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# Virtual State of the $\alpha$ Particle*

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The spectrum of neutrons produced in the forward direction from the breakup reaction T$(d,pn)$T shows a pronounced peak at its high-energy end. Using standard techniques for calculating the energy dependence of cross sections for reactions leading to three-body final states, this peak is analyzed in terms of an $\alpha$-particle resonance slightly above threshold for the unbound T-$p$ system. Reasons are given for believing the virtual state to be $0^+$ with $T=0$.

## I. INTRODUCTION

A GREAT deal of effort has been spent in performing experiments which bear on the existence of bound excited states or low-lying virtual states of the $\alpha$ particle. The four-nucleon system is still simple enough to permit meaningful calculations with specific nucleon-nucleon potentials so that a knowledge of the energy spectrum of $He^4$ can be used as a tool for further study of nuclear forces. Feenberg[1] and Bethe and Bacher[2] have made the only published attempts to derive the positions of excited states from specific nuclear forces. They predicted bound $^1P$ and $^3P$ states. However, their results are based on a sum-rule calculation whose usefulness depends on the absence of space-exchange forces. Since these types of forces are certainly present to a large degree their conclusions are very probably invalid.

The $p-He^4$ elastic and inelastic reactions[3–6] have been used extensively to look for excited states. Separate peaks due to $He^4$, $He^3$, and $d$ have been clearly identified. But no isolated inelastic proton peak indicating a bound state has been found and the inelastic proton continuum discloses no "bumps" due to virtual states. Stewart, Brolley, and Rosen[7] have looked at the proton spectra from $d-He^3$ and $d-$t reactions. If an excited state, $He^{4*}$, does exist, it should have been detected through the observation of a narrow group of protons from the reactions

$$d+He^3 \rightarrow He^{4*}+p \rightarrow He^4+\gamma+p, \qquad \text{(bound state)}$$

$$d+He^3 \rightarrow He^{4*}+p \begin{array}{l} \nearrow (t+p)+p \\ \searrow (He^3+n)+p. \end{array} \qquad \text{(virtual state)}$$

No such group was seen with deuteron energies ranging from 6 to 14 MeV.

On the other hand, there is other evidence which supports the existence of a low-lying virtual state below the threshold for $t(p,n)He^3$. Frank and Gammel[8] made a phase-shift analysis of $t-p$ elastic scattering for which data down to about 0.8 MeV laboratory energy existed. They found a solution in which a resonance occurs in the $^1S_0$ state at about 0.63 MeV in the center-of-mass system. This is 0.13 MeV below the $n-He^3$ threshold. The reduced width is large—2.7 MeV. However, they ignored the $n-He^3$ channel completely and allowed no spin orbit splitting in their final $P$-wave phase shifts. Preliminary measurements by Balashko *et al.*[9] at lower energies of 50, 120, and 175 keV are in agreement with their $S$-wave phase shifts.

The behavior of the elastic cross section at the threshold of this channel is itself consistent with a resonance occurring below the threshold. The singularity is of the $S$ type[10] rather than the cusp type. For $S$ waves the ratio of the derivative of the cross section above threshold to the derivative below is equal to the negative of the tangent of the elastic phase shift.[11] If we assume the triplet $S$-wave phase shift is small and slowly varying, we get $\delta_1 \sim 105°$ from the ratio of the slopes of the cross section curve. This is consistent with $\delta_1 = 90°$ slightly below threshold.

Bergman, Isakov, Popov, and Shapiro[12] have measured the energy dependence in the thermal region of the cross section for the capture process $He^3(n,p)$t. They showed that their curves could be fit by a broad, below-threshold resonance. They obtained the energy dependence by measuring the ratio of the desired cross section to that of the reaction $Li^6(n,t)He^4$ which they took to obey a $1/v$ law. However, Bame and Cubitt[13] have remeasured the $Li^6(n,t)He^4$ cross section and

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1 E. Feenberg, Phys. Rev. **49**, 328 (1936).
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3 J. Benveniste and B. Cork, Phys. Rev. **89**, 422 (1953).
4 R. M. Eisberg, Phys. Rev. **102**, 1104 (1956).
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claim that it falls off slower than $1/v$. This would indicate a nonresonance behavior of the $He^3(n,p)t$ reaction.

