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Nuclear Structure Studies in the 82-Neutron Region with Stripping Reactions*

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The energies of the single-particle states in the $82<N\leqslant 126$ shell are determined by (d,p) reactions on the 82-neutron isotones Ce^{140} and Ba^{138}. The results are: $f_{7/2}-0$: $p_{3/2}-0.83$ MeV; $f_{5/2}-1.88$ MeV; $h_{9/2}-1.9$ MeV; $p_{1/2}-2.25$ MeV. The energies of the single-hole states in the $50<N\leqslant 82$ shell are determined by (d,t) reactions on these nuclei. The results are $d_{3/2}-0$; $s_{1/2}-0.27$ MeV; $h_{11/2}-0.71$ MeV; $g_{7/2}-1.37$ MeV; $d_{5/2}-\simeq 2.3$ MeV. There is good agreement between determinations from the two nuclei, and the slight differences are averaged in the above results. The level structure of the 81 and 83 neutron isotopes of $_{61}Pr$ are similar to those of Ba and Ce; the splitting due to different couplings with the odd proton are small, and there is little evidence for mixing among the low-lying states. New levels are located in various isotopes of Ba, Ce, Pr, and Nd. The $d_{3/2}-s_{1/2}$ splitting in 81-neutron nuclei is measured and discussed. Information is obtained on the shell-model wave functions for the ground states of the two-hole nucleus Ba^{136} and the two-particle nuclei Ce^{142} and Nd^{144}.

INTRODUCTION AND EXPERIMENTAL PROCEDURE

STRIPPING and pickup reactions on nuclei near the 82-neutron closed shell may be expected to provide useful information on the shell structure of this region. Such a stripping reaction, $_{58}Ce^{140}(d,p)Ce^{141}$, has been investigated by Holm and Martin.[1] The purpose of the present paper is to investigate further, with (d,p) and (d,t) reactions, the nuclear structure of Ce^{141} and the isotonic nuclides of Pr, Nd, and Ba, and to make preliminary investigations of other isotopes of these elements.

Self-supporting targets of Ce, Pr, and Nd, each of thickness about 2–3 mg/cm^2, were prepared from natural elements by evaporation onto molybdenum and subsequent peeling. The barium targets, about 8 mg/cm^2 thick, were prepared by rolling. The targets were bombarded with 15-MeV deuterons from the University of Pittsburgh cyclotron. The outgoing particles were magnetically analyzed and detected by photographic plates; the basic method has been described previously.[2] The resolution was about 60 keV for both the (d,p) and (d,t) experiments except those with barium, in which cases the resolution was 100–200 keV.

Errors in the measured relative cross sections are estimated to be less than 10%. In determining absolute cross sections, uncertainties in geometrical factors and target thicknesses introduce an error of about 30%.

The measurements include detailed angular distributions of the protons from $Ce^{140}(d,p)Ce^{141}$ using a natural cerium target (88% Ce^{140}) and from $Ba^{138}(d,p)Ba^{139}$ using natural barium (72% Ba^{138}), measurements of (d,p) spectra from Pr and Nd at three angles chosen to give the maximum information on l values, and measurements of (d,t) spectra from the above four targets at several angles. Typical spectra are shown in Figs. 1 and 2.

Peaks from oxygen contamination were present in all targets, but were a major difficulty only in Ba where the amount of oxygen was large and the energy resolution was poor.

RESULTS AND DISCUSSIONS: (d,p) REACTIONS

$Ce^{140}(d,p)Ce^{141}$

Natural Ce is 88% Ce^{140} and 11% Ce^{142}. This large abundance ratio and the fact that the (d,t) Q values for the two isotopes differ by about 2 MeV permit levels of Ce^{141} to be identified by both (d,p) and (d,t) reactions, at least up to an excitation energy of 2 MeV. The energy spectra for these reactions are shown in Figs. 1 and 2, and the pertinent results are listed in Table I.

Columns (1) and (3) of Table I list the energy levels found in Ce^{141} from (d,p) and (d,t) reactions, respectively; the energy values are in good agreement. Columns (2) and (4) list the cross sections for the levels excited by (d,p) and (d,t) reactions, respectively. Column (5) gives the values we have assigned from the (d,p) angular distributions (see below), and column (6), the j values suggested by shell model. Columns (7), (8), and (9) give the energies and the l and j values assigned to Ce^{141} by reference 1.

The ground and first excited states of Ce^{141} are known to be $f_{7/2}$ and $p_{3/2}$, respectively.[3] Compared with the experimental angular distributions from these levels, shown in Figs. 3 and 4, distorted-wave Born approximation (DWBA) calculations[4] were found to give ap-

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[1] G. B. Holm and H. J. Martin, Jr., Phys. Rev. **122**, 1537 (1961).

[2] B. L. Cohen, J. R. Mead, R. E. Price, K. Quisenberry, and C. Martz, Phys. Rev. **118**, 499 (1960).

[3] *Nuclear Level Schemes*, compiled by K. Way, G. Anderson, G. H. Fuller, N. B. Gove, J. R. Marion, C. S. McGinnis, M. Yamada, National Academy of Sciences (National Research Council, Washington, D. C.).

[4] G. R. Satchler, R. Bassel, R. Drisko, and E. Rost (private communications). The authors are greatly indebted to Dr. Satchler and his group for performing these DWBA calculations for the cases of interest here. They are based on the theory of W. Tobacman [Phys. Rev. **94**, 1655 (1954); **115**, 99 (1959)]. The optical potentials used are of the Saxon form; for the deuterons $V=53$ MeV, $W=10.5$ MeV, $R=1.58A^{1/3}$F, $a=0.6$F, while for the protons $V=(58-\frac{1}{2}E_p)$ MeV, $W=(4+\frac{1}{4}E_p)$ MeV, $R=1.3A^{1/3}$ F, $a=0.5$ F. The matching radius for the captured neutron wave function is $R_N=8.3$ F. See also R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report ORNL-3240 (unpublished).

