VI. CONCLUSIONS

We see that the theoretically predicted μ lifetime agrees with the experimental data for $m \approx M_K$ and that our numerical results are insensitive to the value chosen for the cutoff. Indeed, Eqs. (18), (19), and (22) imply that $T_\mu/T_{O^{14}}$ is cutoff independent if the IVB mass is finite. In the Fermi theory this ratio is logarithmically divergent. The above results were obtained by neglecting the momentum transfer in the diagrams involving boson self-energy parts. An examination of the boson propagators in these diagrams indicates that the leading momentum-transfer term contributing to the μ lifetime for universal coupling is $O(M_\mu M_e/m^2)$. That is, the quadratic divergence in the IVB self-energy parts will yield a significant contribution only when $(\alpha/2\pi)(\Lambda^2/m^2)(M_\mu M_e/m^2) \gtrsim 0.1\%$. For $m \approx M_K$ this implies $\Lambda \gtrsim 35M$. Therefore, the results shown in Fig. 3 are valid for a range of Λ from less than M to approximately $35M$.

Any possible significance of an IVB with a mass equal to that of a K meson must await the description of decay processes by a theory which leads to unique (and finite) results.

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Universal Neutrino Degeneracy

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Modern cosmological theories imply that the universe is filled with a shallow degenerate Fermi sea of neutrinos. In the steady state and oscillating models (and perhaps also the "big bang" theories) it can be shown rigorously that the proportion of filled neutrino levels (plus the proportion of filled antineutrino levels) is precisely one up to a finite Fermi energy E_F. The proof takes into account both absorption and the repressive effects of already filled levels on neutrino emission. Experiment shows that $E_F \lesssim 200$ eV for antineutrinos and $E_F \lesssim 1000$ eV for neutrinos. The degenerate neutrinos could be observed (if $E_F > 10$ eV) by looking for apparent violations of energy conservation in β^- decay. In the steady state and evolutionary cosmologies E_F is much too low to ever be observed, but in the oscillating cosmologies $E_F \simeq 5R_c$ MeV, where R_c is the minimum radius of the universe in units of its present radius; thus experiment already shows that the universe will contract by a factor over 10^3, if at all. Astronomical evidence plus Einstein's field equation (without cosmological constant) require in an oscillating cosmology that $E_F < 2 \times 10^{-3}$ eV (so $R_c < 10^{-9}$) and suggest that higher energy neutrinos may represent the bulk of the energy of the universe. A model universe incorporating this idea is constructed.

I. INTRODUCTION

WE have previously pointed out[1] that neutrinos may be subject to an Olbers paradox even if photons are not. (An Olbers paradox is an infinite value for some total cosmic flux.) Neutrinos carry a quantum number, so that their number density must stay finite, however red shifted they may become. It was shown that the popular modern cosmologies do not lead to a neutrino Olbers paradox, but for very different reasons: In the steady-state cosmology the speed of neutrinos vanishes beyond a certain distance (if we use a time-independent metric). In the evolutionary cosmologies neutrino emission has only been going on for a finite time. In the oscillating cosmologies absorption of neutrinos must become important during the contracted phase.

But neutrinos differ also from photons in that they obey Fermi statistics. The question arises whether any cosmological theories give rise to a degenerate neutrino population.[2] The answer is definitely yes, but again characteristic differences among these theories appear. In any cosmology (such as the steady state or oscillating theories) in which neutrino emission has been going on for an infinite time, it will be shown rigorously that precisely one-half of all neutrinos and antineutrino energy levels are full at very low energy. (The neutrino levels may be full and the antineutrino levels empty, for example.) The same is likely to be true in a "big-bang" evolutionary theory. The calculation takes into account both absorption and the repressive effect of already filled levels.

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[1] S. Weinberg, Nuovo cimento **25**, 15 (1962). The preprint of this work contained some mistaken remarks about degeneracy which should be ignored.

[2] This speculation was first raised by K. M. Watson (private communication).

For the oscillating cosmology the Fermi energy is roughly R_c times 5 MeV, where R_c is the minimum radius of the universe divided by its present radius. For the steady state and evolutionary theories E_F is vastly less, reflecting the fact that absorption and degeneracy are needed to save us from Olbers paradox in an oscillating cosmology, but not in the others.

It seems very worthwhile to look for certain effects of neutrino degeneracy in β^- decay. If they are not found we can set an upper limit on R_c (assuming the universe oscillates), while if they are found it would prove that the universe does oscillate, and would allow us to measure the parameters of the oscillation.

If we are willing to assume that the universe oscillates according to Einstein's dynamical equations, then astronomical evidence allows us to set an upper limit on E_F of 0.02 eV, and hence $R_c \lesssim 2\times 10^{-9}$. If we are also willing to accept some simple guesses about what causes the "bounce," then it turns out that nondegenerate neutrinos are much more important to the dynamics than the degenerate ones. In this case, $R_c \simeq 6\times 10^{-11}$, giving $E_F \simeq 3\times 10^{-4}$ eV. But in any event it seems likely that it is the neutrino energy density that governs the dynamics of an oscillating universe during its contracted phase, and perhaps also at present.

Experiments to search for neutrino degeneracy at very low energy have been begun at the University of Glasgow.

II. THE DEGENERACY FORMULA

In this section we shall derive a general formula for the proportion $X(E,t_0)$ of neutrino energy levels at energy E which are occupied at time t_0:

$$X(E,t_0)=\int_{-\infty}^{t_0}\Omega\left(E\frac{R(t_0)}{R(t)},t\right)\exp\left\{-\int_t^{t_0}\left[\Lambda\left(E\frac{R(t_0)}{R(t')},t'\right)+\Omega\left(E\frac{R(t_0)}{R(t')},t'\right)\right]dt'\right\}dt. \quad (1)$$

Here $\Omega(E,t)$ and $\Lambda(E,t)$ are the neutrino emission and absorption rates, and $R(t)$ is the usual red-shift factor; these quantities are defined more fully below.

We will assume that the universe is isotropic and spatially homogeneous, and hence described by the Robertson-Walker metric,[3]

$$ds^2=c^2dt^2-R^2(t)[dr^2/(1-\kappa r^2)+r^2d\theta^2+r^2\sin^2\theta d\phi^2]. \quad (2)$$

Suppose that $N(W,t)dWdt$ neutrinos are produced per unit coordinate volume between times t and $t+dt$ with energies between W and $W+dW$. Suppose also that the probability of such a neutrino being not yet absorbed at time t_0 is $P(W,t,t_0)$. Then at time t_0 their energy will be red shifted to

$$E=WR(t)/R(t_0)$$

and (as shown in our previous work[1]) their density will be

$$P(W,t,t_0)R^{-3}(t_0)N(W,t)dWdt.$$

Hence at t_0 the density of neutrinos with energies between E and $E+dE$ is $\mathfrak{N}(E,t_0)dE$, where

$$\mathfrak{N}(E,t_0)=\int_{-\infty}^{t_0}dt\int_0^{\infty}dW\,P(W,t,t_0)\times R^{-3}(t_0)N(W,t)\delta\left(E-W\frac{R(t)}{R(t_0)}\right)$$

$$=R^{-2}(t_0)\int_{-\infty}^{t_0}dt\,P\left(E\frac{R(t_0)}{R(t)},t,t_0\right)\times R^{-1}(t)N\left(E\frac{R(t_0)}{R(t)},t\right). \quad (3)$$

The absorption rate of a neutrino with energy E' at time t' is in general a function $\Lambda(E',t')$. If the neutrino was emitted at time t with energy W, then $E'=WR(t)/R(t')$, and hence the probability that it has not yet been absorbed by time t_0 is

$$P(W,t,t_0)=\exp\left[-\int_t^{t_0}\Lambda\left(W\frac{R(t)}{R(t')},t'\right)dt'\right]. \quad (4)$$

This gives

$$\mathfrak{N}(E,t_0)=R^{-2}(t_0)\int_{-\infty}^{t_0}dt\,R^{-1}(t)N\left(E\frac{R(t_0)}{R(t)},t\right)\times\exp\left[-\int_t^{t_0}\Lambda\left(E\frac{R(t_0)}{R(t')},t'\right)dt'\right]. \quad (5)$$

It will be very convenient if we work with a neutrino luminosity *rate* $\Omega(W,t)$ defined by

$$N(W,t)=[2\pi^2(\hbar c)^3]^{-1}R^3(t)[1-X(W,t)]W^2\Omega(W,t). \quad (6)$$

The reasons for this definition are

(1) The factor $R^3(t)$ is inserted to make Ω refer to neutrino luminosity per unit proper volume, as opposed to N which gives the luminosity per coordinate volume. In the steady-state cosmology $\Omega(W,t)$ is independent of t, while in other theories it varies more or less as $R^{-3}(t)$.

(2) The factor $(1-X)$ is inserted because the emission of neutrinos is impeded by this factor if a fraction X of neutrino energy levels are already full. As here defined, Ω may be calculated from ordinary astrophysical considerations without worrying about this effect of the exclusion principle.

(3) The factor W^2 is inserted because most neutrino-emitting processes, such as β decay, have spectrum shapes which vanish like W^2 as $W^2\to 0$. (Some, like pion decay, go faster to zero, but none go slower.[4]) Hence for

[3] H. P. Robertson, Astrophys. J. **82**, 284 (1935); **83**, 187, 257 (1936); A. G. Walker, Proc. London Math. Soc. **42**, 90 (1936).

[4] We are not really certain that none go slower. The matrix element for a decay may have a singularity at $W=0$ due to internal lines with zero mass. Such possible singularities are not well understood today.

small W, $N \sim W^2$, so $\Omega(W)$ approaches a constant limit $\Omega(0)$ as $W \to 0$. This limit plays a key role in our calculations.

(4) The factor $2\pi^2(\hbar c)^3$ is inserted to give Ω the dimensions of a rate, and for general convenience. With this factor, there is a very simple relation (proven in Appendix II) between Ω and Λ.

Using (6), the formula (3) for the neutrino density becomes

$$\mathfrak{N}(E,t_0)=\frac{E^2}{2\pi^2(\hbar c)^3}\int_{-\infty}^{t_0}\Omega\left(E\frac{R(t_0)}{R(t)},t\right)\left[1-X\left(E\frac{R(t_0)}{R(t)},t\right)\right] \times\exp\left[-\int_t^{t_0}\Lambda\left(E\frac{R(t_0)}{R(t')},t'\right)dt'\right]. \quad (7)$$

This is to be compared with the maximum density $\mathfrak{N}_D(E)$ attained at degeneracy when all energy levels are full:

$$\mathfrak{N}_D(E)=E^2/2\pi^2(\hbar c)^3. \quad (8)$$

We thus obtain an integral equation for the proportion $X=\mathfrak{N}/\mathfrak{N}_D$ of filled levels:

$$X(E,t_0)=\int_{-\infty}^{t_0}\Omega\left(E\frac{R(t_0)}{R(t)},t\right)\left[1-X\left(E\frac{R(t_0)}{R(t)},t\right)\right] \times\exp\left[-\int_t^{t_0}\Lambda\left(E\frac{R(t_0)}{R(t')},t'\right)dt'\right]. \quad (9)$$

The solution of this equation is given in Appendix I; the answer is formula (1).

