From Eqs. (23), (24), and (25), we obtain the desired relationship between ξ_0 and $\xi_0\lambda_L^2(0)$ [or ξ_0 and $\lambda_L(0)$]:

$$\xi_0=\hbar^2c^2k^2/4\pi e^2\epsilon_0^2(0)\gamma\xi_0\lambda_L^2(0). \tag{26}$$

If we substitute into Eq. (26), $\xi_0\lambda_L^2(0)=0.319\times10^9$ (Å)^3, obtained by extrapolating to $T=0$°K, the measured values for $\xi_0\lambda_L^2$; $\epsilon_0(0)=1.75kT_c$, where $T_c=3.41$°K; and $\gamma=1.7$ mJ/mole-deg^2, which is the average of calorimetric values obtained by Clement and Quinnell,[17] and Bryant and Keeson,[18] we obtain $\xi_0=2800$ Å, in remarkable agreement with 2600±400 Å obtained by curve-fitting. From Eq. (24), we obtain $\rho l=0.98\times10^{-11}$ Ω-cm^2 for the above values of $\xi_0\lambda_L^2(0)$ and $\epsilon_0(0)$. This value for ρl is quite a bit larger than the value of 0.57×10^{-11} Ω-cm^2 reported by Dheer, but agrees reasonably well with the value of 0.89×10^{-11} Ω-cm^2 obtained by Roberts.

Although thickness effects have been emphasized in this paper, impurity effects can also be calculated through their effect upon ξ and this will be the subject of a subsequent paper. For according to the Pippard theory,

$$1/\xi=1/\xi_0+1/l. \tag{27}$$

In addition, this model can be extended to properties of films other than the critical fields. For through Eqs. (6) and (10), nonlocal equations for δ_0 may easily be obtained. From these, nonlocal relations for surface energy, critical current, etc., might be obtained and compared with experiment. Finally, it is clear that although only the Pippard kernel has been discussed in this paper, calculations could be carried out for other kernels.

[17] J. R. Clement and E. H. Quinnell, Phys. Rev. **92**, 258 (1953).
[18] C. A. Bryant and P. H. Keesom, Phys. Rev. Letters **4**, 460 (1960).

ACKNOWLEDGMENTS

The author would like to express his appreciation to Miss G. Butts for programming Eq. (13) for the IBM 7090 computer and to Dr. R. R. Haering, Dr. P. B. Miller, and Dr. P. M. Marcus for many helpful suggestions concerning the manuscript.

PHYSICAL REVIEW VOLUME 127, NUMBER 2 JULY 15, 1962

Critical Field of Thin Superconducting Shapes

J. J. Hauser and E. Helfand
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received February 5, 1962; revised manuscript received March 20, 1962)

Considerations of the thermodynamics pertaining to the critical field of small superconducting samples of various shapes results in an explicit relation for the ratio of the critical field of small samples to that of bulk in terms of the magnetic moment. The magnetic moment has been calculated using Miller's modification of the Bardeen-Cooper-Schrieffer kernel which includes mean free path. The critical-field ratios of various shapes in decreasing order are sphere, cylinder in parallel field, cylinder in transverse field, and plate in parallel field. The findings are compatible with the fact that dislocations (cylinder like) may be the filaments responsible for hard superconductivity. Under certain conditions the filaments could be numerous and large enough for an appreciable fraction of the sample to appear superconducting in a specific heat measurement. The size of the filaments would also account for the lack of latent heat observed in hard superconductors. Because of the relative orientation of dislocations with respect to the applied field, not all dislocations will serve equally as filaments, thus explaining the current density vs critical-field curve and accounting for an anisotropic critical field when there is a preferred orientation of dislocations.

I. INTRODUCTION

The problem under consideration is the calculation of the stabilization, with respect to transition to normal state in a magnetic field, of a superconducting sample of small size. The transition of a material in a magnetic field is the result of the competition of two effects; the lower Gibbs free energy of electrons in the superconducting state, and the increase in free energy caused by the Meissner effect. When the sample is small, however, a nonvanishing fraction of the sample is penetrated by the magnetic field so that field exclusion is not as strong a destabilization factor.

In this paper, some of the thermodynamics involved in the problem will be reviewed. The relations for the critical fields will be derived in terms of the London and Bardeen-Cooper-Schrieffer (BCS) theories. Finally, explicit results will be obtained for several specimen shapes, and examined with reference to filaments as the possible explanation of hard superconductivity.