The present experiment,[14] which we think strongly indicates a virtual $S$ state at about 0.40 MeV above the $t-p$ threshold, is an observation of the energy spectrum of neutrons produced in the forward direction by the breakup reaction $t(d,np)t$. For convenience, the spectra at four different laboratory energy are reproduced in Fig. 1. At all energies two peaks are present, although the higher-energy one becomes less pronounced with increasing deuteron energy. This effect is probably due to decreasing energy resolution, although the decreasing height of the peak relative to the main one is a real effect. In the analysis which follows, the high-energy peak is ascribed to a resonance of the α particle and its position and width are determined from the data. Cranberg[15] has reported on the same experiment carried out by himself, Smith, and Levin at deuteron energies of 4.25, 5, 5.5, 6.0, and 6.5 MeV. At deuteron energies too low to produce the reaction $t(d,nn)He^3$ only a single peak occurs. With increasing deuteron energy a second low-energy peak appears in the neutron spectra which Cranberg attributes to the onset of $t(d,nn)He^3$. At the higher deuteron energies of Lefevre's experiment it is clear that this second "peak" rapidly broadens out and becomes the main body of the spectrum while the higher-energy peak retains its width.

Vlasov, Bogdanov, Kalinin, Rybakov, and Siderov[16] have also observed the neutron spectra for the same reaction at 18-MeV deuteron energy. They report two peaks, but these apparently do not correspond to Lefevre's. While they claim the high-energy peak is due to an excited state ~2.0 MeV above the $t-p$ threshold, it is possible that their first peak is a combination of the two seen by Lefevre, and the second is due either to onset of the reaction $t(d,d)nd$ or to strong $n-n$, $n-p$ interaction in the singlet state.[17,18]

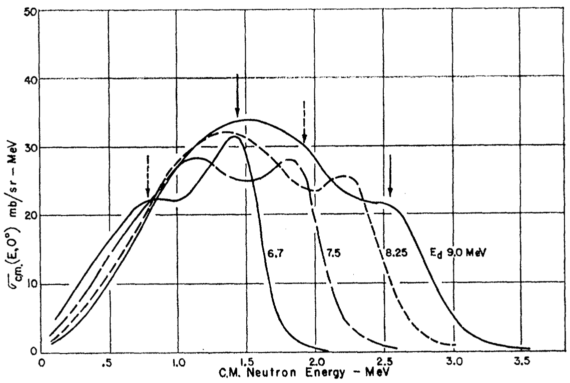


FIG. 1. Neutron spectra in the forward direction as a function of center-of-mass neutron energy. The threshold for the reaction $t(d,np)t$ is indicated by the solid arrow and the threshold for $t(d,nn)He^3$ by the dotted arrow. In order to avoid confusion only the highest and lowest deuteron energy curves are so marked.

## II. THEORY

We have attempted to fit the high-energy peak by using a stripping approximation in which no cutoff is introduced. A triton-proton scattering-state function replaces the final bound-state wave function that usually occurs in stripping theory. One of the $S$-wave phase shifts is assumed to have a resonance behavior at very low energies in the $t-p$ system. Essentially the same approximation has been used by Komarov and Popova[17] in their analysis of forward scattered neutrons from the reaction $p+d\rightarrow n+p+p$ and by Rybakov, Siderov, and Vlasov[18] in their analysis of deuteron three-body breakup reactions in which two of the fragments are produced with a relative energy near to the energy of a known resonance. The stripping-theory matrix element[19] for the breakup reaction $T(d,np)T$ in which the triton and proton are produced in either a singlet or triplet state is given by

$$\underset{(i=1,3)}{M_i}=\left\{\int d\mathbf{r}_j\int d\boldsymbol{\varrho}\,\Psi^{-*}(\mathbf{r}_j,\boldsymbol{\varrho})V_{tp}{}^i(\mathbf{r}_j,\boldsymbol{\varrho})\psi_t(\mathbf{r}_j)\right.$$
$$\left.\times\exp[i(-\mathbf{k}_d+\tfrac{3}{4}\mathbf{k}_n)\cdot\boldsymbol{\varrho}]\right\}$$
$$\times\left\{\int d\mathbf{r}\,\exp[i(\mathbf{k}_d/2-\mathbf{k}_n)\cdot\mathbf{r}]\varphi_d(\mathbf{r})\right\}. \quad (1)$$