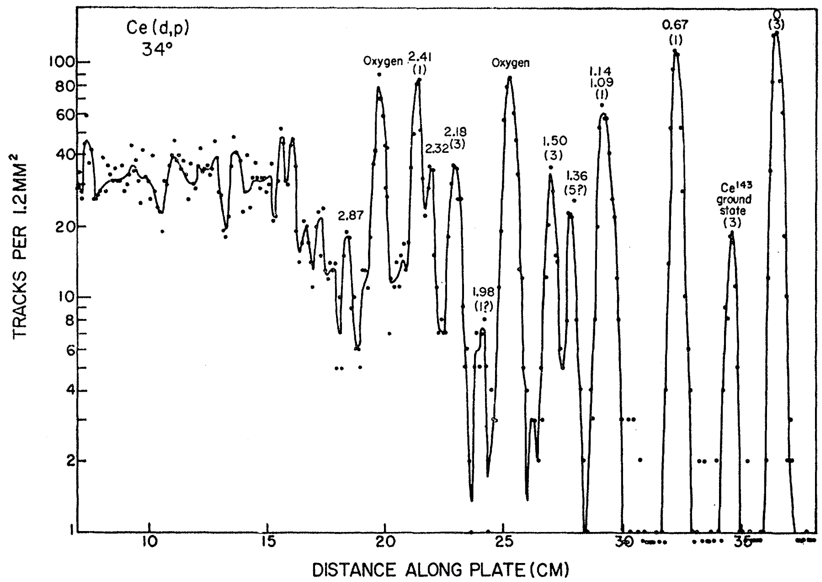

FIG. 1. Energy distribution of protons from (d,p) reactions on a natural Ce target. Numbers above peaks are excitation energies of Ce^{141} in MeV; l values assigned to these peaks are in parentheses. Angle of observation is 34°.

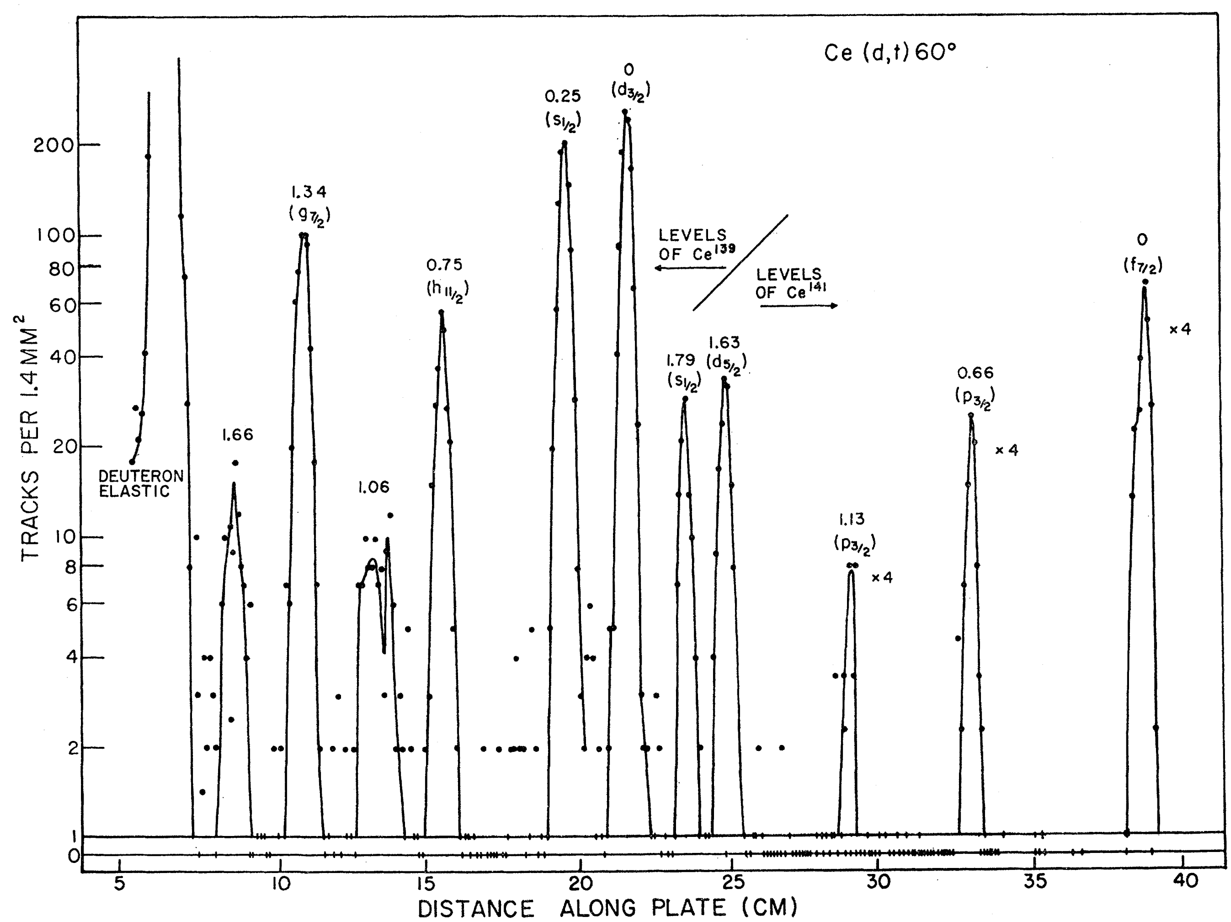

FIG. 2. Energy distribution of tritons from (d,t) reactions on a natural Ce target. Numbers above peaks are excitation energies in MeV; assignments of levels are given in parentheses. The angle of observation is 60°.

TABLE I. Levels of Ce^{141} from $Ce^{140}(d,p)$ and $Ce^{142}(d,t)$ reactions.

$Ce^{140}(d,p)Ce^{141}$ (1) Excitation energy (MeV)	$Ce^{140}(d,p)Ce^{141}$ (2) $(d\sigma/d\omega)_{max}$ (mb/sr)	$Ce^{142}(d,t)Ce^{141}$ (3) Excitation energy (MeV)	$Ce^{142}(d,t)Ce^{141}$ (4) $d\sigma/d\omega(28°)$ (mb/sr)	(5) l	(6) J	Levels of Ce^{141} from (d,p) reference 1 (7) Excitation energy (MeV)	(8) l	(9) J
0	3.83	0	1.42	3	7/2	0	3	7/2
0.67	4.41	0.66	0.89	1	3/2	0.65	1	3/2
1.09 }	3.73	1.08 }	0.19	1	3/2 }	1.12	1	3/2
1.14 }		1.13 }		1	3/2 }			
1.36	0.66	1.37	0.06	(5)	(9/2)	1.35		...
1.50	0.79	1.50	0.08	3		1.47		...
		1.62	1.76					
1.73	0.72			(1)	}	1.77	3	5/2
1.80	1.30	1.78	1.96	(3)	}			
1.98	0.37			(1)	(1/2)			
2.10	(0.5)				...			
2.18	1.39			3	(5/2)	2.15		...
2.32	0.87				...			
2.41	2.22			1	1/2	2.41	1	1/2
2.52	(0.4)			1	1/2			
2.78	0.3			$l \gtrless 3$				
2.87	0.30			$l \gtrless 3$				
2.96	0.42			$l \gtrless 3$				

proximately the correct shape but were shifted by about 6° to lower angles. To assign l values, then, we assumed all DWBA calculations of angular distributions to be shifted by 6°. In addition, DWBA curves were used to determine Q-value dependence and relative cross sections for the various single-particle states. The absolute cross sections obtained from the DWBA calculations were too small by about 40%.