Comparing Eqs. (9) and (1) we see that the effect of neutrino levels becoming partially full can be entirely taken into account by adding a term Ω to the absorption rate Λ.

The greatest possible value for X would be attained if we could neglect absorption altogether. With $\Lambda=0$, Eq. (1) gives

$$X_{\max}(E,t_0)=1-\exp\left[-\int_{-\infty}^{t_0}\Omega\left(E\frac{R(t_0)}{R(t)},t\right)dt\right]. \quad (10)$$

Clearly $X(E,t_0)<1$ for all values of E and t_0, unless the integral in Eq. (10) diverges. But it may diverge, or become very large, as we shall see.

We do not take neutrino scattering into account in this work, but it probably would have little effect on our considerations. The absorption rate is much larger than the scattering rate for very low energy neutrinos, and at least as large as the scattering rate for very high energy neutrinos. At intermediate energies scattering may tend to thermalize the cosmic neutrinos, probably enhancing degeneracy.

There is another sort of "scattering" which we do not treat adequately here. An absorbed neutrino may initiate re-emission processes; for example, $\nu_\mu+N\to\mu+N$ will usually be followed by $\mu\to\nu_\mu+e+\bar{\nu}_e$. Such processes may be very important for ν_μ's, but probably not for ν_e's.

III. DEGENERACY AT ZERO ENERGY

We have defined $\Omega(W,t)$ so that it approaches a finite limit $\Omega(0,t)$ as $W\to 0$. Also, the absorption rate $\Lambda(W,t)$ must approach a finite limit $\Lambda(0,t)$ as $W\to 0$, since for sufficiently low energies the only processes capable of absorbing a neutrino are exothermic ones like $n+\nu\to p+e^-$. If we use labels "+" and "−" to distinguish neutrinos and antineutrinos, then Eq. (1) at $W=0$ gives

$$X_\pm(0,t_0)=\int_{-\infty}^{t_0}dt\,\Omega_\pm(0,t) \times\exp\left\{-\int_t^{t_0}[\Lambda_\pm(0,t')+\Omega_\pm(0,t')]dt'\right\}. \quad (11)$$

Now, every process that can absorb zero-energy neutrinos can also emit zero-energy antineutrinos, and vice versa. It is shown in Appendix II that there is an exact relation between corresponding rates, i.e.,

$$\Omega_+(0,t)=\Lambda_-(0,t),\quad \Omega_-(0,t)=\Lambda_+(0,t). \quad (12)$$

Hence we see from (11) that

$$X_\pm(0,t_0)=\int_{-\infty}^{t_0}dt\,\Omega_\pm(0,t) \times\exp\left\{-\int_t^{t_0}[\Omega_+(0,t')+\Omega_-(0,t')]dt'\right\}, \quad (13)$$

and, therefore,

$$X_+(0,t_0)+X_-(0,t_0) =1-\exp\left\{-\int_{-\infty}^{t_0}[\Omega_+(0,t)+\Omega_-(0,t)]dt\right\}. \quad (14)$$

But in the steady-state cosmology Ω_+ and Ω_- are time independent, so the integral in (14) diverges. In the oscillating cosmological theories Ω_+ and Ω_- are not constant, but they are roughly periodic and, of course, positive, so the integral again diverges. Hence in either sort of cosmological theory,

$$X_+(0,t_0)+X_-(0,t_0)=1. \quad (15)$$

At very low energies exactly half of all neutrino and antineutrino energy levels are full, the entire universe being filled with a dilute but half-degenerate Fermi gas. This remarkable conclusion can only be avoided if we believe that emission of neutrinos and antineutrinos began a finite time ago, or more precisely, if $\Omega_++\Omega_-$ vanishes in the mean faster than $|t|^{-1}$ for times t sufficiently in the past.

TABLE I. Some parameters of interest for the chief neutrino and antineutrino producing processes. The coefficient $g(0)$ is the weighting factor that describes the efficiency of the process for very low energy emission and absorption; it is evidently a very sensitive function of ξ_0, the maximum neutrino energy in units of m_ec^2.

	ξ_0	f	$g(0)$
$n \to p+e^-+\bar{\nu}$	2.535	1.8	65
$p+p \to d+e^++\nu$	1.88	0.16	198
$N^{13} \to C^{13}+e^++\nu$	3.39	7.8	23.7
$O^{15} \to N^{15}+e^++\nu$	4.35	33.8	8.9

We have not said whether the Fermi sea is full for neutrinos and empty for antineutrinos, or vice versa, or somewhere between. More important, we have not said how high in energy Eq. (15) remains valid. These questions can be answered only with respect to specific cosmological theories, and are treated in Secs. V through VIII.

IV. NEUTRINO EMISSION AND ABSORPTION

We will pause here to assemble some formulas which will be useful in calculating the rates $\Omega_\pm(W,t)$ and $\Lambda_\pm(W,t)$, and we will estimate these rates at the present state of the universe. Suppose we know that several processes j produce neutrinos or antineutrinos at rates $\omega_j(t)$ per nucleon. (Usually the ω_j are relatively easy to estimate.) Then the total $\Omega(W,t)$ will be given by

$$\Omega(W,t)=n(t)(\hbar/m_ec)^3\sum_j \omega_j(t)g_j(W/m_ec^2), \quad (16)$$

where $n(t)$ is the average number of nucleons per unit proper volume; the $g_j(W/m_ec^2)$ are a set of dimensionless coefficients calculable from the spectrum shape $S_j(W)$ of process j:

$$g_j(W/m_ec^2)=2\pi^2(m_ec^2)^3S_j(W)\bigg/W^2\int S_j(W')dW'. \quad (17)$$

If the process j is an allowed β decay, or if it is a low energy collision process like $p+p\to d+e^++\nu$, then $S_j(W)$ is just the familiar Fermi shape

$$\begin{aligned}S(W)&=cW^2E_ep_eF(E_e/m_ec^2,Z)\\&=W^2(W_0-W)[(W_0-W)^2-m_e^2c^4]^{1/2}\\&\qquad\times F[(W_0-W)/m_ec^2,Z]. \quad (18)\end{aligned}$$

Here W_0 is the maximum neutrino or electron energy, equal to the nuclear mass difference, and F is the Coulomb correction factor. (We are assuming that initial and final nucleii have essentially zero kinetic energy.) This gives

$$g(\xi)=2\pi^2(\xi_0-\xi)[(\xi_0-\xi)^2-1]^{1/2}F(\xi_0-\xi,Z)/f, \quad (19)$$

where $\xi=W/m_ec^2$, $\xi_0=W_0/m_ec^2$, and f is the conventional phase space factor,

$$f=\int_0^{\xi_0-1}\xi^2(\xi_0-\xi)[(\xi_0-\xi)^2-1]^{1/2}F(\xi_0-\xi,Z)d\xi. \quad (20)$$

The functions F and f have been extensively tabulated.[5] For $Z=0$, $F=1$ and so

$$\begin{aligned}f=&(1/60)(\xi_0^2-1)^{1/2}(2\xi_0^4-9\xi_0^2-8)\\&+\tfrac{1}{4}\xi_0\ln[\xi_0+(\xi_0^2-1)^{1/2}] \quad (21)\end{aligned}$$

Table I gives ξ_0, f, and $g(0)$ for the processes most important in producing neutrinos and antineutrinos on a cosmic scale.

To estimate the present universal rate ω_+ of neutrino production per baryon, we need only note that at the present age of our galaxy, $T\sim6\times10^9$ yr, one-eighth of all nucleons are neutrons.[6] (See Table II.) If our galaxy started with pure hydrogen, the average ω_+ has been

$$\omega_+\simeq1/8T\simeq6.6\times10^{-19}\ \text{sec}^{-1}. \quad (22)$$

We can obtain another estimate of ω_+ by noting that for every helium atom formed from hydrogen there are released 2 neutrinos, plus 26.2 MeV (for the $p-p$ cycle) which eventually appears as radiation. The sun is now releasing 1.99×10^{-18} MeV per nucleon per second, and, therefore, it releases neutrinos at a rate per nucleon of

$$\omega_+(\odot)=1.52\times10^{-19}\ \text{sec}^{-1}.$$

The discrepancy with (22) is not serious; we will use the former estimate (22).

The number density of nucleons present in luminous (neutrino producing) stars is about

$$n\simeq2\times10^{-7}\ \text{cm}^{-3}=1.2\times10^{-38}(m_ec/\hbar)^3. \quad (23)$$

If half of all neutrinos are emitted in the $p-p$ cycle and half in the C$-$N cycle, we see from Table I that the mean effectiveness coefficient at zero energy is

$$\langle g(0)\rangle\simeq\tfrac{1}{2}(198)+\tfrac{1}{4}(24)+\tfrac{1}{4}(9)\simeq107. \quad (24)$$

Putting these factors together in (16) gives at $W=0$,

$$\Omega_+\simeq1.6\times10^{-37}T^{-1}\simeq8.5\times10^{-55}\ \text{sec}^{-1}. \quad (25)$$

TABLE II. Some galactic nuclear abundances[a] useful in calculating the emission rates $\omega_\pm$ of neutrinos and antineutrinos per nucleon.

	Fraction of all nucleons	Average (Z,A)	ν or $\bar{\nu}$ per nucleon
H	0.755	(1,1)	...
He	0.231	(2,4)	0.5ν
Formed by s-process[b]			
$(23\leq A\leq46)$	2.4×10^{-4}	(13,27)	$0.11\bar{\nu}$
$(63\leq A\leq75)$	1.1×10^{-6}	(31,68)	$0.03\bar{\nu}$
$(A>75)$	2.1×10^{-7}	(44,99)	$0.11\bar{\nu}$
Formed by r-process[b]			
(S^{36}, Ca^{46}, Ca^{48}, Ti ?)	1.5×10^{-6}(?)	(25,53)	$0.17\bar{\nu}$
$A>75$	2.6×10^{-7}	(40,91)	$0.15\bar{\nu}$
Rb^{87} (β^-, $t_{1/2}=4.3\times10^8$ yr)	4.4×10^{-11}	(37,87)	0.012ν

[a] See reference 6.
[b] See reference 7.

[5] For F see M. E. Rose, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 876. For f see E. Feenberg and G. Trigg, Revs. Modern Phys. **22**, 399 (1950).

[6] All abundances mentioned are from H. E. Suess and H. C. Urey, Revs. Modern Phys. **28**, 53 (1956), as quoted in reference 7.

