underestimates the true value of ϵ, this cannot be by more than some 30%.

V. RESULTS AND CONCLUSIONS

The results of the g-factor measurements for the 2^+ states of the tungsten isotopes are summarized in Fig. 2. The values corresponding to calculations of Nilsson and Prior[11] and values corresponding to $g=Z/A$ are also shown. It is evident that whereas the values measured by Kegel[4] and Bodenstedt *et al.*[9] are close to Z/A, the values found in the present measurements are considerably lower than even the values calculated in reference 11. However, it is interesting to note that the trend of the g factors as a function of A closely follow the predictions of that theory and there is an almost constant ratio of about 0.8 between the measured g factors and those calculated by Nilsson and Prior.

[11] S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **32**, No. 16 (1960).

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Excitation of Two-Phonon States by Inelastic α-Particle Scattering

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Recent experiments on inelastic scattering of 40-MeV α particles have shown that the angular distributions from the excitation of some known 4^+ states in medium weight nuclei are not in accord with the Blair phase rule. Calculations are reported which indicate that the anomaly arises as an interference effect between two possible mechanisms for exciting the first 4^+ state, i.e., a direct transition and a multiple transition. This conclusion is contrary to previous interpretations based on plane-wave perturbation calculations. It is suggested that experiments of this type, and the analysis given here, promise to be a useful tool for the study of higher excited states.

THE Blair phase rule,[1,2] for the inelastic scattering of strongly absorbed particles, states that the angular distributions corresponding to even values of the angular momentum transfer L are out of phase with those corresponding to odd-L transfer. Furthermore, the elastic scattering angular distributions should be in phase with the odd-L transfer angular distributions. The conditions for the validity of the phase rule should be adequately fulfilled by 40-MeV (α,α') reactions exciting low-lying collective levels of even–even nuclei. The rule has been extensively verified for the excitation of the lowest 2^+ and 3^- collective states.

Recent experiments[3,4] have shown that the rule is not obeyed by the inelastic angular distribution of 43-MeV α particles exciting known 4^+ states of nuclei in the nickel region. The experimental distributions for the 4^+ levels are found to be almost exactly out of phase with the 2^+ distributions.

In the vibrational model, the first 4^+ state is interpreted as an excitation containing two quadrupole phonons. Hence, if quadrupole vibrations alone are considered, the state can only be reached by second-order processes. The processes may be calculated if one assumes that the nuclear surface is capable of quadrupole deformations defined by

$$R(\theta,\phi)=R_0[1+\sum_m \alpha_m Y_2^m(\theta,\phi)]. \qquad (1)$$

Assuming that the nuclear potential V seen by the α particle depends only on the distance from the surface we have, to second order

$$V(r-R)=V(r-R_0)-\sum_m \alpha_m Y_2^m(\theta,\phi)R_0\frac{dV}{dr} +\tfrac{1}{2}[\sum_m \alpha_m Y_2^m(\theta,\phi)]^2R_0^2\frac{d^2V}{dr^2}.$$

The dynamical distortion parameters α_m create or annihilate phonons of angular momentum 2, z component m. $V(r-R_0)$ is taken to be the usual type of

* Operated by Union Carbide Nuclear Company for the U. S. Atomic Energy Commission.
[1] J. S. Blair, Phys. Rev. **115**, 928 (1959).
[2] E. Rost and N. Austern, Phys. Rev. **120**, 1375 (1960).
[3] H. Broek, J. L. Yntema, and B. Zeidman (to be published).
[4] R. Beurtey, P. Catillon, R. Chaminade, M. Crut, H. Farraggi, A. Papineau, J. Saudinos, and J. Thirion, Compt. rend. **252**, 1756 (1961). The results of references 3 and 4 are also contained in the *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Academic Press Inc., New York, 1961).

complex Saxon–Woods potential while dV/dr and d^2V/dr^2 are taken to be the derivatives of the real part of V evaluated for $\alpha_m=0$.