Since Ce^{141} has only one neutron outside the closed shell, according to the simplest form of shell model, only one $p_{3/2}$ state should be excited in the (d,p) reaction. The fact that a second $p_{3/2}$ state is excited is expected from a mixing of the pure single-particle state with the $3/2^-$ state which is formed by a coupling of the $f_{7/2}$ neutron particle with the 2^+ first excited state of Ce^{140}. Because of this mixing, both $p_{3/2}$ states are partly single particle in nature, and both are excited in the (d,p) reaction.

The level at 1.36-MeV excitation energy is assigned as $h_{9/2}$ on the basis of an angular distribution which peaks beyond 40°. This angular distribution is shown in Fig. 5 where it is compared with the 1.50-MeV level ($l=3$), which clearly peaks at less than 35°. From the shape of the distribution, the angular momentum

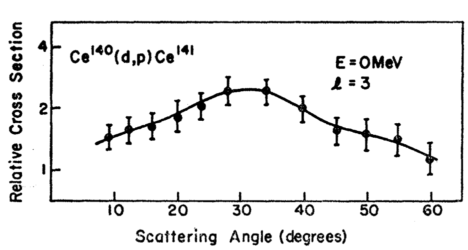

FIG. 3. Experimental angular distribution for the (d,p) reaction leading to the ground state of Ce^{141}. Error bars show statistical errors only.

transfer is more than three; shell-model considerations rule out $l=4$ and predict low-lying $h_{9/2}$ states. An estimate of the single-particle spectroscopic factor may be obtained by extrapolating the relative DWBA cross sections which are available for $l=0,1,2,3$, and 4. This extrapolation gives only $S\approx0.4$ for the 1.36-MeV level under the assumption that it is $h_{9/2}$, although no other clearly resolved level shows an angular distribution suggestive of $l>3$.

This low value of the spectroscopic factor may be explained if there are other unresolved $h_{9/2}$ levels; actually one expects this to be the case as there are other $9/2^-$ levels expected in this energy region from ($f_{7/2}$+phonon) and ($p_{3/2}$+2 phonon) configurations. The energy resolution is such that we would not observe $h_{9/2}$ levels which are closer than about 40 keV to a more intense peak, so that there is a very good chance that they would be missed if they are above 1.4 MeV (see below), but it is considerably less probable for them to be missed if they are at lower energy. We therefore tentatively assume that the 1.36-MeV level is the lowest $h_{9/2}$ state. According to giant resonance theory,[5,6] the single-particle level should be spread over somewhat more than 1 MeV, so that its center of gravity would be at about 1.9 MeV. This is, of course, a very crude estimate.

There is some possibility that the 1.36-MeV level may be $i_{13/2}$. This is considered unlikely because (1) the angular distribution is closer to that expected for $l=5$ ($h_{9/2}$) than to that expected for $l=6$ ($i_{13/2}$); (2) the $h_{9/2}$ state lies considerably lower than $i_{13/2}$ when these shells fill for protons (e.g., Bi^{209}) even though higher

[5] A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).
[6] B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. 126, 698 (1962).

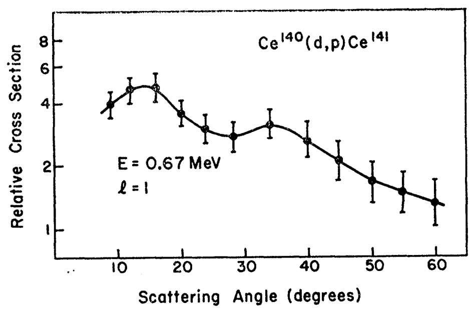

FIG. 4. Experimental angular distribution for the (d,p) reaction leading to the first excited state of Ce^{141}. Error bars are as in Fig. 3.

angular momentum states are lowered more for protons; (3) one expects the $i_{13/2}$ state to be concentrated in a single nuclear level as there are no other positive-parity states expected in this energy region, but the 1.36-MeV state gives only $S\approx 0.6$ when analyzed as $i_{13/2}$.

The three f states between 1.50- and 2.18-MeV excitation energy in Ce^{141} have a combined cross section about as large as is expected for the $f_{5/2}$ single-particle state. This fact plus the wide energy separation between this group of states and the $f_{7/2}$ ground state makes it highly probable that they comprise the $f_{5/2}$ single-particle state; their center of gravity is at about 1.88 MeV. Thus, the $f_{7/2}$-$f_{5/2}$ spin-orbit splitting is 1.88 MeV, which is surprisingly small.[7]

The four p states between 1.73 and 2.52 MeV are almost certainly components of the $p_{1/2}$ single-particle state. The sum of their cross sections is about as expected from this interpretation. The energy difference between the centers of gravity of the $p_{3/2}$ and $p_{1/2}$ states is about 1.37 MeV, which is somewhat larger than expected from the systematics of spin-orbit splitting.[7] In summary, the energies of the single-particle states are $f_{7/2}$-0, $p_{3/2}$-0.88 MeV, $f_{5/2}$-1.88 MeV; $h_{9/2}$-1.9 MeV; and $p_{1/2}$-2.25 MeV. There is no indication of the location of the $i_{13/2}$ state; however, it seems quite certain that it is not between the $f_{7/2}$ and $p_{3/2}$ states as in Pb^{207}, for it would then be the first excited state.

The portion of the energy spectrum above 2.7 MeV is not easily understood. The integrated cross section in this region is very large ($\sim$13 mb/MeV); the angular distribution of the continuum is peaked at relatively large angles characteristic of $l\approx 4$. This region probably includes the $i_{13/2}$ state and the states from the next major shell, and may also include some $h_{9/2}$ states.

$Ce^{142}(d,p)Ce^{143}$

The only $Ce^{142}(d,p)Ce^{143}$ reaction certainly identified was that proceeding to the ground state, which we find to be $l=3$; it is therefore almost certainly an $f_{7/2}$ level. Its maximum (d,p) cross section is 3.1 mb/sr, which is very close to that of the $f_{7/2}$ ground state of Ce^{141} (see Table I). The level at 1.09 MeV listed in Table I may be the $l=1$ state of Ce^{143} corresponding to the 0.67-MeV state of Ce^{141}. If this is the case, its cross section, although not precisely determinable, would have about the same ratio to the ground-state cross section as the 0.67-MeV to ground-state ratio in Ce^{141}. Its excitation energy would be 0.78 MeV. If the level is in Ce^{143}, it is probably the level found by Wall[8] at an excitation energy of 0.90±0.15 MeV.

[7] B. L. Cohen, P. Mukherjee, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. 127, 1678 (1962).

TABLE II. $Pr^{141}(d,p)Pr^{142}$.