We cannot estimate ω_- as directly as ω_+, since antineutrinos are emitted when a tiny fraction of the protons that have turned to neutrons turn back into protons. The nuclear processes[7] responsible are the β^- emissions following a neutron capture (s process) or following a rapid chain of neutron absorptions (r process). If a nucleus (Z,A) is formed from a progenitor (Z_0,A_0) by either process, the number of antineutrons emitted per baryon present in nucleii (Z,A) is $(Z-Z_0)/A$. According to Table II, the most abundant series of nucleii formed by β^- decay are those with $23\leq A\leq 46$ formed by the s process with Ne^{22} as progenitor, so that the number of antineutrinos emitted per nucleon in such a nucleus is $(13-10)/27=0.11$. Hence the contribution to ω_- from this group is

$$\omega_-=0.11\times 2.4\times 10^{-4}\times(1/T)=2.6\times 10^{-5}T^{-1}. \quad (26)$$

and so the ratio of antineutrino to neutrino emission is

$$\omega_-/\omega_+\simeq 2.1\times 10^{-4}. \quad (27)$$

The s and r processes producing the other groups of nucleii listed in Table II could raise ω_- by at most about 1%. There are some long-lived β^- emitters, of which the most abundant[6] is Rb^{87}, but these make a completely negligible contribution to ω_-.

If we assume that the antineutrino producing β^- decays have an average effectiveness coefficient $\langle g(0)\rangle\simeq 200$, then at $W=0$

$$\Omega_-/\Omega_+\simeq 4\times 10^{-4}. \quad (28)$$

These estimates need some minor correction in the steady-state cosmology. The corrections will be discussed in Sec. VII.

We have not assumed any neutrino-antineutrino pair emission in making our estimates, since the chief pair-producing process,[8]

$$e^+ + e^- \rightarrow \nu + \bar{\nu},$$

yields ν and $\bar{\nu}$ with minimum energy m_ec^2, and, therefore, makes no contribution to Ω at zero neutrino energy. (Of course, if the electrons are moving with relativistic velocity, it is possible for lower energy neutrinos to be produced. But to produce 100-eV neutrinos would require 10^4-MeV electrons.) As the neutrino energy W rises from $W=0$, the rates $\Omega_\pm$ remain very roughly constant until $W=m_ec^2$, when Ω_+ roughly doubles[9] and Ω_-/Ω_+ jumps from 4×10^{-4} to about 1/2. Above an energy E_B of order 2 MeV the rates $\Omega_\pm$ fall off very rapidly.

We do not need to make any new estimate of the absorption rates $\Lambda_\pm$ at zero neutrino energy. Any process during which a zero-energy neutrino can be absorbed must be one which could also emit an antineutrino, and vice versa, and furthermore these are the only absorption reactions. In Appendix II we show that this leads to the very simple relation for $W=0$:

$$\Lambda_\pm=\Omega_\mp. \quad (29)$$

Hence the probabilities of a given neutrino or antineutrino being absorbed during a time $T=6\times 10^9$ yr is, respectively, about 6×10^{-41} and 1.6×10^{-37}.

For neutrino energies W above zero the absorption rates remain roughly constant until a threshold for endothermic absorption is reached. For antineutrinos the absorption reaction is

$$p+\bar{\nu}\rightarrow e^+ + n$$

with threshold

$$E_A=1.80 \text{ MeV},$$

or if the density of electrons is sufficiently great,

$$p+e^- + \bar{\nu}\rightarrow n$$

with threshold

$$E_A=0.78 \text{ MeV}. \quad (30)$$

For neutrinos the typical absorption reaction is

$$\nu + Fe^{56}\rightarrow e^- + Co^{56}$$

with threshold

$$E_A=4.61 \text{ MeV}. \quad (31)$$

(There probably is insufficient time during the rapidly contracting phase to re-emit the neutrino; it takes 80 days for Co^{56}.) For W larger than the threshold E_A by several MeV, the absorption cross section σ per nucleon becomes of order 10^{-45} cm^2, so the absorption rate rises to

$$\Lambda_\pm\simeq\sigma nc\sim 10^{-41} \text{ sec}^{-1}. \quad (32)$$

This is a jump by a factor of 10^{13} for antineutrinos and 10^{18} for neutrinos.

For increasing neutrino or antineutrino energy W, the absorption rate rises smoothly, like W^2 for W between 10 MeV and several hundred MeV. About at $W=1$ BeV, the absorption cross section[10] levels off at $\sigma\simeq 10^{-38}$ cm^2 per nucleon, so

$$\Lambda_\pm\simeq 10^{-33} \text{ sec}^{-1}, \quad (33)$$

a further increase by a factor 10^8. Little is known even theoretically for energies much above 10 BeV.

These estimates are obtained for the present, when $R\equiv 1$; at other times $\Omega_\pm$ and $\Lambda_\pm$ are constant in steady-state theories, and vary roughly as R^{-3} in all other cosmologies.

V. OSCILLATORY COSMOLOGIES

If the universe goes through a periodic cycle of expansion and contraction, then during every cycle as many neutrinos must be absorbed as are emitted. Even if the oscillation is only approximately periodic, absorp-

[7] E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, Revs. Modern Phys. **29**, 547 (1957).
[8] For references see H. Y. Chiu, Ann. Phys. (New York) **16**, 321 (1961).
[9] W. A. Fowler (private communication).
[10] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960). See also N. Cabibbo (to be published).

tion must balance emission on the average, or the neutrino population would eventually reach total degeneracy and all neutrino-producing nuclear reactions would cease.

We have already shown[1] that neutrinos liberated now with energies of order 1 MeV will be absorbed in *this* cycle if the universal contraction lowers R to a value R_c for a time t_c such that

$$R_c^{-3}t_c > 10^{26} \text{ yr}.$$

(For example, if the universe contracts by a factor $R_c \sim 10^{-7}$ it must not "bounce" for a time $t_c \gtrsim 10^5$ yr.) The mechanism for absorption is the reaction

$$\nu + n \rightarrow p + e^-,$$

where the violet-shifted neutrino has a high energy (of order R_c^{-1} MeV) and hence may be absorbed by any neutron, bound or free.

However, neutrinos emitted now with low energies (of order R_c MeV) will still have such low energies during the contraction cycle that endothermic absorption is impossible. The mean free time of such neutrinos will be vastly larger than t_c, and they will not be absorbed for many cycles. Therefore, in order to balance emission and absorption, *the density of low-energy neutrinos must be enormous in an oscillating universe.* This will probably also be true for MeV neutrinos since t_c is likely to be very small, as shown in Sec. VI.

To make this quantitative, let us suppose that the properties of the universe are the same at time t_0 as at an earlier time t_1. From (1) or (A4) we see that for any t_0 and t_1:

$$X_\pm(E,t_0) = Y_\pm(E,t_0,t_1) X_\pm\left(E\frac{R(t_0)}{R(t_1)},t_1\right) + \int_{t_1}^{t_0} dt\, \Omega_\pm\left(E\frac{R(t_0)}{R(t)},t\right) Y_\pm(E,t_0,t), \quad (34)$$

where

$$Y_\pm(E,t_0,t) = \exp\left\{-\int_t^{t_0}\left[\Lambda_\pm\left(E\frac{R(t_0)}{R(t')},t'\right) + \Omega_\pm\left(E\frac{R(t_0)}{R(t')},t'\right)\right]dt'\right\}. \quad (35)$$

If $R(t_0)=R(t_1)$ and $X_\pm(E,t_0)=X_\pm(E,t_1)$, then this gives

$$X_\pm(E,t_0) = \left[\int_{t_1}^{t_0} \Omega_\pm\left(E\frac{R(t_0)}{R(t)},t\right) Y_\pm(E,t_0,t)dt\right] \Big/ [1 - Y_\pm(E,t_0,t_1)]. \quad (36)$$

If we define the dimensionless cycle integrals:

$$\bar{\Omega}_\pm(E,t_0) = \int_{t_1}^{t_0} \Omega_\pm\left(E\frac{R(t_0)}{R(t)},t\right) Y_\pm(E,t_0,t)dt, \quad (37)$$

$$\bar{\Lambda}_\pm(E,t_0) = \int_{t_1}^{t_0} \Lambda_\pm\left(E\frac{R(t_0)}{R(t)},t\right) Y_\pm(E,t_0,t)dt, \quad (38)$$

then (39) can be written in a more useful form as

$$X_\pm(E,t_0) = \frac{\bar{\Omega}_\pm(E,t_0)}{\bar{\Omega}_\pm(E,t_0) + \bar{\Lambda}_\pm(E,t_0)}. \quad (40)$$

Now, for neutrino energy W below the threshold $E_A \simeq 5$ MeV for endothermic absorption, we expect $\Lambda_+(W,t)$ to be much smaller than $\Omega_+(W,t)$. We estimated in the last section [Eqs. (28) and (29)] that, at present, Λ_+/Ω_+ is of order 4×10^{-4}, and both Λ_+ and Ω_+ scale roughly as $R^{-3}(t)$ for other times. Hence, if E is so small that even the violet-shifted energy satisfies

$$E[R(t_0)/R(t)] < E_A \simeq 5 \text{ MeV} \quad (41)$$

throughout a cycle, then the cycle integral $\bar{\Lambda}_+$ will be less than $\bar{\Omega}_+$ by a factor of roughly 2500, and so $X_+(E,t_0) \simeq 1-4\times10^{-4}$. At the present when $R\equiv1$, the neutrino levels are 99.96% full up to an energy E_F, which according to (41) has the lower bound:

$$E_F \gtrsim E_A R_c, \quad (42)$$

where R_c is the minimum value of R. *If E_F is not too small the degeneracy of low-energy neutrinos may actually be observable, and would tell us the amount by which the universe will eventually contract.*

Actually we expect that

$$E_F \simeq E_A R_c, \quad (43)$$

unless the contracted phase is so brief that the period during which (41) is satisfied contributes little to the cycle integral $\bar{\Lambda}_+$.

Our conclusion admits of one possible exception. It may be that the matter of the universe during the contracting phase is so shaken up by absorption of high-energy neutrinos or by compression that it contains a large proportion of β^- emitters (like free neutrons) which can absorb neutrinos of arbitrarily low energy, so that $\bar{\Lambda}_+ > \bar{\Omega}_+$ at all energies. We consider this extremely unlikely, since in the conditions of high density and temperature when $R \sim R_c$, any neutron-rich fragment would very rapidly be reincorporated into more stable nucleii like helium or iron. But even if it were the case these β^- emitters would produce antineutrinos which themselves would be degenerate. To see this rigorously, note that when E is so low that the red- or violet-shifted energy $ER(t_0)/R(t)$ is well below any exothermic neutrino or antineutrino absorption threshold throughout a cycle, then as shown in Appendix II,

$$\Omega_\pm\left(E\frac{R(t_0)}{R(t)},t\right) = \Lambda_\mp\left(E\frac{R(t_0)}{R(t)},t\right). \quad (44)$$

For such low E, it follows then that $Y_+ = Y_-$ and $\bar{\Omega}_\pm = \bar{\Lambda}_\mp$, so

$$X_\pm(E,t_0) = \bar{\Omega}_\pm(E,t_0)/[\bar{\Omega}_+(E,t_0) + \bar{\Omega}_-(E,t_0)], \quad (45)$$

and once again,

$$X_+(E,t_0) + X_-(E,t_0) = 1. \quad (46)$$

If absorption keeps the low-energy neutrino levels empty, the corresponding antineutrino levels will have to be full. From our previous estimate of X_+, we actually expect only one antineutrino level in 2500 to be full.

