New Methods in Direct Interaction Theory*

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Methods developed in modern field theory are applied to deuteron stripping. The contribution associated with the direct interaction mechanism is written as the product of two parts, one representing deuteron disassociation in the field of the target and the other nucleon capture by the target. The capture amplitude is written as a dispersion relation and evaluated in terms of the bound-state wave function and the optical model phase shifts for nucleon-nucleus scattering in the capturing channel. The disassociation is also expressed as a dispersion relation and evaluated in terms of the same nucleon-nucleus scattering and also deuteron-target scattering on the energy shell. It is shown that the formalism allows for the distortion of the capture vertex characterized normally by an unusually large nuclear radius, and also explains the persistence of an angular distribution typical of the cutoff Born approximation in spite of a fluctuating yield for the reaction.

I. INTRODUCTION

THE theory and practice of nuclear direct reactions has received much attention.[1] These reactions offer a wide range of striking experimental characteristics, many of which have simple structural interpretations. To the theoretist these reactions offer both the practical challenge of disentangling this structure and also the formal problem of describing the reactions in general. In this paper we present a theory of direct nuclear reactions based on techniques recently developed in the theory of elementary particles which helps to clarify the formal problems and also provides a new basis for analysis of experiment.

The classic nuclear direct reaction is deuteron stripping, and we shall concentrate on it. Much of what we say, however, is of wider validity. In a deuteron stripping reaction of the type $A(d,p)B$ the basic process is the removal of the neutron from the deuteron and its capture directly into the bound state B. This process is described without frills in the Born approximation. The success of this approximation or better of the modified or cutoff Born approximation of Butler[2] in accounting for angular distributions and in aiding the extraction of information on nuclear structure is embarassingly good. However, the success is certainly not perfect and a number of attempts to improve the theory have been made. The most extensive is the distorted-wave Born approximation,[3] in which both incoming and outgoing waves are corrected for elastic rescattering. This calculation involves a number of parameters and assumptions, such as an optical model for deuteron-A scattering and further involves much computation, so that the effect of these parameters is difficult to control. Nevertheless, it gives an even better account of the data, again particularly for angular distributions. The total yield of the reaction is not so easily obtained, particularly as it sometimes shows considerable variation with bombarding energy in spite of the continuing fairly rigid conformity of the angular distribution to the cutoff Born approximation. Of course, the yield predicted by a Born approximation shows no such variation. In particular, this variation often shows some correlation with the variations in the total deuteron-A scattering cross section.[1] Since an optical model fitted to the experimental deuteron-A scattering is used in the distorted wave theory, these oscillations in yield are often accounted for by that theory, but not in a very transparent way. At least part of the problem then of any theory should be to justify the apparent contradiction of an angular distribution fitted at least approximately by the Born approximation and a yield given in terms of something related to deuteron-A scattering. Furthermore, it would be interesting to have some justification for the role of the cutoff in the Butler theory and some method for obtaining it from theory, or from some independent experiment rather than using it as a fitting parameter.

Formally stripping is much more complex than elastic scattering, which we can describe by a potential and hence reduce to simple numerical solution or perhaps even exact solution. No exact solution of stripping exists and even in a potential model there are serious formal problems. In elastic scattering if no exact solution exists, the Born series often serves as a convenient formal solution for investigating the structure of amplitudes. In rearrangement collisions this series has no validity, since both of the two potentials essential to the problem are strong enough to support bound states.[4] In stripping, these are the neutron-proton potential and the neutron-A potential with bound states d and B, respectively. In addition, to making the Born series invalid even as a formal tool, this fact coupled with the identity of the particles in the target (A) and the neutron and proton of the deuteron make the usual methods of adiabatic switching very difficult to apply. A further problem peculiar to the formal theory of rearrangement

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[1] A review and comprehensive list of references will be found in *Proceedings of the International Conference on Nuclear Structure* (University of Toronto Press, Toronto, 1960).
[2] S. T. Butler, *Nuclear Stripping Theory* (John Wiley & Sons, Inc., New York, 1957).
[3] See W. Tobocman, Phys. Rev. **115**, 928 (1959).
[4] R. Aaron, R. D. Amado, and B. W. Lee, Phys. Rev. **121**, 319 (1961).

collisions is the multiplicity of initial and final Hamiltonians describing the initial and final asymptotic states and the concomitant lack of orthogonality of these states.[5]

These problems are very similar to ones which are met in elementary particle theory some even in elastic scattering. For example, in stripping, if we try to reduce the magnitude of the neutron-proton interaction in order to make the Born series valid, the deuteron itself disintegrates. This is similar to the problem of π-nucleon scattering where if one reduces the π-nucleon interaction the nucleon itself disintegrates as it is also in some sense composed of pions and nucleons. These problems have been met in field theory by the methods of asymptotic operators, contraction rules, and dispersion relations.[6] In this paper, we apply these methods to stripping.

In attempting to carry these techniques over from field theory we encounter a number of problems. First, we find that the amplitude we are led to by a straightforward application of the analogy to field theory is zero, while other terms of opaque structure which are zero in field theory contain the entire stripping amplitude. This difficulty can be overcome by making the analogy closer through changing some of the bound particles into elementary particles, particularly B. The method for making such a replacement and the partial justification for it has been given by Vaughn, Aaron, and Amado.[7] We make no further justification of the method here but feel certain that the final answer is correct independent of the use of this construction. One might be concerned whether in doing this, or in making other assumptions about bound states and analytic properties, we encounter anomalous thresholds.[8] We assume that by putting in Schrödinger wave functions, where necessary, and by using nonrelativistic forms throughout, we never meet them.[9]

The approach we shall use to the stripping problem is the same as the one which has proved successful in solving $V-\theta$ scattering in the Lee model.[10] The basic process in stripping is the capture of the neutron from the deuteron by A directly into the state B. In order for this to occur, the deuteron must first disassociate. If it does independently of the field of A, and if the neutron is captured into B independently of the proton, the process can be described by the Born approximation. In our formulation, capture of the neutron by A always occurs independently of the proton but it is not a free neutron that is captured rather one that is allowed to scatter from A as well. The capture process is described in terms of the bound-state wave function of the neutron in B and of the on-the-energy-shell scattering amplitude for neutron-A optical model scattering in the channel with the quantum numbers of B. The bound-state wave function can be obtained from some model. The scattering amplitude is in principle obtainable from experiment. To calculate the capture vertex taking rescattering into account, it is not necessary to solve any Schrödinger equation to obtain the scattering state. The capture amplitude will differ from the Born approximation, which is given just by the wave function, in that rescattering previous to capture is included. In the usual Butler theory of stripping this is taken into account phenomenologically by cutting off the capture at some adjustable radius. No such parameter is necessary here.