(1) Excitation energy (MeV)	(2) $(d\sigma/d\omega)_{max}$ (mb/sr)	(3) l
0, 0.08, 0.14	3.12	3
0.66	1.35	1
0.78	1.54	1
0.91, 1.05, 1.14, 1.18	1.82	1
1.26	0.18	(1)
1.42	0.56	(3)
1.53	0.49	(3)
1.67	0.48	(3)
1.78	0.75	(3)
1.97	0.60	(3)
2.11	1.00	...
2.44	1.15	(1)

$Pr^{141}(d,p)Pr^{142}$

The levels of Pr^{142} were studied with the reaction $Pr^{141}(d,p)Pr^{142}$. The pertinent results are listed in Table II, and shown graphically in Fig. 6 where they are compared with the $Ce^{140}(d,p)Ce^{141}$ results.

Since $_{59}Pr^{142}$ differs from $_{58}Ce^{141}$ only in that it has one more proton, we expect that the levels of Pr^{142} should have approximately the same excitation energies and cross sections as the corresponding levels of Ce^{141}, but that there should be splitting and some mixing of the

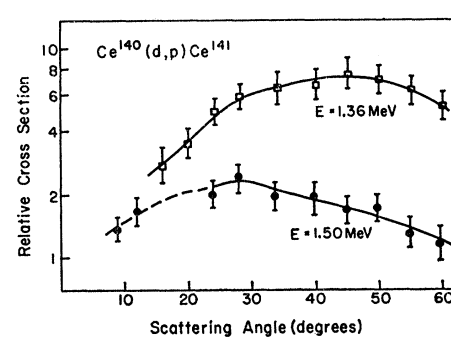

FIG. 5. Comparison of experimental angular distributions for (d,p) reactions leading to states of Ce^{141}. The 1.36-MeV level is assigned as $h_{9/2}$ (see text).

[8] N. S. Wall, Phys. Rev. 96, 664 (1954).

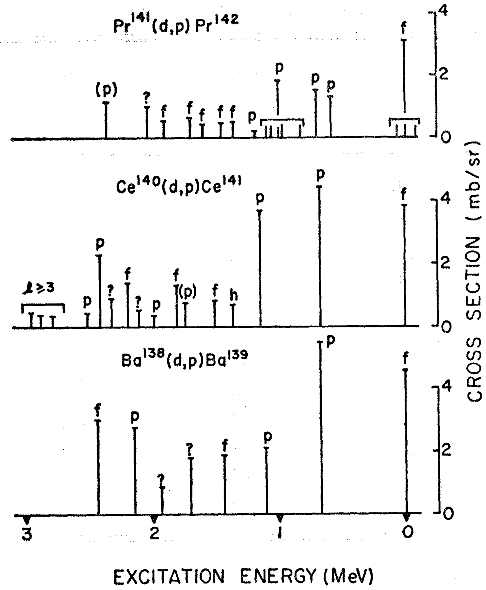

FIG. 6. Comparison of energy spectra of protons from (d,p) reactions. Cross sections for proton groups of the same l value are measured at the same angle, which is the angle corresponding to the maximum of the (d,p) angular distribution.

levels because of the coupling with the extra $(d_{5/2})$ proton.

As an example of this splitting, the $7/2^-$ ground state of Ce^{141} should be split by this coupling into six levels of Pr^{142} having spins within the range $7/2\pm5/2$, i.e., 6^-, 5^-, 4^-, 3^-, 2^-, 1^-; the sum of the cross sections for all of these should be equal to that for the $Ce(d,p)$ ground-state transition. The results indicate that most, if not all, of these states are grouped within about 180 keV. The experimental resolution was sufficient to ascertain the presence of only three distinct peaks in this region, but the summed cross section is just 20% less than that expected for all six states. These states show an $l=3$ angular distribution as expected.

Each of the two $p_{3/2}$ states should be split into four components with spins 1^-, 2^-, 3^-, and 4^-, so that a total of eight states with $l=1$ angular distributions are expected in the region near 0.9-MeV excitation energy. Apparently seven such states are seen in this region but their summed cross section amounts to only about 60% of that expected from the Ce data. Similarly, the group of $f_{5/2}$ states is observed in the expected energy region ($\sim$1.8 MeV), although far more states are expected than are resolved, and the summed cross section is about 75% of that expected from the Ce results.

In addition to the splitting due to the couplings with the odd proton, one expects mixing of states with the same spin and parity which happen to be close in energy. For example, the couplings with the odd $d_{5/2}$ proton give $p_{3/2}$ states with spins 1^-, 2^-, 3^-, and 4^-, and $f_{5/2}$ states with 0^-, 1^-, 2^-, 3^-, 4^-, and 5^-. If, for example, the 2^- states from the two groups lie close together,

TABLE III. $Ba^{138}(d,p)Ba^{139}$.

Excitation energy (MeV)	$(d\sigma/d\omega_{max}$ (mb/sr)	l	J^π
0	4.3	3	$7/2^-$
0.65	5.5	1	$3/2^-$
1.10	2.1	1	$3/2^-$
1.44	1.9	3	$5/2^-$
1.71	1.8	...	...
1.94	0.9	...	...
2.16	2.8	1	$(1/2^-)$
2.45	3.0	3	$(5/2^-)$

they will be mixed by residual interactions to form states of the form

$$a_1(d_{5/2}p_{3/2})_2+b_1(d_{5/2}f_{5/2})_2,$$
$$a_2(d_{5/2}p_{3/2})_2+b_2(d_{5/2}f_{5/2})_2.$$

In stripping reactions, both of these states can be excited by $l=1$ and by $l=3$ transitions (with strengths proportional to a_1^2,a_2^2 and b_1^2,b_2^2, respectively). Thus, one might expect rather complex situations to develop. The results indicate that the separation of the $p_{3/2}$ and $f_{5/2}$ single-particle levels is sufficiently large that there is not a great deal of mixing between these states. On the other hand, mixing would cause the higher energy part of the spectrum to be extremely complicated.

It is somewhat disturbing that in all cases the Pr cross sections are considerably smaller than the corresponding Ce cross sections, although the latter agree well with the Ba cross sections. The same effect was found in the (d,t) reactions on these elements. One is therefore tempted to conclude that there is an error in the thickness of the Pr target. Two separate targets of both Pr and Ce were used at different times and the results were internally consistent; however, no two targets were ever run within two weeks of each other. All three targets are very difficult to prepare and the percentage of successes in their preparation is not high; they last only about one day before disintegrating. Thus, further attempts at rechecking the discrepancy in cross sections were abandoned.

$Ba^{138}(d,p)Ba^{139}$

Table III presents the results obtained from (a,p) reactions leading to the states of Ba^{139}. Of the eight levels we observed, only the ground and first excited states are previously known.[3] The ground state corresponds to an $l=3$ transfer, in agreement with the previous (uncertain) 7/2 spin assignment. The spins of the other states are assigned as in Ce^{141}.

