Electron Capture and Nuclear Matrix Elements of Be⁷†

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We calculate the Be⁷ electron capture rate from continuum orbits as a function of electron temperature and concentration and consider the effect on the decay rate, for some typical stellar conditions, of the statistical distribution of electrons, the electron-nucleus Coulomb interaction, relativistic and nuclear size corrections, the imperfect overlap between initial and final atomic states, and electron screening in bound decay. We also analyze the experimental information on Be⁷ and Li⁷ to obtain accurate experimental Gamow-Teller matrix elements for both the ground- and excited-state transitions and compare these matrix elements with the predictions of some simple nuclear models. This comparison supports the conclusion of other authors that near the beginning of the $1p$ nuclear shell L-S coupling is better satisfied than j-j coupling.

I. INTRODUCTION

THE large value for the $He^3(\alpha,\gamma)Be^7$ cross section found experimentally by Holmgren and Johnston[1] led to studies by Fowler[2] and Cameron[3] of the possibility of completing the proton-proton chain with reactions involving Be⁷. The latter authors[2,3] showed that the proton-proton chain in the sun is completed more frequently through the $He^3(\alpha,\gamma)Be^7$ reaction than through the $He^3(He^3,2p)He^4$ reaction, in regions of high temperature and high helium-to-hydrogen ratio. They emphasized that the rate of energy generation and the effective energy release by the proton-proton chain depend critically on the ratio of proton-capture lifetime to electron-capture lifetime in Be⁷. Fowler[2] and Cameron[3] also showed that the possibility of detecting on earth neutrinos emitted by the sun or other stars depends sensitively on the ratio of the Be⁷ proton-capture lifetime to the electron-capture lifetime. Davis and his collaborators[4] are currently performing an experiment to determine if the solar neutrino flux is detectable by the inverse electron-capture process $Cl^{37}(\nu,e^-)Ar^{37}$.

The present work provides an accurate formula for the rate of capture of continuum electrons by Be⁷ as a function of electron density and temperature.[5] In deriving this formula, we consider the statistical distribution of the electrons, the electron-nucleus Coulomb interaction, relativistic and nuclear size corrections, the imperfect overlap between initial and final atomic states, and electron screening in bound decay. For temperatures of the order of 2×10^{7}°K, our results are in good agreement (14%) with a previous calculation by Cameron,[3] but at higher temperatures it is necessary to consider more accurately the electron-nucleus Coulomb interaction.

Using Hartree-Fock atomic wave functions and accurately determined experimental parameters, we analyze existing information to obtain the experimental Gamow-Teller nuclear matrix elements that govern the Be⁷ decay. We estimate that the experimental value thus determined for the ground-state (excited-state) transition matrix element is accurate to 10% (15%). We calculate the Gamow-Teller matrix elements on the basis of some simple nuclear models and compare with the experimental values. The results support the conclusion by other authors[6] that near the beginning of the $1p$ nuclear shell L-S coupling is better satisfied than j-j coupling.

II. ELECTRON CAPTURE FROM CONTINUUM ORBITS

The allowed continuum electron capture rate for a single nucleus of charge Z, nuclear energy release W_0, corresponding to one electron in a volume V is[7,8]

$$\lambda=\frac{G_V^2(W_0+W)^2\xi F(Z,W)}{2\pi V}, \qquad (1)$$

where W is the total energy of the electron and ξ is the usual allowed combination of nuclear matrix elements,[9]

$$\xi\equiv\langle 1\rangle^2+(C_A^2/C_V^2)\langle\sigma\rangle^2. \qquad (2)$$

The function $F(Z,W)$ is the ratio of the electron density at the nucleus calculated with a Coulomb distorted wave to the density calculated with a plane wave; G_V is the usual beta-decay coupling constant.

Assuming that the electrons in a star constitute a Fermi gas in which interactions among the electrons can

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[1] H. D. Holmgren and R. L. Johnston, Phys. Rev. **113**, 1556 (1959).