It should be mentioned that if there is a muon-type neutrino ν_μ which is different from the electron-type neutrino ν_e, then the threshold E_A for endothermic absorption of ν_μ or $\bar{\nu}_\mu$ is over 100 MeV. Since muons are unstable, ν_μ and $\bar{\nu}_\mu$ are produced in equal numbers, so half their energy levels will be full up to an energy E_F which is 20 times greater than for ν_e. Even though the emission rate of ν_μ and $\bar{\nu}_\mu$ is vastly less than that of ν_e, their absorption rate is so low that at very low energy they pile up to a number density larger by a factor 20^3. (Actually, ν_μ's and $\bar{\nu}_\mu$'s are never absorbed without a subsequent re-emission, and our calculations are really inadequate in this case.)

In Sec. IX we shall quote experimental evidence that shows that $E_F \lesssim 1$ keV if ν's are degenerate, so that the universe must contract by a factor of 5000. (For $\bar{\nu}$, experiment shows $E_F < 200$ eV while for $\bar{\nu}_\mu$, $E_F \lesssim 4$ MeV.) But even for very low E_F, the degenerate neutrinos may have an enormous density. The number density of neutrinos (and antineutrinos) below E_F is

$$\mathfrak{N}_D = (1/6\pi^2)(E_F/\hbar c)^3 = 2.19\times 10^{12}\ \text{cm}^{-3}(E_F/1\ \text{eV})^3. \quad (47)$$

Their energy density is

$$\rho_D = (1/8\pi^2)E_F(E_F/\hbar c)^3 = 1.64\times 10^6\ \text{MeV cm}^{-3}(E_F/1\ \text{eV})^4. \quad (48)$$

If E_F were 1 eV, there would be over 10^{18} neutrinos per baryon, and their energy density would be greater than that of the baryon rest masses by a factor of over 10^9. In the next section we will use this result to show that $E_F < 0.02$ eV if we believe that the oscillation of the universe is governed by Einstein's field equations, and that $E_F \simeq 3\times 10^{-4}$ eV if we use a particular model for the energy density entering in Einstein's equations. In the latter case it is neutrinos of energy $\simeq 1$ MeV that make up the bulk of the energy of the universe.

Before closing this section, it would be well to dispel one possible fallacy. Suppose that both neutrino emission and absorption of neutrinos are very slow compared with the very rapid contraction of the universe during the end of a cycle. We might expect that since the neutrino density is increasing so sharply, a state of degeneracy would eventually be reached. However, Eq. (1) shows that during any period when $\Omega = \Lambda = 0$, the proportion of filled levels is a function only of $ER(t_0)$, and hence if $X < 1$ everywhere at the beginning of contraction it will remain so. As expected for black-body radiation, the shrinking of $R(t_0)$ will only increase the scale of energies, as $R^{-1}(t_0)$.

VI. A MODEL OSCILLATING UNIVERSE

The questions left unanswered by the general considerations of the previous section were

(A) What is the actual value of the Fermi energy E_F?

(B) What does this value tell us about the oscillation? More generally, how is the proportion of filled neutrino levels as a function of energy determined by the parameters of the oscillations?

(C) What is the density of neutrinos with energy well above E_F?

In order to answer these questions we will now make a specific dynamical assumption, that the oscillation is governed by Einstein's field equations, without cosmological constant. Applied to the Robertson-Walker metric (2), these equations give[11]

$$(dR(t)/dt)^2 = -\kappa c^2 + (8\pi G/3c^2)\rho(t)R^2(t), \quad (49)$$

where $\rho(t)$ is the uniform energy density.

Now, there is a well-known difficulty in applying (49). At present,

$$\dot{R}/R = H = (13\times 10^9\ \text{yr})^{-1}, \quad (50)$$

and so from (49),

$$\rho \geq \rho_H, \quad (51)$$

$$\rho_H \equiv 3c^2H^2/8\pi G = 6.0\ \text{keV/cm}^3. \quad (52)$$

But it seems likely[12] that the contribution of nucleon rest masses to ρ is *less* than ρ_H by more than an order of magnitude.

This is not definitely known to be a real discrepancy. Even if the observations are correct, a recent summary by Wheeler[13] makes it clear that there are many ways of making up the deficit, including even gravitational radiation. But we know that since absorption of neutrinos is very slow their numbers in an oscillating universe must be very large, and so it is natural for us to guess that the missing energy is present in the form of neutrinos.[14] If it is degenerate neutrinos (and/or antineutrinos) that predominate, then (51) shows that $E_F > 0.008$ eV.

If the energy density of neutrinos makes the main contribution of ρ now, it certainly would have predominated during the contracted phase of the oscillation, since the energy density is proportional to R^{-4} for neutrinos but only to R^{-3} for nonrelativistic matter. Hence during the past portion of the present cycle,

$$\rho(t) \simeq \rho_0 R^{-4}(t). \quad (53)$$

The problem of finding $R(t)$ with $\rho \propto R^{-4}$ was solved over 30 yr ago by Tolman,[15] who, of course, was interested in

[11] See, e. g., H. Bondi, *Cosmology* (Cambridge University Press, New York, 1960), 2nd ed., Eq. (10.9). The other Einstein equations just give the pressure as a function of ρ and R.

[12] J. H. Oort, in *La Structure et L'Evolution de L'Univers* (Institut International de Physique Solvay, Bruxelles, 1958), p. 163.

[13] J. A. Wheeler, in reference 12, p. 96.

[14] This was recently suggested for very different reasons by B. Pontecorvo and J. Smorodinsky (to be published). It is also mentioned as a possibility by Wheeler, reference 13. These authors were of course considering only nondegenerate neutrinos of energy $\gtrsim 1$ MeV.

[15] R. C. Tolman, Phys. Rev. 38, 1758 (1931).

radiation rather than neutrinos. The solution during an expansion is

$$R/R_m=[1-(1-t/t_m)^2]^{1/2}, \tag{54}$$

$$t_m=H_0/\kappa c^2, \tag{55}$$

$$R_m{}^2=H_0{}^2/\kappa c^2, \tag{56}$$

$$H_0\equiv(8\pi G\rho_0/3c^2)^{1/2}. \tag{57}$$

It is more convenient to use these equations to find ρ_0, H_0, R_m, t_m, and κ in terms of observables, i.e., the present Hubble constant $\dot{R}/R=H$, and the present time T since the last contraction:

$$H_0=(1-HT)/T, \tag{58}$$

$$\rho_0=(8\pi G/3T^2)(1-HT)^2=[(1-HT)/HT]^2\rho_H, \tag{59}$$

$$R_m=(1-HT)/(1-2HT)^{1/2}, \tag{60}$$

$$t_m=[(1-HT)/(1-2HT)]T, \tag{61}$$

$$\kappa c^2=(1-2HT)T^{-2}. \tag{62}$$

It seems from observation that HT is near $\frac{1}{2}$, its theoretical maximum. If T is 6×10^9 yr and H^{-1} is 13×10^9 yr, then $HT=0.46$, and so

$$H_0=1.17H=(11\times10^9\text{ yr})^{-1}, \tag{63}$$

$$\rho_0=1.38\rho_H=8.3\text{ keV/cm}^3, \tag{64}$$

$$R_m=2.1, \tag{65}$$

$$t_m=6.8T=40\times10^9\text{ yr}, \tag{66}$$

$$c\kappa^{-1/2}=0.28T^{-1}=(21\times10^9\text{ yr})^{-1}. \tag{67}$$

If ρ_0 arose entirely from degenerate low-energy neutrinos (and/or antineutrinos), then from (64) and (48) we would have

$$E_F=8.4\times10^{-3}\text{ eV}. \tag{68}$$

If new observations were double H^{-1} we should have ρ_0 greater by a factor 45, but E_F would only be greater by a factor 2.6. We can therefore say with confidence that in a uniform oscillating universe the maximum E_F allowed by Einstein's equations is about 0.02 eV.

The trouble with Tolman's model is that there is no minimum contraction, and no necessary repetitive oscillation. (It is a matter of taste to call this a trouble.) In order to have $\dot{R}=0$, we must have

$$\rho R^2=3\kappa c^4/8\pi G. \tag{69}$$

Since ρR^2 is thus equal at the maximum $R=R_m$ and at the minimum $R=R_c$, the density ρ must be much less for small R than could be possible if ρ scaled as R^{-4}.

We shall therefore assume that there is a negative term in ρ which grows even faster than R^{-4} as $R\to0$. For definiteness, we will write

$$\rho(t)=\rho_0R^{-4}(t)-\rho_1R^{-4-a}(t), \tag{70}$$

where ρ_1 is positive and much less than ρ_0, and $a>0$. Our answers will depend very little on the value of a, and hence presumably are rather independent of the actual form of ρ. If there were an energy independent attractive two-body interaction among neutrinos or electrons or nucleons it would contribute such a negative term to ρ with $a=2$.

Now for $R\gg R_c$ the term ρ_1R^{-4-a} may be neglected in comparison with ρ_0R^{-4}, so Tolman's model can still be used to estimate ρ_0, H_0, and (less reliably) κ, t_m, and R_m.

For $R\ll R_m$ we may neglect the term κc^2 in $\dot{R}^2$, so that

$$\dot{R}^2=(8\pi G/3c^2)(\rho_0R^{-2}-\rho_1R^{-2-a}). \tag{71}$$

Clearly then the minimum value reached by R is

$$R_c=(\rho_1/\rho_0)^{1/a}, \tag{72}$$

and so

$$\dot{R}^2=H_0{}^2R^{-2}[1-(R_c/R)^a]. \tag{73}$$

[The rate H_0 is defined by (57), and estimated above as $(11\times10^9\text{ yr})^{-1}$.]

We will now use this model to calculate the cosmic density of nondegenerate as well as degenerate neutrinos as a function of energy and of the unknown minimum scale factor R_c. We shall use the calculated density to determine R_c, and to decide what sort of neutrino it is that makes up the 8.3 keV/cm^3 required by the Tolman model.

At all energies, the neutrino density is determined by the requirement that there be a perfect balance between emission and absorption per cycle. It is very convenient to divide neutrinos into three groups characterized by different energies and different absorption rates.

(1) Below the top of the Fermi sea there are the degenerate neutrinos, with energies (if measured at present) below about 5 R_c MeV. Even at the end of a cycle their energy is so low (<5 MeV) that they are absorbed extremely slowly. The balance between emission and absorption is achieved through the retardation of emission by the Pauli principle. The neutrino density stays finite, but, with nearly every level full, it is vastly larger than could arise (at these energies) for neutrinos emitted in a single cycle.

(2) "Very low energy" neutrinos are those with present energy between about 5 R_c MeV and R_c BeV. It is very difficult to treat these adequately, but we will try to show that their energy and number density is much less than that of higher energy neutrinos.