The spectra of Ba^{139} and Ce^{141} are compared in Fig. 6. Although the Ba data is somewhat less complete due to experimental difficulties, there is good agreement in both location of levels and in cross sections between these two 83 neutron isotones. The center of gravity of the $p_{3/2}$ state is 0.78 MeV for Ba^{139} as compared with

0.88 MeV for Ce^{141}; the average of these, 0.83 MeV, is listed in the abstract.

$Ba^{135}(d,p)Ba^{136}$ and $Ba^{137}(d,p)Ba^{138}$

The ground states of Ba^{136} and Ba^{138} have been identified from Ba^{135} and $Ba^{137}(d,p)$ reactions. Their cross sections at the maximum of the corresponding angular distributions are 0.34 and 0.54 mb/sr, respectively. The angular distributions for both states are characteristic of an $l=2$ angular momentum transfer. The first excited state of Ba^{136} (2^+) is observed in the (d,p) spectra about half as strongly excited as the ground state.

A proton group was also observed at a Q value corresponding to the known level of Ba^{136} at 2.07-MeV excitation energy. The level is most strongly excited at 27° where its cross section is 0.28 mb/sr. The corresponding angular distribution seems to correspond to $l=2$ or 3.

No other states of Ba^{136} and no excited states of Ba^{138} were identified. However, one other level was observed having a Q value such that it must be assigned to either Ba^{136} or Ba^{138}. The excitation energy and cross section of the level would be 1.54 MeV and 0.32 mb/sr, respectively, for Ba^{136} or 1.06 MeV and 0.19 mb/sr for Ba^{138}. If assigned to Ba^{138}, the level would have a lower excitation energy than the known 2^+ level at 1.43 MeV. Since the 2^+ level should be the first excited state of an even-even nucleus, the unknown level is more likely to be a state of Ba^{136}.

Nd(d,p) Reactions

The levels observed in this preliminary investigation of (d,p) reactions in natural Nd, which is 27% Nd^{142} and 24% Nd^{144} are the ground states of Nd^{143} and Nd^{145}. The cross sections at 45° scattering angle are 1.62 and 1.88 mb/sr, respectively. The lower abundances of the other five isotopes, and lower Q values in the case of the even isotopes, prevented us from making any other identifications.

RESULTS AND DISCUSSIONS: (d,t) REACTIONS

$Ce^{140}(d,t)$ Ce^{139}

The states of Ce^{139} were investigated by the $Ce^{140}(d,t)$ reaction. The spectrum is shown in Fig. 2, and the pertinent results are listed in Table IV. The distinction between states of Ce^{139} and Ce^{141} [from the $Ce^{142}(d,t)$ reaction] is made on the basis of intensities (Ce^{140}/Ce^{142} ratio in target ≈ 8) and energetics.

The ground state of Ce^{139} is known to be $d_{3/2}$. The 0.25-MeV state is assigned as $s_{1/2}$ on the following basis:

(1) The 0.75-MeV isomeric state decays only to the $d_{3/2}$ ground state and not to the 0.25-MeV level; this indicates that the latter is either an s or p state, and only the former is expected from shell model.

(2) From Table IV, the cross section for the 0.25-MeV state is slightly less than that for the $d_{3/2}$ ground state. This fits the cross-section systematics much better for an $s_{1/2}$ state than for any of the other states expected from shell model in this region.

(3) There is a known $s_{1/2}$ state with roughly the same energy spacing from the $d_{3/2}$ state in Ba^{137}.

(4) The $s_{1/2}$ state is known to be in this energy region and is known to have a cross section about equal to that of the 0.25-MeV state. There is no other level in this energy region with nearly the correct cross section.

Considering the available information on the dependence of (d,t) cross sections on l, j, and Q, the $g_{7/2}$ and $h_{11/2}$ states should be about 1/2 and 1/4, respectively, as strongly excited as the ground state. These estimates agree well with the observed cross sections for the 1.34- and 0.75-MeV states, respectively. Moreover, the energies of these states are approximately as expected for the corresponding single-particle states. The 0.75-MeV state is isomeric as expected for an $h_{11/2}$ state, and there are several analogous $h_{11/2}$ isomeric states in this region of the periodic table. We thus conclude that the energies of the single-hole states in Ce^{139} are $d_{3/2}$-0; $s_{1/2}$-0.25 MeV; $h_{11/2}$-0.75 MeV; and $g_{7/2}$-1.34 MeV. The $d_{5/2}$ state must be above 1.8 MeV; this region cannot be studied because of interference from deuterons.

The weakly excited groups at 1.06 and 1.66 MeV could be from $Ce^{142}(d,t)$ or could be minor components of the hole states in Ce^{139}. Actually, one expects the $g_{7/2}$ state to be mixed among more than one nuclear state as there should be a $7/2^+$ state from ($d_{3/2}$+one phonon) in this energy region.

TABLE IV. $Ce^{140}(d,t)Ce^{139}$.

(1) Excitation energy (MeV)	(2) $(d\sigma/d\omega)(60°)$ (mb/sr)	(3) J^π
0	1.33	$3/2^+$
0.25	1.12	$1/2^+$
0.75	0.29	$11/2^-$
1.06 (3.08)[a]	0.10 (0.80)[a]	($7/2^+$, Ce^{141})
1.34	0.69	$7/2^+$
1.66	0.11	

[a] Equivalent value for Ce^{141}.

$Ce^{142}(d,t)Ce^{141}$

The hole states of Ce^{141} were investigated by the $Ce^{142}(d,t)$ reaction. The results are listed in Table I.

From nuclear mass systematics, the $d_{3/2}$ and $s_{1/2}$ hole states of Ce^{141} are expected to appear with about 0.3 MeV higher Q value than the corresponding states of Ce^{139} (see Fig. 2). The levels found in the (d,t)spectra at $E=1.62$ and $E=1.78$ are assigned as these $d_{3/2}$ and $s_{1/2}$ hole states, respectively. They have the proper Q values and relative spacing; the absolute cross section of each is the same as that of the corresponding level in Ce^{139}, and for all scattering angles the ratio of their peak intensities is the same as the intensity ratio of the $d_{3/2}$ and $s_{1/2}$ states in Ce^{139}.

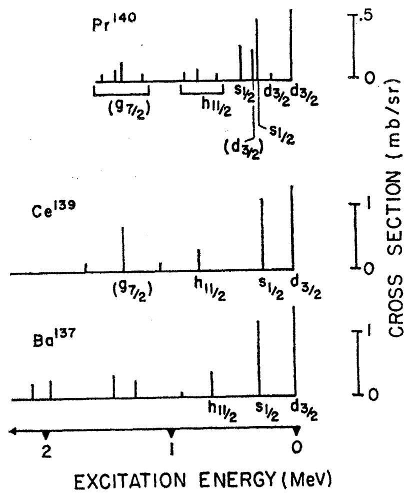

FIG. 7. Comparison of energy spectra of tritons from (d,t) reactions. All cross sections are measured at a scattering angle of 60°.

