$$N=C\int_0^\infty\int d\epsilon d\epsilon'\left\{\frac{(E'-E)^2}{|e^{E'-E}-1|}\left[\epsilon^2\left(1-\frac{\Delta^2}{EE'}\right)+2g\Delta^2-\epsilon\epsilon'\left(1+(g-\tfrac{1}{2})\Delta^2\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon'^2}\right)\right)\right]\right.$$
$$\left.+\frac{(E'+E)^2}{|e^{E'+E}-1|}\left[\epsilon^2\left(1+\frac{\Delta^2}{EE'}\right)+2g\Delta^2+\epsilon\epsilon'\left(1+(g-\tfrac{1}{2})\Delta^2\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon'^2}\right)\right)\right]\right\}$$
$$\equiv C\int_0^\infty\int d\epsilon d\epsilon' F(E,E',\Delta^2), \tag{C6}$$

and from (C2) and (C5)

$$D=2\left\{\int d^3k\epsilon\left(1+g\frac{\Delta^2}{\epsilon^2}\right)\frac{\hbar k}{m}\epsilon f(E)f(-E)\cos^2\theta\right\}^2. \tag{C7}$$

For temperatures $T\approx T_c$ we can expand N and D in powers of $\Delta^2(T)$

$$N=C(\alpha+\beta\Delta^2+\cdots) \tag{C8}$$
$$D=C_2(1+\delta\Delta^2+\cdots). \tag{C9}$$

We get

$$\alpha=\int_0^\infty\int d\epsilon d\epsilon' F(\epsilon,\epsilon',0), \tag{C10}$$

$$\beta=\int_0^\infty\frac{d\epsilon}{\epsilon^2}\int_0^\infty d\epsilon'\,[F(\epsilon,\epsilon',0)-F(0,\epsilon',0)]+2(g-\tfrac{1}{2})$$
$$\times P\int_{-\infty}^\infty\frac{d\epsilon}{\epsilon}\int_0^\infty d\epsilon'\frac{(\epsilon+\epsilon')^3}{|e^{\epsilon+\epsilon'}-1|}(1-f)(1-f'), \tag{C11}$$

$$\delta=0.606(g-\tfrac{1}{2}). \tag{C12}$$

These integrals are evaluated numerically, and we obtain

$$N=C_1[1+\Delta^2(-0.056+0.26(g-\tfrac{1}{2})], \tag{C13}$$
$$D=C_2[1+\Delta^2(0.606(g-\tfrac{1}{2}))], \tag{C14}$$
$$1/K_{es}=(C_1/C_2)[1+\Delta^2(0.124-0.35g)]. \tag{C15}$$

Assuming that C_1/C_2 gives the correct value for the resistivity in the normal state we get

$$K_{es}/K_{en}=1+\Delta^2(0.35g-0.124). \tag{C16}$$

In (C15) g can be made as large as possible, with the restriction that $g\Delta^2$ should remain small enough.

The experimental value of K_{es}/K_{en} is less than unity. Our result (C16) is therefore in disagreement with the experimental data even more so than BRT's "best" result ($g=\frac{1}{2}$ corresponding to $n=1$). This reinforces their conclusion that there must be an "extra mechanism scattering," which adds to the dissipation. As Wentzel[4] has remarked such a mechanism might well be provided by the residual interactions between quasi-particles (see Sec. 1) which were here neglected, or by the many other interaction terms in the complete Hamiltonian which one neglects already when writing down the BCS Hamiltonian. However, the latter interactions would presumably also accelerate the decay of an electric current, and hence make the persistent currents even more difficult to understand.

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Superconductivity in the Case of Overlapping Bands

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The equations for the superconductivity energy gap and transition temperature in the case of overlapping bands are derived by the Nambu-Schrieffer formalism.

THE case when the Fermi surface of a superconductor goes through several overlapping bands has been considered by Suhl, Matthias, and Walker.[1] A simple derivation of their equations for the energy gap and the transition temperature can be given using the formalism of Nambu[2] and Schrieffer.[3]

We label the one electron states by the wave vector $\mathbf{k}$ lying within the first Brillouin zone and by the band

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[1] H. Suhl, B. T. Matthias, and L. R. Walker, Phys. Rev. Letters **3** 552 (1959).
[2] Y. Nambu, Phys. Rev. **117**, 648 (1960).
[3] J. R. Schrieffer, Physica **26**, 124 (1960). See also G. M. Eliashberg, Soviet Phys.—JETP **38**, 966 (1960); *ibid.* **39**, 1437 (1960); and S. Engelsberg, Phys. Rev. **126**, 1251 (1962).