The major part of the calculation is devoted to calculating the amplitude for disassociation of the deuteron in the field of A. This can occur either independently of A (Born approximation) or through interaction with A. The essence of the direct interaction process is that between the final break-up of the deuteron and the capture of the neutron in B neither the neutron nor A are interfered with by the proton. Thus, we neglect neutron-proton rescattering corrections to the disassociation and proton-A rescattering or disassociation contributions. We keep the contribution to both from the neutron-A interaction since it is this that subsequently gives the capture. Furthermore, we keep exactly the effect of deuteron-A rescattering on the disassociation, without assuming this to be purely elastic. This rescattering, as well as the neutron-A interaction, is expressed in terms of the measurable scattering amplitude on the energy shell. There is no need to solve for a wave function for the deuteron-A scattering state. This is a decided advantage, since any reasonably manageable attempt to do this neglects the effects of the very finite size of the deuteron by using a deuteron-A optical model.

It might be thought that by excluding the proton-neutron and proton-A contributions to the disassociation we are leaving out final-state interactions. This is not the case. It is only in a distorted wave treatment that *both* final- and initial-state rescattering must be taken into account,[3] and then only elastic scattering can be included. In the formal theory of scattering[11] it is usually sufficient to include only initial- or final-state interactions so long as all processes, that is elastic and inelastic, are allowed. We do just that in putting in the full deuteron-A scattering corrections from experiment. What we are neglecting are the contributions of the neutron-proton and proton-A interactions to the final deuteron break-up just before the capture of the neutron into B. How some of the proton-A interactions could also be included is discussed in the Appendix.

Thus, finally we are able to express deuteron stripping in terms of deuteron-A and neutron-A scattering on the

[5] Compare S. Sunakawa, Progr. Theoret. Phys, (Kyoto) **24**, 963 (1960).

[6] A review of these methods and extensive references will be found in *Dispersion Relations and Elementary Particles*, edited by DeWitt and Omnes (John Wiley & Sons, New York, 1960).

[7] M. T. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. **124**, 1258 (1961).

[8] Karplus, Sommerfield, and Wichman, Phys. Rev. **111**, 1187 (1958).

[9] R. Blankenbecler and L. F. Cook, Phys. Rev. **119**, 1745 (1960).

[10] R. D. Amado, Phys. Rev. **122**, 696 (1961).

[11] B. A. Lippman and J. Schwinger, Phys. Rev. **79**, 469 (1950)

energy shell, and in terms of the wave function for the neutron in B. The first two are accessible from experiment and the third may be computed fairly well from a model up to a multiplicative constant (essentially the reduced width). This constant we take as a fitting parameter; it is the only one in the theory. The form for the amplitude we obtain is relatively complex, but its essential features can be recognized. In a simple model we see that neutron-A scattering does indeed distort the capture vertex making the nuclear radius apparently increase, but that a part of this distortion effect is canceled in the total stripping amplitude by the corresponding distortion of the amplitude for dis-association of the deuteron. The effect of deuteron-A rescattering can be easily examined if we assume that since the scattering is largely forward, we need only take the forward scattering into account. Doing this, we find an expression for the stripping amplitude in which the usual modified Born approximation, a function of momentum transfer only, appears multiplied by an energy dependent factor which is simply related to the deuteron-A total cross section. To obtain this form it is necessary to simplify the theory enormously, but the qualitative features of this simplified result should still be true of a fuller analysis and hence it is possible to understand the peristence of the form of the angular distribution given in the Born approximation in spite of variations in yield.

The formal theory necessary to obtain these results is set forth in Sec. II. In Sec. III the vertex for capturing the neutron into B in the presence of neutron-A rescattering is computed. The amplitude for disintegration of the deuteron in the field of A is calculated in Sec. IV. It is here that a number of well-defined approximations are made and also that a cavalier approach is taken toward the analtic properties of the amplitudes. In Sec. V the results are discussed and some simplifying assumptions are made in order to examine the properties of the amplitudes in detail.

II. FORMAL METHODS

In this section we will be interested in deriving a Low-type equation[12] describing the nuclear stripping reaction $A+d\rightarrow p+B$. Our attention will be confined to stripping reactions although this in no way exhausts the generality of this approach. In describing the collision of composite particles the powerful tool of contraction rules will be used.[13] Since this is a rather unfamiliar procedure in the present context, the elements will be reviewed where necessary.[14,15]

The S-matrix element of interest is

$$S=\langle pB^{(-)}|dA^{(+)}\rangle. \tag{2.1}$$

The wave functions for these interacting states must be constructed. In order to carry out this program it is convenient to introduce the formalism of second quantization. Commutation relations for the nucleons are chosen to be

$$\begin{gathered}\{\psi(\mathbf{x}),\psi^\dagger(\mathbf{x}')\}=\delta(\mathbf{x}-\mathbf{x}'),\\ \{\psi(\mathbf{x}),\psi(\mathbf{x}')\}=\{\psi^\dagger(\mathbf{x}),\psi^\dagger(\mathbf{x}')\}=0,\end{gathered} \tag{2.2}$$

where spin and isotopic spin indices have been suppressed. We assume the system is described by a Hamiltonian, H, but do not need to prescribe its form for the present. Heisenberg operators are introduced according to

$$\Psi(\mathbf{x},t)=e^{iHt}\psi(\mathbf{x})e^{-iHt}, \tag{2.3}$$

which leads to the equation of motion

$$i\frac{\partial}{\partial t}\Psi(\mathbf{x},t)=[\Psi(\mathbf{x},t),H]. \tag{2.4}$$