(3) "Ordinary" neutrinos are those whose energies are well above R_c BeV, if measured at present. (We shall see that R_c is so small that neutrinos with energies as low as 1 eV are still "ordinary.") At the end of a cycle such neutrinos are violet shifted to above 1 BeV, where the neutrino-absorption cross section per nucleon reaches an asymptotic value

$$\sigma(\infty)=10^{-37}\text{ cm}^2. \tag{74}$$

[Actually, it is believed that the absorption cross section at 1 BeV is about 10^{-38} cm^2, and then rises[16] with the

[16] T. D. Lee, CERN Report 61-30, 1961 (unpublished), p. 73.

square root of the neutrino center-of-mass energy, and hence with the fourth root of its "lab" energy. At some center-of-mass energy of order 30 to 300 BeV, the special features of the weak interactions are believed to disappear, and the cross section presumably levels out to a constant of order 10^{-37} cm^2. We will ignore this very mild energy dependence, and take the cross section as the constant $\sigma(\infty)$.]

For ordinary neutrinos the number emitted can be taken as fixed. The balance between absorption and emission requires then that the number of ordinary neutrinos per unit coordinate volume is

$$\mathfrak{N}=\mathfrak{N}_1/P_A, \tag{75}$$

where $\mathfrak{N}_1$ is the density to be expected for ordinary neutrinos emitted in a single cycle, and P_A is the probability that an ordinary neutrino can be absorbed in a single cycle:

$$P_A=1-\exp(-J), \tag{76}$$

$$J=\oint \Lambda_+ dt. \tag{77}$$

(The precise conditions under which these formulas hold will be examined below.)

If a neutrino is emitted with energy W at time t, its energy when observed at present will be $WR(t)$. All ordinary neutrinos have about the same chance of being absorbed, so the average present energy of an ordinary neutrino is

$$\langle E\rangle=\langle W\rangle\langle R\rangle. \tag{78}$$

(This will also be shown formally below.) The average energy $\langle W\rangle$ when emitted is the average of the mean neutrino energies for the processes $N^{13}\rightarrow C^{13}+e^+ +\nu$ (0.72 MeV), $O^{15}\rightarrow N^{15}+e^+ +\nu$ (0.98 MeV), and $p+p\rightarrow d+e^+ +\nu$ (0.26 MeV), and hence

$$\langle W\rangle\simeq\tfrac{1}{4}(0.72\text{ MeV})+\tfrac{1}{4}(0.98\text{ MeV})+\tfrac{1}{2}(0.26\text{ MeV})\simeq 0.56\text{ MeV}.$$

The average $R(t)$ can be calculated with the Tolman model, using

$$dt=H_0^{-1}(R^{-2}-R_m^{-2})^{1/2}dR. \tag{79}$$

We have then

$$\langle R\rangle=\oint R dt\Big/\oint dt=(\pi/4)R_m\simeq 1.55,$$

and so the average ordinary neutrino energy is

$$\langle E\rangle\simeq 0.87\text{ MeV}. \tag{80}$$

It is also very easy to work out the density $\mathfrak{N}_1$ of ordinary neutrinos emitted per unit coordinate volume in a single cycle. If matter starts at the beginning of a cycle as hydrogen, and turns by the end of a cycle into helium and/or heavier elements, then about $\frac{1}{2}$ ordinary neutrino is emitted per nucleon per cycle. Hence $\mathfrak{N}_1$ is half the nucleon number per coordinate volume, or

$$\mathfrak{N}_1\simeq\tfrac{1}{2}n\simeq 10^{-7}\text{ cm}^{-3}. \tag{82}$$

Now, we have already treated in an earlier work[1] the case of strong absorption, where $J\gg 1$ and $P_A\simeq 1$. We found there that R_c would have to be less than 10^{-14}. Certainly this is an open possibility. However, in this case the ordinary neutrino energy density would be

$$\langle E\rangle\mathfrak{N}_1\sim 10^{-7}\text{ MeV cm}^{-3}.$$

This is far smaller than the $\rho_0=8.3$ keV/cm^3 needed for our model. (Also, for $R_c<10^{-14}$ the energy density of the degenerate neutrinos is twenty orders of magnitude less than ρ_0. In fact, we will show below that if R_c were large enough for degenerate neutrinos to have an energy density ρ_0, the absorption probability P_A would be so low that the ordinary neutrinos would have energy density much greater than ρ_0.)

We conclude, therefore, that in our model it can only be the neutrinos of ordinary energy that serve as the chief source of the cosmic gravitational field. This requires that their energy density is ρ_0, and, hence, their number density is

$$\mathfrak{N}=\rho_0/\langle E\rangle=9.5\times 10^{-3}\text{ cm}^{-3}. \tag{83}$$

Comparing with $\mathfrak{N}_1$, we see that the probability of absorption of an ordinary neutrino in one cycle is

$$P_A\simeq J\simeq 1.05\times 10^{-5}. \tag{84}$$

We will now derive a formula for the proportion of filled levels at various energies, in order to see what this value of P_A implies for the minimum scale factor R_c, and to justify some of the statements made above.

For *all* neutrino energies the proportion of filled levels is given by

$$X_+(E,t_0)=\bar{\Omega}_+(E,t_0)/[\bar{\Omega}_+(E,t_0)+\bar{\Lambda}_+(E,t_0)]. \tag{40}$$

In order to evaluate the cycle integrals $\bar{\Omega}_+$ and $\bar{\Lambda}_+$, we shall assume that essentially all neutrino absorption occurs during a short interval from t_a to t_b, in which $R(t)$ drops steeply from R_a down to R_c and up again to R_a, where

$$R_c\ll R_a\ll 1.$$

This seems very reasonable, as it is for small R that the neutrino energy and the density of absorbers are both high. We shall also assume that during this period neutrino emission is much slower than absorption.

It follows then from (37) and (38) that the cycle integrals just before absorption starts are

$$\bar{\Omega}_+(ER_a^{-1},t_a)=\{1-\exp[-I(E)]\},$$
$$\bar{\Lambda}_+(ER_a^{-1},t_a)=\{1-\exp[-J(E)]\}\exp[-I(E)],$$

and just after absorption ends they are

$$\bar{\Omega}_+(ER_a^{-1},t_b)=\{1-\exp[-I(E)]\}\exp[-J(E)],$$
$$\bar{\Lambda}_+(ER_a^{-1},t_b)=\{1-\exp[-J(E)]\},$$

where

$$I(E)=\oint \Omega_+(ER^{-1}(t),t)dt, \tag{85}$$

$$J(E)=\oint \Lambda_+(ER^{-1}(t),t)dt. \tag{86}$$

Hence the proportion of filled levels at energy E/R_a just before the end of a cycle is

$$X_+(ER_a^{-1},t_a)=\frac{1-\exp[-I(E)]}{1-\exp[-I(E)-J(E)]},$$

and just after the beginning of a cycle it is

$$X_+(ER_a^{-1},t_b)=X_+(ER_a^{-1},t_a)\exp[-J(E)]. \tag{88}$$

These formulas take accurately into account the retardation of emission by the Pauli principle. But we expect (and will show below) that very few levels are full at any time for all neutrinos energies well above the top of the Fermi sea. The condition that $X_+\ll 1$ at t_a is equivalent to the inequalities

$$I(E)\ll 1,\quad I(E)\ll J(E),$$

and we can thus rewrite (87) as

$$X_+(ER_a^{-1},t_a)=\frac{I(E)}{1-\exp[-J(E)]}.$$

We will pause now to verify (75). The number of ordinary neutrinos emitted per coordinate volume during a whole cycle [using (6) with $X\ll 1$] is

$$\mathfrak{N}_1=\frac{1}{2\pi^2(\hbar c)^3}\int_{(WR>R_c\text{ BeV})}dW\oint dt\times W^2R^3(t)\Omega_+(W,t) \tag{89}$$

$$=\frac{1}{2\pi^2(\hbar c)^3}\int_{(E>R_c\text{ BeV})}dE\,E^2I(E), \tag{90}$$

while the number present just before absorption starts is

$$=\frac{R_a^{-3}}{2\pi^2(\hbar c)^3}\int_{(WR_a>R_c\text{ BeV})}dW\,W^2X_+(W,t_a)$$

$$=\frac{1}{2\pi^2(\hbar c)^3}\int_{(E>R_c\text{ BeV})}dE\,E^2\left[\frac{I(E)}{1-\exp[-J(E)]}\right]. \tag{91}$$

We shall see that the absorption integral $J(E)$ is independent of E for ordinary neutrinos; Eq. (75) follows those upon comparison of (90) and (91).

Since we have already decided that $J(E)$ must be quite small, the proportion of filled levels at energy $E/R(t)$ is time independent [compare, e.g., (84) and (85)] and given at all times by a simplified version of (88):

$$X_+(ER^{-1}(t),t)=X(E)=I(E)/J(E). \tag{92}$$

In order to calculate the integral (86) for $J(E)$, we will make the assumption that the absorption cross section per nucleon is constant so

$$\Lambda_+(W,t)=nc\sigma(W)R^{-3}(t), \tag{93}$$

and, hence,

$$J(E)=nc\oint \sigma(E/R(t))R^{-3}(t)dt. \tag{94}$$

This certainly is not the case for very low energy neutrinos, which at most reach energies of order 1 BeV. They can only be absorbed by neutrons in reactions like

$$\nu+n\rightarrow p+e^-.$$

But the fraction of nucleons present as bound or free neutrons is not constant, but drops steeply from about $\frac{1}{2}$ to nearly zero during the brief contracted phase.

However, *ordinary neutrons will be absorbed* (by definition) at energies well above 1 BeV. Such energies are so high that the absorption reaction will involve the production of many pions, and can take place equally well on a proton or neutron. [Of course, the net effect must be that a neutron somewhere is converted to a proton (e.g., by a π^+), or a weak decay will have to take place with re-emission of the neutrino.] The absorption rate will perhaps be a function of the proportion of neutrons, but only weakly dependent on it. [We have calculated $J(E)$ under the extreme assumption that the absorption rate is proportional to the neutron density (and independent of energy), and have found that if a small fraction ϵ of neutrons are left at the beginning of a new cycle, then

$$J\cong\frac{\mathfrak{N}_1c\sigma}{|\ln\epsilon|}\oint R^{-3}(t)dt.$$

Hence, unless ϵ is extremely small the value of J will not be seriously affected.]

