Cyclotron Resonance in Metals with H Perpendicular to the Surface

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(Received March 9, 1962; revised manuscript received April 18, 1962)

Explicit formulas are derived for the surface impedance of a single isotropic carrier in a magnetic field from the zero-field impedance by a scaling procedure originally proposed by Chambers. It is shown that the approximation of a local current-field relation ignores the Doppler effect on the carriers. When the proper nonlocal current-field relation is used, the theoretically predicted surface impedance is altered substantially in the vicinity of the resonance. The nonlocal effects can, in fact, be qualitatively reproduced by using a local theory and replacing ω by $\omega_{\mathrm{eff}}=\omega\pm v_F/\delta_i$, where δ_i is the inductive skin depth. The nonlocal theory leads to: (a) a resonance in the resistance at the Doppler shifted frequency $\omega=|\omega_c|-v_F/\delta_i$; (b) a shift in the threshold field required for the onset of an undamped electromagnetic wave (helicon wave). Recent experiments on Bismuth by Kirsch and Redfield show structure believed to correspond to the Doppler shifted resonance discussed.

INTRODUCTION

MICROWAVE cyclotron resonance with a static magnetic field H perpendicular to the metal surface has been observed in bismuth and its alloys,[1] in zinc,[2] in cadmium,[3] and in antimony.[4] The measured quantity is usually the surface resistance for a circularly polarized incident wave. In general, the surface resistance depends on the number of carriers, the anisotropy of each carrier and the degree to which the current-field relation may be considered to be a local or an anomalous (nonlocal) relation. Galt *et al.*[1] attempted to fit their data on bismuth with a local theory and in this manner deduced the anisotropy of the electron and hole in bismuth, although they recognized that near "resonance" a local theory would be invalid. It is the purpose of the present work to examine in detail the results of a nonlocal theory for a single isotropic carrier, using a scaling procedure originally proposed by Chambers[5] to evaluate the surface impedance. Since our considerations are limited to the case of a single isotropic carrier, our results cannot be directly compared with experiments on bismuth, zinc, cadmium, or antimony. In all these cases, effects due to band structure anisotropy and carrier multiplicity play an important role in the behavior of the surface impedance. However, the nonlocal effects discussed below in terms of a single isotropic carrier are expected to be directly observable in surface impedance measurements on alkali metals and some degenerate semiconductors since they are nearly isotropic.

CALCULATION OF SURFACE IMPEDANCE

We consider an incident wave circularly polarized in the xy plane and the semi-infinite metal occupying the space $z<0$. The current and field are Fourier-analyzed and the wave-number-dependent conductivity tensor defined by

$$\mathbf{J}(q)=\boldsymbol{\sigma}(q)\cdot\mathbf{E}(q). \tag{1}$$

We assume that specular reflection takes place at the surface, which is valid in Bi under some circumstances.[6] Then the semi-infinite metal may be replaced by an infinite medium with a current sheet of strength $2(dE/dz)_{z=0}\delta(z)$ and with H being constant throughout the medium. The surface impedance is defined as

$$Z=(4\pi/c)[E_x(0)/H_y(0)], \tag{2}$$

and may be expressed in terms of the components of the conductivity tensor as

$$Z=\frac{8i\omega}{c^2}\int_0^\infty\frac{dq}{q^2+(4\pi i\omega/c^2)[\sigma_{xx}(q)-i\sigma_{xy}(q)]}. \tag{3}$$

The conductivity tensor in a magnetic field may be found from the Boltzmann equation[7]:

$$\sigma_+(q)\equiv\sigma_{xx}(q)-i\sigma_{xy}(q)=\sigma_0\frac{3}{4}\int_{-1}^{1}\frac{du(1-u^2)}{1+i(\omega+\omega_c+qv_Fu)\tau}, \tag{4}$$

TABLE I. Surface impedance.