The "in" and "out" states may be defined in terms of the asymptotic condition according to

$$\lim_{t\rightarrow-\infty}\langle\gamma|\Psi_\alpha{}^\dagger(t)|\beta\rangle=\langle\gamma|\beta\alpha^{(+)}\rangle \tag{2.5}$$

and

$$\lim_{t\rightarrow\infty}\langle\gamma|\Psi_\alpha(t)|\beta\rangle=\langle\gamma\alpha^{(-)}|\beta\rangle,$$

where, for example, $|\beta\alpha^{(+)}\rangle$ is the interacting state which is a plane wave at $t=-\infty$. The creation and annihilation operators for single-particle states, $\Psi_\alpha(t)$, have been defined as[13,15]

$$\Psi_\alpha(t)=\int d^3x\, f_\alpha{}^*(\mathbf{x},t)\Psi(\mathbf{x},t), \tag{2.6}$$

where we have introduced the function $f_\alpha(\mathbf{x},t)$, which is a properly normalized (in a unit box) solutions of the Schrödinger equation describing the center-of-mass motion of the, in general, composite particle α, i.e.,

$$i\frac{\partial}{\partial t}f_\alpha(\mathbf{x},t)=-\frac{\hbar^2}{2m}\nabla^2 f_\alpha(\mathbf{x},t)-\epsilon_\alpha f_\alpha(\mathbf{x},t), \tag{2.7}$$

where ϵ_α is the (positive) binding energy of particle α. For a composite particle the operators corresponding to (2.6) are defined in an analogous manner,

$$\Phi_\alpha(t)=\int d^3X\, f_\alpha{}^*(\mathbf{X},t)\Phi_\alpha(\mathbf{X},t), \tag{2.8}$$

where $\mathbf{X}$ is the center-of-mass coordinate of the system and

$$\Phi_\alpha(\mathbf{X},t)=\int d^3y_1\cdots d^3y_N\, g_\alpha(\mathbf{y}_1\cdots\mathbf{y}_N)\times\delta\left(\mathbf{X}-\frac{\mathbf{y}_1+\cdots+\mathbf{y}_N}{N}\right)\Psi(\mathbf{y}_1,t)\cdots\Psi(\mathbf{y}_N,t). \tag{2.9}$$

[12] F. W. Low, Phys. Rev. **97**, 1392 (1955).

[13] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo cimento **1**, 205 (1955); R. Haag, Phys. Rev. **112**, 669 (1958); W. Zimmermann, Nuovo cimento **10**, 597 (1958); W. Brenig and R. Haag, Fortschr. Physik **7**, 183 (1959).

[14] Some formal considerations will be found in P. Redmond and J. Uretsky, Ann. Phys. (New York) **9**, 109 (1960).

[15] An account of many of the methods used in this paper will be found in the article by M. L. Goldberger in reference 6.

Here $g_\alpha(\mathbf{y}_1\cdots\mathbf{y}_N)$ is any properly normalized and symmetrized function of the relative coordinates which allows the condition

$$\lim_{t\to\infty}\langle 0|\Phi_\alpha(t)|\alpha\rangle=1 \tag{2.10}$$

to be satisfied.[13] It is otherwise arbitrary. This latitude is possible since our states need only be defined asymptotically. If a Schrödinger Hamiltonian is chosen, then the bound-state wave function is a natural choice for g_α, but even here any function with unit overlap with the bound state will do as well so long as there is only one bound state. If there is more than one a more restricted latitude is still allowed. The operators defined by (2.6), (2.8), and (2.9) have a simple interpretation. First, the required number of free nucleons is created or annihilated by the single-particle operators and then these are folded together with the appropriate weighting function to produce the coherent mixture corresponding to the composite system.

We proceed now to a derivation of the transition amplitude. The S-matrix defined by Eq. (2.1) can be written as

$$S=\lim_{t\to\infty}\langle p|\Phi_B(t)|dA^{(+)}\rangle.$$

Integrating by parts with respect to the time limit, using

$$\lim_{t\to\infty}F(t)=\lim_{t\to-\infty}F(t)+\int_{-\infty}^{\infty}\frac{\partial F(t)}{\partial t}dt, \tag{2.11}$$

and assuming that $f_B(\mathbf{X},t)$ is a plane wave, we find

$$(S-1)_{pB,dA}=-i\int_{-\infty}^{\infty}dt\int d^3X\,\exp(iE_Bt-i\mathbf{K}_B\cdot\mathbf{X})\times\left(i\frac{\partial}{\partial t}-E_B\right)\langle p|\Phi_B(\mathbf{X},t)|dA^{(+)}\rangle. \tag{2.12}$$

Finally, introducing the transition matrix T as

$$(S-1)_{pB,dA}=-2\pi i\delta(E_A+E_d-E_p-E_B)T, \tag{2.13}$$

and using the equations of motion for $\Phi_B(\mathbf{X},t)$ leads to

$$T=\langle p|J_B(0)|dA^{(+)}\rangle, \tag{2.14}$$

where

$$J_B(t)=\int d^3X\,\exp(iE_Bt-i\mathbf{K}_B\cdot\mathbf{X})\times\left(i\frac{\partial}{\partial t}-E_B\right)\Phi_B(\mathbf{X},t). \tag{2.15}$$

In analogy with electromagnetic theory this is called the source current or current for the B field. The form (2.14) for the transition matrix can easily be shown to be equivalent to the usual formulation in the case of a Schrödinger Hamiltonian if the field operators are eliminated in favor of the Schrödinger wave functions.[15]

In order to isolate the most interesting contributions to the stripping reaction, it is convenient to retain the field operator description and to contract a second particle. The choice as to which particle to contract is not unique and leads to several distinct and interesting expansions for the transition matrix. We shall be particularly interested in the form resulting from the contraction of the proton. Use of the asymptotic conditions for the proton leads to

$$T=\lim_{t\to\infty}\langle 0|\Psi_p(t)J_B(0)|dA^{(+)}\rangle. \tag{2.16}$$

It is convenient to rewrite this in terms of an advanced commutator (or anticommutator) and to integrate the time limit by parts; then[16]

$$T=-i\int_{-\infty}^{\infty}dt\int d^3y\,e^{iE_pt-i\mathbf{k}_p\cdot\mathbf{y}}\times\left(i\frac{\partial}{\partial t}-E_p\right)\langle 0|\theta(t)[\Psi_p(\mathbf{y},t),J_B(0)]_\pm|dA^{(+)}\rangle. \tag{2.17}$$

The presence of $\theta(t)$ makes the contribution from the limit at very large negative times zero. The anticommutator is taken if B contains an odd number of nucleons and the commutator otherwise since the proton is a fermion. The extra term introduced by the second ordering in the commutator is

$$\lim_{t\to\infty}\langle 0|J_B(0)\Psi_p(t)|dA^{(+)}\rangle.$$

Inserting a complete set of states, this may be written

$$\sum_S\langle 0|J_B(0)|S\rangle\lim_{t\to\infty}\langle S|\Psi_p(t)|dA^{(+)}\rangle=\sum_S\langle 0|J_B(0)|S\rangle\langle Sp^{(-)}|dA^{(+)}\rangle. \tag{2.18}$$

The only state S with the proper energy and quantum numbers that can contribute is the state $S=B$, since the second matrix element conserves energy. But $\langle 0|J_B(0)|B\rangle=0$ as may be verified from the equations of motion. Thus, the second term gives zero and may be freely added to T.

