the low-frequency, or acoustical, and high-frequency, or optical branches of characteristic vibrations with their respective groups of polarization directions is, necessarily, the only realistic method of description of thermal excitations in crystalline solids, like solid He^4. The practical usefulness of this method of approach necessitates various experimental data. These may comprise the empirical determination of the dispersion laws of the characteristic vibration spectrum of the solid $\nu_\alpha(\mathbf{k})$, $\mathbf{k}$ being the propagation vector of the vibration of frequency ν, along a direction specified by α. The derivation of the associated spectral distribution function $\rho(\nu)$ of the characteristic frequencies, which defines through the Planck thermal distribution of the vibrations the total excitation energy and the corresponding constant volume heat capacity, raises in turn a problem whose solution is again approximate, of semiempirical character, at least at a certain stage of the corresponding knowledge of the empirical $\nu(\mathbf{k})$ function. The approximate heat capacities resulting from the theory lead to temperature variations different from the limiting continuum model T^3 law. These originate with the temperature dependence of the characteristic temperature which in some cases become quite strong.[18] If confirmed, the peculiarities in $\nu(\mathbf{k})$ and $\rho(\nu)$ must be ultimately responsible for the entropy and heat capacity anomalies of solid He^4 at, or near, the melting line and near $T_0(p_m)$. The problem of the negative isobaric expansion coefficients of the solid, both hexagonal and cubic, at $T \lesssim 1.50$–1.55°K is probably another aspect of the apparent thermal anomalies of the solid He^4 phases.

We would like to conclude by stating that while the above-mentioned solid He^4 heat capacity measurements favor a fairly normal behavior of this solid over the explored temperature and pressure ranges, additional experimental work on heat capacities and isobaric expansion coefficients would be of great interest over a region of the phase diagram around the melting pressure line. These should clarify the presence or absence of the various anomalies which present data, however indirect, seem to favor. In view of the possible connection between heat capacities and heat conductivity coefficients, experiments on the latter, in the relevant pressure and temperature intervals, would be also desirable.

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Decay of an Electric Current in a Superconductor*

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A theory of the decay of an electric current in a superconductor is presented. The calculations have been done using a model within the BCS theory of superconductivity, ignoring magnetic effects. The decay of current, from a specially chosen initial state, is found both for low temperatures and for temperatures near the critical temperature. The persistence of currents remains unexplained. The related problem of thermal conductivity is re-examined in an Appendix.

1. INTRODUCTION

THE Bardeen-Cooper-Schrieffer[1] (BCS) theory of superconductivity has been quite successful in explaining the thermodynamic features and the Meissner Effect. There has, however, been no quantitative verification of the transport properties of superconductors. For example, Bardeen-Rickayzen-Tewordt[2] (BRT) applied the theory to the case of thermal conductivity of superconductors but their results were in disagreement with experiment. The basic question of electric conductivity has found no satisfactory answer.

The main purpose of this paper is a study of how an electric current decays in a BCS superconductor. The calculation is based on a simple model in which the interaction between quasi-particles can be neglected without error.

Bogoliubov[3] has observed that a state of the electron gas which results from the BCS ground state by a small displacement in momentum space, is stable, or

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[1] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[2] J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. **113**, 982 (1959).

[3] N. N. Bogoliubov, Nuovo cimento **7**, 794 (1958).

rather metastable, due to the energy gap Δ. In this state, the gas carries an electric current [$(e/m)\times$total momentum] which is truly "persistent." However, if the gas is heated to a small temperature T, there will be excitation of quasi-particles and also of phonons, and their collisions will transfer momentum from the electrons to the phonons and thereby cause the current to decay. The rate of this decay is the main object of the present investigation.

More precisely, we shall take as the initial state of our model, a statistical distribution which, when viewed from a moving coordinate frame F_m, looks like a canonical ensemble, at temperature T, of quasi-particles, whereas the phonons are assumed to form a Debye thermal equilibrium distribution in the rest frame of the crystal, F_r. The idealized nature of this model need not be emphasized, and we are not concerned with the question under what experimental conditions such a situation might be approximately realized. The reason we choose the model is the observation made by Wentzel[4] that the initial decay rate of the current can then be calculated "accurately" in the same sense as the thermal equilibrium properties of the BCS superconductor ("reduced Hamiltonian") can be obtained accurately by using the quasi-particle picture.[5,6] One just has to replace the electron gas by a gas of *free* (i.e., noninteracting) quasi-particles with the properly chosen spectrum of single-particle excitations, namely, with a temperature-dependent energy gap as already derived by BCS.[1] Indeed, it has been shown that the residual interactions among such quasi-particles have no effect, either on the thermodynamic state functions (volume proportional part)[5,6] or in special transport problems[4] like the one treated here. Wentzel's proof was explicitly given for a simpler transport problem (momentum transfer due to a weak force center dragged through the gas), but he pointed out that the argument holds also in our case, provided the initial state of the electron gas is specified as described (canonical ensemble in the frame F_m). A sketch of the proof is given in Appendix B. We shall therefore, in the following, ignore the residual interactions and treat the quasi-particles as a perfect gas.

In the related problem of the electric resistance of stationary currents, a similar simplification is probably not justified, because the deviations of the quasi-particle distribution from equilibrium play an essential role. The same remark applies to the problem of the thermal resistance of superconductors, as treated by BRT.[2] They neglected the residual interactions, and this may, at least partially, account for their failure to reach agreement with the experimental data. We cannot improve their theory in this respect, but there are comments (on their variational calculation) which we want to add in Appendix C.

As will be seen, our results are not encouraging. At very low temperatures, we confirm the expectation that the gap suppresses the current attenuation completely. However, just below the critical temperature, the attenuation turns out to be even larger than in the normal state (at the same temperature). This effect must be attributed to the density of states which, according to the BCS theory, is larger just above the gap than in the normal state. Anyway, our calculation furnishes no indication why the decay should be slow just below the critical temperature.

In appraising this result, it should be kept in mind that our calculation can be claimed to be accurate only for the *initial stage* of the attenuation process. Whether the process leads, at roughly the same fast rate, to a complete decay of the current, or perhaps to a long-lived state carrying a weaker current, as Bohr and Mottelson[7] seem to believe, is much more difficult to decide, because in the later stages the residual interactions between quasi-particles will certainly add to the dissipation, or there may even no longer be any justification for using the simple quasi-particle picture at all.