α	β	Z/R_0
$\ll 1$	$\ll 1$	$\frac{\omega}{\omega+\omega_c}\left[\left(\frac{\alpha}{2\beta}\right)^{\frac{1}{2}}+i\left(\frac{\alpha}{2\beta}\right)^{\frac{1}{2}}\right]$
$\ll 1$	$\gg\alpha^{-\frac{1}{3}}$	$\frac{\omega}{\omega+\omega_c}\left[\left(\frac{4}{3\pi}\right)^{\frac{1}{3}}\frac{2}{3\sqrt{3}}\alpha^{\frac{2}{3}}+i\left(\frac{4}{3\pi}\right)^{\frac{1}{3}}\frac{2}{3}\alpha^{\frac{2}{3}}\right]$
$\gg 1$	$\ll 1/\alpha$	$\frac{\omega}{\omega+\omega_c}\left[\left(\frac{\alpha}{2\beta}\right)^{\frac{1}{2}}+i\left(\frac{\alpha}{2\beta}\right)^{\frac{1}{2}}\right]$
$\gg 1$	$\gg 1/\alpha$	$\frac{\omega}{\omega+\omega_c}\left[\frac{1}{2\beta}+\frac{1}{8\alpha^2}+i\alpha\right]$

[1] J. K. Galt, W. A. Yager, F. R. Merritt, B. B. Cetlin, and A. D. Brailsford, Phys. Rev. **114**, 1396 (1959).
[2] J. K. Galt, F. R. Merritt, W. A. Yager, and H. W. Pail, Jr., Phys. Rev. Letters **2**, 292 (1959).
[3] J. K. Galt, F. R. Merritt, and P. H. Schmidt, Phys. Rev. Letters **6**, 458 (1961).
[4] W. R. Datars and R. N. Dexter, Phys. Rev. **124**, 75 (1961).
[5] R. G. Chambers, Phil. Mag. **1**, 459 (1956).
[6] G. E. Smith, Phys. Rev. **115**, 1561 (1959).
[7] M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. **117**, 937 (1960).

where σ_0 is the dc conductivity$=ne^2\tau/m^*$ and ω_c is the cyclotron frequency eH/m^*c. Define the London skin depth δ_0 and a dimensionless conductivity $\bar{\sigma}_+$:

$$\delta_0^2=m^*c^2/4\pi ne^2;\quad \bar{\sigma}_+=(m^*\omega/ne^2)\sigma_+; \tag{5}$$

then integrating (4) and separating real and imaginary parts, we get

$$\mathrm{Re}\bar{\sigma}_+=\frac{3}{4}\frac{\omega}{qv_F}\left\{\frac{-2}{ql}+\theta\left[1+\frac{1}{q^2l^2}\left(1-(\omega+\omega_c)^2\frac{l^2}{v_F^2}\right)\right]+2\frac{(\omega+_c)\omega}{v_F}\frac{\ln A}{q^2l^2}\right\}, \tag{6}$$

$$\mathrm{Im}\bar{\sigma}_+=\frac{3}{4}\frac{\omega}{qv_F}\left\{\frac{-2(\omega+\omega_c)}{qv_F}+\frac{2\theta}{q^2l}\frac{(\omega+\omega_c)}{v_F}-\ln A\left[1+\frac{1}{q^2l^2}\left(1-(\omega+\omega_c)^2\frac{l^2}{v_F^2}\right)\right]\right\}, \tag{7}$$

where

$$\theta=\tan^{-1}\left[\frac{(\omega+\omega_c)l}{v_F}+ql\right]-\tan^{-1}\left[\frac{(\omega+\omega_c)l}{v_F}-ql\right], \tag{8}$$

$$A=\left(\frac{1+[(\omega+\omega_c)l/v_F+ql]^2}{1+[(\omega+\omega_c)l/v_F-ql]^2}\right)^{\frac{1}{2}}. \tag{9}$$

In Eq. (8), the principal part of the arctangent is to be taken.

A special case of interest is the limit of $l=v_F\tau\rightarrow\infty$. Then from (6) we find that

$$\mathrm{Re}\bar{\sigma}_+=0\quad\text{for}\quad q<|(\omega+\omega_c)/v_F|. \tag{10}$$

This important selection rule may also be derived by considering transitions from a Landau level labeled by the quantum numbers $|n,k_z\rangle$ to the state $|n',k_z'\rangle$; the selection rules $n'-n=\pm1$ and $k_z'=k_z\pm q$ then show that real transitions are not possible for $q<|(\omega+\omega_c)/v_F|$.