A further argument is provided by the (d,p) spectra for Ce^{141}, on which the states below the closed shell should not appear. No level is observed at 1.62-MeV excitation energy, and although a level is observed at 1.80 MeV, its (d,p) cross section is far too small to explain the observed (d,t) intensity. There are undoubtedly two closely spaced levels at this energy; one, a hole level, is excited by the (d,t) reaction, and the other, a particle level, is excited in the (d,p) reaction. We were unable to assign an l value to the 1.80-MeV (d,p) level because of experimental difficulties and oxygen contamination. The level is assigned in reference (1) as $f_{5/2}$.

$Pr^{141}(d,t)Pr^{140}$

As the $Pr^{141}(d,p)Pr^{142}$ spectra should resemble those of Ce^{141} except for splitting and possible mixing of levels, so the $Pr^{141}(d,t)Pr^{140}$ spectra should resemble those of $Ce^{140}(d,t)Ce^{139}$. The Pr^{140} levels are compared with those of Ce^{139} in Fig. 7, and the results are listed in Table V.

TABLE V. $Pr^{141}(d,t)Pr^{140}$.

(1) Excitation energy (MeV)	(2) $(d\sigma/d\omega)$ (60°) (mb/sr)	(3) Assignment of hole state coupled to $d_{5/2}$ proton
0	0.53	$d_{3/2}$
0.17	0.05	$d_{3/2}$
0.27	0.47	$s_{1/2}$
0.30	0.24	$(d_{3/2})$
0.40	0.27	$s_{1/2}$
0.58	0.03	$h_{11/2}$
0.76	0.08	$h_{11/2}$
0.86	0.06	$h_{11/2}$
1.03	0.02	$g_{7/2}$
1.18	0.07	$g_{7/2}$
1.35	0.15	$g_{7/2}$
1.40	0.09	$g_{7/2}$
1.47	0.02	$g_{7/2}$
1.54	0.03	$g_{7/2}$

In Table V, the observed levels of Pr^{140} which are grouped about the excitation energy of a single particle level of Ce^{139} have been assigned as analogs of that Ce level. In addition, angular distribution differences were used in distinguishing $d_{3/2}$ and $s_{1/2}$ levels. The centers of gravity of the level clusters in Pr^{140} then agree within about 0.12 MeV with the excitation energies of corresponding levels in Ce^{139}. Further, at each of the three scattering angles where comparisons were made, the intensities of level clusters in Pr^{140} as a fraction of the intensity of the ground state cluster were within 10% of the value of similarly normalized intensities for the corresponding levels in Ce^{139}. However, the absolute cross sections for Pr are smaller than for Ce. This problem is discussed in connection with the $Pr(d,p)$ reactions above.

$Nd(d,t)$ Reactions

As was the case for the $Nd(d,p)$ reactions, the only levels which could be certainly identified were ones whose Q values were previously known. These are listed in Table VI.

TABLE VI. $_{60}Nd(d,t)$.

Isotope	Excitation energy (MeV)	$d\sigma/d\omega$(45°) (mb/sr)
Nd^{141}	0	1.1
	(0.204)	1.6
Nd^{142}	0	0.68
Nd^{143}	0	0.81
	0.718	0.43
Nd^{144}	0	0.60
	0.694	0.17
	1.268	0.27
Nd^{145}	0	0.41
	0.790	0.25

The spin of the ground state of Nd^{143} is known to be $f_{7/2}$, and there is a level at 0.74-MeV excitation energy which previously was uncertainly assigned as $h_{9/2}$ largely on the basis of β decays from levels which themselves are doubtfully known. Data from the present study suggest that this level ($E=0.72$ MeV in Table VI) is $p_{3/2}$; at almost the same excitation energy in Ce^{141}, which has the same number of neutrons, there is a strong $p_{3/2}$ level. Furthermore, the ratio of cross sections of the excited state of Nd^{143} to its ground state is about the same as the $p_{3/2}$ to ground state ratio in Ce^{141}, whereas a $h_{9/2}$ level in Nd would be expected to have a much smaller cross section.

$Ba^{138}(d,t)Ba^{137}$

Levels of Ba^{137} found from (d,p) and (d,t) reactions, together with cross sections, are listed in Table VII A and B, respectively. The third column of Table VII A lists values of the angular momentum transfer as determined from angular distributions. The 1.80-MeV

TABLE VII. States of Ba^{137} from (d,p) and (d,t) reactions.

A $Ba^{136}(d,p)Ba^{137}$ Excitation energy (MeV)	$(d\sigma/d\omega)_{max}$ (mb/sr)	l	J^π	B $Ba^{138}(d,t)Ba^{137}$ Excitation energy (MeV)	$d\sigma/d\omega(60°)$ (mb/sr)
0	0.78	2	3/2+	0	1.42
0.28	1.2	0	1/2+	0.29	1.11
			11/2−	0.66	0.42
				0.90	0.07
				1.30	0.28
				1.49	0.32
1.80	3.9	3	7/2−		
				1.95	0.24
				2.12	0.24

level found in the (d,p) reaction can be identified from its angular distribution and from the systematics of energy and cross section as the $f_{7/2}$ level of the $N>82$ shell, the analog of the ground state of Ba^{139}. As expected, it is not excited in (d,t) reactions. In determining the levels listed in Table VII B, all levels in the (d,t) spectra with Q values smaller than that of the Ba^{138} $(d,t)Ba^{137}$ ground-state reaction are assumed to be states of Ba^{137}. (Natural barium contains 72% Ba^{138}, the next highest abundance is only 11%.)

The hole states of Ba^{137} and Ce^{139} are compared in Fig. 7. The excitation energies and cross sections of corresponding levels are in good agreement. The 0.29- and 0.66-MeV states were known previously, and the latter has been assigned $h_{11/2}$ on the basis of its isomeric lifetime (2.6 min). Note that in Ba^{137} there are two levels corresponding to the proposed $g_{7/2}$ state of Ce^{139}; the combined cross section of these two levels approximately equals that of the single level in Ce^{139}.

The location of the single-hole states from this data is $d_{3/2}$-0; $s_{1/2}$-0.29 MeV; $h_{11/2}$-0.66 MeV; $g_{7/2}$-1.40 MeV; and $d_{5/2}>2.0$ MeV. Averaging these with the Ce^{139} results gives the values listed in the abstract: $d_{3/2}$-0; $s_{1/2}$-0.27 MeV; $h_{11/2}$-0.71 MeV; $g_{7/2}$-1.37 MeV; and $d_{5/2}>2.0$ MeV.