[2] W. A. Fowler, Astrophys. J. **127**, 551 (1958).

[3] A. G. W. Cameron, Atomic Energy of Canada, Limited Report, CRL-41, 1958 (unpublished), 2nd ed. See also A. G. W. Cameron, Ann. Rev. Nuclear Sci. **8**, 299 (1958).

[4] R. Davis, Jr. (private communication).

[5] At sufficiently low stellar temperatures, not all Be⁷ nuclei will be completely ionized and electron capture from bound orbits must be considered. See reference seven for the combined capture rate formula and for some indication of the temperatures at which bound capture becomes important.

[6] A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953); D. Kurath, Phys. Rev. **101**, 216 (1956). See also J. P. Elliott and A. M. Lane, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 241.

[7] J. N. Bahcall, Phys. Rev. **126**, 1143 (1962).

[8] We use units throughout this paper in which $\hbar=m=c=1$.

[9] All symbols have their usual meaning. For definitions, see E. J. Konopinski, Ann. Rev. Nuclear Sci. **9**, 99 (1959). We use the definition of reduced matrix elements suggested by Konopinski,

$$\langle I'(M')|S_{Jm}{}^+|I(M)\rangle\equiv\langle I'(M')J(m)|I(M)\rangle\langle S_J\rangle.$$

be neglected, the rate of capture of electrons from continuum orbits is[7]

$$\lambda = G_V^2 \xi K / 2\pi^3, \tag{3}$$

where the dimensionless quantity K is given by

$$K \equiv \int_0^\infty dp \; p^2 \frac{F(Z,W)(W_0+W)^2}{1+\exp(-\nu+W/kT)}. \tag{4}$$

If Boltzmann statistics are valid, and if $2\pi\alpha Z \langle v^{-1}\rangle_{\mathrm{av}} \gg 1$, $\alpha^2 Z^2 \ll 1$, and $kT \ll mc^2$, the statistical factor K can be evaluated approximately; the result is[7,10]

$$K_B \cong \pi\alpha Z n_e (2\pi)^{3/2}(kT)^{-1/2}q^2(1-(15/8)kT)$$
$$\times [(1+2\mu_0+2\mu_0^2)$$
$$+ (1+x_m/q)^2(2x_m)^{5/4}e^{-f(x_m)}/(3\alpha Zk^2T^2)^{1/2}], \tag{5a}$$

$$x_m \equiv (2^{-1})[2\pi\alpha ZkT]^{2/3}, \tag{5b}$$

$$f(x) = (x/kT) + \pi\alpha Z(2/x)^{1/2}, \tag{5c}$$

and

$$\mu_0 = kT/q. \tag{5d}$$

The quantity q is the energy of the neutrino emitted in the decay process and is defined by the relation

$$q = W_0 + W$$
$$\cong W_0 + 1. \tag{6}$$

Equations (5) differ from the corresponding Eqs. (17) in reference 7; everywhere Eqs. (17) in reference 7 contain W_0, Eqs. (5) contain q. Moreover, the error function that appears in reference 7 has been replaced by its asymptotic value for small temperatures and/or large nuclear charge.

Equations (5) were obtained by making use of the approximation[7]

$$F(Z,W) \cong 2\pi\eta[1+\exp(-2\pi\eta)], \tag{7}$$

where

$$\eta \equiv \alpha Zv^{-1}. \tag{8}$$

The term in Eq. (5a) involving x_m arose from the term $\exp(-2\pi\eta)$ in Eq. (7). Thus the term involving x_m is of the order of[11]

$$\exp(-2\pi\alpha Zv^{-1})$$
$$\cong \exp[-2\pi\alpha Z(2/\pi kT)^{1/2}]. \tag{9}$$

The central temperature in the sun is less than or of the order of 1.5×10^7 °K,[12] and thus the correction term (9) is about 5% for Be[7] electron capture in the sun.[13] Hence we can omit for Be[7] the term involving x_m in

