to Marshall and Stuart,[29] there is a general tendency for free-ion eigenfunctions to expand when the ion is embedded in a crystal lattice, and this effect would almost certainly increase $(5p|r^2|6p)$ by a greater factor than that by which $(5p|r^{-3}|6p)$ is decreased, thereby affording a possible mechanism for bringing the number -1.1 of Eq. (13) closer to the experimental value of -3.

It is interesting to draw a comparison with the work of Foley, Sternheimer, and Tycko on the quadrupole antishielding factor for Cs^+.[30] They find that the effective quadrupole moment for an external observer should be not Q, but $(1+\gamma)Q$, where $\gamma=86.8$. The greatest contribution to γ comes from the excitation of the $5p$ electrons to higher p orbits. If the effect is treated by perturbation theory, taking the np orbitals as the basis functions (a procedure that Foley *et al.* did not follow), it is not difficult to show that this contribution to γ can be written as

$$-\frac{4e^2}{25}\sum_{n>5}\frac{(5p|r^2|np)(np|r^{-3}|5p)}{E(5p^5np)}. \quad (14)$$

Strictly, the sum should also include states in the continuum. The constituents of the term in the sum for which $n=6$ all occur in Eq. (12), though the greater extension of the $5p$ orbital in the outer reaches of the ion Cs^+, compared to the corresponding orbital of Pr^{3+}, indicates that $(5p|r^2|6p)$ for Cs^+ is larger in magnitude, possibly by an appreciable factor, than the corresponding radial integral for Pr^{+3}. Even if we ignore this increase, the contribution of the term $(np)=(6p)$ to the sum (14) is as large as 12, taking $E(5p^56p)$ as 130 000 cm^{-1}.[31] We may therefore conclude that the special properties of the $5p$ and $6p$ eigenfunctions, properties that permit the radial integrals of r^{-3} and r^2 linking the eigenfunctions to be both large and also opposite in sign, are responsible for the negative value of P for Eu^{3+} and also for a substantial part of the large antishielding factor for Cs^+.

Note added in proof. We wish to acknowledge the receipt of a manuscript by E. G. Wikner and G. Burns. These authors, by considering the direct interaction of the external crystal field with the quadrupole moment induced in the outer shells of the europium ion, come to essentially the same conclusion as us for the kind of mechanism that gives rise to the negative value of P.

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[29] W. Marshall and R. Stuart, Phys. Rev. **122**, 367 (1961).
[30] H. M. Foley, R. M. Sternheimer, and D. Tycko, Phys. Rev. **93**, 734 (1954).
[31] C. E. Moore, *Atomic Energy Levels*, National Bureau of Standards Circular No. 467 (U. S. Government Printing Office, Washington, D. C., 1948).

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Nuclear Spins, Hyperfine Structures, and Nuclear Moments of Scandium-46 and Yttrium-91

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The atomic-beam magnetic-resonance method has been used to study some atomic and nuclear properties of 84-day Sc^{46} and 58-day Y^{91}. The results are

		$^2D_{3/2}$	$^2D_{5/2}$
Sc^{46}	$I=4$	$g_J=-0.7990(8)$	$g_J=-1.1995(18)$
	$(\mu_I)_{\text{uncorr}}=+3.03(2)$ nm	$a=+150.576(9)$ Mc/sec	$a=+60.906(4)$ Mc/sec
	$Q_{\text{uncorr}}=+0.119(6)$ b	$b=+14.38(14)$ Mc/sec	$b=+20.41(10)$ Mc/sec
Y^{91}	$I=1/2$	$a=\pm68.34(2)$ Mc/sec	$a=\pm34.35(3)$ Mc/sec
	$(\mu_I)_{\text{uncorr}}=\pm0.1634(8)$ nm	$\Delta\nu=\pm136.69(3)$ Mc/sec	$\Delta\nu=\pm103.05(4)$ Mc/sec

I. INTRODUCTION

THE atomic-beam magnetic-resonance method has been used to continue hfs investigations of the radioactive yttrium isotopes[1,2] as well as to initiate similar experiments on the radioactive scandium isotopes.[3] Both elements have a $^2D_{3/2}$ electronic ground state which is separated from the next higher state ($^2D_{5/2}$) by only a few hundred wave numbers.[4,5] As a result, transitions

* This research was supported in part by the U. S. Air Force Office of Scientific Research and the U. S. Atomic Energy Commission.
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[1] F. Russell Petersen and Howard A. Shugart, Phys. Rev. **125**, 284 (1962).
[2] F. R. Petersen and H. A. Shugart, Bull. Am. Phys. Soc. **6**, 514 (1961).
[3] F. R. Petersen and H. A. Shugart, Bull. Am. Phys. Soc. **6**, 363 (1961).
[4] Charlotte E. Moore, *Atomic Energy Levels*, National Bureau of Standards Circular No. 467 (U. S. Government Printing Office, Washington, D. C., 1949), Vol. I.
[5] Charlotte E. Moore, *Atomic Energy Levels*, National Bureau of Standards Circular No. 467 (U. S. Government Printing Office, Washington, D. C., 1952), Vol. II.

can easily be observed in both states at the temperatures required to produce an atomic beam.