II. THERMODYNAMICS

The Gibbs free energy for a superconductor may be expressed as

$$dG=-\mathfrak{M}\cdot d\mathbf{H}_0, \tag{1}$$

where $\mathfrak{M}$ is the total magnetic moment of the sample, and $\mathbf{H}_0$ is the externally applied field.

In order to find the difference between a superconducting and a normal small sample in a magnetic field $\mathbf{H}_0$, Eq. (1) may be integrated along the following path (n = normal, s = superconducting, sm = small sample, b = bulk sample, H_c = bulk critical field, V = sample volume):

(a) $n(\mathbf{H}_0,sm) \to n(\mathbf{H}_0,b)$, $\Delta G = \Delta G_{\text{surf}}$;

(b) $n(\mathbf{H}_0,b) \to n(\mathbf{H}_c,b)$, $\Delta G = 0$;

(c) $n(\mathbf{H}_c,b) \to s(\mathbf{H}_c,b)$, $\Delta G = 0$;

(d) $s(\mathbf{H}_c,b) \to s(0,b)$, $\Delta G = -H_c^2 V/8\pi$;

(e) $s(0,b) \to s(0,sm)$, $\Delta G = -\Delta G_{\text{surf}}$;

(f) $s(0,sm) \to s(\mathbf{H}_0,sm)$, $\Delta G = -\mathfrak{M}\cdot\mathbf{H}_0/2$.

The free-energy change of step (b) is zero because the field self-energy has not been included in G, and $\mathfrak{M}=0$. By definition of equilibrium, (c) has ΔG equal to zero. The free-energy change of (d) is obtained by substituting $\mathfrak{M} = -\mathbf{H}_0 V/4\pi$ in Eq. (1). In (e), it is assumed that there is no difference between the surface free energies or normal and field-free superconducting material. Finally, (f) is obtained from Eq. (1) using the proportionality between $\mathfrak{M}$ and $\mathbf{H}_0$. Equation (1) integrates to

$$G_s - G_n = -(1/2)\mathfrak{M}\cdot\mathbf{H}_0 - VH_c^2/8\pi. \qquad (2)$$

The normal state has been chosen as reference, so that the term $-VH_c^2/8\pi$ represents the stabilization associated with superconductivity, while $-(1/2)\mathfrak{M}\cdot\mathbf{H}_0$ represents the destabilization due to the Meissner effect.

It is not generally correct to study transitions exclusively by Eq. (2), since an intermediate state could have a lower free energy than either pure phase. Nevertheless, for small samples where the B field is more or less uniform over the sample one can expect the range of intermediate state to be narrow, and the transition to occur when $G_s - G_n = 0$; i.e., when

$$H_d = -VH_c^2/4\pi, \qquad (3)$$

where H_d is the critical field of a sample of characteristic distance d. This result is equivalent to that used by other authors.

Relation (3) excludes possible nonlinearities as contained, for example, in the Ginzburg-Landau theory.[1] However, the free-energy difference between superconducting and normal states can be formulated as Ginzburg[2] has shown in his Eq. (2.2) by

$$G_s - G_h = -\tfrac{1}{2}\mathfrak{M}\cdot\mathbf{H}_0 - VH_c^2/8\pi + (VH_c^2/8\pi)(\Psi_0^2-1)^2,$$

where $\Psi_0^2 = \Psi^2/\Psi_\infty^2$ is the order parameter. The extra term represents the additional surface free energy of the Ginzburg-Landau theory. For very thin samples, when Ψ_0 can be considered as constant, the magnetic moment is calculated in the same way as in the London theory except that the penetration depth λ which appears in $\mathfrak{M}$ should be replaced by λ/Ψ_0. In the weak-field limit, where Ψ_0 is approximately unity, Eq. (2) is recovered.

On the other hand, in studying critical fields one must employ the strong-field approximation, where Eq. (3) is replaced by

$$\frac{\Psi_0^2(2-\Psi_0^2)}{-4\pi(\mathfrak{M}/V)\cdot\mathbf{H}_0} = \frac{1}{H_c^2}.$$