If the time differentiations in (2.17) are carried out, two terms emerge. The first arises when the time derivative acts on the theta function and yields a delta function on time. This gives an equal time commutator of Ψ_p and J_B which we neglect. The second term gives J_p. Therefore, the final form of the transition matrix is

$$T=-\int_{-\infty}^{\infty}dt\,e^{iE_pt}\langle 0|\theta(t)[J_p(t),J_B(0)]_\pm|dA^{(+)}\rangle. \tag{2.19}$$

By neglecting the equal-time commutator and keeping the form (2.19), we make an assumption about the form of the Hamiltonian. It is shown by Vaughn,

[16] $\theta(t)=0$ for $t<0$; $\theta(t)=1$ for $t>0$.

Aaron, and Amado[7] in a very similar example that if the B state is a bound state in a nonrelativistic potential theory, the T matrix of (2.19) is identically zero since for any state S, $\langle 0|J_B(0)|S\rangle=0$ in that case. Stripping will occur, of course, but is given entirely by the equal-time commutator, which we have neglected. If, however, B is an elementary particle, that is makes its appearance as a basic field in the Hamiltonian, the equal-time commutator is zero and (2.19) the entire amplitude. It is further shown that for elastic scattering, at least, composite-particle theories may be obtained as the limit of theories with elementary particles. We believe it reasonable that this is also true for inelastic amplitudes. Thus, we shall compute T of (2.19) taking B to be elementary when necessary. This freedom will allow us to obtain a nontrivial but soluble equation for T. It is assumed that the final answer will not depend on taking B as elementary, this choice acting only as a catalyst in obtaining equations.

A physical interpretation of the processes contributing to the reaction of interest is simplified, if a complete set of interacting states (eigenstates of H) is inserted into (2.19) and the time integration is performed:

$$T=\sum_S\left\{\frac{\langle 0|J_p(0)|S\rangle\langle S|J_B(0)|dA^{(+)}\rangle}{E_p-E_S+i\epsilon}\pm\frac{\langle 0|J_B(0)|S\rangle\langle S|J_p(0)|dA^{(+)}\rangle}{E_S-E_B+i\epsilon}\right\}. \quad (2.20)$$

Up to this point the treatment has been quite general. Thus, (2.20) may be used to describe any reaction simply by changing labels on the operators since we have thus far used nothing specific to deuteron stripping.

We now begin to put in some of the physics of stripping. The contribution from the first term in (2.20) may be neglected. The states S which contribute must have the quantum numbers of a proton. The one proton state itself is excluded, since $\langle 0|J_p|p\rangle=0$. In a potential theory, this is all there could be. In a meson theory, the state of proton plus one meson is admissible, but this would have a large energy denominator in (2.20) and would thus be negligible.

Consider the second term of T. This term represents a factoring of the amplitude as shown in Fig. 1(a). The states which can contribute to the sum in (2.20) are all those with the quantum numbers of B except B itself since $\langle 0|J_B|B\rangle=0$. There are many such states, but it is essential to the direct reaction method to assume that B is formed from a neutron and A in its ground state. Thus from all the states S, we wish to project just those parts containing a neutron and A in its ground state. This is just what is done in the optical model state for neutron-A scattering.[17] That is, we keep just these projections if we replace S by the state of a neutron scattering from A represented as an optical potential. This state is not quite an eigenstate of the total Hamiltonian, but it should be a fairly good approximation to one for energies below the particle production threshold in neutron-A scattering. Alternately, we can say that the replacement of the sum over S by the neutron-A optical model scattering state consists in replacing the total Hamiltonian of the system by one in which neutron-A scattering is treated via an optical model potential. Such a Hamiltonian is not strictly Hermitian, but for low energies (below the particle threshold) the imaginary part is small. Thus, we proceed by replacing the sum over states in the second term of (2.20) by the optical model scattering state $nA_1^{(+)}$.[18] We obtain

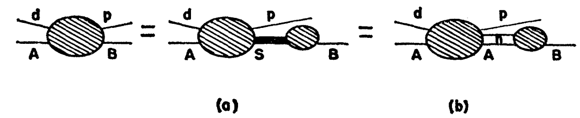

FIG. 1. Schematic representations of the factorization of the deuteron stripping T matrix. (a) Eq. (2.20), second term; (b) Eq. (2.21).

$$T=\sum_{nA_1}\frac{\langle 0|J_B(0)|nA_1^{(+)}\rangle\langle nA_1^{(+)}|J_p(0)|dA^{(+)}\rangle}{E_N+E_{A_1}-E_B}. \quad (2.21)$$

The notation A_1 indicates the nucleus A in its ground state but with, in general, a different asymptotic momentum from the target A. This form for the amplitude is represented in Fig. 1(b). It expresses stripping as the product of two parts. First the amplitude for the breakup of the deuteron $d+A\rightarrow n+p+A$ and then for the capture process $n+A\rightarrow B$. Thus, we must obtain the dependence of the amplitudes

$$\Gamma=\langle 0|J_B(0)|nA_1^{(+)}\rangle, \quad (2.22)$$

and

$$F=\langle nA_1^{(+)}|J_p(0)|dA^{(+)}\rangle, \quad (2.23)$$

on the energy of the nA_1 state. It should be noted that since B will have a definite spin and parity ($J\pi$), only that part of the neutron-A scattering state corresponding to the $J\pi$ partial wave will contribute. In order to simplify the algebra, we will assume that B is an s state and neglect spins. These restrictions are easily relaxed.

Another intermediate state which can contribute to the second term of (2.20) is the state of a deuteron and nucleus C, where C has one less proton than A. The contribution of this term may be written

$$\sum_{Cd_1}\frac{\langle 0|J_B(0)|d_1C^{(+)}\rangle\langle d_1C^{(+)}|J_p(0)|dA^{(+)}\rangle}{E_B-E_{d_1}-E_C}, \quad (2.24)$$

where, as before, d_1 represents the deuteron in a dif-

[17] See N. C. Francis and K. M. Watson, Phys. Rev. 92, 291 (1953); Phys. Rev. 93, 313 (1954).