The electronic g factors are close to the LS coupling values of $g_J(^2D_{3/2})=-0.8$ and $g_J(^2D_{5/2})=-1.2$, and have been previously measured for the stable isotopes by atomic beam and other methods.[6,7]

Investigation of the hyperfine structure and nuclear moments of stable Y^{89} and Sc^{45} has already been carried out by other laboratories,[6,8–10] thus facilitating radioactive experiments. Experimental results for Y^{89} and Y^{90} indicated that Y^{91} could be expected to follow predictions of the shell model with reasonable accuracy. Measurements on Y^{91} should, therefore, tend to test the validity of the shell model in this region of mass numbers. Previous results for Sc^{46}, on the other hand, indicated that shell-model predictions are somewhat anomalous. Thus, an investigation of the properties of this odd-odd nucleus should increase an understanding of the nucleon coupling scheme.

II. THEORY OF THE EXPERIMENT

For a free atom, the noncentral interaction between the electrons and nucleus in the presence of an external magnetic field $\mathbf{H}$ may be represented by the Hamiltonian

$$\mathcal{H}(\mathrm{Mc/sec}) = a\mathbf{I}\cdot\mathbf{J}+b\frac{[3(\mathbf{I}\cdot\mathbf{J})^2+\frac{3}{2}(\mathbf{I}\cdot\mathbf{J})-I(I+1)J(J+1)]}{2I(2I-1)J(2J-1)} - g_J(\mu_0/h)\mathbf{J}\cdot\mathbf{H}-g_I(\mu_0/h)\mathbf{I}\cdot\mathbf{H}, \quad (1)$$

where $\mathbf{I}$ and $\mathbf{J}$ are the nuclear and electronic angular momenta in units of $\hbar$, a and b are the magnetic dipole and electric quadrupole hfs coupling constants, and g_J and g_I are the electronic and nuclear g factors defined by μ_J/J and μ_I/I, respectively, where the magnetic moments μ_J and μ_I are in units of the absolute value of the Bohr magneton, μ_0. The terms in this Hamiltonian are, from left to right: the magnetic dipole interaction between the nuclear magnetic moment and the electronic magnetic field; the electric quadrupole interaction between the nuclear electric quadrupole moment and the gradient of the electric field produced by the electrons; the interaction between the electronic magnetic moment and the applied external field; and the interaction between the nuclear magnetic moment and the applied external field.

Because of symmetry considerations in the special case I or $J=1/2$, b is 0 and the second term in the Hamiltonian vanishes. In this case, the energy for any hyperfine level specified by the quantum numbers F, m may be expressed as a function of the magnetic field $\mathbf{H}$ by the familiar Breit-Rabi equation (for the case $I=1/2$)

$$\frac{W(F,m)}{h}=-\frac{\Delta\nu}{2(2J+1)}-g_J\frac{\mu_0}{h}Hm \pm\frac{\Delta\nu}{2}\left(1+\frac{4mx}{2J+1}+x^2\right)^{1/2}, \quad (2)$$

where

$$x=(-g_I+g_J)\mu_0 H/h\Delta\nu \quad \text{and} \quad \Delta\nu=aF_{\max}. \quad (3)$$

Here, the positive sign is taken with the $F=J+1/2$ levels and the negative sign with the $F=J-1/2$ levels. Figures 1 and 2 show the behavior of the hyperfine levels for Y^{91}, for which the nuclear moment has been assumed to be negative.

In the general case, including the second term of Eq. (1), a closed-form expression for the energy cannot be found, and digital computing techniques must be used

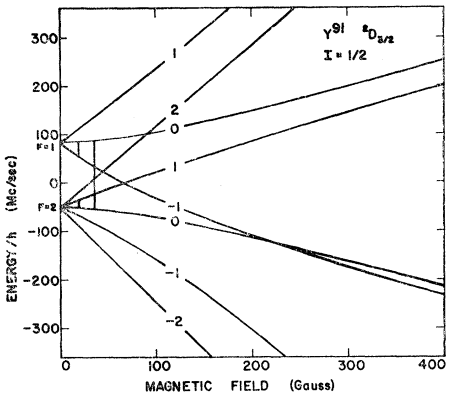

FIG. 1. Energy-level diagram of the hyperfine structure in the $^2D_{3/2}$ state of Y^{91}. The nuclear magnetic moment has been assumed to be negative ($a=-68.34$ Mc/sec).