For very thin shapes, one may make an expansion of $\mathfrak{M}$ in terms of Ψ_0, which shall be shown below to start with a quadratic term of the form:

$$\mathfrak{M}/V = -(k^2\Psi_0^2 d^2/4\pi\lambda^2)\mathbf{H}_0.$$

Therefore, the transition will occur when

$$H_0^2/H_c^2 = k^2[\Psi_0^2(2-\Psi_0^2)\lambda^2/\Psi_0^2 d^2],$$

where k is a proportionality constant depending on the shape. In the weak-field limit, when $\Psi_0 \simeq 1$, $H_d/H_c = k\lambda/d$ but in the strong-field limit, when $\Psi_0 \to 0$, $H_d/H_c = k\sqrt{2}\lambda/d$. In other words, the nonlinearities may be taken into account for thin samples by introducing a factor of 2 in the right-hand side of Eq. (3), replacing it by

$$H_d = -VH_c^2/2\pi\mathfrak{M}. \qquad (3')$$

III. LONDON THEORY

Equation (3) may be applied to various shapes. The resulting critical fields can be compared with that obtained by London[3] for a thin film of thickness d with a magnetic field parallel to the plane of the film

$$H_d/H_c = [1 - 2\lambda/d \tanh(d/2\lambda)]^{-\frac{1}{2}}. \qquad (4)$$

When $d/2\lambda \ll 1$, Eq. (4) reduces to

$$H_d/H_c = \sqrt{3}/x, \qquad (5)$$

where $x = d/2\lambda$ and λ is the London penetration depth. In the strong-field limit,

$$H_d/H_c = \sqrt{6}/x. \qquad (5')$$

The field outside a cylinder of radius R in a transverse magnetic field is generally given by

$$\mathbf{B} = [H_0(1 - aR^2/r^2)\cos\theta]\mathbf{e}_r - [H_0(1 + aR^2/r^2)\sin\theta]\mathbf{e}_\theta, \qquad (6)$$

where a is related to the magnetic moment per unit length of the cylinder by

$$\mathfrak{M} = -1/2(R^2 a H_0). \qquad (7)$$

By solving the London equation for this geometry,

[1] V. L. Ginzburg and L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 1064 (1950).

[2] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 113 1(958); [translation: Soviet Phys.—JETP **7**, 78 (1958)].

[3] F. London, *Superfluids* (John Wiley & Son, Inc., New York, 1950), Vol. 1.

Schafroth[4] finds

$$a=1-2\lambda I_1(R/\lambda)/RI_0(R/\lambda), \quad (8)$$

where I_0 and I_1 are, respectively, zero- and first-order modified Bessel functions. Substituting (7) into Eq. (3) yields

$$H_d/H_c=\{2[1-2I_1(x)/xI_0(x)]\}^{-\frac{1}{2}}, \quad (9)$$

where $x=R/\lambda$. As x tends to infinity, $H_d/H_c\simeq 1/\sqrt{2}$. This artifact, at first surprising, may be considered to be the result of averaging the **B** field on the surface of the cylinder. If instead, one considers the local field at $\theta=\pi/2$ and $r=R$, then $\mathbf{B}=-2H_0\mathbf{e}_\theta$ and the intermediate state sets in when $H_d=H_c/2$.

When $x\ll 1$, relation (9) reduces to

$$H_d/H_c=2/x; \quad (10)$$

and in the strong-field limit,

$$H_d/H_c=(\sqrt{8})/x. \quad (10')$$

One might question the exact validity of relation (10) on the basis that an intermediate state could develop at a slightly lower critical field if one considered local rather than average magnetic induction. Referring to relation (6)

$$|\mathbf{B}(\theta=\pi/2;\, r=R)|=2H_0[I_1(x)/xI_0(x)-1],$$

which is equal to $-2H_0$, when x is large. When $x\ll 1$, however, $|\mathbf{B}|=H_0(1+\frac{1}{8}x^2)$ and the value actually used in relation (10) was $|\mathbf{B}|=H_0$. Therefore, the maximum deviation from the externally applied field H_0 is only $\frac{1}{8}x^2H_0$ and intermediate state would set in when

$$H_d/H_c=(2/x)[1-\tfrac{1}{8}x^2].$$

Consequently, relation (10) is quite accurate for small enough cylinders.

The critical field of a small cylinder in a parallel field is found, in the same manner, to be

$$H_d/H_c=[1-2I_1(x)/xI_0(x)]^{-\frac{1}{2}}; \quad (11)$$

and when $x\ll 1$, relation (11) reduces to

$$H_d/H_c=(\sqrt{8})/x; \quad (12)$$

and in the strong-field limit,

$$H_d/H_c=4/x. \quad (12')$$

Finally, this calculation can be applied to the case of a sphere where[3]

$$\mathfrak{M}=-1/2(H_0R^3)[1-(3/x)\coth x+(3/x^2)].$$

Substitution of this moment in relation (3) yields

$$H_d/H_c=\{\tfrac{3}{2}[1-(3/x)\coth x+(3/x^2)]\}^{-\frac{1}{2}}. \quad (13)$$

Again as x approaches infinity, $H_d/H_c\simeq\sqrt{(2/3)}$, as a result of the averaging of **B** on the surface of the sphere.