The neutrino-absorption cross section $\sigma(W)$ is an increasing function of W, and hence both terms in the integrand of (94) are largest for small R. We can, therefore, use a simple formula to represent the behavior of $\sigma(W)$ at $W\sim E/R_c$:

$$\sigma(W)=AW^b. \tag{95}$$

This gives then

$$J(E)=ncAE^b\oint_{R<R_a}R(t)^{-3-b}dt.$$

The integral can be evaluated using (73):

$$J(E)=\frac{2ncAE^m}{H_0}\int_{R_c}^{R_a}R^{-2-b}[1-(R_c/R)^a]^{-1/2}dR.$$

Since the upper limit is much greater than R_c, it can be

TABLE III. Values of the absorption integral $B_{a,b}$ [see (97)] for various choices of the parameter a describing the behavior of the cosmic energy density for small R, and of the parameter b describing the behavior of the neutrino absorption cross section at energy E/R_c in Eq. (95).

a	b	B_{ab}
1	0	2
2	0	$\pi/2$
∞	0	1
2	1	1
2	2	$\pi/4$

replaced by infinity, and we obtain

$$J(E)=[2nc\sigma(E/R_c)/H_0R_c]B_{a,b}, \tag{96}$$

$$B_{a,b}=\int_1^\infty R^{-2-b}(1-R^{-a})^{-1/2}dR$$
$$=\frac{1}{a}\frac{\Gamma[(1+b)/a]\sqrt{\pi}}{\Gamma[\frac{1}{2}+(1+b)/a]}. \tag{97}$$

Some values of this integral are listed in Table III. It clearly does not matter very much which value we choose; in numerical calculations for definiteness we will take $B=\pi/2$.

For the ordinary neutrino absorption cross section per nucleon, we shall take the value (74). Then (90) gives the absorption probability per cycle of ordinary neutrinos as

$$J\simeq 6\times 10^{-16}R_c^{-1}. \tag{98}$$

Comparing with (84) we have for the minimum scale factor

$$R_c\simeq 6\times 10^{-11}. \tag{99}$$

This is such an enormous contraction that the Fermi energy of the degenerate neutrinos is about

$$E_F\simeq 3\times 10^{-4}\text{ eV}. \tag{100}$$

Comparing with (68), we see that the degenerate neutrino energy density is less than ρ_0 by a factor of 6×10^5.

We shall now calculate the proportion of filled levels at various energies. To evaluate the integral (85) for $I(E)$ we shall assume that the emission rate per nucleon is not time dependent, so that

$$\Omega_+(W,t)=\lambda W^{-2}S(W)R^{-3}(t). \tag{101}$$

Here $S(W)$ is the average spectrum shape (presumably the average of the Fermi shapes for the N^{13} and O^{15} β^+ decays and $p+p\rightarrow d+e^++\nu$), normalized so that

$$\int S(W)dW=1. \tag{102}$$

The constant λ must be chosen to give the correct $\mathfrak{N}_1$ in (87), so

$$\lambda=2\pi^2(\hbar c)^3\mathfrak{N}_1/2t_m,$$

where $t_m\simeq 40\times 10^9$ yr is half the length of a cycle.

Inserting (101) in (85) gives now

$$I(E)=[2\pi^2(\hbar c)^3\mathfrak{N}_1/2t_m]E^{-2}\oint S(E/R)R^{-1}dt,$$

so that the proportion of filled levels at energy $E/R(t)$ is

$$X(E)=\frac{\pi^2(\hbar c)^3H_0R_c}{4t_mc\sigma(E/R_c)B_{a,b}}E^{-2}\oint S(E/R)R^{-1}dt. \tag{103}$$

We have used $\mathfrak{N}_1=\frac{1}{2}n$. It is characteristic of the weak neutrino absorption case that the density of nucleons cancels out in $X(E)$. [We can now verify (78). The average neutrino energy observed at present is

$$\langle E\rangle=\left[\int E^3X(E)dE\right]\Big/\left[\int E^2X(E)dE\right].$$

For ordinary neutrinos the absorption cross section $\sigma(E/R_c)$ is constant, so (103) gives

$$\langle E\rangle=\left[\int dE\oint dtER^{-1}S(E/R)dt\right]\Big/\left[\int dE\oint dtR^{-1}S(E/R)dt\right]$$
$$=\left[\int dWWS(W)\oint Rdt\right]\Big/\left[\int dWS(W)\oint dt\right]$$
$$=\langle W\rangle\langle R\rangle.$$

Only with a constant absorption cross section do we expect this result.]

In order to get a quantitative idea of the value and variations in $X(E)$, let us assume for the spectrum shape the simplest form consistent with its known low-energy behavior:

$$S(W)=(1/3E_B^3)W^2\quad (W<E_B)$$
$$=0\quad (W>E_B). \tag{104}$$

The maximum energy E_B can be guessed to be about 2 MeV. Inserting this S into (103) gives

$$X(E)=\frac{\pi^2(\hbar c/E_B)^3H_0R_c}{12t_mc\sigma(E/R_c)B_{ab}}\int_{R>E/E_B}R^{-3}dt. \tag{105}$$

The integral can be evaluated with the Tolman model. We get

$$X(E)=\eta[\sigma(\infty)/\sigma(E/R_c)][(E_m/E)^2-1]^{1/2}, \tag{106}$$

with

$$\eta=\pi^2(\hbar c/E_B)^3R_c/6R_mct_m\sigma(\infty)B_{a,b}=10^{-36}, \tag{107}$$

$$E_m=R_mE_B\simeq 4\text{ MeV}, \tag{108}$$

$$\sigma(\infty)=10^{-37}\text{ cm}^2.$$

For $E>E_m$, $X(E)$ vanishes.

We see that even at the lowest nondegenerate energies, where

$$E \simeq 5R_c \text{ MeV} = 10^{-10} E_m,$$

$$\sigma(E/R_c) \simeq 10^{-45} \text{ cm}^2 \simeq 10^{-8} \sigma(\infty),$$

the proportion of filled levels is 10^{-19}. Thus there really is no degeneracy for E/R_c even slightly above the lowest endothermic absorption threshold. At the lowest "ordinary" energies, where

$$E \simeq R_c \text{ BeV} \simeq 10^{-8} E_m,$$

$$\sigma(E/R_c) \simeq 10^{-38} \text{ cm}^2 \simeq (1/10)\sigma(\infty),$$

the proportion of filled levels is 10^{-27}. It drops roughly as E^{-1} to about 10^{-37} at $\frac{1}{2}$ MeV, and then more steeply to zero at E_m.

Let us now use (106) to calculate the relative number of neutrinos of "ordinary" and of "very low" energy. For ordinary neutrinos we have $\sigma(E/R_c) = \sigma(\infty)$, so the number density is proportional to

$$\int_{R_c \text{ BeV}}^{E_m} E^2 X(E) dE = \eta \int_0^{E_m} E[E_m^2 - E^2]^{1/2} dE = \tfrac{1}{3} \eta E_m^3.$$

For very low energy neutrinos we have (very roughly)

$$\sigma(E/R_c) \simeq \sigma(\infty)(E/R_c \text{ BeV})^2,$$

and $E \ll E_m$, so the number density is proportional to

$$\int_{5R_c \text{ MeV}}^{R_c \text{ BeV}} E^2 X(E) dE = \int_{5R_c \text{ MeV}}^{R_c \text{ BeV}} E(R_c \text{ BeV}/E)^2 E_m dE = E_m (R_c \text{ BeV})^2 \ln 200.$$

The ratio is

$$\frac{\mathfrak{N}(\text{very low})}{\mathfrak{N}(\text{ordinary})} \simeq 3 \ln 200 \left[\frac{R_c \text{ BeV}}{E_m}\right]^2 \simeq 1.6 \times 10^{-15}. \tag{109}$$

The ratio of the energy densities is even smaller. This is probably a gross underestimate, because we have ignored the fact that the neutrons needed to absorb the very low energy neutrinos are disappearing rapidly at the end of a cycle due to absorption of the more numerous ordinary neutrinos. Also, the ordinary neutrino absorptions often result (though π^+ decay, or nuclear β^+ decay) in a neutrino re-emission. The re-emitted neutrino when observed at present would have energy from R_c MeV to $100R_c$ MeV, and hence would fall in the "very low" energy band. Both effects can increase the density of very low energy neutrinos quite considerably, but it is hard to imagine that their energy density or even their number density could be made comparable with the ordinary neutrinos.

In summary, our model has led to the conclusion that if neutrinos dominate as the source of the cosmic gravitational field, it must be ordinary neutrinos of mean energy 0.9 MeV, with number density about 10^{-2} cm^{-2}, and energy density 8 keV cm^{-3}. The corresponding fluxes are 3×10^8 neutrinos cm^{-2} sec^{-1} and 2.5×10^7 MeV cm^{-2} sec^{-1}. (This is about $\frac{1}{2}\%$ of the neutrino flux from the sun.)

Thus there are about 10^5 neutrinos per baryon, whereas both the steady state and evalutionary cosmology would lead us to expect less than one neutrino per baryon.

Finally, the minimum universal scale factor R_c in this model must be 6×10^{-11}.

VII. THE STEADY-STATE COSMOLOGY

In the cosmological theories[17] of Bondi, Gold, and Hoyle the properties of the universe do not change with time (although the coordinate system corresponding to the Robertson-Walker metric tends to obscure this). It follows that

$$\Omega(W,t) = \Omega(W), \tag{110}$$

$$\Lambda(W,t) = \Lambda(W), \tag{111}$$

$$R(t) = \exp(Ht). \tag{112}$$

An immediate consequence noted above is that the integral in (14) diverges so that the proportion of filled neutrino and antineutrino levels at zero energy is precisely $\frac{1}{2}$. We will calculate here whether it is the neutrino or antineutrino levels that are full, and up to what energy the degeneracy persists.

Using (110), (111), and (112) with Eq. (1), we have the proportion of filled ν and $\bar{\nu}$ levels $X_\pm(E)$ constant in time and given by

$$X_\pm(E) = \frac{1}{H} \int_E^\infty \Omega_\pm(W) \exp\left\{ -\frac{1}{H} \int_E^W W'^{-1} [\Lambda_\pm(W') + \Omega_\pm(W')] dW' \right\} W^{-1} dW. \tag{113}$$

Since $\Lambda_\pm(W)$ and $\Omega_\pm(W)$ approach constants $\Lambda_\pm$, $\Omega_\pm$ as $W \to 0$, and $\Lambda_\pm = \Omega_\mp$, we have for $E=0$:

$$X_\pm(0) = \Omega_\pm / (\Omega_\pm + \Lambda_\pm) = \Omega_\pm / (\Omega_+ + \Omega_-). \tag{114}$$

We see again that $X_+(0) + X_-(0) = 1$. Clearly the neutrino levels are full if $\Omega_+ \gg \Omega_-$, the antineutrino levels are full if $\Omega_- \gg \Omega_+$, and both are about half full if $\Omega_+ \simeq \Omega_-$.