Eq. (5a), although this term must be included in order to obtain accurate results for the electron-capture lifetime of hydrogen or helium, Equation (5a) can be further simplified by observing that kT is less than or of the order of 1 keV for the sun while q is 863(385) keV for the ground-state (excited-state) decay of Be[7]. Thus we may neglect kT everywhere with respect to q in Eq. (5a). The dimensionless quantity K can then be written in the form

$$K_B \cong \pi\alpha Z n_v (2\pi)^{3/2}(kT)^{-1/2}q^2, \tag{10}$$

and the continuum electron-capture rate is given simply by

$$\lambda = (2/\pi kT)^{1/2}G_V^2\alpha Z n_e q^2 \xi. \tag{11}$$

Expression (11) for the allowed electron-capture rate in a nondegenerate Fermi gas of electrons can be obtained heuristically in a simple way by substituting in Eq. (1)

$$n_e = 1/V, \tag{12a}$$

$$F(Z,W) \cong 2\pi\alpha Z\langle v^{-1}\rangle, \tag{12b}$$

and [11]

$$\langle v^{-1}\rangle = (2/\pi kT)^{1/2}. \tag{12c}$$

Formula (11) is then obtained immediately.

III. BOUND ELECTRON CAPTURE

The allowed capture rate for an electron bound in an atomic orbit with quantum numbers[14] n, κ, μ is

$$\lambda_{n,\kappa,\mu} = G_V^2 q^2\xi|\psi_{n,\kappa}(0)|^2/2\pi, \tag{13}$$

where[14,15]

$$|\psi_{n,-1}(0)|^2 = (4\pi)^{-1}g_{n,-1}^2(0), \tag{13a}$$

$$|\psi_{n,+1}(0)|^2 = (4\pi)^{-1}f_{n,-1}^2(0), \tag{13b}$$

The quantity $\Delta b_{n,\kappa}$ is the change in the atomic binding energy when an electron with quantum numbers n, κ is captured; this binding energy change is negligible for Be[7]. Only electrons with κ equal to plus or minus one undergo allowed decay; electrons with κ different from one possess orbital angular momentum and hence have their decay rates retarded by centrifugal repulsion. The form of the functions f and g is well known.[14]

Equation (13) neglects the difference between initial and final atomic states due to the change of nuclear charge by one unit. This effect has been studied for Be[7] by Benoist-Gueutal[16] who concludes that the Be[7] capture rate might be decreased by as much as 34% compared to the value given in Eq. (13), due to the imperfect overlap of initial and final atomic states. The estimate of Benoist-Gueutal only indicates the need for further study, since no accurate atomic wave functions were available for the excited states of Li[7]. The Li[7]

[10] Equations (5) are valid for low temperatures and/or large nuclear charge, since the formulas were obtained by using Eq. (7). The term involving x_m is not given very accurately by Eq. (5); if this term is appreciable, then numerical integration is probably necessary in order to obtain an accurate value for K.

[11] Formula (12c) is a general result for systems obeying Maxwell-Boltzmann statistics.

[12] R. L. Sears, Mém. soc. roy. sci. Liege **3**, 479 (1960).

[13] Equation (5a) yields a similar estimate for the correction term.

[14] We use the notation of M. E. Rose, *Relativistic Electron Theory* (John Wiley & Sons Inc., New York, 1961) for relativistic electron wave functions and quantum numbers.

[15] J. N. Bahcall, Phys. Rev. **124**, 495 (1961).

[16] P. Benoist-Gueutal, Ann. Phys. (New York) **8**, 593 (1953).

atom is usually left in the $1s2s^2$ excited state following Be⁷ electron capture.