The surface impedance in a magnetic field may be obtained from the zero-field impedance by a scaling procedure originally used by Chambers.[5] This method is based on noting that in Eqs. (6) and (7) the conductivity in a magnetic field is obtained from the zero-field conductivity by replacing ω by $(\omega+\omega_c)$ except for the multiplying factor $\omega\delta_0/v_F$. It is convenient to consider the zero-field impedance to be a function of the two parameters l/δ_0 and $\omega\delta_0/v_F$. Then one may show that the impedance in a magnetic field is related to the zero-field impedance by[5]

$$\frac{Z(H;l/\delta_0;\omega\delta_0/v_F)}{R_0}=\frac{\omega}{\omega+\omega_c}\frac{Z(0;\beta;\alpha)}{R_0}, \tag{11}$$

where

$$\beta=\frac{l}{\delta_0}\left(\frac{\omega}{\omega+\omega_c}\right)^{\frac{1}{3}};\quad \alpha=\frac{\omega\delta_0}{v_F}\left(\frac{\omega+\omega_c}{\omega}\right)^{\frac{2}{3}};\qquad R_0=(4\pi/c^2)v_F. \tag{12}$$

Formulas for the zero-field impedance have been given by Dingle.[8] When $(\omega+\omega_c)$ is negative one must replace α by $i\alpha$ and β by $i\beta$. In this case, the scaling procedure given by Eq. (11) gives correctly the magnitudes of the real and imaginary parts, R and X, of the surface impedance Z. The signs of R and X must be chosen so that both R and X are positive.

An alternate method to the scaling procedure of Chambers is direct integration of Eq. (3). The direct integration method also shows more explicitly the difference between a local and a nonlocal theory. We express the integral in terms of a dimensionless variable $u=q\delta_0|(\omega+\omega_c)/\omega|^{\frac{1}{3}}$:

$$Z=\frac{8i\omega\delta_0}{c^2}\left|\frac{\omega+\omega_c}{\omega}\right|^{\frac{1}{3}}\int_0^\infty\frac{du}{u^2\pm i\bar{\sigma}_+'(u;\beta;\alpha)};\qquad \bar{\sigma}_+'=[(\omega+\omega_c)/\omega]\bar{\sigma}_+. \tag{13}$$

The negative sign is to be taken when $(\omega+\omega_c)$ is negative. We subdivide the region of integration into two regions. In the case $\alpha\beta\ll1$, the two regions are defined by $u\ll1/\beta$ and $u\gg1/\beta$, whereas if $\alpha\beta\gg1$ the two regions are defined by $u\ll\alpha$ and $u\gg\alpha$. In region I the conductivity $\bar{\sigma}_+'$ is local (independent of u):

$$\bar{\sigma}_+(u;\beta;\alpha)=(\alpha\beta-i\alpha^2\beta^2)/(1+\alpha^2\beta^2). \tag{14}$$

In region II, the conductivity is

$$\bar{\sigma}_+'(u;\beta;\alpha)=\frac{3\pi}{4}\frac{\alpha}{u}-\frac{3i}{u^2\beta^2}\frac{\alpha^2\beta^2}{1+\alpha^2\beta^2}. \tag{15}$$

If the contribution to the integral is largest from region I, then the result is equivalent to a local (classical) result. In Table I, we summarize the results of the direct integration or equivalently of the scaling procedure of Chambers. Formulas for negative $(\omega+\omega_c)$ may be obtained from Table I by the substitution $\alpha\rightarrow i\alpha$ and $\beta\rightarrow i\beta$, except for the previously mentioned arbitrariness of sign.

It is interesting to note that the approximate criterion suggested by Galt *et al.*,[1] for determining whether the surface impedance is anomalous, agrees approximately

[8] R. B. Dingle, Physica **19**, 311 (1953).

with the results of Table I. This criterion may be written

$$1+(\alpha\beta)^2<\beta^2. \tag{16}$$

For example, if we apply the above criterion to the first entry in Table I, we find $\alpha\beta \ll 1$ and $\beta \ll 1$ so that the inequality is not satisfied. The relations $R,X=[\omega/(\omega+\omega_c)](\alpha/2\beta)^{\frac{1}{2}}$ are thus local relations and may be obtained from a classical theory.