The expression (2) defines two possible second-order excitation mechanisms for the 4^+ state which may be designated multiple excitation and direct two-phonon excitation. Multiple excitation arises from the second term in (2), operating twice, and describes a process which goes via the first 2^+ state, whereas direct two-phonon excitation comes from the third term of (2), operating once.

Lemmer, de-Shalit, and Wall,[5] using plane-wave perturbation theory, concluded that the multiple excitation process is negligible and that the direct two-phonon mechanism, by itself, does indeed give the correct (i.e., the experimental) angular distribution both in phase and magnitude. The more complete calculations reported here show these conclusions to be incorrect. The trouble lies in their use of plane waves for the incident and scattered α particles.

The processes have now been calculated exactly from the Schrödinger equation for the collectively coupled scattering states coming from the 0^+ ground state and the 2^+ and 4^+ levels. The resulting sets of coupled differential equations were solved numerically on the IBM 7090 at Oak Ridge.

The value of the root-mean-square deformation, $\beta=[\langle\sum_m|\alpha_m|^2\rangle]^{\frac{1}{2}}=0.2$, was derived from Coulomb excitation $B(E2)$ measurements and the optical potentials chosen so that the elastic and 2^+ differential cross sections for $Ni^{58}(\alpha,\alpha')$ at 43 MeV were in agreement with experiment. The results are shown in Fig. 1, where the curves are labeled 0^+ and 2^+. Everything in the model is now fixed, and so the 4^+ differential cross section is automatically predicted.

However, it is of interest to study the multiple process and the direct two-phonon process by themselves, as well as their combined effect. The results are presented in Fig. 2. The upper solid curve is the 2^+ angular distribution. The dotted curve is the result of including only the direct two phonon mechanism for reaching the 4^+ state, while the dashed curve shows the result of using only the multiple process for exciting this level. It should be noted that the multiple process actually has a larger over-all cross section than the direct process. Also, both curves are shifted slightly in phase relative to the 2^+ curve and in opposite directions; but neither can be said to be out of phase with the 2^+ angular distribution. Both of these results are contrary to the conclusions of Lemmer *et al.*

The lower solid curve in Fig. 2 shows the effect of including both excitation mechanisms. Here we find the interesting result that the amplitudes for the two mechanisms combine in such a way as to give a distribution out of phase with the 2^+ distribution and in agreement with the experimental indications. Hence, we conclude that the anomalous phase of the 4^+ angular distribution arises as a destructive interference effect between two contributing mechanisms. A more extended discussion of this effect has been given by Austern.[6] The simple Fraunhofer diffraction model[7,8] also predicts that the 4^+ curve should be out of phase with the 2^+ distribution. The reason is that this model, in effect, uses distorted waves rather than plane waves and implicitly includes both the multiple and the direct two-phonon contributions.