The final integral is the Fourier transform of the deuteron internal wave function. The factor $\Psi_i^-(\mathbf{r}_j,\boldsymbol{\varrho})$ is the $t-p$ scattering wave function which asymptotically consists of a plane proton wave plus distorted incoming spherical waves and a triton. The potential energy factor $V_{tp}{}^i(\mathbf{r}_j,\boldsymbol{\varrho})$ is the sum of the two body potentials which are acting between each of the three nucleons of the triton and the proton. The vectors $\mathbf{r}_j$ are the internal position vectors of the triton and $\boldsymbol{\varrho}$ is the distance between the center of mass of the triton and the proton ($\boldsymbol{\varrho}=\mathbf{r}_p-\mathbf{r}_t$). If we ignore all but $S$ waves, $\Psi_i^-$ takes on the form (omitting spinors)

$$\Psi_i^-(\mathbf{r}_j,\boldsymbol{\varrho})=\psi_t(\mathbf{r}_j)\frac{e^{-i\delta_i}\sin\delta_i}{k_p\rho}[G_0(k_p\rho)+\mathrm{ctn}\delta_iF_0(k_p\rho)] \quad (2)$$

for $|\boldsymbol{\varrho}|$ sufficiently large that $V_{tp}{}^i(\mathbf{r}_j,\boldsymbol{\varrho})$ reduces to the

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[14] Harlan Lefevre (to be published).

[15] L. Cranberg, *Proceedings of the International Conference Held at the Physics Department, University College, London on July 8–11, 1959* (Pergamon Press, New York, London, Paris, Oxford, 1960), Vol. II.

[16] G. F. Bogdanov, N. A. Vlasov, S. P. Kalinin, B. V. Rybakov, V. A. Siderov, Zhur. Eksp. i. Teoret. Fiz. **30**, 981 (1956) [translation: Soviet Phys.—JETP **3**, 793 (1956)].

[17] V. V. Komarov and A. M. Popova, Nuclear Phys. **18**, 296 (1960).

[18] B. V. Rybakov, V. A. Siderov, and N. A. Vlasov, Nuclear Phys. **23**, 491 (1961).

[19] E. Gerduoy, Phys. Rev. **91**, 645 (1953).

repulsive Coulomb potential. In the above, $\psi_t(\mathbf{r}_j)$ is the ground state triton wave function, $G_0(k_p\rho)$ and $F_0(k_p\rho)$ are the irregular and regular Coulomb wave functions, respectively. The wave vectors which appear are defined by

$$k_d=(12mE_d/5\hbar^2)^{1/2},\quad k_n=(8mE_n/5\hbar^2)^{1/2},$$
$$k_p=(3mE_p/2\hbar^2)^{1/2}, \tag{3}$$
$$E_n+E_p=E_d-2.2\text{ MeV}.$$

The energies $E_d$, $E_n$, and $E_p$ are the center-of-mass energies in the $t-d$, $(tp)-n$, and $t-p$ systems. Still ignoring all but $S$ waves, the center-of-mass differential cross section for observing neutrons within a solid angle $d\Omega_n$ and within an energy interval $dE_n'$ is

$$\frac{d\sigma}{d\Omega_n dE_n'}=\frac{2\pi}{\hbar}\frac{6m}{5\hbar k_d}[\tfrac{1}{4}|M_1|^2+\tfrac{3}{4}|M_3|^2]\rho(E_n'),$$
$$\rho(E_n')=\frac{8\pi(\frac{3}{4}m^2)^{3/2}(5/4)^{1/2}}{(2\pi)^6\hbar^6}(E_n')^{1/2}(\tfrac{4}{5}E_{\text{tot}}-E_n')^{1/2}. \tag{4}$$
$$E_{\text{tot}}=E_d-2.2\text{ MeV},\quad E_n'=\text{Neutron c.m. energy}.$$