When $x\ll 1$, relation (13) reduces to

$$H_d/H_c=(\sqrt{10})/x, \quad (14)$$

and if one takes into account the correction for intermediate state

$$H_d/H_c=[(\sqrt{10})/x][1-(x^2/30)].$$

In the strong-field limit,[5]

$$H_d/H_c=(\sqrt{20})/x. \quad (14')$$

IV. NONLOCAL THEORY

One is aware of the fact that the London theory is not a proper description of the problem. Schafroth[4] has formulated the magnetic moment of a slab and cylinder in parallel field in terms of an arbitrary kernel. The BCS kernel $K(q)$ is defined by the equation

$$\mathbf{j}(\mathbf{q})=-(c/4\pi)K(q)\mathbf{a}(\mathbf{q}), \quad (15)$$

where $\mathbf{j}(\mathbf{q})$ and $\mathbf{a}(\mathbf{q})$ are, respectively, the Fourier transforms of the supercurrent and the vector potential. The Schafroth[4] kernel $\chi(q)$ is defined by the relation between the Fourier transforms of the magnetization and the induction

$$\mathbf{M}(\mathbf{q})=\chi(q)\mathbf{B}(\mathbf{q}). \quad (16)$$

With the use of the equation $\mathbf{j}=c\nabla\times\mathbf{M}$ one can easily show that, for bulk,

$$\chi(q)=-K(q)/4\pi q^2. \quad (17)$$

In the limit of small q, i.e., the London limit, $K(q)=\lambda_L^{-2}$, and consequently $\chi(q)=-(4\pi q^2\lambda_L^2)^{-1}$.

In the large-q limit, one can use a modification of the BCS kernel introduced by Miller[6] which takes into account the mean free path, l. Therefore, when $\pi q\xi_0\gg 1$, $ql\gg 1$,

$$\chi(q)=-\frac{3}{16q^3\lambda_L^2(T)\xi_0}\left\{1-\frac{16}{\pi^3 q\xi_0}\ln(\pi q\xi_0)-\frac{4}{\pi ql}\right\}, \quad (18)$$

where $\lambda_L(T)$ is the London penetration depth at T°K, and ξ_0 is the coherence distance. Following a theory developed by May and Schafroth,[7] **B** and $\mathfrak{M}$ are expanded in terms of a complete orthonormal set $\mathbf{u}_k$, such that

$$\nabla\times\nabla\times\mathbf{u}_k=q_k^2\mathbf{u}_k, \quad (19)$$

$$\nabla\cdot\mathbf{u}_k=0, \quad (20)$$

$$(\mathbf{u}_k)_{||}=0 \quad \text{on} \quad \Sigma. \quad (21)$$

Here, $(\mathbf{u}_k)_{||}$ denotes the tangential components of $\mathbf{u}_k$ on the surface Σ of the superconductor.

[4] M. R. Schafroth, *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1960), Vol. 10.

[5] The strong-field limits given in relations (5′), (10′), (12′), and (14′) have been derived in a different fashion by V. P. Silin [J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 1330 (1951)].

[6] P. B. Miller, Phys. Rev. **113**, 1209 (1959).

[7] R. M. May and M. R. Schafroth, Proc. Phys. Soc. (London) **74**, 153 (1959).

In the case of a plane slab, where $-\infty<y, z<+\infty$ and $0<x<d$, Schafroth[4] has shown that

$$\mathbf{u}_k=(2/V)^{\frac{1}{2}}\sin(\pi kx/d)\mathbf{e}_z, \tag{22}$$

where $\mathbf{e}_z$ is a unit vector along the z axis, $V=dL^2$ is the volume of the slab, L is the periodicity length along the y and z axes, and $k=1, 2, 3, \cdots$.