We conclude that the transport properties of superconductors still pose a difficult problem. In particular, it is conceivable that magnetic effects help to stabilize the persistent currents. This aspect of the problem is not treated here.

2. COLLISION PROBABILITIES

The probabilities of various transitions involving quasi-particles and phonons, caused by the Bloch-Fröhlich interaction, have already been calculated by BRT (p. 986 of reference 2) and we summarize their results in this section.

Note that, in the BRT formulas, all momenta ($\mathbf{k}$) and energies (ϵ,E) refer to the rest frame of the crystal, and it is in this frame that the Cooper pairing is assumed to take place (momenta $+\mathbf{k}$ and $-\mathbf{k}$). This will no longer be the case in our model, but the later discussion (in Sec. 3) will show why we can use the same formulas, with some change in interpretation.

We now refer to formulas (4.3), (4.4) of BRT. f_{k0} and f_{k1} denote (single) quasi-particle distribution functions in momentum space for two spin orientations called 0 and 1 (or "up" and "down"). The rate of their change in time, caused by collisions with phonons, is expressed as follows:

$$\frac{\partial f_{k0}}{\partial t}=\sum_{\mathbf{k}'}[-Q_0(\mathbf{k},\mathbf{k}')+Q_0(\mathbf{k}',\mathbf{k})+Q_c(\mathbf{k},\mathbf{k}')-Q_d(\mathbf{k},\mathbf{k}')] \tag{2.1}$$

$$\frac{\partial f_{k1}}{\partial t}=\sum_{\mathbf{k}'}[-Q_1(\mathbf{k},\mathbf{k}')+Q_1(\mathbf{k}',\mathbf{k})+Q_c(\mathbf{k}',\mathbf{k})-Q_d(\mathbf{k}',\mathbf{k})]. \tag{2.2}$$

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[4] G. Wentzel, *Werner Heisenberg und die Physik unserer Zeit* (Friedrich Vieweg und Sohn, Braunschweig, 1961), p. 189.
[5] N. N. Bogoliubov. D. B. Zubarev, and Iu. A. Tserkovnikov, Soviet Phys.—Doklady **2**, 535 (1957).
[6] G. Wentzel, Phys. Rev. **120**, 1572 (1960).
[7] A. Bohr and B. R. Mottelson, Phys. Rev. **125**, 495 (1962).

$Q_0(\mathbf{k},\mathbf{k}')$ gives the total probability that the quasi-particle of type "0" is scattered from the state $(\mathbf{k},0)$ to $(\mathbf{k}',0)$ with either absorption or emission of a phonon of wave vector $\pm\mathbf{q}=\pm(\mathbf{k}'-\mathbf{k})$

$$Q_0(\mathbf{k},\mathbf{k}')=\left\{\frac{2\pi}{\hbar}|V_q|^2\times\tfrac{1}{2}\left(1+\frac{\epsilon\epsilon'-\Delta^2}{EE'}\right)[\delta(E'-E-h\nu_q)N_q+\delta(E'-E+h\nu_{-q})(N_{-q}+1)]\times f_{k0}(1-f_{k'0})\right\}_{\mathbf{q}=\mathbf{k}'-\mathbf{k}}$$
$$=P_0(\mathbf{k},\mathbf{k}')f_{k0}(1-f_{k'0}), \tag{2.3}$$

where V_q = electron-phonon interaction matrix element ($\sim q^{1/2}$); ϵ = energy of a free electron of wave vector $\mathbf{k}$ measured relative to Fermi energy E_f (c.f., 3.2). $E_F\equiv\mu$; E = quasi-particle energy; Δ = energy gap (temperature dependent); $h\nu_q$ = energy of a phonon wave vector $\mathbf{q}$; f_{k0} = mean occupation number of quasi-particle state $(\mathbf{k},0)$; f_{k1} = mean occupation number of quasi-particle state $(-\mathbf{k},1)$; $Q_1(\mathbf{k},\mathbf{k}')$ similarly gives the scattering probability for quasi-particle of type "1". But note that for type "1" the momentum is denoted by $-\mathbf{k}$; $Q_c(\mathbf{k},\mathbf{k}')$ and $Q_d(\mathbf{k},\mathbf{k}')$ give the probability that the quasi-particles $(\mathbf{k},0)$ and $(\mathbf{k}',1)$ are simultaneously created, (Q_c) or destroyed (Q_d), while emitting or absorbing a phonon.

$$Q_c(\mathbf{k},\mathbf{k}')=\left\{\frac{2\pi}{\hbar}|V_q|^2\times\tfrac{1}{2}\left(1-\frac{\epsilon\epsilon'-\Delta^2}{EE'}\right)\times[\delta(E'+E-h\nu_{-q})N_{-q}+\delta(E'+E+h\nu_q)\times(N_q+1)](1-f_{k0})(1-f_{k'1})\right\}_{\mathbf{q}=\mathbf{k}'-\mathbf{k}}$$
$$=P_c(\mathbf{k},\mathbf{k}')(1-f_{k0})(1-f_{k'1}). \tag{2.4}$$

$$Q_d(\mathbf{k},\mathbf{k}')=\left\{\frac{2\pi}{\hbar}|V_q|^2\times\tfrac{1}{2}\left(1-\frac{\epsilon\epsilon'-\Delta^2}{EE'}\right)\times[\delta(E'+E-h\nu_{-q})(N_{-q}+1)+\delta(E'+E+h\nu_q)\times(N_q+1)]f_{k0}f_{k'1}\right\}_{\mathbf{q}=\mathbf{k}'-\mathbf{k}}$$
$$=P_d(\mathbf{k},\mathbf{k}')f_{k0}f_{k'1}. \tag{2.5}$$

We are interested in the total momentum transferred from the phonons to the electrons, or quasi-particles, per unit time.

$$\frac{\partial\mathbf{P}_{\mathrm{el}}}{\partial t}=\frac{\partial}{\partial t}\sum_{\mathbf{k}}\hbar\mathbf{k}(f_{k0}-f_{k1}).$$

Making use of (2.1) and (2.2) in this formula, we get

$$\frac{\partial\mathbf{P}_{\mathrm{el}}}{\partial t}=-\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}=\mathbf{k}-\mathbf{k}}\frac{2\pi}{\hbar}|V_q|^2\hbar\mathbf{k}(N_q-N_{-q})\times\left\{\tfrac{1}{2}\left(1+\frac{\epsilon\epsilon'-\Delta^2}{EE'}\right)[\delta(E-E'-h\nu_q)-\delta(E'-E-h\nu_q)](f-f')+\tfrac{1}{2}\left(1-\frac{\epsilon\epsilon'-\Delta^2}{EE'}\right)[\delta(E+E'+h\nu_q)-\delta(E+E'-h\nu_q)](1-f-f')\right\}, \tag{2.6}$$

(where we have written $f=f_{k0}=f_{k1}$ and $f'=f_{k'0}=f_{k'1}$).