Our only information about the wave function $\Psi_i^-$ in the internal region is that it must be normalized to be equal to the asymptotic form [Eq. (2)] on a surface defined by $|\boldsymbol{\varrho}|=a$ where $a$ is large enough to insure that the proton is out of the range of nuclear forces. Thus,

$$\Psi_i^<(\mathbf{r}_j,\mathbf{a})=\psi_t(\mathbf{r}_j)\frac{e^{-i\delta_i}\sin\delta_i}{k_pa}[G_0(k_pa)+\cot\delta_iF_0(k_pa)]. \tag{5}$$

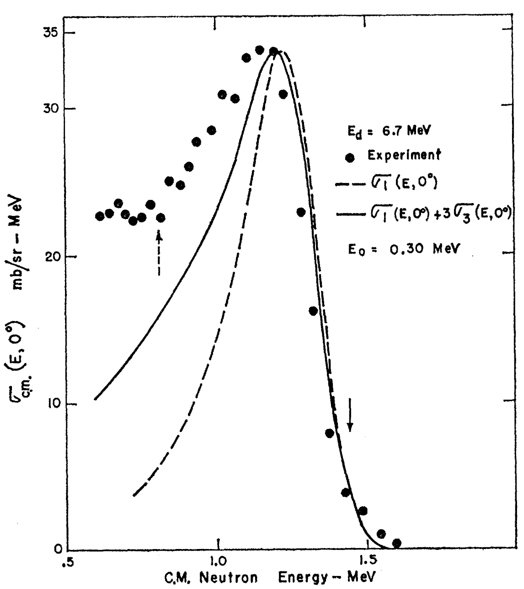


FIG. 2. Comparison of experimental and theoretical curves of neutron spectrum in forward direction for $E_d=6.7$ MeV. The resonance energy, $E_0$, has been chosen to be 0.30 MeV and the reduced widths satisfy $\gamma_p^2/a=\gamma_n^2/a=4.2$ MeV, $a=3.0$ F. The experimental curve has been shifted to the left so that the minimum between peaks occurs at the threshold for $t(d,nn)He^3$.

To lowest order in $k$ the logarithmic derivative at the nuclear surface is a constant near threshold for a charged or neutral channel. Therefore, it may be assumed that the shape of the interior wave function changes slowly, even while its magnitude is rapidly varying. Over a small energy range the principal energy dependence of the contribution to the matrix element from the interior region should be determined by the normalization conditions. Since it can be shown that the main contribution to the multiple integral in Eq. (1) comes from the region $|\boldsymbol{\varrho}|<a$, the energy dependence of the matrix element should be given by

$$M_i\propto\varphi_d(|\mathbf{k}_d/2-\tfrac{3}{4}\mathbf{k}_n|)\frac{e^{-i\delta_i}\sin\delta_i}{k_pa}\times[G_0(k_pa)+\cot\delta_iF_0(k_pa)]. \tag{6}$$

### III. RESONANCE PARAMETERS

In the neighborhood of a resonance below the $n-He^3$ channel threshold the energy dependence of the phase shift is given explicitly by

$$\delta_i(E_p)=\alpha+\xi_i,$$
$$\alpha=\tan^{-1}\left(\frac{-F_0(k_pa)}{G_0(k_pa)}\right), \tag{7}$$
$$\xi_i=\tan^{-1}\left(\frac{\frac{1}{2}\Gamma(E_p)}{E_0+\Delta(E_p)-E_p}\right).$$

Equation (6) can then be rewritten as

$$M_i\sim\varphi_d(|\mathbf{k}_d/2-\mathbf{k}_n|)[G_0^2(k_pa)+F_0^2(k_pa)]^{1/2}\frac{\sin\xi_i}{k_pa}. \tag{8}$$

Aside from the first factor, this particular form for such a matrix element was first given by Watson.[20] It is the typical energy dependence which one expects for interactions leading to a three-body final state in which a pair of charged particles have a strongly attractive force acting between them. The resonance energy, $E_0$, is defined as the energy at which $\delta=\pi/2$. As usual, $\Gamma(E_p)$ is the full width and $\Delta(E_p)$ is the level shift; both are functions of energy.