The present writer has reinvestigated the effect on weak interaction decay rates of a different nuclear charge in initial and final atomic states. Our method differs from the one used by Benoist-Gueutal and has been briefly described in a previous publication.[15] Preliminary unpublished results by the present writer suggest that the imperfect overlap between individual atomic states does not inhibit the total Be⁷ decay rate by nearly as much as the 34% upper bound estimated by Benoist-Gueutal.[17] The numerical results presented in this paper ignore the imperfect overlap between initial and final atomic states since the present writer's estimate for the smallness of this effect is not contradicted by the experimental comparisons described in Sec. V of this paper.

In terrestrial experiments, Be⁷ decays by allowed electron capture to both the ground and the first excited states of Li⁷. We denote by q (q^*), ξ (ξ^*), and λ (λ^*), the ground- (excited-) state neutrino energy, nuclear matrix elements, and transition probability respectively. The laboratory transition probability can then be written,

$$\lambda_{lab} = \lambda + \lambda^*$$
$$= G_V^2 A (q^2\xi + q^{*2}\xi^*), \qquad (14)$$

where

$$A = (4\pi^2)^{-1}[g_{1,-1}^2(0) + g_{2,-1}^2(0)]. \qquad (15)$$

IV. NUMERICAL RESULTS

In this section, we present some numerical results for the Be⁷ electron capture rate from continuum orbits under typical stellar conditions. The stellar capture rate can be written as a sum of two terms, analogous to Eq. (14) for the laboratory decay rate, by making use of Eq. (10). Forming the ratio of λ_{star} and λ_{lab}, we obtain

$$\lambda_{star}/\lambda_{lab} = \tau_{lab}/\tau_{star}$$
$$= A^{-1}(2/\pi kT)^{1/2}\alpha Z n_e. \qquad (16)$$

Since the laboratory half-life, τ_{lab}, is accurately known, the atomic factor A is the only quantity that must still be determined before Eq. (16) can be used. Brysk and Rose[18] have shown that nuclear size effects on the wave functions (13a) and (13b) are negligible for nuclei as small as Be⁷. Relativistic effects can easily be estimated by examining the form of g appropriate to a pure Coulomb field. The Dirac $g_{1,-1}$, correct to terms of order $\alpha^2 Z^2$, has the value[19] (at the nuclear radius R)

$$g_{1,-1}(R) \cong 2(\alpha Z)^{3/2}e^{-x}\{1 + \alpha^2 Z^2[(5/4) - \gamma - \ln 2x]\}, \qquad (17)$$

where

$$x = \alpha ZR,$$

and γ is the Euler-Mascheroni constant. The dependence of $g_{1,-1}(R)$ on the nuclear radius is weak and relativistic corrections, represented by the terms proportional to $\alpha^2 Z^2$, are much less than one percent. Similar results are valid for $g_{2,-1}(R)$.

Electronic screening causes the beryllium atomic wave function to differ appreciably at the origin from the value computed using a pure Coulomb field. Hartree and Hartree[20] computed accurate beryllium radial wave functions using the method of the self-consistent field with exchange; they found (atomic units):

$$g_{1,-1}(0) = 14.67, \qquad (18a)$$

$$g_{2,-1}(0) = 2.67. \qquad (18b)$$

The value of $g_{1,-1}(0)$ given above is probably accurate to better than 1%. It differs by much less than 1% from the $g_{1,-1}(0)$ computed by Hartree and Hartree[21] using a self-consistent field without exchange and also agrees to better than 1% with their[21] $g_{1,-1}(0)$ computed, ignoring exchange, for Be⁺⁺. The values of $g_{1,-1}(0)$ computed by Hartree and Hartree for neutral Be, including exchange, and for Be⁺⁺, ignoring exchange, agree to within a few tenths of a percent with the value determined by Pekeris[22] for Be⁺⁺ using an elaborate numerical technique.