It then follows that

$$\begin{aligned} M_k&=[(8V)^{\frac{1}{2}}/\pi k]H_0\chi(q_k)/[1-4\pi\chi(q_k)], \quad k \text{ odd}, \\ M_k&=0, \quad k \text{ even}, \end{aligned} \tag{23}$$

where $q_k=\pi k/d$. The total magnetic moment $\mathfrak{M}$, in the case of specular reflection, can be obtained using

$$\mathbf{M}(\mathbf{r})=\textstyle\sum_k M_k\mathbf{u}_k(\mathbf{r}),$$

and

$$\begin{aligned} M&=\int d^3r\,\mathbf{M}(\mathbf{r})\cdot(\mathbf{H}_0/H_0) \\ &=\sum_{k\text{ odd}}\frac{8V}{(\pi k)^2}H_0\frac{\chi(q_k)}{1-4\pi\chi(q_k)}. \end{aligned} \tag{24}$$

Substituting (24) into Eq. (3) will yield the critical field of a slab for any chosen kernel. If one chooses the London kernel of Eq. (17), the summation in Eq. (24) may be evaluated by contour integration, and one finds again the result described by Eq. (4). If, however, one substitutes in Eq. (24) the BCS-Miller kernel of Eq. (18), the summation can only be performed numerically. After some algebra and substitution in Eq. (3′), one finds

$$H_d/H_c=\frac{2}{x(d/2\xi_0)^{\frac{1}{2}}}[1+0.0825(d/\xi_0)+0.2(d/l)], \tag{25}$$

where $x=d/2\lambda_L$.[8]

For the cylinder in parallel field, the total magnetic moment as given by Schafroth[4] is

$$\mathfrak{M}=\frac{4H_0V}{L}\sum_k\frac{1}{z_k^2}\frac{\chi(z_k/R)}{1-4\pi\chi(z_k/R)}, \tag{27}$$

where $V=\pi R^2L$ is the volume of the period of the cylinder. The z_k's must be chosen so that the boundary condition (21) is satisfied, and consequently are the roots of the equation $J_0(z)=0$. Using the BCS-Miller kernel (18) in Eq. (27) and summing numerically yields, after substitution in Eq. (3′),

$$H_d/H_c=\frac{4.05}{x(R/\xi_0)^{\frac{1}{2}}}[1+0.1(R/\xi_0)+0.25(R/l)], \tag{28}$$

where $x=R/\lambda_L$.

For the cylinder in a transverse field, the same approach is used as the one developed by May and Schafroth[7] for the case of the sphere. The z_k's are now the solutions of $J_1(z)/z-J_0(z)=0$ and the total magnetic moment is given by

$$\frac{2\mathfrak{M}}{R^2H_0-2\mathfrak{M}}=\sum_{k=1}^{\infty}\frac{1}{(z_k^2-1)}\left[\frac{4\pi\chi(z_k/R)}{1-4\pi\chi(z_k/R)}\right]. \tag{29}$$

Substituting relation (29) into Eq. (3′) and following the same technique as before yields

$$H_d/H_c=\frac{2.52}{x(R/\xi_0)^{\frac{1}{2}}}[1+0.14(R/\xi_0)+0.35(R/l)]. \tag{30}$$

In the case of the sphere where May and Schafroth[7] calculated the susceptibility using a Bogoliubov kernel, one obtains, using instead the BCS-Miller kernel:

$$H_d/H_c=\frac{4.87}{x(R/\xi_0)^{\frac{1}{2}}}[1+0.085R/\xi_0+0.21(R/l)]. \tag{31}$$

V. DISCUSSION

If defects such as dislocations are indeed the filaments responsible for the phenomenon of hard superconductivity, one may be able to approximate qualitatively these dislocations by circular cylinders. The small cylinder is a more stable shape with respect to transitions in a magnetic field than a thin film of comparable thickness, as may be seen by comparing the numerical coefficients of Eq. (5), (10), and (12) for H_d/H_c according to the London theory. The greater penetration which one finds in the nonlocal theories enhances this difference, as may be seen from Eqs. (25), (28), and (30).

It is true, however, that the calculations made previously apply only to thin shapes in vacuum. The cylinders that are now considered are eventually surrounded by normal material. One is aware of the fact that the superconducting properties of a thin film are affected by the presence of normal metal in contact with it, as discussed theoretically by Cooper[9] and shown experimentally by Meissner[10] and later on by Smith and his co-workers.[11] However, some doubt is still present in such experiments, as conceivably the effect could be due to diffusion as pointed out by Rose-Innes and Serin.[12] Furthermore, all these experiments were conducted on soft superconductors. In a hard superconductor, where the theory of dirty superconductors is more likely to apply, the superconductivity associated with a dislocation could conceivably be so strongly localized by the strain

[8] Since our calculation, a formula for H_d/H_c in the case of a thin film has been reported by R. A. Ferrell and A. J. Glick at the American Physical Society, New York Meeting, January, 1962 [Bull. Am. Phys. Soc. 7, 63 (1962)]. They (and, independently, A. M. Toxen) obtain the same functional dependence in d, ξ_0, and λ_L as Eq. (25).