It should be remarked that all spatial variation of the distribution function f will be ignored. Since, in reality, the current density $[\sim\sum_{\mathbf{k}}\mathbf{k}(f_{k0}-f_{k1})]$ varies through distances of the order of the London penetration depth λ, the volumes considered here should be confined to layers of thickness $<\lambda$. The magnetic field associated with the currents, though important in other respects, shall be disregarded with the understanding that we intend only to examine the current attenuation (2.6) due to phonon-electron interactions. [To avoid misunderstanding, in the presence of a vector potential $\mathbf{A}$, the "momentum" $\hbar\mathbf{k}$ does not refer to the "canonical momentum" $(-i\hbar\nabla)$ but to the gauge invariant quantity $(-i\hbar\nabla-e\mathbf{A})$ = mass × velocity.]

3. APPLICATION TO MOVING FRAME

Let $\mathbf{v}$ be the velocity of the frame F_m relative to the crystal rest frame F_r, so that the electronic momenta in the two frames are related by

$$\mathbf{k}_m=\mathbf{k}_r-m\mathbf{v}. \tag{3.1}$$

Regarding the kinetic energies, we define

$$\epsilon_r=k_r^2/2m-\mu=(k_m^2/2m-\mu)+\mathbf{v}\cdot\mathbf{k}_m+\tfrac{1}{2}mv^2$$
$$=\epsilon_m+\mathbf{v}\cdot\mathbf{k}_m+\tfrac{1}{2}mv^2. \tag{3.2}$$

Our model is characterized by the assumption that (in the initial state) the Cooper pairing interaction operates in the frame F_m, i.e., it couples the momenta $\mathbf{k}_m$ and $-\mathbf{k}_m$. The ground state of the system is then just the metastable, current carrying state considered by Bogoliubov.[3] The excitation energies are

$$E_m=(\epsilon_m^2+\Delta^2)^{1/2} \tag{3.3}$$

and the initial distribution, according to our assumption, is the canonical one when viewed from the frame F_m, i.e.,

$$f=[\exp(E_m/KT)+1]^{-1} \tag{3.4}$$

is the number of quasi-particles of momentum $\mathbf{k}_m$ in F_m, or $\mathbf{k}_r$ in F_r. Such a distribution may be thought to be generated, from the *normal* thermal equilibrium in F_r, by a sudden accelerating pulse which displaces the whole Fermi sphere in the momentum space by $\delta\mathbf{k}=m\mathbf{v}$. Of course, $\mathbf{v}$ is meant to be very small.

The effects of the electron-phonon interactions will then be most conveniently studied in the frame F_m. The Bloch-Fröhlich matrix elements will have the same values in both frames, so in Eqs. (2.1)–(2.6) we can identify $\mathbf{k}$ with $\mathbf{k}_m$. The delta functions expressing energy conservation need not be changed if we interpret E as E_m (3.3) and also ν_q as the Doppler shifted frequency ν_m

$$h\nu_m=h\nu_r-\mathbf{v}\cdot\mathbf{q} \tag{3.5}$$

because, then, the $\mathbf{v}$-dependent terms in the total energy change cancel as a consequence of momentum conservation (Galilei invariance).

As mentioned in the introduction, we shall assume the phonons initially to have their thermal equilibrium distribution in the crystal rest frame F_r.

$$N_q=[\exp(h\nu_r/KT)-1]^{-1}. \tag{3.6}$$

Actually, one might admit a more general assumption without changing the main result. The essential point is that N_q differs from the canonical distribution in the moving frame F_m by an anisotropic term

$$N_q=N_q^0+\delta N_q \tag{3.7}$$

$$\begin{aligned} &N_q^0=[\exp(h\nu_m/KT)-1]^{-1}\\ &\delta N_q\neq\delta N_{-q}. \end{aligned} \tag{3.8}$$

According to (3.6), (3.7), and (3.8), in particular if $|\mathbf{v}|$ is much smaller than the velocity of sound,

$$\delta N_q=\mathbf{v}\cdot\mathbf{q}(\partial N_q^0/\partial(h\nu_q))\quad(\delta N_{-q}=-\delta N_q). \tag{3.9}$$

If we insert this into (2.6), the factor $\mathbf{v}$ in (3.9) guarantees that $\partial\mathbf{P}_{\mathrm{el}}/\partial t$ is proportional to the initial momentum $\mathbf{P}_{\mathrm{el}}$. In all other factors, we may then neglect $\mathbf{v}$; in particular, the Doppler shift of the phonon frequencies (in the δ functions) may be disregarded, so that we can write $h\nu_q=\omega_q=c_0q$. The resulting expression may, for convenience, be symmetrized in $\mathbf{k}$ and $\mathbf{k}'$.