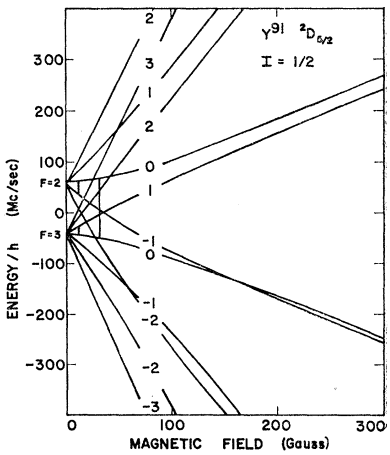

FIG. 2. Energy-level diagram of the hyperfine structure in the $^2D_{5/2}$ state of Y^{91}. The nuclear magnetic moment has been assumed to be negative ($a=-34.35$ Mc/sec).

[6] See, for example, G. Fricke, H. Kopfermann, S. Penselin, and K. Schlüpmann, Z. Physik **156**, 416 (1959).
[7] Siegfried Penselin, Z. Physik **154**, 231 (1959).
[8] G. Fricke, H. Kopfermann, and S. Penselin, Z. Physik **154**, 218 (1959).
[9] E. Brun, J. Oeser, H. H. Staub, and C. G. Telschow, Phys. Rev. **93**, 172 (1954).
[10] D. M. Hunten, Can. J. Phys. **29**, 463 (1951).

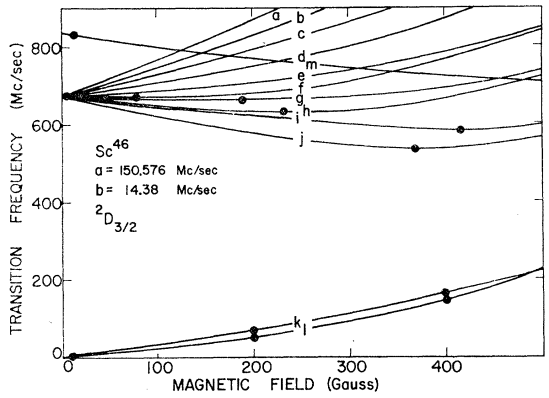

FIG. 3. Frequency dependence of transitions in the $^2D_{3/2}$ state of Sc^{46}. The identification of levels is given in Tables II and IV.

to obtain the energy levels as a function of the external magnetic field.[1] The energy-level diagrams for scandium are not shown because of the large multiplicity of levels; however, the magnetic field dependences of several observable transitions are given in Figs. 3 and 4.

The interaction constants for a d electron are theoretically related to the nuclear moments by the expressions[11]

$$\begin{aligned} a(\mathrm{Mc/sec}) &= [2\mu_0^2 g_I L(L+1)/10^6 hJ(J+1)] \\ &\quad \times \langle 1/r^3\rangle_{\mathrm{av}} F_r(J,Z_i), \\ b(\mathrm{Mc/sec}) &= [e^2Q(2J-1)/10^6 h(2J+2)] \\ &\quad \times \langle 1/r^3\rangle_{\mathrm{av}} R_r(L,J,Z_i). \end{aligned} \tag{4}$$

Here, L is the electronic orbital angular momentum in units of $\hbar$, Q is the nuclear electric quadrupole moment in cm^2, e is the electronic charge, and $F_r(J,Z_i)$ and $R_r(L,J,Z_i)$ are relativistic correction factors given by Casimir[12] and tabulated by Kopfermann.[11]

Since the factor $\langle 1/r^3\rangle_{\mathrm{av}}$ is difficult to estimate accurately, and since the nuclear constants for the stable isotopes are known, the nuclear-magnetic moment can best be calculated from the Fermi-Segrè-type relation

$$(g_I)_1 = (a_1/a_2)(g_I)_2. \tag{5}$$

An uncertainty in this equation, which is usually less than 0.5%, may result from assuming similar electronic properties for the d electron of the two isotopes and from neglecting hfs anomalies. In a like manner, the nuclear quadrupole moment may be calculated from the expression

$$Q_1 = (b_1/b_2)Q_2. \tag{6}$$

If the quadrupole moment for the stable isotope is not known accurately, a better calculation consists of taking the ratio of the interaction constants for a given electronic state, with the resultant relation

$$Q = \frac{4g_I\mu_0^2}{e^2} \frac{F_r(J,Z_i)}{R_r(L,J,Z_i)} \frac{L(L+1)}{J(2J-1)} \frac{b}{a}. \tag{7}$$

The foregoing theory has assumed that the electronic states are not mixed by the interaction. These effects have been discussed by Schwartz[13] and the necessary corrections to the interaction constants are contained in reference 1.

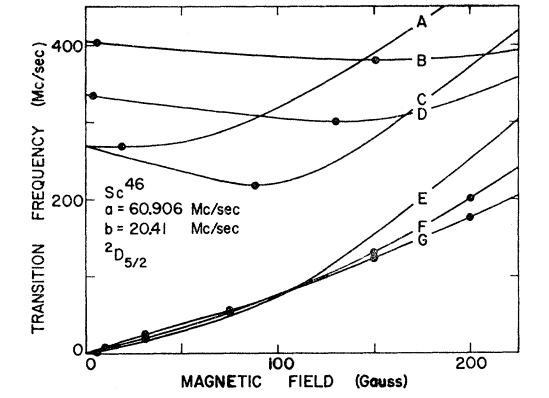

FIG. 4. Frequency dependence of transitions in the $^2D_{5/2}$ state of Sc^{46}. The identification of levels is given in Tables III and V.