index n. Using the wave function

$$\Psi_{nk}=\begin{pmatrix} C_{nk\uparrow} \\ C_{n-k\downarrow}^{\dagger} \end{pmatrix}=\begin{pmatrix} \Psi_{nk1} \\ \Psi_{nk2} \end{pmatrix} \tag{1}$$

and the Pauli matrices $\tau_0=\mathbf{1}$, τ_1, τ_2, τ_3, the Hamiltonian can be written in the form

$$H=\sum_{nk}\epsilon_{nk}\Psi_{nk}^{\dagger}\tau_3\Psi_{nk} -\tfrac{1}{2}\sum_{nkn'k'}(n'\mathbf{k}', n'-\mathbf{k}'|V|n\mathbf{k}, n-\mathbf{k}) \times(\Psi_{n'k'}^{\dagger}\tau_3\Psi_{nk})(\Psi_{nk}^{\dagger}\tau_3\Psi_{n'k'})+\text{const}, \tag{2}$$

with the reduced two-particle interaction allowing for interband scattering

$$(n'\mathbf{k}', n'-\mathbf{k}|V|n\mathbf{k}, n-\mathbf{k})=V_{nk,n'k'}.$$

We use the Green's functions

$$G_{\alpha\beta}(n\mathbf{k},t)=i\langle T\Psi_{n\mathbf{k}}(t)_{\alpha}\Psi_{n\mathbf{k}}^{\dagger}(0)_{\beta}\rangle, \quad \alpha,\beta=1,2. \tag{3}$$

The zero-order Green's function corresponding to the Hamiltonian

$$H^{(0)}=\sum_{nk}\epsilon_{nk}\Psi_{nk}^{\dagger}\tau_3\Psi_{nk}$$

is given by

$$G^{(0)}(nk)=-(\omega\tau_0+\epsilon_{nk}\tau_3)(\omega^2-\epsilon_{nk}^2+i\eta)^{-1}. \tag{4}$$

Here $\eta=0+$ and $k=\mathbf{k}$, ω. We consider the diagonal in n matrix elements of the full Green's function, $G(nk)$. It satisfies the Dyson equation[4]

$$[G(nk)]^{-1}=[G^{(0)}(nk)]^{-1}+\Sigma(nk). \tag{5}$$

The self-energy operator $\Sigma(nk)$ can be expanded in the full set of Pauli matrices

$$\Sigma(nk)=(1-Z_{nk})\omega\tau_0+\chi_{nk}\tau_3+\Delta_{nk}\tau_1. \tag{6}$$

We set the coefficient of the τ_2 matrix equal to zero. Further we will set $Z_{nk}=1$ and choose $\chi_{nk}=0$. Thus $G(nk)$ expressed in terms of Δ_{nk} is

$$G(nk)=-(\omega+\epsilon_{nk}\tau_3+\Delta_{nk}\tau_1)(\omega^2-E_{nk}^2+i\eta)^{-1}, \tag{7}$$

with $E_{nk}=(\epsilon_{nk}^2+\Delta_{nk}^2)^{1/2}$. For a finite temperature $T=(k_B\beta)^{-1}$ the corresponding Green's function becomes

$$G(nk,B)=-(\omega+\epsilon_{nk}\tau_3+\Delta_{nk}\tau_1) [P(\omega^2-E_{nk}^2)-i\pi\tanh(\beta/2)\omega\delta(\omega^2-E_{nk}^2)]. \tag{8}$$

[4] J. R. Schrieffer, Argonne National Laboratory Report No. 6527 1962 (unpublished).

The superconducting solution in the case of overlapping bands is obtained if the diagonal in n matrix element of the self-energy operator is approximated by

$$\Sigma(nk)=i\sum_{n'}\int\tau_3G(n'k')\tau_3V_{nkn'k'}\frac{d^4k'}{(2\pi)^4}. \tag{9}$$

This approximation leads to the set of the energy-gap equations

$$\Delta_{nk}=\sum_{n'}\int\frac{\Delta_{n'k'}V_{nkn'k'}\tanh(\beta/2)E_{n'k'}}{2E_{n'k'}}\frac{d^3k'}{(2\pi)^3}. \tag{10}$$

We approximate the density of states at the Fermi surface in the nth band by a constant N_n and each interaction matrix element $V_{nkn'k'}$ by a constant $V_{nn'}$ different from zero only within a shell of width $2\omega_D$ near the Fermi energy. Neglecting any dependence of Δ_{nk} on $\mathbf{k}$ one has the equations of Suhl *et al.*[1]

$$\Delta_n=\sum_{n'}V_{nn'}\Delta_{n'}N_{n'}F(\Delta_{n'},\beta), \tag{11}$$

with

$$F(\Delta_n,\beta)=\int_0^{\omega_D}\frac{\tanh(\beta/2)(\epsilon^2+\Delta_n^2)^{1/2}}{(\epsilon^2+\Delta_n^2)^{1/2}}d\epsilon. \tag{12}$$

Near the transition temperature β_c one can put $\Delta_n=0$ in F obtaining the set of linear in Δ_n equations. To find the transition temperature, one has to solve the secular equation

$$\det(V_{nn'}N_{n'}F(0,\beta_c)-\delta_{nn'})=0.$$

The evaluation of the gaps Δ_n requires a numerical procedure. The knowledge of the Δ_n's gives all the one-particle and pair-distribution Green's functions. The present derivation exhibits the approximations involved in the equations of Suhl *et al.* More satisfactory integral equations for $\Sigma(nk)$ may be studied by the present method but the resulting Eqs. (10) are already complicated enough. Particular cases with certain $V_{nn'}$ independent of n may be of importance in some instances.

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