We obtain

$$\begin{aligned}\frac{\partial\mathbf{P}_{\mathrm{el}}}{\partial t}=-\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}=\mathbf{k}-\mathbf{k}'}2\pi|V_q|^2(\mathbf{k}-\mathbf{k}')\delta N_q\bigg\{&\tfrac{1}{2}\bigg(1+\frac{\epsilon\epsilon'-\Delta^2}{EE'}\bigg)\\ &\times[\delta(E-E'-\omega_q)-\delta(E'-E-\omega_q)](f-f')\\ &+\tfrac{1}{2}\bigg(1-\frac{\epsilon\epsilon'-\Delta^2}{EE'}\bigg)[\delta(E+E'+\omega_q)\\ &-\delta(E+E'-\omega_q)](1-f-f')\bigg\}.\end{aligned} \tag{3.10}$$

4. REDUCTION OF EQUATION (3.10)

In this section we evaluate Eq. (3.10) by changing sums over $\mathbf{k}$, $\mathbf{k}'$ into integrals. [$\sum_{\mathbf{k}}\rightarrow(2\pi)^{-3}\int d^3k$, per unit volume.] $(\partial\mathbf{P}_{\mathrm{el}}/\partial t)_z$ is the only nonzero component of $\partial\mathbf{P}_{\mathrm{el}}/\partial t$ if we define the direction of the velocity $\mathbf{v}$ as z axis.

From (3.10) we therefore get,

$$\begin{aligned}\left(\frac{\partial\mathbf{P}_{\mathrm{el}}}{\partial t}\right)_z=\frac{\partial P_{\mathrm{el}}}{\partial t}=-\frac{1}{(2\pi)^6}\int d^3k\int d^3k'(2\pi)|V_q|^2(k_z-k_z')\left[v(k_z-k_z')\frac{\partial N_q^0}{\partial\omega_q}\right]\bigg\{\tfrac{1}{2}\bigg(1+\frac{\epsilon\epsilon'-\Delta^2}{EE'}\bigg)\\ \times[\delta(E-E'-\omega_q)-\delta(E'-E-\omega_q)](f-f')+\tfrac{1}{2}\bigg(1-\frac{\epsilon\epsilon'-\Delta^2}{EE'}\bigg)[\delta(E+E'+\omega_q)-\delta(E+E'-\omega_q)](1-f-f')\bigg\}.\end{aligned} \tag{4.1}$$

Except for the factor $(k_z-k_z')^2$ the integrand is a function of $|\mathbf{k}|$, $|\mathbf{k}'|$, and $|\mathbf{q}|$ only. We can therefore change z to x and again z to y without changing the integral. On adding and dividing by three, we get

$$\begin{aligned}\frac{\partial P_{\mathrm{el}}}{\partial t}=-\frac{1}{(2\pi)^6}\int d^3k\int d^3k'(2\pi)|V_q|^2\frac{(\mathbf{k}-\mathbf{k}')^2}{3}\left[v\frac{\partial N_q^0}{\partial\omega_q}\right]\bigg\{\tfrac{1}{2}\bigg(1+\frac{\epsilon\epsilon'-\Delta^2}{EE'}\bigg)[\delta(E-E'-\omega_q)-\delta(E'-E-\omega_q)](f-f')\\ +\tfrac{1}{2}\bigg(1-\frac{\epsilon\epsilon'-\Delta^2}{EE'}\bigg)[\delta(E+E'+\omega_q)-\delta(E+E'-\omega_q)](1-f-f')\bigg\}.\end{aligned} \tag{4.2}$$

While keeping $\mathbf{k}$ fixed the volume element d^3k' may be written $d^3k'=k'^2dk'\sin\psi d\psi d\phi'$ where ψ is the angle between $\mathbf{k}'$ and $\mathbf{k}$. We express ψ in terms of $q=|\mathbf{q}|$ (keeping $\mathbf{k}$ and $\mathbf{k}'$ fixed):

$$\mathbf{q}=\mathbf{k}-\mathbf{k}',\quad q^2=k^2+k'^2-2kk'\cos\psi,\quad qdq=kk'\sin\psi d\psi.$$

We then obtain

$$\begin{aligned}\frac{\partial P_{\mathrm{el}}}{\partial t}=\frac{\partial 1}{(2\pi)^3}\tfrac{2}{3}v\int_0^\infty kdk\int_0^\infty k'dk'\int_{|k-k'|}^{k+k'}dq\,q^3|V_q|^2\left[-\frac{\partial N_q^0}{\partial\omega_q}\right]\bigg\{\tfrac{1}{2}\bigg(1+\frac{\epsilon\epsilon'-\Delta^2}{EE'}\bigg)[\delta(E-E'-\omega_q)-\delta(E'-E-\omega_q)(f-f')\\ +\tfrac{1}{2}\bigg(1-\frac{\epsilon\epsilon'-\Delta^2}{EE'}\bigg)[\delta(E+E'+\omega_q)-\delta(E+E'-\omega_q)](1-f-f')\bigg\}.\end{aligned} \tag{4.3}$$

Now $k\approx k'\approx k_F$ and so we can take the upper limit $k+k'$ in the q integration to be infinite. The lower limit $|k-k'|$ can be replaced by zero because at the temperatures of interest the contributions of the interval $0<q<|k-k'|$ are negligible (in the main contributions, $|k-k'|\sim 10^{-4}k_F$, $q\sim 10^{-2}k_F$; see the discussion of this point in BRT, p. 988.

We change the variable q to $\omega=qc_0$ (c_0 velocity of sound). Also $|V_q|^2=g^2\omega/2$ (g=coupling constant). Note

$$-\left(\frac{\partial N_q^0}{\partial\omega_q}\right)=-\frac{\partial}{\partial\omega}\left(\frac{1}{e^{\beta\omega}-1}\right)=\frac{\beta}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})}.$$

Instead of k and k' we introduce ϵ and ϵ', respectively:

$$\epsilon=(1/2m)(k^2-k_F^2),\quad d\epsilon=kdk/m.$$

The lower limit for ϵ (or ϵ') may be replaced by $-\infty$. We will also express all energies in units of $KT=1/\beta$ so that the integrals are all pure numbers.