III. ISOTOPE PRODUCTION AND IDENTIFICATION

Scandium-46 was produced by an (n,γ) reaction on the pure stable metal. Initial irradiations were done in the Livermore pool-type reactor, and later, in the General Electric test reactor at the Vallecitos Atomic Laboratory. Each sample, consisting of approximately 250 mg of metal, was bombarded for 1 to 2 weeks with a flux of (2 to 9)$\times 10^{13}$ n/cm^2-sec.

Yttrium-91, a fission product, was purchased as carrier-free YCl_3 in HCl solution from Oak Ridge National Laboratory. To each 250-mC sample of Y^{91}, approximately 30 mg of stable yttrium in HCl solution were added. Hydrated YCl_3 crystals were then formed by vacuum evaporation of the solution. The water of hydration was removed by slight heating in a high vacuum. The anhydrous YCl_3 was then reduced by calcium in a tantalum oven at elevated temperatures. An increase in temperature evaporated the $CaCl_2$ slag, leaving the pure yttrium metal.

The decay schemes and half-lives of both isotopes have been established by previous experimenters.[14] As a result, pulse-height analysis of the γ-ray spectrum and observation of the decay rate over several half-lives were used to verify identity of each isotope.

IV. APPARATUS

The atomic-beam magnetic-resonance apparatus and radioactive technique used have been described previ-

[11] Hans Kopfermann, *Nuclear Moments*, English version by E. E. Schneider (Academic Press Inc., New York, 1958), 2nd ed.

[12] H. B. G. Casimir, *On the Interaction Between Atomic Nuclei and Electrons* (Teyler's Tweede Genootschap, Haarlem, 1936).

[13] Charles Schwartz, Phys. Rev. **97**, 380 (1955).

[14] D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. **30**, 585 (1958).

ously,[15] so that only a brief description is included here.

The atomic-beam apparatus used the "flop-in" geometry consisting of the usual three-magnet system (A and B magnets inhomogeneous, with gradients in the same direction, C magnet homogeneous). Atoms in the beam were not refocused unless transitions were induced in the C-field region corresponding to the change $m_J=\pm 1/2 \leftrightarrow \mp 1/2$ in the high fields of the A and B magnets. This condition, together with the transition selection rules ($\Delta F=0, \pm 1$; $\Delta m=0, \pm 1$), limits the number of observable transitions. For yttrium-91 the possible transitions are shown in Figs. 1 and 2. The behavior of selected Sc^{46} resonance frequencies with magnetic field is shown in Figs. 3 and 4. The strength of the magnetic C field was measured by observation of the $\Delta F=0$ "flop-in" transitions in Rb^{85} and Rb^{87}.

Scandium beams were produced from small (3/8 in. square) tantalum ovens heated by electron bombardment. Because of the bulkiness of the reduction reaction ingredients, the yttrium ovens were slightly larger (1/2 in. sq) but of similar design. In both cases, sharp-edged tantalum crucibles were used inside the oven block to prevent the molten metal from creeping up the oven walls. The slits through which the beam emerged were adjusted to be 4 to 5 mils wide.

The radioactive beam was detected by collecting the atoms on sulfur-coated "buttons" which were subsequently counted in low-background continuous-flow-methane β counters. Undeflected scandium beam intensities varied from 1000 to 2000 counts/min for 1-min exposures; undeflected yttrium beam intensities varied from 100 to 300 counts/min for the same exposure time. Normally, 60 to 70% of the scandium beam could be deflected by the magnet system. The deflectability for yttrium usually ranged from 30 to 60%; the reason it was lower was probably an incomplete reduction reaction. Resonance-signal intensities ranged from 10 to 30 counts/min with a 30% apparatus background for 10-min exposures for scandium and from 5 to 10 counts/min with a 30% apparatus background for 15-min exposures for yttrium. Counter backgrounds were normally about 2.5 counts/min.

Radio frequencies from 1 to 850 Mc/sec were produced by four signal generators in conjunction with suitable wide-band or traveling-wave-tube amplifiers. The oscillators and their ranges were as follows: Tektronix, type 190, 0.35 to 50 Mc/sec; Hewlett-Packard, model 608C, 10 to 480 Mc/sec; Airborne Instruments, type 124C, 200 to 2500 Mc/sec; and Gertsch, model FM-4, 500 to 1000 Mc/sec. Frequencies were measured by using a Hewlett-Packard transfer oscillator, model 540A, in conjunction with a model 524B electronic counter. The 100-kc/sec internal reference crystal in the counter was checked against a separate laboratory standard which was monitored weekly against WWV time signals and against a National Company atomichron. All frequencies were recorded to the nearest kilocycle per second.