[18] We may take either the "plus" or "minus" states, since they each independently form a complete set.

ferent state of asymptotic momentum from d. It is clear that in the lowest-order approximation in which

$$\langle d_1C^{(+)}|J_p(0)|dA^{(+)}\rangle \cong \delta_{d,d_1}\langle C|J_p(0)|A\rangle,$$

this term represents the contribution of "heavy particle" stripping or exchange stripping.[19] We will not consider it further in our analysis but it can be included by the techniques which we shall outline in the following sections.

III. THE VERTEX FUNCTION

We now turn to the problem of evaluating the vertex function (2.22). Recall it is assumed that B is an s state so that only the s-wave part of $nA_1^{(+)}$ is needed. If the neutron is contracted, we find

$$\Gamma = -i\int_{-\infty}^{\infty} dt \int d^3y\, e^{-i(E_nt-\mathbf{n}\cdot\mathbf{y})} \times\left(i\frac{\partial}{\partial t}+E_n\right)\langle 0|\theta(-t)[J_B(0),\Psi_n(\mathbf{y},t)]_\pm|A_1\rangle, \quad (3.1)$$

where we have called the neutron momentum $\mathbf{n}$,[20] and where now the neutron field is defined with respect to an optical model Hamiltonian for the neutron-A interaction. The extra term introduced by the commutator gives zero. Carrying out the time differentiation yields two terms:

$$\Gamma = -\int d^3y\, e^{i\mathbf{n}\cdot\mathbf{y}}\langle 0|[J_B(0),\psi_n^\dagger(\mathbf{y})]_\pm|A_1\rangle - \int_{-\infty}^{\infty} dt\, e^{-iE_nt}\langle 0|[J_B(0),J_n^\dagger(t)]_\pm|A_1\rangle. \quad (3.2)$$

To study Γ further it is convenient to use states normalized in a box of unit volume and to work in the rest frame of B, that is, the center-of-mass system of the neutron and A_1. We can then factor a momentum-conserving Kronecker delta function from Γ. In this frame the interaction of the neutron, A and B, recalling that B is considered to be elementary, can be written:

$$H_{\text{int}} = \sum_{\mathbf{n}} \Phi(n^2)[\tilde{\psi}_B^\dagger(0)\tilde{\psi}_N(\mathbf{n})\tilde{\psi}_A(-\mathbf{n}) + \tilde{\psi}_B(0)\tilde{\psi}_N^\dagger(\mathbf{n})\tilde{\psi}_A^\dagger(-\mathbf{n})], \quad (3.3)$$

where the momentum space operators are defined by

$$\tilde{\psi}(\mathbf{k}) = \int d^3x\,\psi(\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}}, \quad (3.4)$$

and $\Phi(n^2)$ is the form factor describing the interaction. Up to self-mass terms, which do not contribute in our matrix element, the B current J_B is then

$$J_B(0) = \frac{1}{Z}\sum_{\mathbf{n}} \Phi(n^2)\tilde{\psi}_n(\mathbf{n})\tilde{\psi}_A(-\mathbf{n}), \quad (3.5)$$

where Z is the wave function renormalization constant associated with the B field. Thus, for the first term of Γ in (3.2) we obtain

$$-\int d^3y\, e^{i\mathbf{n}\cdot\mathbf{y}}\langle 0|[J_B(0),\psi_n^\dagger(\mathbf{y})]_\pm|A_1\rangle = -\frac{1}{Z}\Phi(n^2) \quad (3.6)$$

If the elementary particle theory is really to be used to represent a bound state, then it is shown by Vaughn, Aaron, and Amado[7] that $\Phi(n^2)$ should be related to the Fourier transform of the bound-state wave function. That is,

$$\Phi(n^2) = (n^2+\epsilon_B)\int d^3x\, e^{i\mathbf{n}\cdot\mathbf{x}}\phi_B(\mathbf{x}), \quad (3.7)$$

where ϵ_B is the (positive) binding energy of the neutron in B and $\phi_B(\mathbf{x})$ is the normalized wave function of the state. We shall assume later that $\Phi(n^2)$ is an analytic function of n^2 in the entire n^2 plane except for a cut running from $n^2=-\epsilon_B-\lambda$ to $-\infty$; λ a positive real number depending on the details of the strength and range of the potential which binds B.[9] For the present, all we need is that $\Phi(n^2)$ is real for positive n^2 and goes to zero sufficiently rapidly for large n^2. The factor $n^2+\epsilon_B$ in (3.7) removes the pole that is otherwise present in the Fourier transform of ϕ_B at $n^2=-\epsilon_B$. Hence, $\Phi(-\epsilon_B)$ is the residue at that pole, we shall call it Γ_0. This quantity is the natural invariant quantity associated with the strength of the process $B \rightleftarrows n+A$, that is to say the coupling constant for the process. It is related by kinematical factors to the reduced width of B. Although we might be able to calculate the form of $\Phi(n^2)$, at least for small n^2 which is all we really need, from some model, we do not expect to be able to calculate Γ_0. We shall keep it as a parameter to be discovered by measuring stripping. Thus, we write

$$\Phi(n^2) = \Gamma_0 f(n^2); \quad f(-\epsilon_B) = 1.$$

In the bound-state limit, $Z\to 0$; we shall side-step this difficulty by assuming Γ_0 to be slightly less than the value associated with the bound state limit, and therefore Z small but not zero. We can then compute Γ and T and finally let Z be zero at the end, when in fact it shall have canceled out.

We can now write for (3.2)

$$\Gamma(n^2) = -\frac{\Gamma_0}{Z}f(n^2) + i\int_{-\infty}^{\infty} dt \times\theta(-t)e^{-in^2t}\langle 0|[J_B(0),J_n^\dagger(t)]_\pm|A_1\rangle. \quad (3.8)$$

It is natural to assume that this defines a function

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[19] L. Madansky and G. E. Owen, Phys. Rev. **99**, 1608 (1955); G. E. Owen and L. Madansky, Phys. Rev. **105**, 1766 (1957).