We now get,

$$\frac{\partial P_{\rm el}}{\partial t}=\frac{1}{(2\pi)^3}\tfrac{1}{6}v\frac{1}{c_0^4}g^2\beta^{-5}m^2\int_0^\infty d\omega\,\frac{\omega^4}{(e^\omega-1)(1-e^{-\omega})}\int_{-\infty}^\infty d\epsilon\int_{-\infty}^\infty d\epsilon'\left\{\left(1+\frac{\epsilon\epsilon'-\Delta^2}{EE'}\right)\right.$$
$$\left.\times[\delta(E-E'-\omega)-\delta(E'-E-\omega)](f-f')+\left(1-\frac{\epsilon\epsilon'-\Delta^2}{EE'}\right)[\delta(E+E'+\omega)-\delta(E+E'-\omega)](1-f-f')\right\}.\quad(4.4)$$

The terms $\sim\epsilon\epsilon'$ give zero; also $\delta(E+E'+\omega)\equiv 0$. If in the term $\sim\delta(E'-E-\omega)$, we interchange ϵ and ϵ', then

$$\frac{\partial P_{\rm el}}{\partial t}=2A\int_0^\infty d\omega\,\frac{\omega^4}{(e^\omega-1)(1-e^{-\omega})}\int_0^\infty d\epsilon\int_0^\epsilon d\epsilon'\left(1-\frac{\Delta^2}{EE'}\right)\delta(E-E'-\omega)(f-f')$$
$$-A\int_0^\infty d\omega\,\frac{\omega^4}{(e^\omega-1)(1-e^{-\omega})}\int_0^\infty d\epsilon\int_0^\infty d\epsilon'\left(1+\frac{\Delta^2}{EE'}\right)\delta(E+E'-\omega)(1-f-f')\quad(4.5)$$

$$A=\tfrac{2}{3}\frac{1}{(2\pi)^3}g^2\frac{1}{c_0^4}vm^2(KT)^5.\quad(4.6)$$

Note that $A\sim T^5$; this determines the temperature dependence in the normal state ($\Delta=0$).

Integrating over ω

$$\frac{\partial P_{\rm el}}{\partial t}=2A\int_0^\infty d\epsilon\int_0^\epsilon d\epsilon'\,\frac{(E-E')^4}{(e^{E-E'}-1)(1-e^{-(E-E')})}\left(1-\frac{\Delta^2}{EE'}\right)(f-f')$$
$$-A\int_0^\infty d\epsilon\int_0^\infty d\epsilon'\,\frac{(E+E')^4}{(e^{E+E'}-1)(1-e^{-(E+E')})}\left(1+\frac{\Delta^2}{EE'}\right)(1-f-f').\quad(4.7)$$

In Appendix A we evaluate the above integrals in two limiting cases of interest

$$\text{(a) } T\approx T_c,\quad \text{(b) } T\to 0.$$

5. DISCUSSION

The results of (4.7) are [see Eqs. (A9) and (A15) of Appendix A]

Case (a): $T\approx T_c$

$$\partial P_{\rm el}/\partial t=-124.5A\{1+0.78[(T_c-T)/T_c]\}.\quad(5.1)$$

Case (b): $T\approx 0$

$$\partial P_{\rm el}/\partial t=-25.8e^{-1.75(T_c/T)}.\quad(5.2)$$

The unsatisfactory character of these results has been already stressed in the Introduction. Particularly disturbing is the fact that just below T_c the initial decay rate turns out to be even larger than that of the normal state (given by $-124.5A$). As we have mentioned, it is conceivable that the process, in its later stages, leads to a metastable state which still carries a reduced current, but out method is not adequate for examining such a possibility.

Finally it should be observed that our result remains essentially unchanged if the "slab" of material we have been considering is bent into a torus-shaped object, like a thin cylindrical shell, with a macroscopic circumference $L=2\pi R$. [The condition of single valuedness of the wave function ($\varphi\to\varphi+2\pi$) is equivalent to the usual periodicity requirement $(Z\to Z+L,\ Z=R\varphi)$.] Our initial state then has an angular momentum whose average value per electron is $Rmv=L\delta k/2\pi$ which is $\hbar$ times an integer. [This integer is called ν in reference 7; it determines the quantized magnetic flux.] This initial angular momentum is transferred to the phonon gas at a rate given approximately by our formulas (5.1) and (5.2). Again, the initial current attenuation fails to show any sudden change when the temperature is lowered through T_c. Various reaction effects will certainly become important at a later stage, but here also a near-zero gap can hardly make much of a difference.

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APPENDIX A. EVALUATION OF INTEGRAL

Case (a):

Since the energy gap $\Delta(T)\to 0$ as $T\to T_c$ we expand $(1/A)(\partial P_{\rm el}/\partial t)$ (Eq. (4.7)) in powers of Δ^2 [to $O(\Delta^2)$]. The procedure is similar to that of BRT, Appendix A.

$$(1/A)(\partial P_{\rm el}/\partial t)=a+b\Delta^2+\cdots,$$

where a=decay rate in normal metals, $b\Delta^2$=term proportional to Δ^2; b is a constant.

We get from (4.7)

$$a=2\int_0^\infty d\epsilon\int_0^\epsilon d\epsilon'\frac{(\epsilon-\epsilon')^4}{(e^{\epsilon-\epsilon'}-1)(1-e^{-(\epsilon-\epsilon')})}(f-f')-\int_0^\infty d\epsilon\int_0^\infty d\epsilon'\frac{(\epsilon+\epsilon')^4}{(e^{\epsilon+\epsilon'}-1)(1-e^{-(\epsilon+\epsilon')})}(1-f-f') \quad \text{(A1)}$$

and

$$b=2\int_0^\infty d\epsilon\int_0^\epsilon d\epsilon'\left[\frac{dF_1}{d\Delta^2}\right]_{\Delta=0}-\int_0^\infty d\epsilon\int_0^\infty d\epsilon'\left[\frac{dF_2}{d\Delta^2}\right]_{\Delta=0}, \quad \text{(A2)}$$

where

$$\begin{aligned}F_1(E,E',\Delta^2)&=\frac{(E-E')^4}{(e^{E-E'}-1)(1-e^{-(E-E')})}\left(1-\frac{\Delta^2}{EE'}\right)(f-f')\\ F_2(E,E',\Delta^2)&=\frac{(E+E')^4}{(e^{E+E'}-1)(1-e^{-(E+E')})}\left(1+\frac{\Delta^2}{EE'}\right)(1-f-f').\end{aligned} \quad \text{(A3)}$$

We will now evaluate a and then b.