To calculate the degeneracy at nonzero energy we will make the simplifying assumption that for neutrino energies less than some upper bound E_B of order 2 MeV,

$$\Omega_\pm(W) = \Omega_\pm, \quad \Lambda_\pm(W) = \Lambda_\pm = \Omega_\mp, \tag{115}$$

[17] H. Bondi and T. Gold, Monthly Notices Roy. Astron. Soc. **108**, 252 (1948); F. Hoyle, *ibid.* **108**, 372 (1948).

and that $\Omega_\pm(W)$ vanishes for $W>E_B$. Then (40) gives

$$X_\pm(E)=\frac{\Omega_\pm}{\Omega_++\Omega_-}[1-(E/E_B)^{(\Omega_++\Omega_-/H)}]. \quad (116)$$

Hence the proportion of filled levels falls to negligible values for $E\gg E_F$, where

$$E_F=E_B\exp[-H/(\Omega_++\Omega_-)]. \quad (117)$$

We must now estimate $\Omega_\pm/H$. In the steady-state cosmology protons are created at a rate (per baryon) equal to $3H$, either directly or by the decay of newly created neutrons. These protons then get cooked by stellar nuclear reactions, and wind up eventually bound into nucleii near the top of the binding energy curve, i.e., near iron. Hence the rate of neutrino production per baryon is

$$\omega_+\simeq\tfrac{3}{2}H. \quad (118)$$

This is about six times greater than the estimate (22) made in Sec. IV (if $HT\simeq\frac{1}{2}$) so we have here

$$\Omega_+\simeq 1.9\times 10^{-36}H. \quad (119)$$

If it is hydrogen that is created continually then we can use the estimate of Ω_-/Ω_+ in Sec. IV, i.e.,

$$\Omega_-/\Omega_+\simeq 4\times 10^{-4}. \quad (120)$$

On the other hand, if matter is created as neutrons, then antineutrinos are produced by neutron decay at a rate $\omega_-=3H$ per baryon. The effectiveness coefficient of neutron decay is 65 (see Table I) as compared with an average of 107 for neutrino emission, so in this case

$$\Omega_-/\Omega_+\simeq(65/107)\times 2\simeq 1.2. \quad (121)$$

Of course, if baryons and antibaryons are continually created in equal numbers, then

$$\Omega_-/\Omega_+=1. \quad (122)$$

We see, therefore, that the proportions $X_+(0)$ and $X_-(0)$ of filled neutrino and antineutrino levels at zero energy are 0.45 and 0.55 if neutrons are continually created; 1 and 4×10^{-4} if hydrogen is continuously created; and 0.5 and 0.5 if baryons and antibaryons are continually created.

However $\Omega_+\simeq 10^{-36}H$ and Ω_- cannot be greater by more than about 10%, so the energy above which degeneracy becomes negligible has the ridiculously low value

$$E_F\simeq 1\text{ MeV}\times\exp(-10^{36}). \quad (123)$$

This is so low that the degeneracy at zero energy, while it does exist, is of only academic interest for the steady-state cosmology. Only if neutrino-antineutrino pair emission contributed a factor $\sim 10^{34}$ to $\Omega_\pm$ could we hope to get any observable effects.

The reason for the tremendous difference for the energy E_F at the top of the Fermi sea for the steady state and oscillating cosmology is essentially given in our earlier work. For the steady-state theory no Olbers paradox for neutrinos can occur, even if we neglect absorption altogether. This can be thought of as an effect of neutrinos getting lost in the expanding coordinate mesh, or better, as an effect of non-Euclidean geometry if we transform to a coordinate system in which the metric is time independent. But for the oscillating cosmologies nothing but absorption (or total degeneracy) can save us from Olber's paradox. For reasonably low-energy emission and absorption are related by crossing in such a way that we cannot escape degeneracy.

VIII. EVOLUTIONARY COSMOLOGIES

Suppose the universe started with a "big bang" at time $-T$. We will also suppose for simplicity that neutrino emission and absorption began immediately, and remained constant (at least in the early stages) except for the effect of a rapidly shrinking density, and were constant in energy below some maximum neutrino energy E_B, of order 1 MeV. Thus, for $W<E_B$

$$\Omega(W,t)=\Omega R^{-3}(t), \quad (124)$$

$$\Lambda(W,t)=\Lambda R^{-3}(t), \quad (125)$$

and $\Omega(W,t)=0$ for $W>E_B$.

Equation (1) then gives the proportion of filled levels as

$$X_\pm(E,t_0)=\int_{t_1[ER(t_0)]}^{t_0}\Omega_\pm R^{-3}(t)\times\exp\left[-\int_t^{t_0}(\Omega_\pm+\Lambda_\pm)R^{-3}(t')dt'\right]dt$$
$$=\frac{\Omega_\pm}{\Omega_\pm+\Lambda_\pm}\left\{1-\exp\left[-(\Omega_\pm+\Lambda_\pm)\int_{t_B[ER(t_0)]}^{t_0}R^{-3}(t)dt\right]\right\}, \quad (126)$$

where $t_B(W)$ is the solution of

$$W/R(t_B)=E_B.$$

To go any further, we shall have to estimate the behavior of $R(t)$ near the beginning. Einstein's field equations give[11] for the Robertson-Walker metric [Eq. (2)]:

$$8\pi G\rho=-\lambda+3[(\kappa c^2+\dot{R}^2)/c^2R^2] \quad (127)$$

where λ is the cosmological constant and ρ the energy density (per proper volume). For R near zero we may presumably neglect λ and κ, obtaining

$$\dot{R}^2=8\pi G\rho R^2/3. \quad (128)$$

We can presumably also set

$$\rho(R)=\rho_0R^{-n}, \quad (129)$$

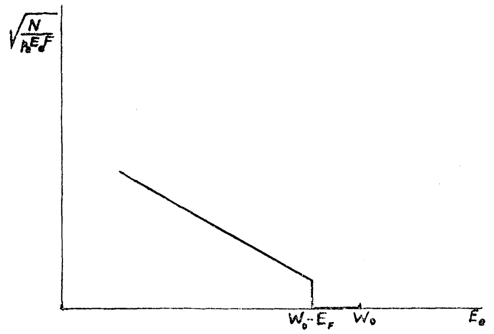

FIG. 1. Shape of the upper end of an allowed Kurie plot to be expected in a β^+ decay if neutrinos are degenerate up to energy E_F, or in a β^- decay if antineutrinos are degenerate.

where $n=3$ or $n=4$ if nonrelativistic matter or radiation (including neutrinos), respectively, make the dominant contribution to the energy density. This then allows us to write

$$X_\pm(E,t_0)=\frac{\Omega_\pm}{\Omega_\pm+\Lambda_\pm}\times\left[1-\exp\left(\frac{-(\Omega_\pm+\Lambda_\pm)[E_B/ER(t_0)]^{3-n/2}}{(3-n/2)H_0}\right)\right], \quad (130)$$

where

$$H_0=(8\pi G\rho_0/3c^2)^{1/2}. \quad (131)$$

Our conclusion then is that at present (when $R\equiv 1$) neutrinos are degenerate[18] at energies $E<E_F$, where

$$E_F=E_B[(\Omega_++\Omega_-)/H_0]^{1/(3-n/2)}. \quad (132)$$

For $E<E_F$,

$$X_\pm(E,0)=\Omega_\pm/(\Omega_++\Omega_-), \quad (133)$$

so again

$$X_+(E,0)+X_-(E,0)=1. \quad (134)$$

(We are using $\Omega_\pm=\Lambda_\mp$.)

In order to make a rough numerical estimate of E_F we can give ρ_0 its present value, so that H_0 is about the same as the present Hubble's constant, and also give $\Omega_++\Omega_-$ the present value of Ω_+. We have shown in Sec. IV that

$$\Omega_+/H\sim 10^{-36}, \quad (135)$$

and so

$$E_F\sim 10^{-36}\text{ MeV}\quad (n=4), \quad (136)$$

or

$$E_F\sim 10^{-24}\text{ MeV}\quad (n=3). \quad (137)$$

The Fermi level is much higher than in the steady-state cosmology, but still much lower than is likely in an oscillating cosmology and much too low for any hope of detection.

[18] The author's attention was called to the possibility of neutrino degeneracy in a "big bang" theory by F. Hoyle (private communication).

IX. EXPERIMENTAL TESTS

The total flux of neutrinos from the sun is equal to the solar constant divided by the radiant energy (13.1 MeV) released per neutrino:

$$\psi(\odot)=6.5\times 10^{10}\text{ cm}^{-2}\text{ sec}^{-1}. \quad (138)$$

But if neutrino levels are all full up to a Fermi energy E_F, the flux will be (according to (47)),

$$\psi_D=6\times 10^{20}(E_F/1\text{ eV})^3\text{ cm}^{-2}\text{ sec}^{-1}. \quad (139)$$

Hence, if $E_F>10^{-3}$ eV we can at least hope to detect the degenerate low-energy neutrinos without being swamped by solar background.

The experiments[19] of Cowan and Reines and Davis have shown that the energy flux of antineutrinos and neutrinos is less than about 10^{12} MeV/cm^2 sec. However their experiments could detect only particles with energies above a few MeV, and hence tell us nothing about the very low energy degenerate neutrinos and antineutrinos that concern us here. What is needed is a detection scheme with no energy threshold.

Consider a β^- emitter with an allowed spectrum shape. We would normally expect the number of events for an electron energy between E_e and E_e+dE_e to be

$$N(E_e)dE_e=aP_eE_e(W_0-E_e)^2FdE_e,\quad (E_e<W_0) \quad (140)$$

where W_0 is the maximum electron energy, F is the Coulomb factor, and a is some constant. This gives a straight line when we make the usual Kurie plot of $(N/P_eE_eF)^{1/2}$ vs E_e.

But if the fraction of occupied antineutrino levels is $X_-(W)$ at neutrino energy W, these occupied levels will impede the decay, giving

$$N(E_e)dE_e=aP_eE_e(W_0-E_e)^2F[1-X_-(W_0-E_e)]dE_e,\quad (E_e<W_0). \quad (141)$$

The Kurie plot will not continue to be a straight line very near W_0, but will dip below the line at an electron energy $E_e\simeq W_0-E_F$, provided that it is the antineutrinos that are degenerate (see Fig. 1).

If it is the neutrinos that are degenerate there will occur an even more striking effect. Instead of an antineutrino being emitted, the decay can take place with a neutrino being absorbed from the Fermi sea. The electron will then be created with an energy greater than the nuclear mass difference W_0, giving the appearance of a violation of the conservation of energy! The number of such events between E_e and E_e+dE_e is

$$N(E_e)dE_e=[(2\pi\hbar c)^3/4\pi]aP_eE_eF\mathfrak{N}_+(E_e-W_0)dE_e,$$

or

$$N(E_e)dE_e=aP_eE_e(E_e-W_0)^2FX_+(E_e-W_0)dE_e,\quad (E_e>W_0). \quad (142)$$

[19] F. Reines and C. L. Cowan, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958). R. Davis, Bull. Am. Phys. Soc. 4, 217 (1959).

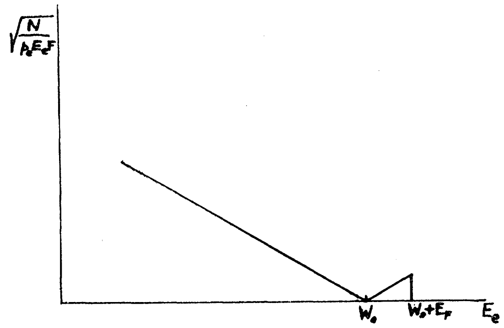

FIG. 2. Shape of the upper end of an allowed Kurie plot to be expected in a β^- decay if neutrinos are degenerate up to energy E_F, or in a β^+ decay if antineutrinos are degenerate.