[20] We take units in which $\hbar=1$ and $2m=1$ where m is the nucleon mass.

analytic in the upper-half n^2 plane. Thus, we may write

$$\Gamma(n^2)=-\frac{\Gamma_0}{Z}f(n^2)+\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Gamma(x)}{x-n^2-i\epsilon}dx \tag{3.9}$$

where by using the limits 0 to ∞ we have anticipated the fact that the absorptive part of Γ coming from the second term of (3.2) is nonzero only for $n^2>0$. Since $f(n^2)$ is real in that region, the absorptive part is indeed the imaginary part. We deliberately leave out of consideration the question of anomalous thresholds since these occur only in a relativistic theory and we believe they are completely avoided in our work by explicit use of the bound-state wave function in $f(n^2)$.[9]

In the usual way we evaluate the imaginary part of Γ by writing $\theta(-t)=\frac{1}{2}+\frac{1}{2}\epsilon(-t)$,[21] and associating the imaginary part with the first term[15]. We have

$$\mathrm{Im}\Gamma(n^2)=\frac{i}{2}\int_{-\infty}^{\infty} dt\, e^{-in^2t}\langle 0|[J_B(0),J_n^\dagger(t)]_\pm|A_1\rangle. \tag{3.10}$$

Inserting a complete set of interacting states and using the fact that $\langle 0|J_n^\dagger|S\rangle=0$ for any state in a nonrelativistic model, we have

$$\mathrm{Im}\Gamma(n^2)=-\pi\sum_S \delta(n^2+E_{A_1}-E_S)\times\langle 0|J_B(0)|S\rangle\langle S|J_n^\dagger(0)|A_1\rangle. \tag{3.11}$$

The only state which can contribute to the sum is the nA scattering state, and we take the "plus" state. The threshold for its contribution begins at $E=0$. We obtain

$$\mathrm{Im}\Gamma(n^2)=-\pi\sum_{n'A'} \delta(n^2+E_{A_1}-E_{n'}-E_{A'})\times\langle 0|J_B(0)|n'A'^{(+)}\rangle\langle n'A'^{(+)}|J_n^\dagger(0)|A_1\rangle, \tag{3.12}$$

which relates $\mathrm{Im}\Gamma$ to Γ itself through a factor which is the n-A scattering T matrix. This scattering amplitude is on the energy shell by virtue of the delta function. Further, since B is an s-state, the factor $\langle 0|J_B(0)|n'A'^{(+)}\rangle$ projects out the s-wave part of the $n'A'$ state so that knowledge of the scattering is needed only in that partial wave. $\mathrm{Im}\Gamma$ must be real and therefore Γ itself must have the phase ϕ of s wave neutron A scattering in the optical model. Thus,

$$\mathrm{Im}\Gamma(x)=\Gamma(x)\sin\phi(x)\exp[-i\phi(x)], \tag{3.13}$$

or

$$\Gamma(n^2)=-\frac{\Gamma_0 f(n^2)}{Z}+\frac{1}{\pi}\int_0^\infty \frac{e^{-i\phi(x)}\sin\phi(x)\Gamma(x)dx}{x-n^2-i\epsilon}. \tag{3.14}$$

The phase ϕ is related to the, in general complex, phase shift δ by[15,22]

$$\tan\phi=\mathrm{Re}\beta/(1-\mathrm{Im}\beta), \tag{3.15}$$

where β is given in terms of the s-wave scattering amplitude by $\beta=\sin\delta\exp i\delta$. If δ is real $\delta=\phi$. The integral equation (3.14) (or mapping problem) is well known in dispersion theory. The solution is[23]

$$\Gamma(n^2)=-\frac{\Gamma_0}{Z}\left(f(n^2)+\frac{e^{\rho(n^2)+i\phi(n^2)}}{\pi}\times\int_0^\infty \frac{f(x)\sin\phi(x)e^{-\rho(x)}dx}{x-n^2-i\epsilon}\right), \tag{3.16}$$

where

$$\rho(x)=\frac{P}{\pi}\int_0^\infty \frac{\phi(y)}{y-x}dy \tag{3.17}$$

and where we have made some assumptions about the asymptotic behavior of Γ for large n^2. These assumptions, particularly that the integral for ρ converges, may not be valid, at least in the bound-state limit. They may be put right by a subtraction. We define

$$\sigma(x)=(x+\epsilon_B)\frac{P}{\pi}\int_0^\infty \frac{\phi(y)dy}{(y-x)(y+\epsilon_B)} \tag{3.18}$$

and then

$$\Gamma(n^2)=-\frac{\Gamma_0}{Z}\left(f(n^2)+\frac{e^{\sigma(n^2)+i\phi(n^2)}}{\pi}\times\int_0^\infty \frac{\sin\phi(x)e^{-\sigma(x)}f(x)dx}{x-n^2-i\epsilon}\right). \tag{3.19}$$

The two terms of (3.19) are easily interpreted. The first is just the Fourier transform of the bound-state wave function and represents the probability amplitude for a neutron capturing into A to form B in the absence of distortion. The second term gives the effect of distortion on the neutron as it comes in to be captured.

Having obtained (3.19) by taking B to be elementary, we can drop this replacement and assume that this form of the vertex is still valid in the limit that B is composite, except of course we do not yet let Z tend to 0.

IV. BREAKUP AMPLITUDE

We now attach the more formidable task of calculating the breakup amplitude of (2.23), defined as

$$F=\langle nA_1^{(+)}|J_p(0)|dA^{(+)}\rangle.$$

In order to calculate the sum in (2.21) we need to know the dependence of this factor on the energy of the nA_1 state only, keeping the momenta of the proton, deuteron, and A fixed. As it stands, F does not depend only on the energy of the nA_1 state but also on the momenta in this state. This dependence on momenta is removed when F is folded with the s-wave vertex function in the

[21] $\epsilon(t)=1$ for $t>0$; $\epsilon(t)=-1$ for $t<0$.

[22] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

[23] N. I. Muskhelishvili, *Singular Integral Equations* (P. Noordhoff, N. V. Groningen, Holland, 1953); R. Omnes, Nuovo cimento **8**, 316 (1958).

sum; that is, when the s wave part of the nA_1 state is projected out. If we continue to work in the center-of-mass frame of the $n-A_1$ system, that is, the rest frame of B, we can easily perform this s-wave projection and we need only study

$$\tilde{F}(E_{A_1})=P_s\langle nA_1{}^{(+)}|J_p(0)|dA^{(+)}\rangle$$
$$=\frac{1}{4\pi}\int d\Omega_n\langle nA_1{}^{(+)}|J_p(0)|dA^{(+)}\rangle, \quad (4.1)$$

where Ω_n is the solid angle of $\mathbf{n}$ and we have written $\tilde{F}(E_{A_1})$ understanding that all the other momenta are kept fixed save that of the neutron and A, which are $\mathbf{n}$ and $-\mathbf{n}$ respectively by momentum conservation. Again a Kronecker delta for momentum conservation is factored out.