The electron density calculated from Eq. (18a) is 16% less than the pure Coulomb value,

$$[g_{1,-1}^2(0)]_{coulomb} \equiv 4Z^3$$
$$= 256. \qquad (19)$$

The fact that $g_{1,-1}(0)$ is essentially the same for Be and for Be⁺⁺ shows that the 16% decrease in the self-consistent field decay rate compared to the pure Coulomb decay rate is due to the mutual screening of the two $1s$ electrons. The ratio of L to K capture determined solely from Eqs. (18) is 3.3×10^{-2}; this is much less than the pure Coulomb ratio of 12.5×10^{-2}, although it has frequently been stated that for light atoms the L to K capture ratio is given accurately by pure Coulomb wave functions.

The value[8] of A computed from Eqs. (18) is 2.19×10^{-6}; this estimate of A is probably accurate to one percent. The half-life τ_{lab} is $(4.61 \pm 0.01) \times 10^6$ sec.[23] Substituting these numbers in Eq. (16), we find:

$$\tau_{star}^{-1} \equiv \lambda_{star}/\ln 2$$
$$= 1.02 n_e T_6^{-1/2} \times 10^{-32} \text{ sec}^{-1}, \qquad (20)$$

[17] It is hoped that a complete account of this work will be available in the near future.

[18] H. Brysk and M. E. Rose, Revs. Modern Phys. **30**, 1169 (1958).

[19] D. Layzer and J. Bahcall, Ann. Phys. (New York) **17**, 177 (1962).

[20] D. R. Hartree and W. Hartree, Proc. Roy. Soc. (London) **A150**, 9 (1935).

[21] D. R. Hartree and W. Hartree, Proc. Roy. Soc. (London) **A149**, 210 (1935).

[22] C. L. Pekeris, Phys. Rev. **112**, 1649 (1958).

[23] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959).

TABLE I. Experimental parameters.

Quantity	Value[a]	Quantity	Value[a]
q^2	2.85	G_V^2	$(8.82\pm0.12)\times10^{-24}$
$(q^*)^2$	0.568	C_A^2/C_V^2	1.41 ± 0.05
λ^*/λ	0.130 ± 0.013	A	2.19×10^{-6}
$\tau_{1/2}$	$3.58\times10^{+27}$		

[a] We use units in which $\hbar=m=c=1$. Except where shown explicitly, experimental errors are assumed negligible.

where τ_{star} is the half-life for capture of an electron by a Be[7] nucleus, n_e is the number of electrons per cm³, and T_6 is the temperature in units of 10^6 °K.

Let

$$n_e \equiv \rho/(\mu_e m_p), \qquad (21)$$

where ρ is the density of the stellar matter in g/cm³ and m_p is the mass of the proton. Then,

$$\tau_{star}^{-1} = 6.12\times10^{-9}\rho/\mu_e T_6^{+1/2} \text{ sec}^{-1}. \qquad (22)$$

Equations (20) and (22) are accurate to 1 or 2% if we ignore higher order correction terms in the expansion[24] of $F(Z,W)$ and the imperfect overlap of initial and final atomic states.

Equation (22) is in good agreement with a heuristic estimate given by Cameron.[3] His result differs by only 14% from the value given by Eq. (22), the difference being due almost entirely to the neglect of electron screening in his calculation of A. For temperatures of the order of the central temperature in the sun, i.e., T_6 about 15, Eq. (29) yields a lifetime three times shorter than the order-of-magnitude estimate derived by Bethe and used by Fowler[2] in his estimates of the solar neutrino flux. As a first approximation, Bethe assumed that $F(Z,W)$ was equal to one; it is about three for solar temperatures.

V. NUCLEAR MATRIX ELEMENTS

We can extract experimental values for the nuclear matrix elements involved in Be[7] electron capture by making use of the parameters listed in Table I. The values for q, q^*, λ^*/λ, and $\tau_{1/2}$ were taken from the compilation of Ajzenberg-Selove and Lauritsen.[23] The value of G_V^2 has recently been determined very accurately[25]; our knowledge of C_A/C_V is much less precise.[26] The value of A adopted here has been discussed in Sec. III. Unless explicitly shown in Table I, it is assumed that experimental uncertainties are unimportant in our considerations.