$$\begin{aligned}a=&2\int_0^\infty d\epsilon f(\epsilon)\int_0^\epsilon d\epsilon'\frac{(\epsilon-\epsilon')^4}{(e^{\epsilon-\epsilon'}-1)(1-e^{-(\epsilon-\epsilon')})}-2\int_0^\infty d\epsilon' f(\epsilon')\int_{\epsilon'}^\infty d\epsilon\frac{(\epsilon-\epsilon')^4}{(e^{\epsilon-\epsilon'}-1)(1-e^{-(\epsilon-\epsilon')})}\\ &-\int_0^\infty d\epsilon\int_0^\infty d\epsilon'\frac{(\epsilon+\epsilon')^4}{(e^{\epsilon+\epsilon'}-1)(1-e^{-(\epsilon+\epsilon')})}+2\int_0^\infty d\epsilon f(\epsilon)\int_0^\infty d\epsilon'\frac{(\epsilon+\epsilon')^4}{(e^{\epsilon+\epsilon'}-1)(1-e^{-(\epsilon+\epsilon')})}\\ =&2\int_0^\infty d\epsilon f(\epsilon)\int_0^\epsilon dx\frac{x^4}{(e^x-1)(1-e^{-x})}-2\int_0^\infty d\epsilon f(\epsilon)\int_0^\infty dx\frac{x^4}{(e^x-1)(1-e^{-x})}\\ &-\int_0^\infty d\epsilon\int_\epsilon^\infty dx\frac{x^4}{(e^x-1)(1-e^{-x})}+2\int_0^\infty d\epsilon f(\epsilon)\int_\epsilon^\infty dx\frac{x^4}{(e^x-1)(1-e^{-x})}\\ =&-\int_0^\infty d\epsilon\int_\epsilon^\infty dx\frac{x^4}{(e^x-1)(1-e^{-x})}=-\int_0^\infty dx\frac{x^4}{(e^x-1)(1-e^{-x})}\int_0^x d\epsilon\\ =&-\int_0^\infty dx\frac{x^5}{(e^x-1)(1-e^{-x})}\\ =&-124.5.\end{aligned}$$

To calculate b according to (A2), (A3), one has to take the derivatives of F_1, F_2, not only with respect to the explicitly appearing Δ^2, but also via $E=(\epsilon^2+\Delta^2)^{1/2}$ and E'.

$$\left[\frac{dF_1}{d\Delta^2}\right]_{\Delta=0}=\left[\frac{\partial F_1}{\partial\Delta^2}+\frac{1}{2E}\frac{\partial F_1}{\partial E}+\frac{1}{2E'}\frac{\partial F_1}{\partial E'}\right]_{\Delta=0}. \quad \text{(A4)}$$

In (A4) we add all the terms and simplify. In performing the ϵ, ϵ' integration, the terms involving $\partial/\partial\epsilon$ or $\partial/\partial\epsilon'$ may be simplified by partial integration. This finally leads to

$$2\int_0^\infty d\epsilon\int_0^\epsilon d\epsilon'\left[\frac{dF_1}{d\Delta^2}\right]_{\Delta=0}=-2\int_0^\infty\frac{d\epsilon'}{\epsilon'}\int_{\epsilon'}^\infty d\epsilon\frac{(\epsilon-\epsilon')^3}{(e^{\epsilon-\epsilon'}-1)(1-e^{-(\epsilon-\epsilon')})}(f-f')$$
$$+2\int_0^\infty\frac{d\epsilon}{\epsilon}\int_0^\epsilon d\epsilon'\frac{(\epsilon-\epsilon')^3}{(e^{\epsilon-\epsilon'}-1)(1-e^{-(\epsilon-\epsilon')})}(f-f')+\int_0^\infty\frac{d\epsilon}{\epsilon^2}\int_0^\epsilon d\epsilon'\frac{(\epsilon-\epsilon')^4}{(e^{\epsilon-\epsilon'}-1)(1-e^{-(\epsilon-\epsilon')})}(f-f')$$
$$+\int_0^\infty\frac{d\epsilon'}{\epsilon'^2}\int_{\epsilon'}^\infty d\epsilon\,[F_1(\epsilon,\epsilon',0)-F_1(\epsilon,0,0)]-\int_0^\infty d\epsilon\frac{\epsilon^3}{(e^\epsilon-1)(1-e^{-\epsilon})}(f-\tfrac{1}{2}).\quad\text{(A5)}$$

There will be no trouble at $\epsilon=0$ or $\epsilon'=0$ after properly grouping the above integrals with those coming from the second term in b [from $(dF_2/d\Delta^2)_{\Delta=0}$]. Similarly one finds

$$-\int_0^\infty d\epsilon\int_0^\infty d\epsilon'\left[\frac{dF_2}{d\Delta^2}\right]_{\Delta=0}=-2\int_0^\infty\frac{d\epsilon}{\epsilon}\int_0^\infty d\epsilon'\frac{(\epsilon+\epsilon')^3}{(e^{\epsilon+\epsilon'}-1)(1-e^{-(\epsilon+\epsilon')})}(1-f-f')$$
$$-\int_0^\infty\frac{d\epsilon}{\epsilon^2}\int_0^\infty d\epsilon'\,[F_2(\epsilon,\epsilon',0)-F_2(0,\epsilon',0)].\quad\text{(A6)}$$

We obtain b by adding the two terms (A5) and (A6), and after some simplification,

$$b=-2\int_0^\infty\frac{d\epsilon}{\epsilon}\int_0^\infty dx\frac{x^3}{(e^x-1)(1-e^{-x})}\times[f(\epsilon+x)+f(\epsilon-x)-2f(\epsilon)]$$
$$+\int_0^\infty\frac{d\epsilon}{\epsilon^2}\int_0^\infty dx\frac{x^4}{(e^x-1)(1-e^{-x})}\times[f(x+\epsilon)+f(x-\epsilon)-2f(x)].\quad\text{(A7)}$$

Note that in (A7) there is no longer divergence at $\epsilon=0$. After numerical integration $b=-9.4$. We therefore get

$$\frac{\partial P_{\text{el}}}{\partial t}=-A[124.5+9.4\Delta^2]=-124.5A[1+0.0755\Delta^2].\quad\text{(A8)}$$

We use $\Delta=3.2(1-T/T_c)^{1/2}$ then

$$\partial P_{\text{el}}/\partial t=-124.5A\{1+0.78[(T_c-T)/T_c]\}.\quad\text{(A9)}$$

Case (b): $T\approx0$.