DISCUSSION

The formulas in Table I have been used to plot the graphs of surface resistance R, its derivative dR/dH, and the resistive skin depth δ_r, shown in Figs. 1–3. Intermediate ranges for Figs. 1–2 have been obtained by numerical integration. Plots of the surface reactance X may also be obtained from Table I. Such plots are similar to those for R except that the sense of the magnetic field must be reversed. The resistive and inductive skin depths have been defined so that the field near $z=0$ falls off as $\exp(-|z|/\delta_r+i|z|/\delta_i)$, where

$$\delta_r=\frac{c^2}{4\omega}\left(\frac{X^2+R^2}{X}\right);\quad \delta_i=\frac{c^2}{4\omega}\frac{X^2+R^2}{R}. \tag{17}$$

The results of the local theory have also been plotted for comparison. Chambers[5] has also discussed the surface impedance using a nonlocal theory. His results cover a restricted range of magnetic fields and for this reason they do not show the qualitative behavior exhibited in Figs. 1–3. For example, for the case considered in the figures ($l/\delta_0=10^3$, $\omega\delta_0/v_F=0.1$) the corresponding curve given by Chambers covers only the range $-1.1\leq\omega_c/\omega\leq-0.9$. In this range, where the two calculations can be compared, they agree except for a numerical factor arising from a different assumption about the nature of surface reflection.

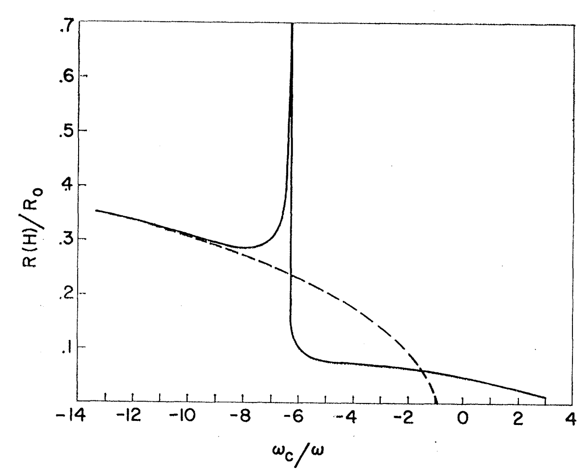

FIG. 1. Plot of surface resistance $R(H)/R_0$ vs ω_c/ω for the parameters $\omega\delta_0/v_F=0.1$ and $l/\delta_0=10^3$. The dashed curve is the local theory.

The main physical difference between the local and nonlocal theories is that the local theory neglects the Doppler effect on the carriers. A carrier moving with velocity v_z sees an effective frequency $\omega_{\mathrm{eff}}=\omega\pm v_z/\delta_i$, depending on whether it moves towards or away from the surface. That this effect is ignored in a local theory is readily seen from Eq. (4). This equation yields the local result if we put $q=0$. It is clear that this is completely equivalent to putting $v_z=0$, i.e., to ignoring the Doppler effect. The nonlocal effects can in fact be qualitatively reproduced by using a local theory and replacing ω by ω_{eff}.

In the local theory in the limit of $\tau\to\infty$ the resistive and inductive skin depths are given by

$$\left(\frac{1}{\delta_r}-\frac{i}{\delta_i}\right)^2=\frac{1}{\delta_0{}^2}\frac{\omega}{\omega+\omega_c}. \tag{18}$$

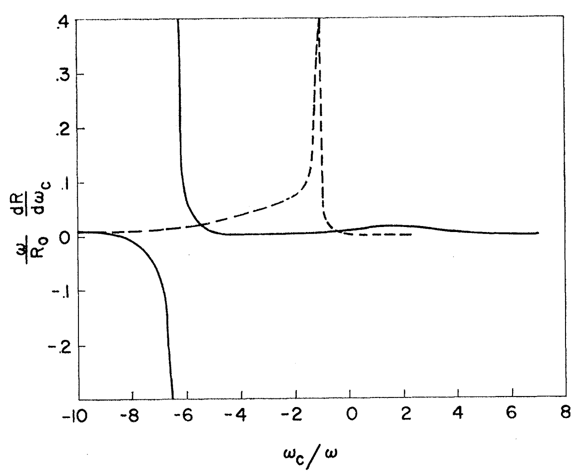

FIG. 2. Plot of the derivative $dR/d\omega_c$ vs ω_c/ω for the same parameters as Fig. 1. The dashed curve is the local theory.