The relations in (7) have been obtained from an $R$ matrix[21] for the $t-p$ system in which only one pole is retained. Below threshold, $Et$, for the reaction $t(p,n)He^3$ we have

$$R_{pp}\cong\gamma_p^2/a/[E_0+\Delta_n(E_p)-E_p], \tag{9}$$

where $\gamma_p^2/a$ is the reduced proton width. The level shift due to the closed neutron channel is given by

$$\Delta_n(E_p)=(\gamma_n^2/a)(K_na-K_0a),$$

[20] K. M. Watson, Phys. Rev. **88**, 1163 (1952).
[21] A. M. Lane and R. G. Thomas, Revs. Modern Phys. **30**, 257 (1958); see particularly Sec. X.

where

$$K_n=[(3m(E_t-E_p))/2\hbar^2]^{1/2},$$
$$K_0=\{[3m(E_t-E_0)]/2\hbar^2\}^{1/2}. \tag{10}$$

We note that $\Delta n(E_0)=0$. The quantity $\gamma_n^2/a$ is the reduced neutron width. Above threshold the $R$-matrix elements have the form

$$R_{pp}\cong\frac{\gamma_p^2/a}{E_0'-E_p},\quad R_{pn}\cong\frac{\gamma_n\gamma_p/a}{E_0'-E_p},$$

and

$$R_{nn}\cong\frac{\gamma_n^2/a}{E_0'-E_p}, \tag{11}$$

where

$$E_0'=E_0-\gamma_n^3/a(k_pa).$$

The quantities $\Gamma(E_p)$ and $\Delta(E_p)$ which appear in Eq. (7) can now be defined in terms of the reduced widths by the relations

$$\Gamma(E_p)=2(\gamma_p^2/a)\frac{k_pa}{G_0^2(k_pa)+F_0^2(k_pa)},$$
$$\Delta(E_p)=\Delta_n(E_p)+\Delta_p(E_p),\quad E_p<E_t,$$
$$\Delta(E_p)=\Delta_n(E_t)+\Delta_p(E_p),\quad E_p\geqslant E_t, \tag{12}$$
$$\Delta_p(E_p)=\frac{\gamma_p^2}{a}\left\{\frac{dG_0(\rho)/d\rho}{G_0(\rho)}-\frac{[F_0(\rho)dF_0(\rho)/d\rho+G_0(\rho)dG_0(\rho)/d\rho]}{G_0^2(\rho)+F_0^2(\rho)}\right\}\Bigg|_{\rho=k_pa}.$$

As a start toward finding suitable resonance parameters we used the values of energy and reduced proton width which Frank and Gammel[8] found gave the best fit to the $^1S_0$ phase shift obtained in their phase shift analysis of $t-p$ scattering. For $a=3.0$ F, they chose[22] $E_0=0.63$ MeV, $\gamma_p^2/a=2.85$ MeV, $\gamma_n^2/a=0$. Upon using their parameters we obtained a peaked curve whose maximum occurred at much too low a neutron energy to be acceptable as a good fit to the high-energy peak. Therefore, we used values of $E_0$ which were considerably smaller.

From charge independence of nuclear forces one expects $\gamma_p^2/a\cong\gamma_n^2/a$. (However, $a$ must be chosen as small as possible.) Furthermore, the sum of the reduced widths should be less than the Wigner limit[23]: $\gamma_p^2/a+\gamma_n^2/a<2\hbar^2/ma^2$. It was found that the maximum widths allowable gave the best fit to the high-energy side of the experimental peak. However, the widths could be considerably reduced without producing a poor fit. The theoretical curve rises rapidly from zero at threshold to a maximum value whose position is determined by the resonant energy $E_0$. If this peak is near the threshold, upon folding in the experimental width one obtains a curve whose behavior on the high-energy side depends chiefly on $E_0$. In any case, the theoretical curve drops off faster on the low-energy side of the first peak than does the experimental curve for all choices of $\gamma_p^2/a$ and $\gamma_n^2/a$.

