/\partial x_\mu)\boldsymbol{\varrho}_\mu(xyz)_{ab}=i\boldsymbol{\omega}_{ab}[&K_y\delta(x-y)K_z\Delta_F'(x-z)\\&-K_z\delta(x-z)K_y\Delta_F'(x-y)].\end{aligned} \quad \text{(II10)}$$

If we rewrite this equation in momentum space and adopt the arguments in reference 7, we get

$$\begin{aligned}i(q-p)_\mu\boldsymbol{\mathfrak{G}}_\mu(p,q)_{ab}&=i\boldsymbol{\omega}_{ab}\left[\left(1+(q^2+m^2)\int\frac{\sigma(\kappa^2)d\kappa^2}{q^2+\kappa^2-i\epsilon}\right)(p^2+m^2)\right.\\&\quad\left.-\left(1+(p^2+m^2)\int\frac{\sigma(\kappa^2)d\kappa^2}{q^2+\kappa^2-i\epsilon}\right)(q^2+m^2)\right],\end{aligned} \quad \text{(II11)}$$

where σ is the Källén-Lehmann weight function for Δ_F' and $\boldsymbol{\mathfrak{G}}_\mu$ is defined by

$$\boldsymbol{\varrho}_\mu(xyz)_{ab}=\frac{-i}{(2\pi)^8}\int d^4p\,d^4q\times e^{ip(y-x)+iq(x-z)}\boldsymbol{\mathfrak{G}}_\mu(p,q)_{ab}. \quad \text{(II12)}$$

We can write $\boldsymbol{\mathfrak{G}}_\mu$ in the following form:

$$\begin{aligned}\boldsymbol{\mathfrak{G}}_\mu(p,q)_{ab}=\{(p+q)_\mu\mathfrak{G}_A+[&(p+q)_\mu(p-q)^2\\&-(p-q)_\mu(p^2-q^2)]\mathfrak{G}_B\}\boldsymbol{\omega}_{ab},\end{aligned} \quad \text{(II13)}$$

where $\mathfrak{G}_A$ and $\mathfrak{G}_B$ are functions of scalar products p^2, q^2,

[33] A similar conclusion was obtained in the study of the decay process $K^+\to\pi^++e^++e^-$ by N. Cabibbo and E. Ferrari, Nuovo cimento (to be published).

and $(p-q)^2$. Inserting (II13) into (II11), we find

$$\mathfrak{G}_A=-\left[1+(p^2+m^2)(q^2+m^2)\times\int\frac{\sigma(\kappa^2)d\kappa^2}{(p^2+\kappa^2-i\epsilon)(q^2+\kappa^2-i\epsilon)}\right]. \quad \text{(II14)}$$

This result shows that $\mathfrak{G}_A$ cannot satisfy an unsubtracted parametric disperion relation no matter whether $\mathfrak{G}_B$ satisfies an unsubtracted dispersion relation or not, since $\mathfrak{G}_A$ is independent of $(p-q)^2$.

On the mass shell, where $p^2+m^2=q^2+m^2=0$, (II13) reduces to

$$\mathfrak{G}_\mu(p,q)_{ab}=-\omega_{ab}(p+q)_\mu \times\{1-(p-q)^2\mathfrak{G}_B[(p-q)^2]\}. \quad \text{(II15)}$$

In this Appendix we showed that, if the vector part of the β decay amplitude is proportional to the isospin current $\mathfrak{J}_\mu{}^\dagger$, we need a subtraction if the electromagnetic interactions are discarded. In other words the decay amplitude partly involves the two-point function like structure as was the case for $\pi\to\mu+\nu$ and $K\to\mu+\nu$, and this difficulty is supposed to be removed by the introduction of the electromagnetic interactions. It is clear, however, that the third component $\mathfrak{J}_{\mu3}$ is still conserved in strong interactions and, hence, requires a subtraction. This probably forbids the existence of a decay amplitude of the form

$$\mathfrak{J}_{\mu3}(0)\bar{u}_\nu\gamma_\mu(1+\gamma_5)u_\nu,$$

which is unfortunately unobservable.

The isospin current density certainly satisfies the dynamical equations in the dispersion theory in the absence of the electromagnetic interactions, since the equations are exactly the same as those for the electromagnetic form factor. The remaining question is whether the introduction of the electromagnetic coupling between the isospin current density and the charged lepton really makes the decay amplitude fall off more rapidly at high energies than all other possible solutions. Unless this problem is solved, we cannot understand why the vector part of the β-decay amplitude is universal and proportional to the isospin current density.

There are further complicated problems concerning leptonic decays. In those leptonic decays that are described by means of unsubtracted dispersion relations even in the absence of the electromagnetic interactions we can understand the symmetry between μ mesons and electrons, since the eigenvalue equations are independent of the lepton mass in this approximation, i.e., if the μ-decay mode is allowed then the electron decay mode is also allowed and the strong interaction part of the decay amplitude is common to both modes. When the introduction of the electromagnetic interactions is needed in order to avoid subtractions, the eigenvalue equations will involve the fine structure constant, as well as the lepton mass, and, hence, the observed symmetry between μ meson and electron will not follow automatically. But instead, the eigenvalue equation for the μ mode and the electron mode will be different and serve to determine more fundamental constants.

In the discussions of the selection rule $|\Delta I|=1/2$ in Sec. V we assumed the presence of both the S-wave and P-wave decay amplitudes since they are phenomenologically necessary, but we have no deeper reason for doing this. The same thing occurred in the leptonic decays and we simply have to assume that, if the μ mode of decay is present, the electron mode is also allowed without explanation. Perhaps we have to put this symmetry into our postulates.

The derivation of the linear equations for the leptonic decay constants is relatively easy if the amplitudes satisfy unsubtracted dispersion relations in the absence of the electromagnetic interactions. The Goldberger-Treiman relation falls into this category. However, it is extremely hard to derive such relations for those amplitudes that require the introduction of the electromagnetic interactions, since we have to deal with equations with a larger number of independent variables or with three-body problems. Therefore in practice we will be able to derive only part of the complete set of linear relations, i.e., those which are independent of the fine structure constant and lepton masses.