We will make here the following approximation

$$E=(\epsilon^2+\Delta^2)^{1/2}\approx\Delta(1+(\epsilon^2/2\Delta^2)).\quad\text{(A10)}$$

Also we take $e^E\gg1$ so that

$$f(E)=(1+e^E)^{-1}\approx e^{-E}.\quad\text{(A11)}$$

The following is a rough estimate in as much as we will be calculating only the most dominant term. From (4.7) we get

$$\frac{\partial P_{\text{el}}}{\partial t}=2A\int_0^\infty d\epsilon\int_0^\epsilon d\epsilon'\frac{(E-E')^4}{(e^{E-E'}-1)(1-e^{-(E-E')})}\times\left(1-\frac{\Delta^2}{EE'}\right)[f(E)-f(E')].\quad\text{(A12)}$$

We have neglected the second integral in (4.7) because it behaves as $e^{-2\Delta}$, whereas the term we have retained is $\sim e^{-\Delta}$. It can be easily checked that the main contribution to (A12) comes from a region around the diagonal in the plane of ϵ and ϵ'. The region near the origin, and a narrow strip along the ϵ' (or ϵ) axis does not contribute to the most dominant term. We can therefore further simplify (A12) and we get

$$\frac{\partial P_{\text{el}}}{\partial t}\approx2A\int_0^\infty d\epsilon\int_0^\epsilon d\epsilon'\,(E-E')^4\left(1-\frac{\Delta^2}{EE'}\right)\times e^{-(E-E')}[e^{-E}-e^{-E'}]\quad\text{(A13)}$$
$$\approx-25.8Ae^{-\Delta},\quad\text{(A14)}$$

where $\Delta\approx$ energy gap at $T=0$ in units $KT=1.75\times(KT_c/KT)=1.75(T_c/T)$. On substituting in (A14)

$$\partial P_{\text{el}}/\partial t=-25.8Ae^{-1.75(T_c/T)}.\quad\text{(A15)}$$

APPENDIX B. RESIDUAL INTERACTIONS

The Bloch-Fröhlich interaction which gives rise to the collisions discussed in Sec. 2 may be written, in terms of free-electron and free-phonon creation and annihilation operators

$$H_{\text{int}}=\sum_{\mathbf{kk}'}H_{kk'},$$

where

$$H_{kk'} \sim a_{k'}^* a_k (b_{k-k'}^* + b_{k'-k}), \tag{B1}$$

and the transition amplitude resulting from the term $H_{kk'}$ is, to first order,

$$A_{kk'} = \int_0^{\delta t} dt \exp(itH) H_{kk'} \exp(-itH). \tag{B2}$$

In the spirit of the BCS theory, we include in H, besides the phonon energy, only the reduced BCS Hamiltonian which may be split into the free quasi-particle energy H^0 $(=\sum_{\mathbf{k}s} E_k \alpha_{ks}^* \alpha_{ks} + \text{const}$ in the moving frame $F_m)$ and the residual interactions H'. [See, for instance, reference 6, Eq. (4), with (3) and (13).] The phonon energy in the moving frame is

$$H_{\text{ph}} = \sum_{\mathbf{q}} \omega_q b_q^* b_q,$$

where ω_q stands for the Doppler-shifted energy [$\omega_q = h\nu_m$, see Eq. (3.5)]. Of course, it does not matter whether H in (B2) refers to frame F_m or F_r because the difference involves only the total momentum

$$\mathbf{P} = \mathbf{P}_{\text{el}} + \mathbf{P}_{\text{ph}} = \sum_{\mathbf{k}s} \mathbf{k} a_{ks}^* a_{ks} + \sum_{\mathbf{q}} \mathbf{q} b_q^* b_q$$

which commutes with $H_{kk'}$.

The momentum transfer in a transition with amplitude (B2) is proportional to $(\mathbf{k}'-\mathbf{k}) A_{k'k} A_{kk'}$, averaged over the statistical ensemble which we assumed to be a canonical ensemble when viewed from the frame F_m as far as the electrons are concerned, whereas the phonon distribution can remain unspecified. In other words, we have to study the canonical average.

$$\mathrm{Tr} A_{k'k} A_{kk'} \exp[-\beta(H^0+H')] / \mathrm{Tr} \exp[-\beta(H^0+H')], \tag{B3}$$

where "Tr" is the trace with regard to the states of the electron gas only (with $H_{\text{int}} \to 0$).

The expression (B3) has, in all respects of importance, the same structure as the corresponding expression which was analyzed by Wentzel in context with a simpler transport problem [see Eq. (15) in reference 4]. His proof which is based on a perturbation expansion in powers of H' both in $\exp[-(H^0+H')]$ and in $\exp(\mp itH)$ can be taken over without any essential change. (An average over an arbitrary phonon distribution may be carried out eventually.) Provided the quasi-particles are properly defined, namely, as in a thermal equilibrium at temperature $T = 1/K\beta$, a mere counting of nonvanishing terms leads to the result that neglecting H' in (B2,3) causes no errors in the limit of infinite volume. The only important assumption is the convergence of the perturbation expansion.

APPENDIX C. THERMAL CONDUCTIVITY OF SUPERCONDUCTORS

We re-examine the problem of thermal conductivity in this section. We will discuss here only the electronic contribution to it, extending somewhat the work of BRT[2].

Making use of the variational method, these authors obtain the expression $1/K_{\text{es}} = N/D$ for the thermal resistivity, where

$$N = T \int d^3k \int d^3k' W(\mathbf{k},\mathbf{k}') \chi_k (\chi_k - \chi_{k'}), \tag{C1}$$

$$D = 2 \left\{ \int \chi_k \frac{\hbar k_z}{m} \epsilon_k \frac{df_k}{dE} d^3k \right\}^2, \tag{C2}$$

and

$$W(\mathbf{k},\mathbf{k}') = (KT)^{-1} \{ P_0(\mathbf{k},\mathbf{k}') f_k (1-f_{k'}) + P_c(\mathbf{k},\mathbf{k}') (1-f_k)(1-f_{k'}) \}. \tag{C3}$$

$P_0(\mathbf{k},\mathbf{k}')$ and $P_c(\mathbf{k},\mathbf{k}')$ are defined by Eqs. (2.3) and (2.4); also throughout this section f_k, $f_{k'}$, refer to thermal equilibrium distribution functions for the quasi-particles. χ_k essentially characterizes the departure from equilibrium of the quasi-particle distribution function. The variational method consists in choosing the "best" trial function χ_k. The "true" thermal resistance is smaller than the smallest value we obtain by use of various trial functions.