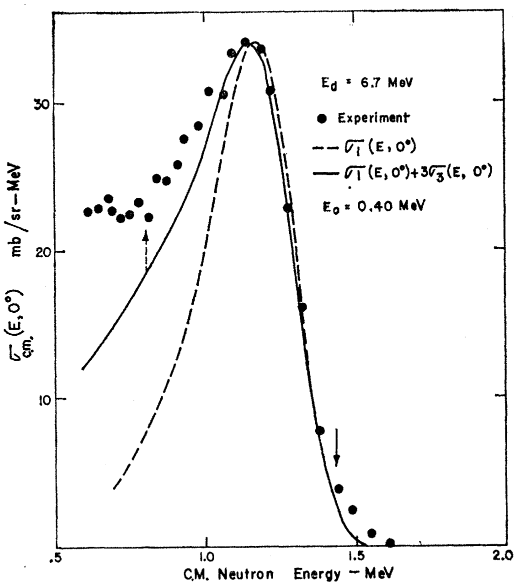


FIG. 3. Comparison of experimental and theoretical curves of neutron spectrum in forward direction for $E_d=6.7$ MeV. The resonance parameters have the values $E_0=0.40$ MeV, $\gamma_p^2/a=\gamma_n^2/a=4.2$ MeV, and $a=3.0$ F.

The contribution of the $^1S_0$ state to the cross section for $\gamma_p^2/a=\gamma_n^2/a=\frac{1}{2}(2\hbar^2/ma^2)$ $(=4.2$ MeV$)$, $E_0=0.30$, 0.40, and 0.50 MeV are shown by the dotted curves in Figs. 2, 3, and 4. The experimental width has been folded in and the curve has been normalized so as to yield the correct maximum. An $a=3.0$ F has been used in each case. This value is larger than the sum of the mean-square-charge radii of the triton and the proton.

An attempt was made to improve the fit on the low-energy side by taking into account the $^3S_1$ state. The triplet phase shift, $\delta_3$, was calculated from a scattering-length approximation with the constant[24] $f(0)$ chosen such as to yield a triplet phase shift near to the one listed by Frank and Gammel at $E_p=0.90$ MeV. In their solution $\delta_3$ starts from 0° and decreases to around −30° at the given energy. Equation (6) instead of Eq. (8) was then used to compute the energy dependence. $M_3$ was normalized relative to $M_1$ by requiring the following ratio to be satisfied:

$$\frac{3|M_3|^2}{|M_1|^2}\Bigg|_{0.40\text{ MeV}}=\frac{3|\sin(\delta_3+\eta_0)|^2}{|\sin(\delta_1+\eta_0)|^2}\Bigg|_{E_p=0.40\text{ MeV}}. \tag{13}$$

[22] These authors chose $E_0$ to be the energy at which $(dw/dr)_a=0$. Equations 7–12 are still valid if the term $G_0'/G_0$ is removed from the expression for $\Delta(E_p)$. We have not used the remainder term because it should not change the energy dependence near resonance.

[23] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953), pp. 310–311.

[24] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).

$\eta_0$ is the $S$-wave Coulomb phase shift. The right-hand side comes from noting that in Eq. (1) we essentially have, aside from the deuteron factor, an off-the-energy shell value of the $T$ matrix for $t-p$ scattering. In Eq. (13) this has been approximated by the physical matrix element. The solid curves in Figs. 2, 3, and 4 represent the sum of the contributions from the singlet and triplet states. Once again, the theoretical curves have been normalized so as to have the correct maximum.

The curve for $E_0=0.40$ MeV seemed to be the best fit to the data. Therefore, the calculations were repeated for the three other deuteron energies using this value. In our model the dependence of the cross section on $E_d$ and $E_n$ is assumed to be contained entirely in the factor,

$$(1/k_d)\,|\,\varphi(\mathbf{k}_d/2-\mathbf{k}_n)\,|^2\rho(E_n'). \tag{14}$$

The resulting curves, after folding in the experimental width, appear in Fig. 5. It should be stressed that the normalization of $|M_1|^2+3|M_3|^2$ has not been changed from what it was at $E_d=6.7$ MeV.