[15] J. P. Hobson, J. C. Hubbs, W. A. Nierenberg, H. B. Silsbee, and R. J. Sunderland, Phys. Rev. **104**, 101 (1956); also W. B. Ewbank, L. L. Marino, W. A. Nierenberg, H. A. Shugart, and H. B. Silsbee, *ibid.* **120**, 1406 (1960).

TABLE I. Summary of physical constants used in analysis of the Sc^{46} and Y^{91} experimental data.

$M_p/M_e=1836.12$ $\mu_0/h=1.399677$ Mc/G-sec	
Rb^{85} [a]	
$I=5/2$	$^2S_{1/2}\{g_J=-2.00238(4)$ $\Delta\nu=3035.735(2)$ Mc/sec
$(\mu_I)_{\text{uncorr}}=+1.348190(5)$ nm	
Rb^{87} [a]	
$I=3/2$	$^2S_{1/2}\{g_J=-2.00238(4)$ $\Delta\nu=6834.685(2)$ Mc/sec
$(\mu_I)_{\text{uncorr}}=+2.7413970(47)$ nm	
Sc^{45} [b]	
$I=7/2$	$^2D_{3/2}\{g_J=-0.800(4)$ $a=+269.560(20)$ Mc/sec $b=-26.37(10)$ Mc/sec
$(\mu_I)_{\text{uncorr}}=+4.74916(12)$ nm	
$Q_{\text{uncorr}}=-0.22(1)$ b	$^2D_{5/2}\{g_J=-1.20(1)$ $a=+109.034(10)$ Mc/sec $b=-37.31(10)$ Mc/sec
Y^{89} [c]	
$I=1/2$	$^2D_{3/2}\{g_J=-0.79927(11)$ $a=-57.217(15)$ Mc/sec
$(\mu_I)_{\text{uncorr}}=-0.136825(4)$ nm	$^2D_{5/2}\{g_J=-1.20028(19)$ $a=-28.749(30)$ Mc/sec

[a] See references 11 and 16.
[b] See references 6, 10, 17, and 18.
[c] See references 7, 8, 9, 19, and 20.

V. RESULTS

For transitions of the type $F=I+J$, $\Delta F=0$, and $\Delta m=\pm 1$, the frequency separations at low magnetic fields for the two observable electronic states are given by

$$\nu(^2D_{3/2})=-g_J\frac{\mu_0 H}{h}\frac{3}{2I+3},$$
$$\text{and} \qquad (8)$$
$$\nu(^2D_{5/2})=-g_J\frac{\mu_0 H}{h}\frac{5}{2I+5},$$

where higher-order terms in H have been neglected.

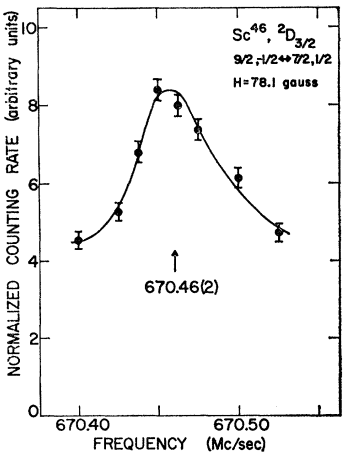

FIG. 5. A sample resonance corresponding to the transition $F, m=9/2, -1/2 \leftrightarrow 7/2, 1/2$ in the $^2D_{3/2}$ state of Sc^{46} at $H=78.1$ G.

TABLE II. Regions of minimum field dependence of some of the observable $\Delta F=\pm 1$ transitions in the $^2D_{3/2}$ electronic state of Sc^{46}. The calculations were performed for $a=150.576$ Mc/sec and $b=14.38$ Mc/sec.

Designation in Fig. 3.	Transition $F_1, m_1 \leftrightarrow F_2, m_2$	$(\partial\nu/\partial H)_{\min}$ (Mc/sec-G)	H(approx) (G)	$\nu(g_I^+)$ (Mc/sec)	$\nu(g_I^-)$ (Mc/sec)
j	9/2, −5/2 ↔ 7/2, −3/2	0	368.8	535.904	535.477
i	9/2, −3/2 ↔ 7/2, −5/2	0	414.9	584.494	584.975
h	9/2, −3/2 ↔ 7/2, −1/2	0	231.8	631.084	630.816
g	9/2, −1/2 ↔ 7/2, −3/2	0	189.1	664.264	664.483
f	9/2, −1/2 ↔ 7/2, 1/2	0	78.1	670.460	670.369

TABLE III. Regions of minimum field dependence of some of the observable $\Delta F=\pm 1$ transitions in the $^2D_{5/2}$ electronic state of Sc^{46}. The calculations were performed for $a=60.906$ Mc/sec and $b=20.41$ Mc/sec.