For positive $(\omega+\omega_c)$ this implies

$$\delta_i\to\infty \text{ and } \delta_r=\delta_0|(\omega+\omega_c)/\omega|^{\frac{1}{2}}, \tag{19}$$

whereas for negative $(\omega+\omega_c)$ we obtain

$$\delta_r\to\infty \quad\text{and}\quad \delta_i=\delta_0|(\omega+\omega_c)/\omega|^{\frac{1}{2}}. \tag{20}$$

The expression for δ_r resulting from the nonlocal theory may be qualitatively reproduced by replacing ω by $\omega+v_F/\delta_i$ in Eq. (18).[9] As a result, δ_r now rises rapidly for $(\omega+\omega_c)/\omega<-v_F/\omega\delta_i$. The inductive skin depth in this region obtained from Table I is $\delta_i\simeq\delta_0(v_F/\omega\delta_0)^{\frac{1}{3}}$. Hence the rapid rise of δ_r occurs at

$$|(\omega+\omega_c)/\omega|^{\frac{3}{2}}(\omega\delta_0/v_F)\equiv\alpha\sim 1. \tag{21}$$

The nonlocal theory thus results in a shift of the position of the abrupt rise in δ_r, from the point $\omega_c/\omega\sim-1$ to the point $\alpha\sim 1$. This behavior is shown in Fig. 3.

[9] A rise in δ_r corresponds to the absence of real energy-conserving transitions. The choice of the plus sign in ω_{eff} ensures that this condition is satisfied for all electrons.

The large resistive skin depth obtained for $\omega_c/\omega < -1$ in the local theory and for $\alpha \gtrsim 1$ in the nonlocal theory corresponds to the existence of an essentially undamped electromagnetic wave in the metal. The wave vector q_c of this wave is given for both theories by

$$q_c = |\omega/(\omega+\omega_c)|^{\frac{1}{2}}(1/\delta_0). \tag{22}$$

In the nonlocal theory this wave manifests itself in the formalism as a pole in the integrand of Eq. (13), which appears at q_c when $\alpha \gtrsim 1$.

The properties of undamped electromagnetic waves in a *two*-carrier system such as Bi (i.e., Alfvén waves) have been discussed by Buchsbaum and Galt on the basis of a local theory.[10] For high magnetic fields they showed that the experimentally observed absorption in Bi could be interpreted in terms of the properties of such waves. The present analysis of a single isotropic carrier shows that the local theory becomes valid at high magnetic fields ($\alpha > 1$) and, hence, supports the use of such a theory in this region.

The abrupt rise in δ_r is also reflected in the plots of the surface resistance and its field derivative as shown in Figs. 1 and 2. Plots of the surface reactance and its field derivative are not shown but are qualitatively similar to Figs. 1 and 2 except that the sense of magnetic field must be reversed. In addition to the sharp resonance at $\alpha \sim 1$, the curve in Fig. 1 shows a plateau resulting from the Doppler broadening of the cyclotron resonance. The width of this plateau corresponds approximately to a Doppler broadening $\Delta\omega_D \sim v_F/\delta_i$, where $\delta_i \simeq \delta_0(v_F/\omega\delta_0)^{\frac{1}{3}}$. This broadening is normally large compared to the collision broadening $\Delta\omega_s$, which is of order $1/\tau$.

The field derivative of the surface resistance is shown in Fig. 2. For the local theory, a single peak whose Q value $\omega/\Delta\omega_s \sim \omega\tau$ is obtained at $\omega = -\omega_c$. When nonlocal effects are included, this peak is broadened so that the Q value becomes $\omega/\Delta\omega_D \sim \omega\delta_i/v_F$. For the parameter values used in Fig. 2 the Doppler effect is in fact large enough to result in a double-humped peak. However, one of the two peaks is much larger than the other.

We have shown that, for a single isotropic carrier, nonlocal effects become important when

$$1 + (\alpha\beta)^2 < \beta^2.$$

For pure materials $\beta \equiv (l/\delta_0)[\omega/(\omega+\omega_c)]^{\frac{1}{2}}$ is large and the above criterion reduces to $\alpha \lesssim 1$. The nonlocal theory takes into account the Doppler effect on the carriers. This effect, which is not included in the local theory, results in: (a) a sharp resonance in $R(H)$ at the Doppler shifted frequency, in addition to the Doppler broadened plateau, and (b) a shift in the threshold field required for the onset of an essentially undamped electromagnetic wave. The above situation complicates the interpretation of experimental curves of the absorption on its field derivative, and makes it difficult to obtain effective masses from such curves.

[10] S. J. Buchsbaum and J. K. Galt, Phys. Fluids 4, 1514 (1961).