[9] L. N. Cooper, Phys. Rev. Letters 6, 689 (1961).

[10] H. Meissner, Phys. Rev. 117, 672 (1960).

[11] P. H. Smith, S. Shapiro, J. L. Miles, and J. Nicol, Phys. Rev. Letters 6, 686 (1961).

[12] A. C. Rose-Innes and B. Serin, Phys. Rev. Letters 7, 278 (1961).

field of that defect, that the surrounding normal material would not influence it appreciably.

In order to obtain an order of magnitude estimate of the effective size of the filaments take $H_d/H_c=30$, which is a typical figure for hard superconductors, $\lambda_L=5\times10^{-6}$ cm and $\xi_0=2.5\times10^{-5}$ cm. For the most favorable orientation of filaments, i.e., parallel to the field, the first term of Eq. (28) yields $R\simeq2\times10^{-6}$ cm. Thus, for a well deformed sample or for an inhomogeneous intermetallic compound, the density of dislocations could be as high as 4×10^{10} per cm^2, which corresponds to a mean separation of 5×10^{-6} cm, approximately the diameter of the filaments calculated above. This may explain why at 70 kG Morin *et al.*[13] have found that the electronic specific heat corresponds to a large fraction of the electrons still being superconducting. In the case of niobium it was found[14] that the bulk critical field is approximately 1 kG at 4.2°K while the ultimate critical field under zero current, probably due to the best filaments, is about 12 kG. For an H_d/H_c ratio of 12, using the same penetration depth and coherence length as before, one finds an effective radius of 4×10^{-6} cm. Morin has conducted specific heat measurements on samples which were only deformed by tension, and consequently may be expected to have a mean dislocation separation of the order of 10^{-4} cm. An overwhelming fraction of the material is therefore bulk, and Morin[13] finds predominantly normal electronic specific heat curves in fields exceeding the bulk critical field.

The concept of filaments also predicts the absence of a latent heat for the superconducting to normal transition in hard superconductors, such as V_3Ga, where the filaments are nearly overlapping. On the basis of the Ginzburg-Landau theory, Ginzburg[15] and Douglass[15] have shown that for small samples, of characteristic distance less than $(\sqrt{5})\lambda$, the energy gap goes continuously to zero as the critical field is approached, so that the transition is second order. The filaments definitely satisfy this size condition so that they will not release any latent heat when they turn normal.

[13] F. J. Morin *et al.*, Bull. Am. Phys. Soc. **7**, 190 (1962).
[14] J. J. Hauser and E. Buehler, Phys. Rev. **125**, 142 (1962).

Comparison of relations (28) and (29) for cylinders in parallel and transverse field indicates that cylindrical filaments will have a different critical field depending on their relative orientation with respect to the externally applied field. Consequently, not all dislocations will have the same stability with respect to transition in the field, and the current will tend finally to run in the dislocations closest to the direction of the applied field. A preferred alignment of dislocations resulting from some mechanical treatment would, therefore, lead to anisotropic properties of the critical fields such as the one reported by LeBlanc and Little as well as by Berlincourt, Hake, and Leslie[16] on Nb-Zr alloys. The anisotropy in critical field as related to anisotropy in plastic deformation has actually been observed by Hauser[17] on Re single crystals which can be deformed predominantly on one slip system.

ACKNOWLEDGMENT

The authors are indebted to Dr. R. L. Garwin for a helpful discussion.

[15] V. L. Ginzburg, Doklady Akad. Nauk. S.S.S.R. **83**, 385 (1952); D. H. Douglass, Jr., Phys. Rev. Letters **6**, 346, (1961).
[16] M. A. R. LeBlanc and W. A. Little, *Proceedings of the Seventh International Conference on Low-Temperature Physics, Toronto, 1960* (University of Toronto Press, Toronto, 1961), p. 362. T. G. Berlincourt, R. R. Hake, and D. H. Leslie, Phys. Rev. Letters **6**, 671 (1961).
[17] J. J. Hauser, J. Appl. Phys. (to be published).