To get an equation for $\tilde{F}$, we proceed as in V-θ scattering in the Lee model.[10] We shall not have to assume that B is elementary in this calculation. Continuing the convention of box normalization, we may write

$$F=\lim_{t\to-\infty}\langle n|\psi_{A_1}(t)J_p(0)|dA^{(+)}\rangle$$
$$=\lim_{t\to-\infty}\langle n|[\psi_{A_1}(t),J_p(0)]|dA^{(+)}\rangle$$
$$+\langle n|J_p(0)|d\rangle\delta_{\mathbf{A},\mathbf{A}_1}. \quad (4.2)$$

The second term comes from the extra factor introduced by the commutator and follows from the definition of the "in" state. For simplicity, we assume that A is a boson. Using the now familiar techniques, we obtain for (4.2)

$$F=\langle n|J_p(0)|d\rangle\delta_{\mathbf{A},\mathbf{A}_1}+\langle n|[\psi_{A_1}(0),J_p(0)]|dA^{(+)}\rangle$$
$$+i\int_{-\infty}^{\infty}dt\,\theta(-t)e^{iE_{A_1}t}\langle n|[J_{A_1}(t),J_p(0)]|dA^{(+)}\rangle, \quad (4.3)$$

where the equal time commutator arises as usual from the time differentiation of the θ function. It represents breakup of the deuteron through an explicit proton-A interaction. If no such interaction exists, it should be noted that it is not essential to the occurrence of stripping, the commutator will be zero. We shall assume we may neglect it. This does not mean that we are neglecting initial or final interactions. In fact, since we shall later use the exact experimental deuteron-A scattering amplitude to take into account these interactions, the actual contribution of a proton-A interaction to the initial and final state rescattering will be taken into account. What is left out by neglecting the commutator is the contribution of this interaction to the process $d+A\to p+n+A$. This is consistent with the direct interaction idea that the neutron is captured directly by A so that the proton is essentially free during the deuteron breakup and the neutron capture. In the Appendix we will discuss this approximation in more detail and describe how it may be relaxed to some extent.

Dropping the equal-time commutator, we can proceed to studying the analytic properties implied by (4.3). The immediate temptation to guess that the third term defines a function analytic in the lower half E_{A_1} plane is not justified since this term is not a function of E_{A_1} only. This hurdle is easily passed, however, by introducing the projection operator of (4.1), we then get

$$\tilde{F}(E_{A_1})=P_s\langle n|J_p(0)|d\rangle\delta_{\mathbf{A},\mathbf{A}_1}$$
$$+i\int_{-\infty}^{\infty}dt\,\theta(-t)e^{iE_{A_1}t}P_s\langle n|[J_{A_1}(t),J_p(0)]dA^{(+)}\rangle. \quad (4.4)$$

We shall now presume that the second term does indeed define a function of E_{A_1} analytic in the lower half plane which further has the proper asymptotic properties to allow us to write

$$\tilde{F}(E_{A_1})=P_s\langle n|J_p(0)|d\rangle\delta_{\mathbf{A},\mathbf{A}_1}$$
$$-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{A(x)}{x-E_{A_1}+i\epsilon}dx, \quad (4.5)$$

where the absorptive part of $\tilde{F}$, is given as before by

$$A(E_{A_1})=\frac{1}{2}\int_{-\infty}^{\infty}dt$$
$$\times e^{iE_{A_1}t}P_s\langle n|[J_{A_1}(t),J_p(0)]|dA^{(+)}\rangle. \quad (4.6)$$

Inserting a complete set of interacting states and doing the time integrals, we obtain

$$A(E_{A_1})$$
$$=\pi P_s\sum_S\langle n|J_{A_1}(0)|S\rangle\langle S|J_p(0)|dA^{(+)}\rangle$$
$$\times\delta(E_{A_1}+E_n-E_s)-\pi P_s\sum_S\langle n|J_p(0)|S\rangle$$
$$\times\langle S|J_{A_1}(0)|dA^{(+)}\rangle\delta(E_{A_1}+E_s-E_d-E_A). \quad (4.7)$$

In the first term the permissible states are just B and nA. The nA term will make a phase correction to F just as described in the previous section for the vertex. The term with B will give back the stripping T matrix, but in the coordinate system we have chosen (rest frame of B), the T matrix will not depend in E_{A_1}, and hence will not need to be known to do the sum in (2.21). Finally, we shall obtain an algebraic equation for T, which we can solve. In the second term of (4.7) the states which can occur are d and n-p. The first will represent the effect of the initial state rescattering on the breakup, and through it on the full amplitude. In this formulation there is no need to include final-state interactions as well. This asymmetry is well known in the usual formal theory of scattering.[11] The n-p intermediate state represents neutron-proton rescattering corrections to the breakup. We neglect this as we neglect proton-A corrections. This approximation is surely in the direct-interaction spirit both since the neutron-proton rescattering interferes with the direct capture of the neutron and because the neutron-proton interaction is much less effective than the neutron-A interaction. In this approximation we may write for A

$$A(E_{A_1})=\pi P_s\langle n|J_{A_1}(0)|B\rangle\langle B|J_p(0)|dA^{(+)}\rangle\delta(E_{A_1}+E_n-E_B)$$
$$+\pi P_s\sum_{n'A'}[\langle n|J_{A_1}(0)|n'A'^{(+)}\rangle\langle n'A'^{(+)}|J_p(0)|dA^{(+)}\rangle\delta(E_{A_1}+E_n-E_{A'}-E_{n'})]$$
$$-\pi P_s[\langle n|J_p(0)|d_1\rangle\langle d_1|J_{A_1}(0)|dA^{(+)}\rangle\delta(E_{A_1}+E_{d_1}-E_A-E_d)]. \quad (4.8)$$

There is no sum in the first or third term since momentum conservation fixes the momentum of B to be zero and of d_1 to be $\mathbf{n}+\mathbf{p}$. We are neglecting the trivial but algebraically annoying complication of the spin of the deuteron, and of the neutron and proton for that matter. The projection operator is unnecessary in the first term, this already being a function of E_{A_1} only. In the second term it picks out the s-wave part of n-A scattering, as we would expect. In the third term its function is not so trivial, and it is largely for this term that it is introduced.