## IV. DISCUSSION

The calculated curves fit the data fairly well with the choice of parameters $E_0=0.40$ MeV, $a=3.0$ F, and $\gamma_p^2/a=\gamma_n^2/a=4.2$ MeV. It should be stressed that the data has been shifted toward lower energies by an amount[25] varying from 0.20 to 0.40 MeV so that the minimum between peaks falls at the $2n-He^3$ threshold. There could easily be an error of $\pm 0.10$ MeV in the "repositioning" of the data. Furthermore, the curve for $E_0=0.30$ MeV is very near to the curve for $E_0=0.40$ MeV on the high-energy side. Thus, we would say that the energy of the resonance is given by

$$E_0=0.40_{-0.20}^{+0.10}\text{ MeV}. \tag{15}$$

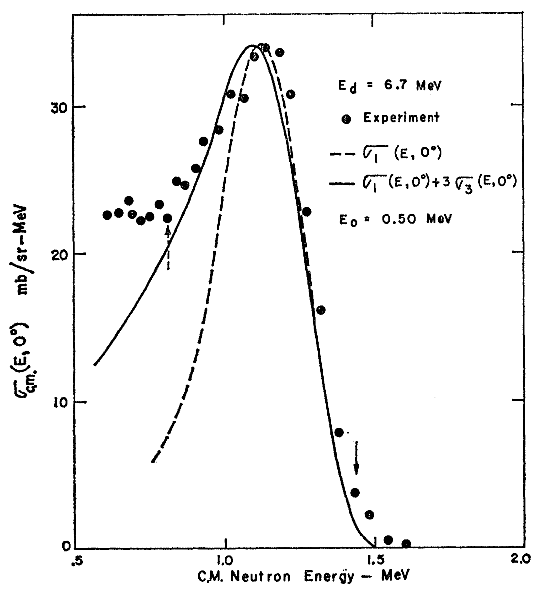


FIG. 4. Comparison of experimental and theoretical curves of neutron spectrum in forward direction for $E_d=6.7$ MeV. The resonance parameters have the values $E_0=0.50$ MeV, $\gamma_p^2/a=\gamma_n^2/a=4.2$ MeV, and $a=3.0$ F.

[25] Later measurements by Carl Poppe of the University of Wisconsin indicate that an error of this magnitude was probably made in the absolute energy calibration.

The widths given above are extremely large—the proton width is almost as large as that for potential scattering. Under such conditions it might seem questionable to talk about the existence of a virtual state. However, below threshold for $t(p,n)He^3$ one can eliminate all explicit reference to the closed channel[21] from the $R$ matrix. If one expands the $\Delta_n(E_p)$ in the denominator of Eq. (9) around $E_0$ and retains the linear term, one finds

$$\begin{aligned}&R_{pp}\cong\gamma_p'^2/a/(E_0-E_p),\\&\gamma_p'^2/a=\gamma_p^2/a/(1-d\Delta_n/dE_p)\,|_{E0}.\end{aligned} \tag{16}$$

In our case we obtain $\gamma_p'^2/a=1.4$ MeV. This means that the reduced proton width which would be measured in a $t-p$ scattering experiment below threshold is much smaller than the reduced width which appears in our calculations. The 1.4 MeV should be considered as an upper limit for the single-channel proton width.

An $^1S_0$ assignment has been given to the resonance on the basis of the phase-shift analysis of Frank and Gammel. There are two other pieces of information which support the choice of $J=0$ over $J=1$.

Bergman, Isakov, Popov, and Shapiro[12] have shown that the energy dependence of the neutron-capture cross section of $He^3$ at thermal energies can be fit by a broad $S$-wave resonance. The total capture rate at thermal energy is 5400 b. Therefore, the following inequality must be fulfilled by the resonance-capture cross section:

$$\sigma_{pn}{}^i=C_i\frac{4\pi}{k_n^2}\frac{\Gamma_p(E_p)\Gamma_n(E_p)}{[E_0'+\Delta_p(E_p)-E_p]^2+\frac{1}{4}(\Gamma_n+\Gamma_p)^2}$$

$$\leqslant 5400\text{ b};\quad C_1=\tfrac{1}{4},\quad C_3=\tfrac{3}{4},\quad \Gamma_n=2(\gamma_n^2/a)k_na. \tag{17}$$

Using $\gamma_p^2/a=\gamma_n^2/a=4.2$ MeV, $E_0'=-1.03$ MeV, $[E_0=0.40,\ \Delta_n(E_t)=-1.43\text{ MeV}]$, we get

$$\sigma_{pn}{}^2=2940\ \mathbf{f},\quad \sigma_{pn}{}^3=8820\ \mathbf{b}.$$

A much weaker support for $J=0$ comes from a calculation of the absolute magnitude of the cross section by using the physical $T$ matrix in Eq. (6) for both $M_1$ and $M_3$. For $J=0$ one finds that $|M_1|^2$ give the major contribution to the cross section; it contributes 38 mb/sr MeV. With the choice $J=1$, $3|M_3|^2$ would contribute 114 mb/sr MeV. The observed value is 34 mb/sr MeV.