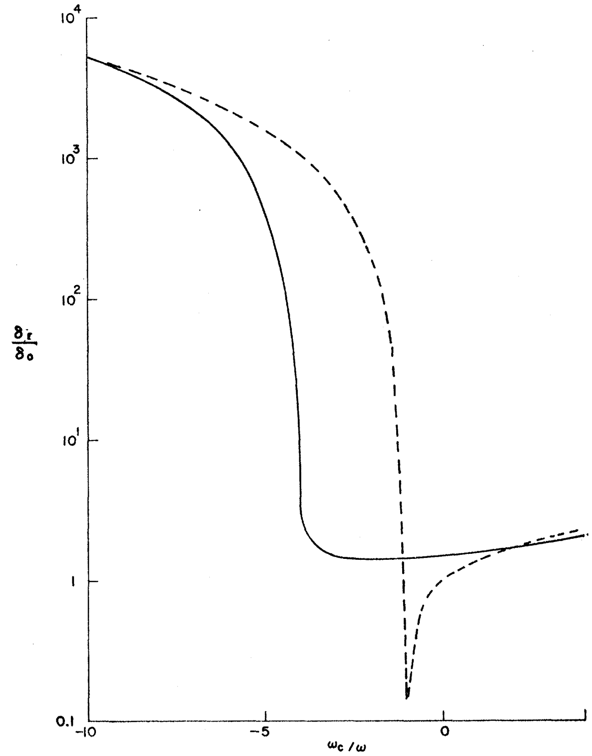

FIG. 3. Plot of the resistive skin depth δ_r/δ_0 vs ω_c/ω for the parameters of Fig. 1. The dashed curve is the local theory. Intermediate regions of the solid graph have been found by interpolation.

CONCLUSIONS

Previous discussions of cyclotron resonance with H perpendicular to the surface have been based on either: (a) a local theory,[1] valid only in high magnetic fields, or (b) an anomalous theory which has been evaluated for small magnetic fields[5] only, where the surface impedance is nearly independent of field. The present calculation shows where the transition from type (a) behavior to type (b) behavior occurs and interprets the transition in terms of a Doppler effect. For a local theory a cyclotron resonance, i.e., a peak in dR/dH, occurs at $\omega \simeq |\omega_c|$. The present work shows that there is a resonance at a Doppler shifted frequency so that the resonant condition is

$$\omega \simeq |\omega_c| - v_F q, \tag{23}$$

where q is the wave vector of the microwave field in the metal. The resonance relation (23) differs from the classical resonance relation $\omega = |\omega_c|$ in that: (a) the resonance of (23) occurs at a larger magnetic field, and (b) *a plot of resonant magnetic field versus frequency does not extrapolate through the origin.* A large kink recently observed in Bi at 9 kMc/sec[11] with H along the binary axis has both properties (a) and (b) and, hence, is believed to be a Doppler shifted resonance described by Eq. (23). Since Bi has three nonequivalent anisotropic carriers with H along the binary axis, the q value entering into Eq. (23) is, however, not simply given by the single isotropic carrier model [Eq. (22)].

If a local theory is used to interpret cyclotron resonance data of the above type, the effective masses derived from such data may be seriously in error. In general, the magnitude of the error (in the single isotropic carrier model) will depend on both parameters $\omega\tau$ and $\omega\delta_0/v_F$. In the limit of large τ the error will decrease with increasing $\omega\delta_0/v_F$. For the parameters chosen in Figs. 1–3, it is evident that use of the local theory leads to an effective mass which is about six times too large.

[11] J. Kirsch and A. G. Redfield, Bull. Am. Phys. Soc. 7, 477 (1962).

The Doppler effect is also closely related to the existence of undamped electromagnetic waves in metals. Such waves have been discussed by Buchsbaum and Galt[10] for a charge neutral system with two carriers (Alfvén waves) and by Aigrain[12] for a charged plasma ("helicon" waves). These discussions are based on the local theory approximation and hence are valid in the high-magnetic-field region. However, in order to find the magnetic field onset of such undamped waves, one must take into account the Doppler effect, as has been carried out here for the single isotropic carrier model.

ACKNOWLEDGMENTS

We thank E. N. Adams, W. P. Dumke, J. Kirsch, and A. G. Redfield for helpful discussions of the present work. We also thank J. Kirsch for carrying out a computer integration of $R(H)$ given by Eq. (13).

[12] P. Aigrain, *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Czechoslovakia Academy of Sciences, Prague, 1961), p. 225.