Note on an Isomeric State of Po^{212}†

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A shell-model calculation for Po^{212} involving the four nucleons outside the Pb^{208} core was performed to show how the shell model can account for a recently discovered high-spin isomer of Po^{212}.

IN the preceding paper, the discovery of an isomeric state of Po^{212} was reported.[1] The isomer, which lies at 2.93 MeV, has a half-life of 45.1 sec and α decays to the ground state of Pb^{208} with α-particle energy 11.65 MeV. No internal de-excitation of the isomer was detected. As discussed in that paper, the large α-hindrance factor suggests that the spin of the isomer is very large and the absence of γ rays or converted electrons corresponding to its internal decay suggests that no levels of spin differing by less than five units lie below the isomer.

In this note we show how the shell model can account for the existence of such an isomer.

Po^{212} has two protons and two neutrons beyond the doubly magic Po^{208} core. Levels in Pb^{208} up to 4 MeV are known, and all have negative parity. We therefore expect that the low-lying positive-parity states of Po^{212} can be described to good approximation in terms of the four extra-core nucleons. The single-particle levels beyond the magic core are shown in Fig. 1. The various four-particle configurations of Po^{212} are therefore $(h_{9/2}^2 g_{9/2}^2)$, $(h_{9/2}^2 i_{11/2}^2)$, etc. From the single-particle states shown, 20 such *basic* particle configurations are generated for positive-parity states. Each basic configuration itself generates a multitude of *sub*configurations for any given total angular momentum J, corresponding to the different J_n and J_p in the configuration $|(j_n^2)J_n, (j_p^2)J_p; J\rangle$. Thus, for example, the spin $J=6$ can be formed in twenty different ways from the configuration $(h_{9/2}^2)J_n$, $(i_{11/2}^2)J_p$. Hence, for some spin states, one would have to compute and diagonalize matrixes of dimension approx 400 to solve the problem as so far stated. Fortunately this is not necessary, as will be explained.

The most important matrix elements of the interaction are those that connect members of the same *basic* configuration. They are [see Appendix]

$$\langle (j_p^2)J_p,(j_n^2)J_n; J|\sum V_{ij}|(j_p^2)J_p',(j_n^2)J_n'; J\rangle$$
$$=\{\langle j_p^2; J_p|V_{12}|j_p^2; J_p\rangle + \langle j_n^2; J_n|V_{34}|j_n^2; J_n\rangle\}\delta_{J_pJ_p'}\delta_{J_nJ_n'}$$
$$+4\sum_{I,K}\begin{Bmatrix} j_p & j_n & I \\ j_p & j_n & K \\ J_p & J_n & J \end{Bmatrix}\begin{Bmatrix} j_p & j_n & I \\ j_p & j_n & K \\ J_p' & J_n' & J \end{Bmatrix} \times\langle j_pj_n; I|V_{13}|j_pj_n; I\rangle. \quad (1)$$

Since there are four particles, the diagonal terms contain six interactions, and the off-diagonal terms four. The energy scale is therefore about six times as great as for a two-particle spectrum. Matrix elements connecting different basic configurations have only one term and are given by

$$\langle (j_p^2)J_p,(j_n^2)J_n; J|\sum V_{ij}|(j_p'^2)J_p',(j_n^2)J_n'; J\rangle = \langle j_p^2; J_p|V_{12}|j_p'^2; J_p\rangle\delta_{J_pJ_p'}\delta_{J_nJ_n'}. \quad (2)$$

Whereas all terms in (1) contain diagonal two-particle matrix elements, (2) contains only an off-diagonal matrix element. We, therefore, ignore at first the matrix elements (2) connecting two different basic configurations, since they are small compared with (1). The Hamiltonian is thus reduced to block form, each block corresponding to one of the basic particle configurations. The result of diagonalizing the part of the Hamiltonian corresponding to the configuration $h_{9/2}^2 i_{11/2}^2$ is shown in Fig. 2. Notice how only a few levels form such coherent superpositions that they become separated from the multitude of levels generated by this configuration. Proceeding thus for each configuration, one generates a new set of basic states which diagonalize H within the separate subspaces of the basic configurations initially considered. The lowest few such levels are shown in Fig. 3. Now we could include the neglected matrix elements connecting the lowest few levels of different basic configurations. The matrix element connecting the lowest two 0+ levels, which arise, respectively, from $h_{9/2}^2 i_{11/2}^2$ and $h_{9/2}^2 g_{9/2}^2$, is, however, only ~0.23 MeV and shifts the levels by only about 0.1 MeV. Such shifts cannot affect our conclusion, and we, therefore, are justified in ignoring the matrix elements (2) altogether.

The calculations represented in the figures are based

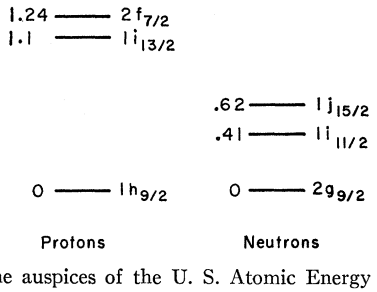

FIG. 1. The single-particle levels beyond the magic Pb^{206} core are shown. The spacings (shown in MeV) are taken from B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 1, No. 8 (1959).