Designation in Fig. 4.	Transition $F_1, m_1 \leftrightarrow F_2, m_2$	$(\partial\nu/\partial H)_{\min}$ (Mc/sec-G)	H(approx) (G)	$\nu(g_I^+)$ (Mc/sec)	$\nu(g_I^-)$ (Mc/sec)
B	13/2, −7/2 ↔ 11/2, −7/2	0	151.0	380.960	380.960
D	11/2, −5/2 ↔ 9/2, −5/2	0	130.3	301.825	301.825
A	9/2, 1/2 ↔ 7/2, 3/2	0	19.4	268.652	268.629
···	9/2, −1/2 ↔ 7/2, 1/2	0	55.1	255.143	255.079
C	9/2, −3/2 ↔ 7/2, −1/2	0	88.6	219.023	218.921

Thus, at a given magnetic field H, observation of an increase in signal intensity at a frequency corresponding to one of the assumed values for I gives a preliminary indication of the value of the nuclear spin. The results were $I(Sc^{46})=4$ and $I(Y^{91})=1/2$, in units of $\hbar$. These preliminary results were subsequently verified by observation of the proper field dependence for the $\Delta F=0$ transitions.

As the magnetic field H was increased, the higher-order terms which have been omitted in Eq. (8) become significant. Since these terms are functions of the zero-field hfs separations, the quadratic shift was used to obtain preliminary estimates of the hyperfine interaction constants a and b. These preliminary estimates were used in the HYPERFINE computer routine, which uses a least-squares procedure to fit any combination of the constants a, b, g_I, and g_J to the observed data.[1] Table I summarizes the various physical constants[6–11,16–20] used in the computer routine and for subsequent calculations.

For Sc^{46}, 18 resonances corresponding to 13 transitions out of 23 possible ones in the $^2D_{3/2}$ state and 18 resonances corresponding to 7 transitions out of 22 possible ones in the $^2D_{5/2}$ state were observed. A sample $\Delta F=\pm 1$ transition is shown in Fig. 5. Because of health problems associated with the high-energy γ-ray activity of this isotope, we decided to concentrate on only those transitions which could provide the most information with a given amount of effort—namely, $\Delta F=\pm 1$, $\Delta m=\pm 1$ transitions which were field-independent at high magnetic fields. By this method, we were able to determine the signs of the nuclear moments as well as measure the constants a, b, and g_J. Tables II and III summarize the field-independent points of these transitions and show the transition frequencies for both a positive and a negative nuclear magnetic moment as predicted by the final values of the interaction constants.

A summary of all observed experimental data as fitted by the HYPERFINE routine is shown in Tables IV and V. The residual represents the difference between the experimental resonance frequency and the value of this frequency predicted by the final computed atomic and nuclear constants. The weight factor is the reciprocal of the sum of the squares of the frequency uncertainties due to resonance linewidth and magnetic field uncertainty.

From the computer analysis, the final values for the $^2D_{3/2}$ state are $g_J=-0.7990(8)$, $a=+150.576(9)$ Mc/sec, and $b=+14.38(14)$ Mc/sec; and, for the $^2D_{5/2}$ state, $g_J=-1.1995(18)$, $a=+60.906(4)$ Mc/sec, and $b=+20.41(10)$ Mc/sec. From these coupling constants, the zero-field hyperfine separations (in Mc/sec) are:

$^2D_{3/2}$

$$\Delta\nu_{11/2-9/2}=838.06(11),$$
$$\Delta\nu_{9/2-7/2}=674.12(6),$$
$$\Delta\nu_{7/2-5/2}=517.13(10),$$

$^2D_{5/2}$

$$\Delta\nu_{13/2-11/2}=405.84(6),$$
$$\Delta\nu_{11/2-9/2}=336.19(2),$$
$$\Delta\nu_{9/2-7/2}=270.14(3),$$
$$\Delta\nu_{7/2-5/2}=207.05(3),$$
$$\Delta\nu_{5/2-3/2}=146.25(3).$$

[16] Norman F. Ramsey, *Molecular Beams* (Oxford University Press, New York, 1956).
[17] H. Kopfermann and E. Rasmussen, Z. Physik **92**, 82 (1934).
[18] H. Schüler and T. Schmidt, Naturwissenschaften **22**, 758 (1934).
[19] M. F. Crawford and A. L. Schawlow, Phys. Rev. **76**, 1310 (1949).
[20] H. Kuhn and G. K. Woodgate, Proc. Phys. Soc. (London) **A63**, 830 (1950).

TABLE IV. Summary of Sc^{46} resonances in the $^2D_{3/2}$ state.