Note added in proof. In a further search for the $d_{5/2}$ hole state with (d,t) reactions, the region between 1.5–4 MeV excitation energy has been studied using the spectrograph as a spectrometer and distinguishing between deuteron and triton reaction products by use of the pulse height in CsI(Th) scintillation detector. The method has been described in detail by P. Mukherjee and B. L. Cohen, Phys. Rev. **127**, 1284 (1962) (see Fig. 11 of that paper). Groups of levels were found in both Ba^{137} and Ce^{139} between 1.6 and 3 MeV, including the 1.66-Mev level of Ce^{139} in Table IV and the 1.95- and 2.12-MeV levels of Ba^{137} in Table VII. These groups are each tentatively assigned to the $d_{5/2}$ hole state, because in each case (1) the combined cross section of the group is that expected for the $d_{5/2}$ state; (2) no other levels were observed up to 4-MeV excitation energy although the $d_{5/2}$ state is expected to appear at about 2-MeV excitation energy; and (3) the group appears at about the same energy and with the same cross section in both Ce^{139} and Ba^{137}. If these assignments are correct, the $d_{5/2}$ center of gravity is at ~2.2 MeV for Ce^{139} and at ~2.4 MeV in Ba^{137}. The average of these values is listed in the abstract.

The $d_{3/2}-s_{1/2}$ Energy Gap

The data on the energy gap between the $d_{3/2}$ and $s_{1/2}$ hole states in nuclei with 81 neutrons is summarized in Table VIII.

The $s_{1/2}$ assignment is not completely certain in the case of Nd^{141} because the target was not isotopically pure, but the intensity and position of the level are those expected of the $s_{1/2}$ state in Nd^{141} and there are no other nearby peaks of similar intensity.

Assuming that the spin assignment in Nd^{141} is correct, there is a decrease in the $d_{3/2}$–$s_{1/2}$ energy gap as the $d_{5/2}$ proton shell is filling. This decrease in the energy gap is equivalent to a lowering of the $d_{3/2}$ neutron particle level, and hence could be explained by an attractive interaction between protons and neutrons with the same l value.[9]

TABLE VIII. Energy gap between $d_{3/2}$ and $s_{1/2}$ hole states in nuclei with 81 neutrons.

Nuclide	Excitation energy of $d_{3/2}$ level	Energy of level assigned as $s_{1/2}$ (MeV)
$_{56}Ba^{137}$	0	0.28
$_{58}Ce^{139}$	0	0.25
$_{60}Nd^{141}$	0	0.20

Wave Functions for Two-Particle and Two-Hole Nuclei

The measured ratios of various (d,p) and (d,t) cross sections can be used to determine the wave functions of the two-particle nuclei Ce^{142} and Nd^{144} and of the two-hole nucleus Ba^{136}. We first consider the Ba case. We take the wave function of the ground state of Ba^{136} to be

$$\psi(136)=a(d_{3/2}^{-2})+b(s_{1/2}^{-2})+c(h_{11/2}^{-2})+\cdots. \quad (1)$$

We write the (d,p) and (d,t) cross section as[10]

$$\begin{aligned}\sigma(d,p)&=(2I_f+1/2I_i+1)P_l(Q,\theta)S(i,f),\\ \sigma(d,t)&=T_l(Q,\theta)S(i,f),\end{aligned} \quad (2)$$

where P and T contain the information from the reaction mechanism and S is the fractional parentage coefficient times the number of particles. French has shown[11] that, for a transition between $(j^n)_0$ and $(j^{n-1})_j$, $S=n$ for even n, and $S=[1-n-1/(2j+1)]$ for odd n.

The cross sections for several pertinent reactions are

[9] B. L. Cohen, Phys. Rev. **127**, 597 (1962).
[10] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).
[11] J. B. French, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960).

then (figures are masses of Ba isotopes):

$$\begin{aligned}\sigma_1&=\sigma[136(d,p)137-d_{3/2}]=2P_2a^2,\\ \sigma_2&=\sigma[137(d,p)138-0^+]=P_2,\\ \sigma_3&=\sigma[138(d,t)137-d_{3/2}]=4T_2,\\ \sigma_4&=\sigma[137(d,t)136-0^+]=\tfrac{1}{2}T_2a^2.\end{aligned}\tag{3}$$

By taking ratios of the above at any given angle, we find

$$\begin{aligned}\sigma_1/\sigma_2&=2a^2,\\ \sigma_4/\sigma_3&=\tfrac{1}{8}a^2.\end{aligned}\tag{4}$$

These experimental cross-section ratios thus give independent determinations of a^2, and independent determinations from each may be obtained at each angle. In deriving (4), we have ignored the Q dependence of P_2 and T_2; this is corrected for by using the DWBA calculations.

The wave function (1) can also be expressed in terms of pairing theory. The occupation numbers, V_j^2, and their complements, U_j^2, are just

$$\begin{aligned}U_{3/2}^2&=1-V_{3/2}^2=\tfrac{1}{2}a^2,\\ U_{1/2}^2&=1-V_{1/2}^2=b^2,\\ U_{11/2}^2&=1-V_{11/2}^2=\tfrac{1}{6}c^2.\end{aligned}\tag{5}$$

From pairing theory, these may be obtained[11] as solutions of

$$\begin{aligned}U_j^2&=\tfrac{1}{2}\left\{1+\frac{\epsilon_j-\lambda}{[(\epsilon_j-\lambda)^2+\Delta^2]^{1/2}}\right\},\\ &\textstyle\sum_j (2j+1)U_j^2=n,\end{aligned}\tag{6}$$

where ϵ_j are the energies of the single-particle states (or the negative of the energies of the single-hole states obtained in this paper); n is the number of holes ($=2$ for Ba^{136}); and Δ is an important parameter in pairing theory. The solutions of (6) for various values of Δ are given in Table IX; from (5), they give a theoretical prediction for a^2.