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[1] I. Perlman, F. Asaro, A. Ghiorso, A. Larsh, and R. Latimer, preceding paper [Phys. Rev. 127, 917 (1962)].

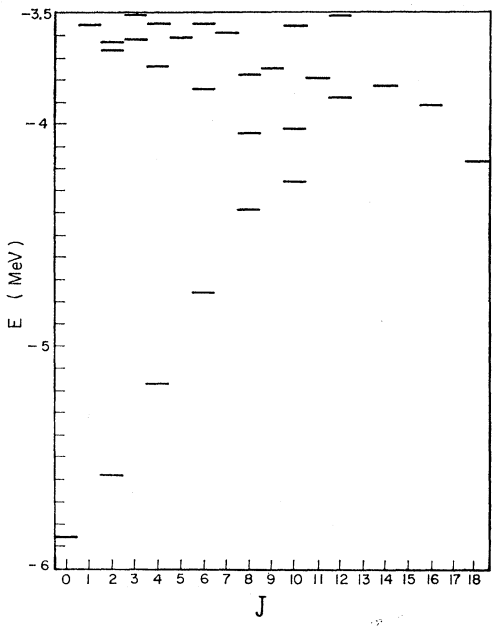

FIG. 2. The lowest levels resulting from the diagonalization of the two-particle interaction among states generated by $h_{9/2}i_{11/2}^2$ are shown. The density of levels at energies above that shown becomes enormous. Notice that the ordering of levels with spin is at first normal for a short-range force, i.e., $J=0, 2, 4, \cdots$.

on the two-body force,

$$V(r)=-(32.9P_{SE}+51.9P_{TE}) \times\exp[-(r/1.732)^2]\ \text{MeV}, \quad (3)$$

where P_{SE} and P_{TE} are projection operators for the singlet-even and triplet-even states.[2] The residual two-body force in nuclei is not well known. However, the odd-state forces for isolated nucleons are weak and possibly repulsive. We therefore neglect them altogether. The triplet-even force for free nucleons is certainly stronger than the singlet-even force, and the interpretation of the Nordheim coupling rules in terms of the residual interaction[3] suggests that the ratio of singlet to triplet is about 0.6. Referring to Fig. 2, we see that the level ordering of the lowest levels is normal for a short-range force, i.e., $J=0, 2, 4, \cdots$. However, the higher spin states appear at a lower energy than some of the intermediate spin states. This happens because the interaction energy between like nucleons becomes very small with increasing spin while the interaction energy between neutron and proton increases as the spin approaches (in steps of two) either of the extreme values $|j_n \pm j_p|$ from intermediate ones.

[2] This is the even part of the Ferrell-Visscher force. One parameter more has to be defined, the oscillator parameter. We adopt the prescription $\nu=M\omega/\hbar=(2N+3)/(1.3A^{\frac{1}{3}})^2$, which makes the classical turning point of the oscillator shell N correspond to the nuclear radius $1.3A^{\frac{1}{3}}$. Since two oscillator shells are involved here, we use an average value, $\nu=0.232$ F^{-2}.

[3] See, for example, M. H. Brennan and A. M. Bernstein, Phys. Rev. 120, 927 (1960) and references contained in this work.

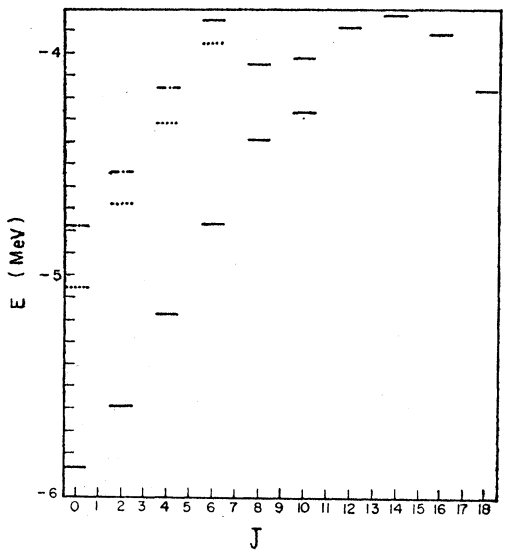

FIG. 3. The lowest levels arising from the configurations $h_{9/2}^2i_{11/2}^2$ (solid lines), $h_{9/2}^2g_{9/2}^2$ (dotted lines), and $h_{9/2}^2j_{15/2}^2$ (dash-dot) are shown. None of the other configurations that can be formed from the single-particle states shown in Fig. 1 appear at energies as low as shown in this figure.

This statement is of course most rigorous when j_n and j_p are large.

Referring again to Fig. 3, we identify the $J=18$ level as the observed isomeric state. No levels of spin greater than 10 occur below it. It appears at an energy $\sim$2 MeV, whereas the observed level appears at $\sim$3 MeV. This is not a serious discrepancy, however, since the residual interaction is not well known. Increasing the singlet-even depth would have the desired effect of depressing the ground state relative to the isomeric state.