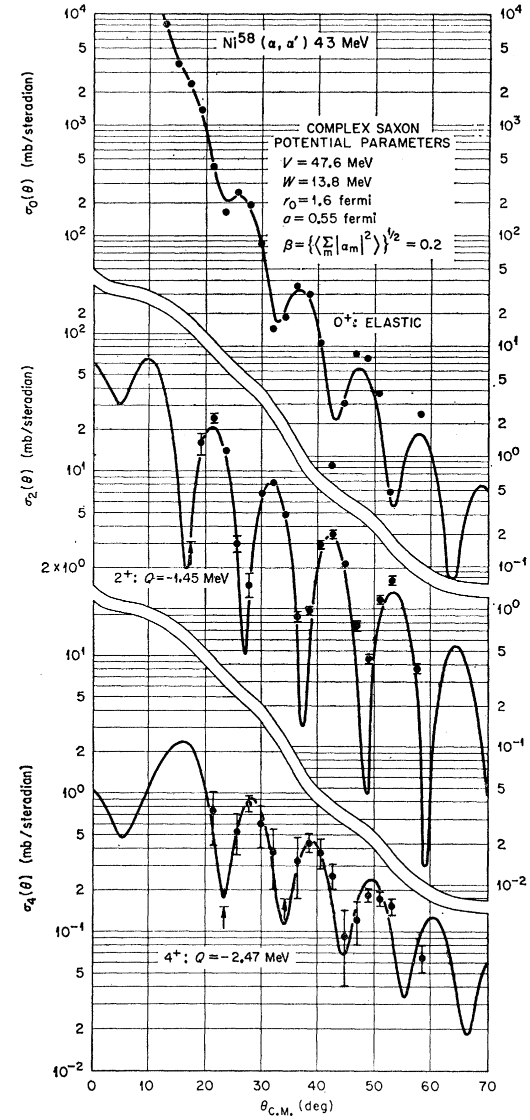

FIG. 1. Comparison of theory and experiment for 43-MeV α-particles incident on Ni^{58}. The 4^+ curve is the prediction of the vibrational model except that the contribution of the direct two-phonon amplitude has been increased by a factor of 1.5. The experimental points are taken from reference 3.

[5] R. H. Lemmer, A. de-Shalit, and N. S. Wall, Phys. Rev. **124**, 1155 (1961).

[6] N. Austern (to be published).

[7] S. I. Drozdov, Soviet Phys.—JETP **38**, 499 (1960).

[8] J. S. Blair (private communication to J. Thirion).

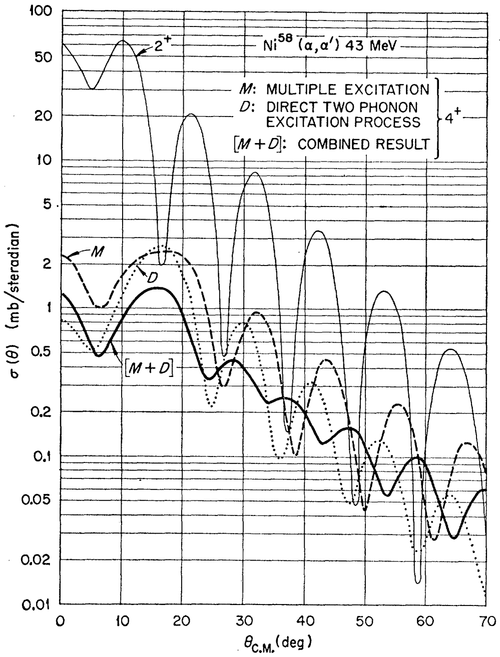

FIG. 2. Comparison of the multiple and the direct two-phonon mechanisms for exciting the first 4^+ state in Ni^{58}, as predicted by the simple vibrational model. The 2^+ distribution is included as a reference curve for exhibiting phase relations. The lower solid curve is the result of including both excitation mechanisms.

The theoretical 4^+ curve in Fig. 2 does not appear to oscillate as strongly as indicated by experiment. But excellent agreement between theory and observation is obtained if the contribution of the direct two-phonon mechanism is increased by a factor of 1.5 relative to the prediction of the simple vibrational model. The theoretical 4^+ distribution now oscillates more strongly but retains nearly the same phase as before relative to the 2^+ curve. The final comparison with experiment is presented as the curve labeled 4^+ in Fig. 1. It should be pointed out that the magnitude of the 4^+ cross section, as well as its angular distribution is given correctly.

The necessity for increasing the direct two-phonon contribution indicates some breakdown of the simple vibrational model for nuclei in this mass region. This is also suggested by the fact that the energy ratio for the 4^+ and 2^+ levels in Ni^{58} is not equal to 2.

One possible explanation of the deviation from the predictions of the pure quadrupole vibration model is that the first 4^+ state includes some admixture of the state containing a single 4^+ phonon. The main component of the single 4^+ phonon usually occurs at a higher energy ($\sim$5 MeV). This suggested admixture would account for the above factor of 1.5, because (i) the direct transition would then appear to be enhanced and (ii) the coupling between the 2^+ and 4^+ states in the multiple process would be decreased by state normalization effects. If one continues to employ the pure quadrupole vibration picture, then the combination of these admixture effects would result in an apparently increased contribution of the direct two-phonon mechanism over the multiple mechanism.

Because of the sensitivity to interference between the various contributions, this kind of reaction, together with the type of analysis reported here, promises to be a valuable way to study and interpret the higher collective states.

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