The experimental nuclear matrix elements can be computed from the following version of formula (13),

$$\xi = \langle1\rangle^2 + (C_A^2/C_V^2)\langle\sigma\rangle^2$$
$$= \ln2[G_V^2 q^2 A \tau_{1/2}(1+\lambda^*/\lambda)]^{-1}, \qquad (23)$$

and an analogous expression for ξ^*. For mirror nuclei, it is an excellent approximation to assume that $\langle1\rangle=1$ and thus Eq. (23) can be used to obtain an experimental value for the reduced Gamow-Teller matrix element $\langle\sigma\rangle$; the Fermi matrix element $\langle1\rangle^*$ vanishes for the excited state decay. The experimental matrix elements are shown in the second column of Table II; the largest uncertainty arises from the estimated experimental error in the determination of λ^*/λ. The estimated value of excited state Gamow-Teller matrix element $\langle\sigma\rangle^*$ depends strongly upon the branching ratio λ^*/λ; thus $\langle\sigma\rangle^*$ is not as well known as $\langle\sigma\rangle$.

Table II also lists several theoretical predictions of the Gamow-Teller matrix elements. The third column of Table II contains the single-particle j-j coupling values. A number of authors[27] have emphasized that the correct j-j wave functions to use for light nuclei are eigenfunctions of the total angular momentum J and the total isotopic spin T. Thus column four lists values computed with eigenfunctions of the total isotopic spin; the entries in this column are explained in more detail below. The supermultiplet theory of Wigner[28] predicts the same matrix elements as the pure single-particle values; the supermultiplet predictions are shown in column five.

The ground state of Be[7] has a spin of 3/2 and a total isotopic spin of 1/2; the ground-state configuration is assumed to be (in the notation of Mayer and Jensen[29]): $(\pi p_{3/2})^2(\nu p_{3/2})$. The ground state of Li[7] has the same total spin and isotopic spin and an analogous configuration. Mayer and Jensen have constructed the isotopic spin wave function in j-j coupling for the Be[7] and Li[7] ground states.

The first excited state of Li[7] has a total spin and isotopic spin both equal to one-half; the configuration of the excited state may be either $(\nu p_{3/2})^2(\pi p_{3/2})$ or $(p_{3/2})^2(p_{1/2})$, according to the shell model.[29] The Li[7]* configuration involving $(p_{1/2})$ can be described by two independent wave functions which correspond to the two $p_{3/2}$ nucleons being coupled to an intermediate angular momentum of zero or one. The antisymmetrized eigenfunctions of total spin and isotopic spin can be found by the method of Mayer and Jensen[29] or by direct construc-

[24] For temperatures of the order of 2×10^7 °K, these terms will decrease the predicted lifetime by about 5%, as we have seen in Sec. II. At lower temperatures, the correction terms in the expansion of $F(Z,W)$ are completely negligible. For low densities and high temperatures, such as exist in our sun, electron screening in continuum orbits will be much less than in bound atomic orbits.

[25] R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger, Phys. Rev. 127, 583 (1962); J. W. Butler and R. O. Bondelid, ibid. 121, 1770 (1961).

[26] A. Sosnovskii, P. Spivak, Yu. Prokoviev, I. Kutikov, and Yu. Dobrynin, Soviet Phys.—JETP 35, 739 (1959).

[27] See, for example, M. Mizushima and M. Umezawa, Phys. Rev. 85, 37 (1952) and B. H. Flowers, Phil. Mag. 43, 1330 (1952). The first entry in column four of Table II has also been obtained by A. Winther and O. Hofoed-Hansen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 27, no. 14 (1953). The latter authors pointed out the necessity of using isotopic spin eigenfunctions for the calculation of Gamow-Teller matrix elements.

[28] E. P. Wigner, Phys. Rev. 56, 519 (1939).

[29] M. G. Mayer and J. H. D. Jensen, Elementary Theory of Nuclear Shell Structure (John Wiley & Sons, Inc., New York, 1953).

