Simple Method for Calculating the Tunneling Current of an Esaki Diode

JAROSLAV KARLOVSKÝ
Popov Research Institute of Radio Communications, Praha, Czechoslovakia
(Received January 3, 1962)

The exact solution of Esaki's equation for the tunneling current is extremely complicated. A method is presented allowing a simple solution. Under the assumption that the distances between the Fermi level and the band edges E_1 and E_2 on both sides of the junction do not exceed $2kT$, the Fermi function is approximated by a straight line. The resulting equation indicating the voltage dependence of the tunneling current is as follows: $I=A'''V(E_1+E_2-qV)^2$.

In a recent article Bates published a method[1] for calculating the Esaki diode's characteristic and he compared the calculated curves with those published by Esaki.[2] However, the calculation may be much simplified. Let us assume, as the authors named above did, that the tunneling probabilities in both directions $Z_{c\to v}$ and $Z_{v\to c}$ have the same value and do not depend upon the external voltage V suppressed. Further we assume that the densities of states in the conduction band ρ_c on the n side and in the valence band ρ_v on the p side of the junction vary as $(E-E_c)^{\frac{1}{2}}$ and $(E_v-E)^{\frac{1}{2}}$, respectively. If the distance between the Fermi level and the band edge on both sides of the junction does not exceed $2kT$ (see Fig. 1),

$$E_1=\xi_n-E_c\leqq 2kT,\quad E_2=E_v-\xi_p\leqq 2kT, \tag{1}$$

we are allowed to approximate the Fermi distribution by a straight line:

$$f_c=\tfrac{1}{2}-(E-\xi_n)/4kT,\quad f_v=\tfrac{1}{2}+(\xi_p-E)/4kT. \tag{2}$$

Starting from the original Esaki equation,[2]

$$I=A'\int_{E_c}^{E_v}[f_c(E)-f_v(E)]Z\rho_c(E)\rho_v(E)dE, \tag{3}$$

and using the previous assumptions, we obtain a simple relation:

$$I=A''\int_{E_c}^{E_v}\frac{\xi_n-\xi_p}{4kT}(E-E_c)^{\frac{1}{2}}(E_v-E)^{\frac{1}{2}}dE. \tag{4}$$

For simplifying the integration we transform the zero energy level so that $E_c=0$. Considering that $\xi_n-\xi_p=qV$, we get the final form for the tunneling current component:

$$I=A''\int_0^{E_1+E_2-qV}\frac{qV}{4kT}[E(E_1+E_2-qV-E)]^{\frac{1}{2}}dE =A'''V(E_1+E_2-qV)^2. \tag{5}$$

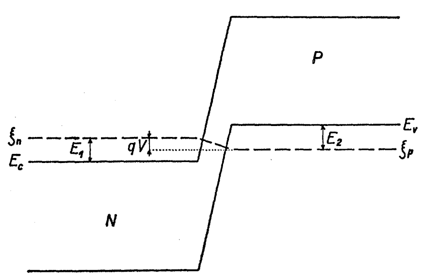

FIG. 1. The band scheme of the junction of an Esaki diode.

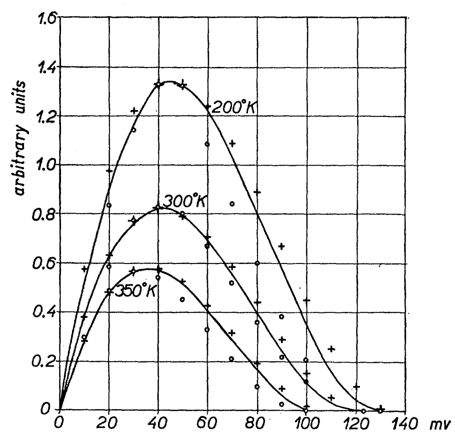

FIG. 2. Comparison of the curves published by Esaki with the points calculated by Bates (circles) and by the author (crosses) at various temperatures. The author's points are derived from the peak points with the following coordinates: For 200°K, $V_p=45$ mv, $I_p=1.33$ (in arbitrary units); for 300°K, $V_p=41$ mv, $I_p=0.82$; for 350°K, $V_p=36$ mv, $I_p=0.58$.

Figure 2 shows a comparison of the curves published by Esaki[2] and by Bates,[1] with the curve calculated with the help of Eq. (5). Good agreement is evident; especially at normal and elevated temperatures our curve is more closely adjacent to that of Esaki. The constant in Eq. (5) is chosen so that the peak of all the three curves be the same.

[1] C. W. Bates, Phys. Rev. **121**, 1070 (1961).
[2] L. Esaki, Phys. Rev. **109**, 603 (1958).