Designation in Fig. 3	H[a] (G)	δH[a] (G)	F_1	m_1	F_2	m_2	ν_{obs} (Mc/sec)	$\delta\nu_{obs}$ (Mc/sec)	Residual (Mc/sec)	Weight factor
k	10.021	0.041	11/2	−7/2	11/2	−9/2	3.085	0.035	+0.009	721.3
k	200.080	0.089	11/2	−7/2	11/2	−9/2	71.000	0.075	+0.015	144.0
k	399.907	0.120	11/2	−7/2	11/2	−9/2	164.450	0.200	−0.035	22.7
l	10.055	0.041	9/2	−5/2	9/2	−7/2	1.950	0.050	−0.012	389.6
l	200.122	0.079	9/2	−5/2	9/2	−7/2	53.800	0.080	+0.006	139.2
l	399.842	0.210	9/2	−5/2	9/2	−7/2	150.050	0.225	−0.028	14.4
m	5.000	0.075	11/2	−7/2	9/2	−7/2	836.080	0.070	−0.010	173.5
a	5.033	0.082	9/2	9/2	7/2	7/2	678.750	0.250	−0.047	14.6
b	5.033	0.082	9/2	7/2	7/2	5/2	677.800	0.200	+0.062	23.0
c	5.033	0.082	9/2	5/2	7/2	3/2	676.800	0.200	+0.119	24.0
d	5.033	0.082	9/2	3/2	7/2	1/2	675.800	0.200	+0.175	24.6
e	5.033	0.082	9/2	1/2	7/2	−1/2	674.680	0.075	+0.108	176.0
f	5.033	0.082	9/2	−1/2	7/2	1/2	673.600	0.100	−0.100	99.5
f	78.109	0.075	9/2	−1/2	7/2	1/2	670.460	0.020	+0.000	2500.0
g	189.210	0.161	9/2	−1/2	7/2	−3/2	664.275	0.020	+0.012	2500.0
h	231.738	0.079	9/2	−3/2	7/2	−1/2	631.075	0.020	−0.009	2500.0
i	415.108	0.163	9/2	−3/2	7/2	−5/2	584.500	0.030	+0.007	1111.1
j	368.912	0.163	9/2	−5/2	7/2	−3/2	535.895	0.015	−0.009	4444.0

[a] Magnetic field and uncertainties are the averages of those computed from Rb^{85} and Rb^{87} calibrations.

TABLE V. Summary of Sc^{46} resonances in the $^2D_{5/2}$ state.

Designation in Fig. 4	H[a] (G)	δH[a] (G)	F_1	m_1	F_2	m_2	ν_{obs} (Mc/sec)	$\delta\nu_{obs}$ (Mc/sec)	Residual (Mc/sec)	Weight factor
G	10.021	0.064	13/2	−7/2	13/2	−9/2	6.600	0.035	+0.023	327.6
G	30.857	0.061	13/2	−7/2	13/2	−9/2	21.125	0.200	+0.161	23.9
G	75.346	0.125	13/2	−7/2	13/2	−9/2	54.900	0.250	−0.068	13.7
G	150.414	0.068	13/2	−7/2	13/2	−9/2	123.400	0.250	−0.054	14.9
G	199.988	0.063	13/2	−7/2	13/2	−9/2	177.300	0.375	−0.066	6.9
F	10.056	0.064	11/2	−5/2	11/2	−7/2	5.975	0.050	+0.022	248.6
F	30.854	0.061	11/2	−5/2	11/2	−7/2	19.430	0.100	+0.069	85.3
F	75.275	0.113	11/2	−5/2	11/2	−7/2	53.200	0.200	−0.005	20.3
F	150.419	0.068	11/2	−5/2	11/2	−7/2	131.600	0.300	+0.070	10.3
F	200.030	0.070	11/2	−5/2	11/2	−7/2	200.800	0.425	−0.082	5.2
E	10.056	0.064	9/2	−3/2	9/2	−5/2	4.815	0.050	−0.033	281.5
B	6.058	0.074	13/2	−7/2	11/2	−7/2	404.300	0.150	−0.070	43.8
B	151.062	0.154	13/2	−7/2	11/2	−7/2	380.965	0.020	+0.003	2499.9
D	4.707	0.085	11/2	−5/2	9/2	−5/2	334.750	0.050	−0.067	322.4
D	130.373	0.127	11/2	−5/2	9/2	−5/2	301.835	0.020	+0.010	2499.9
A	19.955	0.042	9/2	1/2	7/2	3/2	268.660	0.030	+0.007	1111.1
A	19.982	0.042	9/2	1/2	7/2	3/2	268.660	0.030	+0.007	1111.0
C	88.521	0.083	9/2	−3/2	7/2	−1/2	219.012	0.020	−0.011	2499.9

[a] Magnetic field and uncertainty are averages of those computed from Rb^{85} and Rb^{87} calibrations.

Since the observable transitions in Y^{91} have minimum field dependence at zero field, the sign of the nuclear magnetic moment could not be determined by the technique used for Sc^{46}. Tables VI and VII summarize the observed data as fitted by the HYPERFINE routine. A sample $\Delta F=\pm1$ transition is shown in Fig. 6. The final values of the magnetic dipole interaction constants of yttrium-91 are: $a(^2D_{3/2})=\pm68.34(2)$ Mc/sec and $a(^2D_{5/2})=\pm34.35(3)$ Mc/sec. The zero-field hyperfine-structure separations for each electronic state are $\Delta\nu(^2D_{3/2})=\pm136.69(3)$ Mc/sec and $\Delta\nu(^2D_{5/2})=\pm103.05(4)$ Mc/sec.