The trial functions that BRT use are

$$\chi_k = b\epsilon (E/\epsilon)^n \cos\theta, \quad n = 0, 1, -1. \tag{C4}$$

θ is the angle between the vector $\mathbf{k}$ and the direction of the temperature gradient $\boldsymbol{\nabla} T$ (z axis). The function with the highest n (viz., $n=1$) gives the lowest value for thermal resistivity and so it is the best trial function of the type (C4), according to the variation principle. This suggests that we might use a trial function with n even greater than unity. We expect then to get a still lower value for thermal resistivity, but there is the limitation that no divergence should occur at $\epsilon = 0$. ($\int d\epsilon\, \chi_k$ finite). Since in any case BRT expand $|E/\epsilon| = |(\epsilon^2+\Delta^2)^{1/2}/\epsilon|$ in powers of Δ^2/ϵ^2 and retain terms up to $O(\Delta^2/\epsilon^2)$ only, we can allow the exponent n in (C4) to take large values, provided we also terminate the expansion of $|E/\epsilon|$ after the second term. This will not lead to divergences, any worse than in BRT; again the integral $P \int_{-\infty}^{\infty} d\epsilon\, \chi_k$ will be convergent. The most general trial function then is

$$\chi_k = b\epsilon [1 + g(\Delta^2/\epsilon^2)] \cos\theta, \tag{C5}$$

where g is a constant ($g = n/2$) and can be used as a parameter in our variational calculation.

With this trial function (C5) we now calculate the thermal resistivity near the critical temperature. We will not discuss any details of the calculation, since we essentially follow BRT[2] in this respect. From (C1), (C3), and (C5), we get, after simplification,

$$N=C\int_0^\infty\int d\epsilon d\epsilon'\left\{\frac{(E'-E)^2}{|e^{E'-E}-1|}\left[\epsilon^2\left(1-\frac{\Delta^2}{EE'}\right)+2g\Delta^2-\epsilon\epsilon'\left(1+(g-\tfrac{1}{2})\Delta^2\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon'^2}\right)\right)\right]\right.$$
$$\left.+\frac{(E'+E)^2}{|e^{E'+E}-1|}\left[\epsilon^2\left(1+\frac{\Delta^2}{EE'}\right)+2g\Delta^2+\epsilon\epsilon'\left(1+(g-\tfrac{1}{2})\Delta^2\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon'^2}\right)\right)\right]\right\}$$
$$\equiv C\int_0^\infty\int d\epsilon d\epsilon'\,F(E,E',\Delta^2), \quad \text{(C6)}$$

and from (C2) and (C5)

$$D=2\left\{\int d^3k\epsilon\left(1+g\frac{\Delta^2}{\epsilon^2}\right)\frac{\hbar k}{m}\epsilon f(E)f(-E)\cos^2\theta\right\}^2. \quad \text{(C7)}$$

For temperatures $T\approx T_c$ we can expand N and D in powers of $\Delta^2(T)$

$$N=C(\alpha+\beta\Delta^2+\cdots) \quad \text{(C8)}$$

$$D=C_2(1+\delta\Delta^2+\cdots). \quad \text{(C9)}$$

We get

$$\alpha=\int_0^\infty\int d\epsilon d\epsilon'\,F(\epsilon,\epsilon',0), \quad \text{(C10)}$$

$$\beta=\int_0^\infty\frac{d\epsilon}{\epsilon^2}\int_0^\infty d\epsilon'\,[F(\epsilon,\epsilon',0)-F(0,\epsilon',0)]+2(g-\tfrac{1}{2})$$
$$\times P\int_{-\infty}^\infty\frac{d\epsilon}{\epsilon}\int_0^\infty d\epsilon'\,\frac{(\epsilon+\epsilon')^3}{|e^{\epsilon+\epsilon'}-1|}(1-f)(1-f'), \quad \text{(C11)}$$

$$\delta=0.606(g-\tfrac{1}{2}). \quad \text{(C12)}$$

These integrals are evaluated numerically, and we obtain

$$N=C_1[1+\Delta^2(-0.056+0.26(g-\tfrac{1}{2})], \quad \text{(C13)}$$

$$D=C_2[1+\Delta^2(0.606(g-\tfrac{1}{2}))], \quad \text{(C14)}$$

$$1/K_{\text{es}}=(C_1/C_2)[1+\Delta^2(0.124-0.35g)]. \quad \text{(C15)}$$

Assuming that C_1/C_2 gives the correct value for the resistivity in the normal state we get

$$K_{\text{es}}/K_{\text{en}}=1+\Delta^2(0.35g-0.124). \quad \text{(C16)}$$

In (C15) g can be made as large as possible, with the restriction that $g\Delta^2$ should remain small enough.

The experimental value of $K_{\text{es}}/K_{\text{en}}$ is less than unity. Our result (C16) is therefore in disagreement with the experimental data even more so than BRT's "best" result ($g=\frac{1}{2}$ corresponding to $n=1$). This reinforces their conclusion that there must be an "extra mechanism scattering," which adds to the dissipation. As Wentzel[4] has remarked such a mechanism might well be provided by the residual interactions between quasi-particles (see Sec. 1) which were here neglected, or by the many other interaction terms in the complete Hamiltonian which one neglects already when writing down the BCS Hamiltonian. However, the latter interactions would presumably also accelerate the decay of an electric current, and hence make the persistent currents even more difficult to understand.

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Superconductivity in the Case of Overlapping Bands

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The equations for the superconductivity energy gap and transition temperature in the case of overlapping bands are derived by the Nambu-Schrieffer formalism.

THE case when the Fermi surface of a superconductor goes through several overlapping bands has been considered by Suhl, Matthias, and Walker.[1] A simple derivation of their equations for the energy gap and the transition temperature can be given using the formalism of Nambu[2] and Schrieffer.[3]

We label the one electron states by the wave vector **k** lying within the first Brillouin zone and by the band

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