Since the ground state of the triton has positive parity, the virtual state is $0^+$. An isotopic spin of 0 is indicated since $T=1$ would predict the existence of a bound $H^4$. Thus the quantum numbers are the same as those of the ground state of $He^4$. This is certainly a reflection of the strong spin and parity dependence of the nucleon-nucleon forces. The particular combination of potentials which binds the $\alpha$ particle so strongly almost brings about a second bound state in spite of the introduction of another node into the spatial part of the wave function.

The virtual state of the $\alpha$ particle under discussion falling as it does just below the threshold of a new channel, is of the type discussed by Baz.[26] He showed that if one assumes the existence of a real potential tail extending out from a nucleus and assumes a constant $R$ matrix on a surface with a much smaller radius than usual, one can show that resonances should occur frequently near the thresholds of new channels. A resonance of this type has the same quantum numbers as the associated threshold state. In the same article Baz points out that charge independence of nuclear forces does not necessarily mean that neutron and proton reduced widths should be the same for states with the same isotopic spin. Such an assumption has been made in the present paper. However, it must be pointed out that we have tried to match mainly the position of the experimental maxima with our curves. As mentioned before, this determines only the resonant energy. The shape of the theoretical curve below the neutron peak is sensitive to the widths but presumably $p$-wave effects become important in this region.

The question naturally arises of why the supposed virtual state has not been observed before in $p-\alpha$ and $d-He^3$ experiments. As mentioned before, the state should show up as a peak in the high energy end of the continuous proton spectra in each experiment. As far as the $p-\alpha$ experiments are concerned the answer is certainly that the energy resolution was not high enough. An experimental width of 0.25 MeV or less is required. In a private communication from Louis Rosen of Los Alamos the author has been informed that the $d-He^3$ breakup experiment of Stewart *et al.*[7] was not designed specifically for observing an excited state of the $\alpha$ particle. As a result the number of tracks of inelastic protons is very small and the results of the experiment are neutral with regard to the possible existence of a virtual state.

[26] A. I. Baz, in *Advances In Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1959), Vol. 8, p. 349.

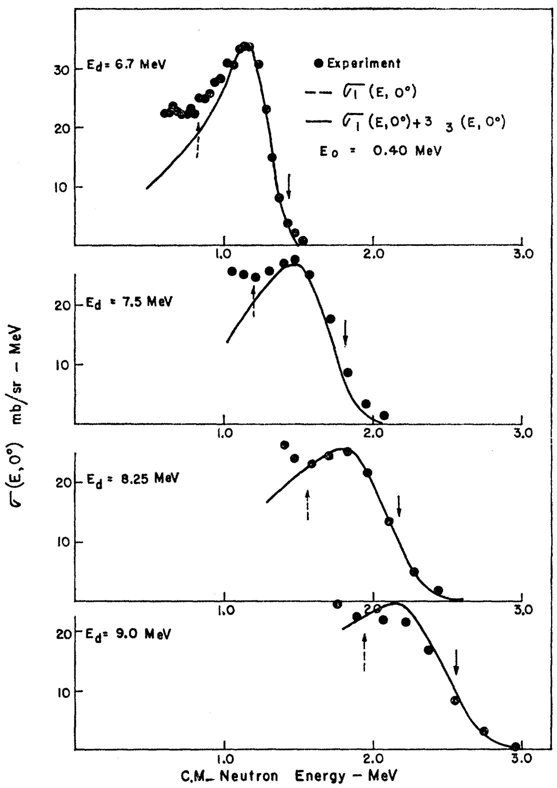


FIG. 5. Comparison of experimental and theoretical curves of neutron spectra in forward direction for indicated deuteron energies. The resonance parameters are $E_0=0.40$ MeV, $\gamma_p^2/a=\gamma_n^2/a=4.2$ MeV. Only the curve for the lowest deuteron energy has been normalized.

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