In applying (4) there is an experimental difficulty in that the resolution is not sufficient to separate the $d_{3/2}$ state in Ba^{135} and Ba^{137} from (d,p) reactions in Ba^{134} and Ba^{136}, respectively. A similar difficulty arises between the ground states of Ba^{136} and Ba^{134} from (d,t) reactions in Ba^{137} and Ba^{135}, respectively. The observed cross sections for the sums was divided between the two reactions as follows: It has been shown[11] that one may write

$$\begin{aligned}\sigma_1&=\sigma[136(d,p)137-d_{3/2}]=4P_2U_{3/2}^2(136),\\ \sigma_5&=\sigma[134(d,p)135-d_{3/2}]=4P_2U_{3/2}^2(134),\\ \sigma_4&=\sigma[137(d,t)136-0^+]=T_2U_{3/2}^2(136),\\ \sigma_6&=\sigma[135(d,t)134-0^+]=T_2U_{3/2}^2(134).\end{aligned}\tag{7}$$

Taking ratios,

$$\sigma_1/\sigma_5=\sigma_4/\sigma_6=U_{3/2}^2(136)/U_{3/2}^2(134).\tag{8}$$

Equations (6) were solved for Ba^{134} ($n=4$); the results for $U_{3/2}^2(134)$ are listed in Table IX. It is remarkable that the calculations for the ratio appearing in (8)—shown in the last line of Table IX—give essentially the same result for the broad range of Δ used. This result was therefore used to determine σ_1 and σ_4 from the summed cross sections $\sigma_1+\sigma_5$ and $\sigma_4+\sigma_6$ which are observed experimentally.

Application of the first of (4) averaged over five angles gives

$$a^2=0.52\ [\text{from } (d,p)].$$

The value at different angles varies between 0.42 and 0.64. Application of the second of (4) at 30°, 45°, and 60° gives $a^2=0.67$, 0.52, and 0.56, respectively. The average of these is

$$a^2=0.58\ [\text{from } (d,t)].$$

The agreement between results from (d,p) and (d,t) reactions is satisfactory, so we adopt their average, $a^2=0.55$, as our final result. From Table IX we see that this corresponds to $\Delta\approx0.41$ MeV, a surprisingly small result. Odd-even mass differences[12] give $\Delta\approx1.0$ MeV, and Kisslinger and Sorenson[13] used $\Delta=0.83$ MeV for the proton states in Ba^{138}.

From Table IX, the wave function for the ground state of Ba^{136} is

$$\psi(136)=(0.55)^{1/2}(d_{3/2}^{-2})+(0.115)^{1/2}(s_{1/2}^{-2})+(0.26)^{1/2}(h_{11/2}^{-2})+(0.06)^{1/2}(g_{7/2}^{-2}).$$

The methods used in Ce^{142} and Nd^{144} are similar to those in Ba^{136} except that we are dealing with particle states rather than hole states, so that the roles of U_j^2 and V_j^2 are reversed.

We assume that the ground-state wave function of Ce^{142} is

$$\psi(142)=m(f_{7/2}^2)+n(p_{3/2}^2)+\cdots.\tag{9}$$

TABLE IX. Pairing theory solutions for Ba^{136}.

	$\Delta=1.00$	$\Delta=0.80$	$\Delta=0.60$	$\Delta=0.45$	$\Delta=0.35$
$U_{3/2}^2$	0.15	0.175	0.21	0.255	0.31
$U_{1/2}^2$	0.11	0.115	0.115	0.115	0.115
$U_{11/2}^2$	0.07	0.065	0.058	0.05	0.036
$U_{7/2}^2$	0.04	0.035	0.05	0.017	0.012
λ	0.97	0.68	0.41	0.25	0.14
a^2	0.30	0.35	0.42	0.51	0.62
$U_{3/2}^2(134)$	0.335	0.385	...	0.555	0.64
$\dfrac{U_{3/2}^2(134)}{U_{3/2}^2(136)}$	2.23	2.20	...	2.17	2.07

[12] S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 16 (1961).

[13] L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 9 (1960).

In analogy with (3)

$$\sigma_7=\sigma[142(d,t)141-f_{7/2}]=2m^2T_3,$$
$$\sigma_8=\sigma[142(d,t)141-p_{3/2}]=2n^2T_1, \tag{10}$$
$$\sigma_9=\sigma[140(d,p)141-f_{7/2}]=8P_3.$$

The ratio of the first two of (10) gives

$$\sigma_7/\sigma_8=(m^2/n^2)(T_3/T_1). \tag{11}$$

The ratio T_3/T_1 is determined from DWBA calculations to be approximately 4.0, averaged over angles. Using this in (11) with the experimental ratio σ_7/σ_8 gives m^2/n^2; the results averaged over angles is 6.7. A pairing theory calculation analogous to the solution of (6) gives this ratio for $\Delta\approx 0.72$ MeV. For $\Delta=1.0$ and 0.45 MeV, the calculation gives $m^2/n^2=4.8$ and 11.6, respectively.

If we accept $\Delta=0.72$ MeV, the pairing theory calculation gives $m^2=0.73$, and the complete wave function is

$$\psi(142)=(0.73)^{1/2}(f_{7/2}{}^2)+(0.11)^{1/2}(p_{3/2}{}^2)+(0.05)^{1/2}(f_{5/2}{}^2)+(0.017)^{1/2}(p_{1/2}{}^2)+(0.087)^{1/2}(h_{9/2}{}^2).$$

The value of m^2 can also be determined from the $Ce^{142}(d,p)Ce^{143}$ reaction if we assume that the ground state of Ce^{143} is the only $7/2^-$ state appreciably excited in this reaction. Under this assumption, (2) and (9) give

$$\sigma_{10}=\sigma[142(d,p)143-f_{7/2}]=6P_3m^2+8P_3(1-m^2),$$

so that, using the last of (10)

$$\sigma_{10}/\sigma_9=1\tfrac{1}{4}m^2.$$

This is clearly a rather insensitive method for determining m^2. The result is $m^2=0.77\pm0.23$. Although the error is large, the agreement with the other method is very good.

For the two-neutron nucleus Nd^{144}, a method exactly analogous to the first method described for Ce^{142} can also be used. If we take the wave function of Nd^{144} as

$$\psi(144)=q(f_{7/2}{}^2)+r(p_{3/2}{}^2)+\cdots,$$

the experimental value for the ratio q^2/r^2 is 6.2, in close agreement with the result for Ce^{142}. This ratio is reproduced by pairing theory for $\Delta\approx0.76$. The pairing theory calculation gives $q^2=0.73$ and a wave function essentially identical to that given above for Ce^{142}.

A more direct and reliable method is available for determing q^2, using the following cross sections:

$$\sigma_{11}=\sigma[144(d,t)143-f_{7/2}]=2T_3q^2,$$
$$\sigma_{12}=\sigma[143(d,t)142-0^+]=T_3.$$

The ratio of these is

$$\sigma_{11}/\sigma_{12}=2q^2.$$

When corrections are made for Q-value dependence of T_3, the result averaged over these angles is

$$q^2=0.78.$$

The agreement with the result of the first method is very satisfactory.

The discrepancy between the values of Δ obtained for Ba^{136} (0.41 MeV) and for Ce^{142} and Nd^{144} (0.72 and 0.76 MeV, respectively) is perhaps disturbingly large. However, the errors in these determinations are rather large, and one does not necessarily expect the same Δ for both cases. The wave functions are relatively insensitive to the value of Δ over the range of uncertainty.

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