The positions of a few levels in Po^{212} are known. However, only the ground-state spin is certain. In view of the large number of parameters in our calculation we do not consider it meaningful to attempt to adjust them to fit the known levels.

APPENDIX

The square brackets in Eq. (1) are the recoupling coefficients for four angular momenta related to the usual $9-j$ coefficients by

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = ([c][f][g][h])^{\frac{1}{2}} \begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix},$$

$$[c]\equiv 2c+1.$$

The sum appearing in Eq. (1) can be written in a form better suited to machine calculations by writing out the $9-j$ symbol in terms of the $6-j$ symbols, and

using the orthonormality of the latter. One finds

$$\langle (j_p^2)J_p,(j_n^2)J_n;J|\sum V_{ij}|(j_p^2)J_p',(j_n^2)J_n';J\rangle=(\langle j_p^2;J_p|V_{12}|j_p^2;J_p\rangle+\langle j_n^2;J_n|V_{34}|j_n^2;J_n\rangle)\delta_{J_pJ_p'}\delta_{J_nJ_n'}$$
$$+4([J_p][J_p'][J_n][J_n'])^{\frac{1}{2}}\sum_\nu[\nu]\begin{Bmatrix} j_p & j_p & J_p \\ J_n & J & \nu \end{Bmatrix}\begin{Bmatrix} j_p & j_p & J_p' \\ J_n' & J & \nu \end{Bmatrix}$$
$$\times\sum_I[I]\begin{Bmatrix} j_n & j_n & J_n \\ j_p & \nu & I \end{Bmatrix}\begin{Bmatrix} j_n & j_n & J_n' \\ j_p & \nu & I \end{Bmatrix}\langle j_pj_n;I|V_{13}|j_pj_n;I\rangle.$$

For identical nucleons the matrix element of a general central force is

$$\langle j^2;J|V|j'^2;J\rangle=[l][l']\sum_{LS}(-)^L\begin{bmatrix} l & \frac{1}{2} & j \\ l & \frac{1}{2} & j \\ L & S & J \end{bmatrix}\begin{bmatrix} l' & \frac{1}{2} & j' \\ l' & \frac{1}{2} & j' \\ L & S & J \end{bmatrix}\frac{1+(-)^{L+S}}{2}\sum_k\begin{Bmatrix} l & l & L \\ l' & l' & k \end{Bmatrix}\begin{pmatrix} l & k & l' \\ 0 & 0 & 0 \end{pmatrix}^2 R_S{}^k(ll|l'l'),$$

where the subscript $S=0$, 1 on the Slater integral refers to the singlet-even and triplet-odd force, respectively.

For nonidentical nucleons,

$$\langle j_pj_n;J|V|j_p'j_n';J\rangle=(\tfrac{1}{2})(-)^{l_p+l_p'}([l_p][l_n][l_p'][l_n'])^{\frac{1}{2}}\sum_{LS}\begin{bmatrix} l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \\ L & S & J \end{bmatrix}\begin{bmatrix} l_p' & \frac{1}{2} & j_p' \\ l_n' & \frac{1}{2} & j_n' \\ L & S & J \end{bmatrix}$$
$$\times\sum_k\Bigg[(-)^L\begin{Bmatrix} l_p & l_n & L \\ l_n' & l_p' & k \end{Bmatrix}\begin{pmatrix} l_p & k & l_p' \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l_n & k & l_n' \\ 0 & 0 & 0 \end{pmatrix}R_{S+}{}^{(k)}(l_pl_n|l_p'l_n')$$
$$+\begin{Bmatrix} l_p & l_n & L \\ l_p' & l_n' & k \end{Bmatrix}\begin{pmatrix} l_p & k & l_n' \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l_n & k & l_p' \\ 0 & 0 & 0 \end{pmatrix}R_{S-}{}^{(k)}(l_pl_n|l_n'l_p')\Bigg],$$

where $R_{S\pm}$ are the Slater integrals of the various combinations of the singlet-even ($V_0{}^+$), singlet-odd ($V_0{}^-$), triplet-even ($V_1{}^+$) and triplet-odd ($V_1{}^-$) parts of the central force;

$$V_{S\pm}=V_0{}^+\pm V_0{}^-, \quad \text{for} \quad S=0,$$
$$=V_1{}^+\pm V_1{}^-, \quad \text{for} \quad S=1.$$

Finally, the Slater integrals are

$$R^{(k)}(l_1l_2|l_3l_4)=\int\int R_{l_1}{}^*(r_1)R_{l_2}{}^*(r_2)f_k(r_1r_2)R_{l_3}(r_1)R_{l_4}(r_2)r_1{}^2r_2{}^2dr_1dr_2,$$

$$f_k(r_1,r_2)=\frac{2k+1}{2}\int V(\mathbf{r}_1-\mathbf{r}_2)P_k(\cos\omega_{12})d(\cos\omega_{12}).$$

Harmonic oscillator wave functions were used in evaluating the Slater integrals.