TABLE VI. Summary of Y^{91} resonances in the $^2D_{3/2}$ state.

H[a] (G)	δH[a] (G)	F_1	m_1	F_2	m_2	ν_{obs} (Mc/sec)	$\delta\nu_{obs}$ (Mc/sec)	Residual (Mc/sec)	Weight factor
4.645	0.085	2	1	2	0	3.900	0.100	−0.011	66.0
30.447	0.061	2	1	2	0	26.275	0.100	+0.040	77.3
49.915	0.104	2	1	2	0	43.850	0.150	−0.038	31.5
100.242	0.126	2	1	2	0	92.200	0.300	−0.426	9.4
199.803	0.155	2	1	2	0	197.100	0.300	+0.052	8.5
49.848	0.050	1	0	2	0	147.550	0.200	−0.069	24.7
49.902	0.149	1	0	2	0	147.500	0.200	−0.141	22.8
5.051	0.106	1	0	2	0	136.775	0.050	−0.027	396.2
3.088	0.107	1	0	2	0	136.735	0.025	+0.006	1577.1
3.014	0.107	1	0	2	0	136.730	0.025	+0.003	1578.2

[a] Magnetic field and uncertainty are the averages of those computed from Rb^{85} and Rb^{87} calibrations.

TABLE VII. Summary of Y^{91} resonances in the $^2D_{5/2}$ state.

H[a] (G)	δH[a] (G)	F_1	m_1	F_2	m_2	ν_{obs} (Mc/sec)	$\delta\nu_{obs}$ (Mc/sec)	Residual (Mc/sec)	Weight factor
5.058	0.106	2	0	3	0	103.375	0.100	−0.021	97.9
4.964	0.106	2	0	3	0	103.350	0.100	−0.033	98.0
3.022	0.107	2	0	3	0	103.185	0.050	+0.014	388.0

[a] Magnetic field and uncertainty are averages of those computed from Rb^{85} and Rb^{87} calibrations.

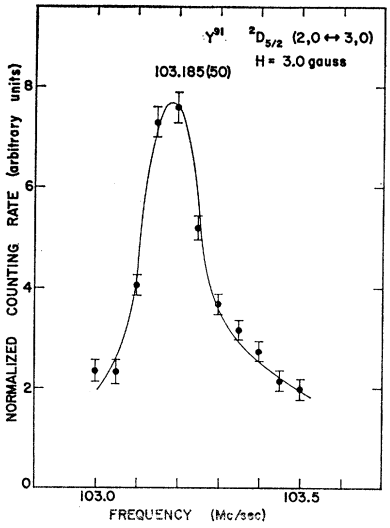

FIG. 6. A sample resonance corresponding to the transition F, $m=2,\ 0\leftrightarrow 3,\ 0$ in the $^2D_{5/2}$ state of Y^{91} at $H=3.0$ G.

With the aid of Eqs. (5), (6), and (7), the following uncorrected nuclear moments were calculated from the measured interaction constants and the appropriate physical constants for Sc^{45} and Y^{89}:

$$(\mu_I)_{\text{uncorr}}(Sc^{46})=+3.03(2)\text{ nm},$$
$$Q_{\text{uncorr}}(Sc^{46})=+0.119(6)\text{ b},$$
$$(\mu_I)_{\text{uncorr}}(Y^{91})=\pm 0.1634(8)\text{ nm}.$$

VI. DISCUSSION

For Sc^{46}, the shell model predicts an $f_{7/2}$ proton configuration and a $(f_{7/2}^{-3})_{5/2}$ neutron configuration. The revised coupling rule proposed by Brennan and Bernstein[21] predicts $I=I_p+I_n-1=5$. Thus, the measured spin $I=4$ is a violation of this rule but not of the much weaker rule $|I_p+I_n|\geqslant I\geqslant|I_p-I_n|$ proposed by Nordheim.[22] The directly measured spin agrees with previous results inferred from β- and γ-ray spectroscopy. If we use jj coupling to combine the magnetic-moment effects of the proton and neutron groups and use the g factors from the nucleon groups of neighboring odd-Z nuclei, we obtain $\mu_s=+3.26$ nm. This result agrees reasonably well with the corrected experimental value $\mu_I=+3.03$ nm.

Since the 39th proton in Y^{91} is a single particle in a $p_{1/2}$ level, we would expect the shell model to predict the nuclear spin of this isotope with a great deal of certainty. The measured spin $I=1/2$ confirms this prediction. We should also expect the nuclear magnetic moment to be very nearly equal to the moment of Y^{89}. If we assume the sign of the moment to be negative, then the slightly more negative experimental value follows a trend similar, for instance, to the odd-A silver isotopes.

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[22] L. W. Nordheim, Phys. Rev. **78**, 294 (1950); Revs. Modern Phys. **23**, 322 (1951).