The events with $E_e > W_0$ will fall on an extension of the Kurie plot that rises with precisely the same slope [if $X_+(0)=1$] with which the plot fell up to W_0. (See Fig. 2.)

If $X_+(W)=1$ for $W \leq E_F$, and $X_+(W)=0$ for $W > E_F$, then this extension of the Kurie plot will extend to $E_e = E_F + W_0$, and the function of all events with $E_e > W_0$ will be

$$N(E_e > W_0)/N = \int_{W_0}^{W_0+E_F} P_e E_e (E_e - W_0)^2 F dE_e \Big/ \int_{m_e c^2}^{W_0+E_F} P_e E_e (E_e - W_0)^2 F dE_e$$

$$\simeq [g(0)/6\pi^2](E_F/m_e c^2)^3. \quad (143)$$

Here $g(0)$ is the effectiveness coefficient defined in Sec. IV. If the decay has a Q value well below $m_e c^2$, and if we neglect Coulomb effects, then

$$g(0) = (105/8)\pi^2 (m_e c^2/Q)^3,$$

so

$$N(E_e > W_0)/N \simeq (35/16)(E_F/Q)^3. \quad (144)$$

On the other hand, if $Q \gg m_e c^2$, then

$$g(0) = 60\pi^2 (m_e c^2/Q)^3,$$

so

$$N(E_e > W_0)/N \simeq 10 (E_F/Q)^3. \quad (145)$$

If it is antineutrinos that are degenerate with $X_- = 1$ for $W < E_F$, and $X_- = 0$ for $W > E_F$, then precisely the same formula gives the number of missing events between $W_0 - E_F$ and W_0:

$$\frac{N(\text{missing})}{N} \simeq \frac{(E_F/m_e c^2)^3 g(0)}{6\pi^2}.$$

If both neutrinos and antineutrinos were partly degenerate, it would be the sum of $N(E_e > W_0)$ and N(missing) that would be correctly given by (143). (Of course the effects of ν and $\bar{\nu}$ degeneracy are interchanged for β^+ decay.)

Now, the best experiments[20] on the shape of the right end of an allowed Kurie plot were performed on the tritium β^- decay with the object of setting an upper limit on the $\bar{\nu}$ mass. A mass effect would look very much like the effect of degenerate antineutrinos, and so we can use the results of these experiments to say that if antineutrinos are degenerate the Fermi energy E_F must be below about 200 eV.

But in an oscillating or evolutionary cosmology it would be the neutrinos rather than the antineutrinos that are degenerate at low energy, and the anomaly in β^- decay would come *beyond* the end point. No apparent violations of energy conservation have ever been reported, but it is not clear whether they would have been. In reports of the more recent experiments[20] the plotted points stop about 200 eV short of the tritium end point, so (since the energy resolution was about 120 eV) it is not certain whether a rise beyond the end point would have been noticed. In an earlier experiment[21] the reported histogram extended about 1 keV beyond the end point and continued dropping (in a manner consistent with the finite energy resolution), so presumably we can conclude that if neutrinos are degenerate, $E_F < 1$ keV. (Hence if the universe oscillates, $R_c \lesssim 2\times 10^{-4}$.)

Neutrino degeneracy would give an effect appearing like a finite neutrino mass in a β^+ decay. The Kurie plot of the β^+ decay mode[22] of Cu^{64} is linear to within 60 keV of its end point, so this gives an independent upper bound on the Fermi energy, $E_F < 60$ keV.

Much less is known about the spectrum end point in muon decay. Experiments[23] on μ^+ decay indicate that the $\bar{\nu}_\mu$ mass is less than about 4 MeV, so if $\bar{\nu}_\mu$'s are degenerate the same experiments would also indicate that $E_F \lesssim 4$ MeV. We expect E_F to be about 20 times greater for $\bar{\nu}_\mu$ than for ν_e (if they are different) but even so these experiments are much less informative than those on β-decay spectrum shapes.

It would evidently be very worthwhile to do a counter experiment specifically designed to look for electrons with energies just above the end point in a β^- decay. Tritium might be preferable because of background problems and because it has an accurately known end point; a decay process with a higher Q value would give more counts above the end point, though a smaller proportion. For tritium $Q = 17.95$ keV and the half-life is 12.5 yr, so (using 144) the number of events above the end point per gram of tritium is 76/sec if $E_F = 1$ eV. It varies as E_F^3 (and for other decays roughly as Q^2). The limiting factor on such an experiment is energy resolution and our imperfect knowledge of β^- end points rather than the rarity of absorption events. Probably it

[20] L. M. Langer and R. J. D. Moffat, Phys. Rev. **88**, 689 (1952); D. R. Hamilton, W. D. Alford, and L. Gross, *ibid.* **92**, 1521 (1953).
[21] S. C. Curran, J. Angus, and A. L. Cockcroft, Phil. Mag. **40**, 53 (1949).
[22] C. S. Wu and R. D. Albert, Phys. Rev. **75**, 315, 1107 (1949).
[23] See, for example, W. F. Dudziak, R. Sagane, and J. Vedder, Phys. Rev. **114**, 336 (1959).

will not be possible[24] by studying β^- decay to detect absorption of neutrinos from the Fermi sea if $E_F < 10$ eV. However, even setting an upper limit $E_F < 100$ eV would imply that an oscillating universe must contract by a factor 5×10^4. Of course, if degenerate low energy neutrinos were discovered the problems of cosmology would be brought much closer to solution.

An experiment to search for absorption of low-energy neutrinos from the Fermi sea by tritium is now being performed by R. W. P. Drever at the University of Glasgow.

Note added in proof. A preliminary result of the Drever experiment gives $E_F < 500$ eV for neutrinos (private communication). Hence, if the universe is eternally oscillating, the oscillation must be by more than a factor of 10^4.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor K. M. Watson, who first speculated that neutrinos might be degenerate in some cosmologies. The author would also like to thank Professor William A. Fowler and Professor Fred Hoyle for their advice on calculating antineutrino emission rates, Dr. R. W. P. Drever and Dr. M. A. Grace for a discussion of possible experiments, and Professor A. Salam for his hospitality at Imperial College.

APPENDIX I

Solution of the Integral Equation

The integral equation (9) for X may be written

$$X_R(E,t_0)=\int_{-\infty}^{t_0}\Omega_R(E,t)[1-X_R(E,t)]\times\exp\left[-\int_t^{t_0}\Lambda_R(E,t')dt'\right]dt, \quad \text{(A1)}$$

where for any function $f(E,t)$

$$f_R(E,t)=f(E/R(t),t). \quad \text{(A2)}$$

Differentiating with respect to t_0, we have

$$(d/dt_0)X_R(E,t_0)=\Omega_R(E,t_0)[1-X_R(E,t_0)]-\Lambda_R(E,t_0)X_R(E,t_0). \quad \text{(A3)}$$

This is just an ordinary first-order differential equation, which can be solved to give

$$X_R(E,t_0)=\int_{t_1}^{t_0}dt\,\Omega_R(E,t_0)\times\exp\left\{-\int_t^{t_0}[\Omega_R(E,t')+\Lambda_R(E,t')]dt'\right\}+X_R(E,t_1)\times\exp\left\{-\int_{t_1}^{t_0}[\Omega_R(E,t')+\Lambda_R(E,t')]dt'\right\}. \quad \text{(A4)}$$

[24] M. A. Grace (private communication).

Now we must use the boundary condition implied by the integral equation. We have for the second term on the right side of (A4),

$$X_R(E,t_1)\exp\left\{-\int_{t_1}^{t_0}[\Omega_R(E,t')+\Lambda_R(E,t')]dt'\right\}=\exp\left[-\int_{t_1}^{t_0}\Omega_R(E,t')dt'\right]\int_{-\infty}^{t_1}\Omega_R(E,t)[1-X_R(E,t)]\times\exp\left[-\int_t^{t_0}\Lambda_R(E,t')dt'\right]dt. \quad \text{(A5)}$$

Since the integral converges this clearly vanishes as $t_1\to-\infty$. Then letting $t_1\to-\infty$, Eq. (A4) becomes

$$X_R(E,t_0)=\int_{-\infty}^{t_0}dt\,\Omega_R(E,t_0)\times\exp\left\{-\int_t^{t_0}[\Omega_R(E,t')+\Lambda_R(E,t')]dt'\right\}, \quad \text{(A6)}$$

which is just the same as Eq. (1).

It should be noted that although (A5) vanishes as $t_1\to-\infty$, it is not necessarily true that $X_R(E,-\infty)$ vanishes. In fact, it does vanish in the steady-state cosmology (except for $E=0$) and in the evolutionary cosmologies, but not in the oscillating ones.

APPENDIX II

The Crossing Formula

Suppose that there is a system A that can absorb a zero-energy neutrino, and turn into a system B. The simplest example is $n+\nu\to p+e^-$ (where n is free) but we can consider systems containing any numbers of particles. The rate of the process $A+\nu\to B$ with one zero energy neutrino in a box of proper volume V will be

$$\Lambda_+(A\to B)=(2\pi/\hbar)\rho(B)|\langle B|M|A+\nu\rangle|^2.$$

Here $\rho(B)$ is the density of states B per unit energy interval, and $\langle|M|\rangle$ is the usual matrix element; the label "+" on Λ_+ is to remind us that this is a process that absorbs neutrinos rather than antineutrinos.

If $A+\nu\to B$ is possible at zero neutrino energy, then the decay process $A\to B+\bar\nu$ must also take place, with a rate

$$\omega_-(A\to B)=(2\pi/\hbar)\rho(B+\bar\nu)|\langle B+\bar\nu|M|A\rangle|^2\times[1-X_-(0)],$$

where $1-X_-(0)$ is the proportion of unfilled antineutrino levels at zero energy. If the antineutrino is emitted with a very small energy W, then, by "crossing,"

$$\langle B+\bar\nu|M|A\rangle=\langle B|M|A+\nu\rangle,$$

and also

$$\rho(B+\bar{\nu})=\frac{4\pi W^2 V}{(2\pi\hbar c)^3}\rho(B).$$

Hence the emission rate per unit coordinate volume is

$$N_-(A\rightarrow B)=\frac{\omega_-(A\rightarrow B)}{VR^{-3}(t)}$$

$$=\frac{W^2R^3(t)}{2\pi^2(\hbar c)^3}\Lambda_+(A\rightarrow B)[1-X_-(0)].$$

Comparing with the definition (6) of Ω, we see that for zero energy

$$\Omega_-(A\rightarrow B)=\Lambda_+(A\rightarrow B).$$

The same is true if we compare absorption of a zero-energy *antineutrino* in a process like $p+p+\bar{\nu}\rightarrow d+e^+$ with emission of zero-energy *neutrinos* in the crossed reaction $p+p+X\rightarrow d+e^++\nu$. Summing over all systems A and B, we have our theorem:

For $W=0$,

$$\Omega_\mp=\Lambda_\pm.$$