TABLE II. Comparison of experimental and theoretical
nuclear matrix elements.

Matrix element	Experimental	Single-particle j-j	Isotopic spin[a] j-j	Supermultiplet theory
$\langle\sigma\rangle^2$	1.48 ± 0.15	1.67	0.90 $(p_{3/2})^3$: 0.44	1.67
$\langle\sigma\rangle^{*2}$	1.42 ± 0.22	1.33	$J_{\text{in}}=0(p_{1/2})$: 0.19 $J_{\text{in}}=1(p_{1/2})$: 0.12	1.33

[a] Entries in this column are explained in the text.

tion. The excited state Gamow-Teller matrix element
was computed using each of the three possible j-j
isotopic spin wave functions; the results are listed below.

$$(\nu p_{3/2})^2(\pi p_{3/2}): \quad \langle\sigma\rangle^{*2}=4/9; \quad\quad (24a)$$

$$(p_{3/2})^2{}_{J\text{in}=0}(p_{1/2}): \quad \langle\sigma\rangle^{*2}=5/27; \quad\quad (24b)$$

$$(p_{3/2})^2{}_{J\text{in}=1}(p_{1/2}): \quad \langle\sigma\rangle^{*2}=12/100. \quad (24c)$$

The configuration label indicates which of the three
Li⁷* j-j eigenfunctions was used; the quantity J_{in} gives
the value of the intermediate angular momentum to
which the two $p_{3/2}$ nucleons are coupled in the
$(p_{3/2})^2(p_{1/2})$ configuration. A similar notation has been
used in Table II.

The $p_{1/2}$ configuration yields in both cases a smaller
value of $\langle\sigma\rangle^{*2}$ than the $(p_{3/2})^3$ configuration. This is
because the orthogonality of the $p_{3/2}$ and $p_{1/2}$ single-
particle wave functions permits only parts of the Be⁷ and
Li⁷* wave functions to contribute to the matrix element
if the Li⁷* state contains, unlike the Be⁷ state, a $p_{1/2}$
single-particle component.

The trend of the numbers in Table II is in agreement
with results of extensive intermediate coupling calcu-
lations[6] of other nuclear parameters for light nuclei.
The intermediate coupling calculations show that L-S
coupling is better satisfied than j-j coupling for nuclei
near the beginning of the $1p$ nuclear shell.[30] This fact

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[30] C. W. Kim has confirmed this result by analyzing the ft
values of light even-A nuclei (private communication).

is reflected in Table II by the excellent agreement be-
tween the supermultiplet predictions and the experi-
mental values; the isotopic spin j-j eigenfunctions yield
matrix elements that are too small. It would be inter-
esting to see how well the intermediate coupling wave
functions that have been used to compute other nuclear
parameters for mass number seven reproduce the ex-
perimental Gamow-Teller matrix elements.

If the imperfect overlap between initial and final
atomic states caused the atomic factor A to be decreased
significantly from the value adopted in Sec. III, the
theoretical matrix elements listed in Table II would ap-
pear to be too small when compared with the experi-
mental values. However, the agreement between theory
and experiment is similar to the agreement found for
nuclear magnetic moments,[27,29] and for magnetic mo-
ments there is, of course, no atomic overlap correction.
We conclude that there is no evidence for a large de-
crease in the total electron capture probability due to
an imperfect overlap between initial and final atomic
states. An intermediate coupling calculation with the
same wave functions that were used to calculate the
Be⁷ and Li⁷ magnetic moments would allow one to
place quantitative limits on the decrease in A, and thus
the total electron capture probability, due to the im-
perfect atomic overlap.

Note added in proof. R. W. Kavanagh [Nuclear Phys.
15, 411 (1960)] measured the Be⁷(p,γ)B⁸ cross section
and W. A. Fowler [Mem. Soc. Roy. Liege **3**, 207 (1960),
Ser. 5] used Kavanagh's results to show that proton
capture by Be⁷ is rare in the sun.