Vertex functions of the form $\langle n|J_p(0)|d\rangle$ or the corresponding $\langle n|J_{A_1}(0)|B\rangle$ appear in this analysis. We must relate these to the quantities defined in the previous section. Let us study

$$V=\langle n|J_{A_1}(0)|B\rangle. \quad (4.9)$$

If we express J_{A_1} in terms of the commutator with the Hamiltonian, we obtain

$$V=(E_B-E_n-E_{A_1})\int d^3xe^{i\mathbf{n}\cdot\mathbf{x}}\langle n|\psi_A(\mathbf{x})|B\rangle. \quad (4.10)$$

If one goes over to Schrödinger wave functions, we see that V is just the Fourier transform of the bound state. That is, in the rest frame of B, $V=-\Gamma_0 f(n^2)$, where we have factored out a Kronecker delta for momentum conservation. Furthermore, we know that $V(-\epsilon_B)=-\Gamma_0$. Correspondingly $\langle n|J_p(0)|d\rangle$ is the Fourier transform of the deuteron wave function. Let us call

$$\langle n|J_p(0)|d\rangle|_{E_n+E_p=E_d}=-\gamma_0,$$

the effective coupling constant or invariant reduced width of the deuteron. It is known in terms of the effective range and scattering length of neutron-proton triplet scattering.[24] We may write

$$\langle n|J_p(0)|d\rangle=-\gamma_0 f_d[(\mathbf{n}-\mathbf{p})^2/4]. \quad (4.11)$$

f_d is the Fourier transform of the deuteron wave function with the pole removed normalized so that $f_d=1$ at $E_n+E_p=E_d$. Since the deuteron is very spread out, $f_d(q^2)$ will be approximately one for a large range of q^2 near the binding energy. If the deuteron were completely asymptotic f_d would be 1. For simplicity we shall put it equal to 1, that is, take the deuteron to be all asymptotic. This approximation is on the same footing as neglecting spins. We make it to clarify the algebra, it is easily relaxed in as much as the actual deuteron wave function in momentum space may be taken as known.

We may now write for (4.8)

$$A(E_{A_1})=-\pi P_s\Gamma_0 T\delta(E_{A_1}+E_n-E_B)+\pi P_s\sum_{n'A'}[\langle n|J_{A_1}(0)|n'A'^{(+)}\rangle\tilde{F}(E_{A'})\delta(E_{A_1}+E_n-E_{A'}-E_{n'})]$$
$$+\pi P_s\gamma_0\langle d_1|J_{A_1}(0)|dA^{(+)}\rangle\delta(E_{d_1}+E_{A_1}-E_d-E_A), \quad (4.12)$$

where we have set $\langle B|J_p(0)|dA^{(+)}\rangle=T$, the stripping amplitude, and have recognized that with the projection factor present, the second factor in the second term is $\tilde{F}$. Only Γ_0 appears in the first term since

$$f(n^2)\delta(E_{A_1}+E_n-E_B)=\delta(E_{A_1}+E_n-E_B).$$

The other two delta functions serve to put n-A and d-A scattering on to the energy shell. We define the s-wave n-A scattering amplitude by

$$P_s\langle n|J_{A_1}(0)|n'A'^{(+)}\rangle\delta(E_{A_1}+E_n-E_{A'}-E_{n'})$$
$$=t_s(E_{A_1})\delta(E_{A_1}+E_n-E_{A'}-E_{n'}), \quad (4.13)$$

and also define the d-A scattering amplitude by

$$\langle d_1|J_{A_1}(0)|d'A'\rangle\delta(E_{A_1}+E_{d_1}-E_A-E_d)$$
$$=\tau(E_{dA},(\mathbf{d}-\mathbf{d}_1)^2)\delta(E_{A_1}+E_{d_1}-E_A-E_d). \quad (4.14)$$

We have written τ explicitly as a function of energy E_{dA}, and of momentum transfer $(\mathbf{d}-\mathbf{d}_1)^2=(\mathbf{d}-\mathbf{n}-\mathbf{p})^2$. It is only in the momentum transfer dependence that $\mathbf{n}$ or E_{A_1} enters.

Substituting (4.12) into (4.5) and writing $\tilde{F}$ now as a function of n^2, we have

$$\tilde{F}(n^2)=-\gamma_0 P_s\delta_{\mathbf{A},\mathbf{A}_1}-\frac{\Gamma_0 T}{n^2+\epsilon_B}-\sum_{n'}\frac{t_s(n'^2)\tilde{F}(n'^2)}{n'^2-n^2+i\epsilon}$$
$$-\gamma_0 P_s\frac{\tau(E_{dA},(\mathbf{d}-\mathbf{n}-\mathbf{p})^2)}{E_d+E_A-E_{A_1}-E_{d_1}}. \quad (4.15)$$

[24] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 611.

Expressing the third term in terms of the phase ϕ of (3.15) and changing the sum to an integral, we get

$$\tilde{F}(n^2)=g(n^2)+\frac{1}{\pi}\int_0^\infty \frac{e^{i\phi(x)}\sin\phi(x)\tilde{F}(x)dx}{x-n^2+i\epsilon}, \quad (4.16)$$

where

$$g(n^2)=P_s\left[-\gamma_0\delta_{\mathbf{A},\mathbf{A}_1}-\frac{\Gamma_0 T}{n^2+\epsilon_B}-\frac{\gamma_0\tau(E_{dA},(\mathbf{d}-\mathbf{n}-\mathbf{p})^2)}{E_d+E_A-E_{d_1}-E_{A_1}}\right]. \quad (4.17)$$

This is just the complex conjugate form of (3.14) and its solution reflects only this difference. The three terms making up $g(n^2)$ have a known dependence on n^2. The first depends on n^2 through the delta function, and in a more complete theory on the known internal structure of the deuteron. The second term depends on n^2 only through the denominator, $\Gamma_0 T$ being constants so far as dependence on n^2 is concerned. The third term depends on n^2 both through the denominator and through the assumed known momentum transfer dependence of τ. The solution of this equation, making the usual assumptions about asymptotic behavior, is[23]

$$\tilde{F}(n^2)=g(n^2)+e^{\sigma(n^2)-i\phi(n^2)}\frac{1}{\pi}\int_0^\infty \frac{e^{-\sigma(x)}\sin\phi(x)g(x)dx}{x-n^2+i\epsilon} \quad (4.18)$$

where σ is defined in (3.18). When this is written out and the trivial integral involving the delta-function term in g is performed, we obtain

$$\tilde{F}(n^2)=-\gamma_0 P_s\left[\delta_{\mathbf{n},\mathbf{q}}+\frac{\tau(E_{dA},(\mathbf{q}-\mathbf{n})^2)}{E_d+E_A-E_{d_1}-E_{A_1}}+\frac{2\pi\sin\phi(q^2)e^{-\sigma(q^2)+\sigma(n^2)-i\phi(n^2)}}{q(q^2-n^2+i\epsilon)}\right.$$
$$\left.-\frac{e^{\sigma(n^2)-i\phi(n^2)}}{\pi}\int_0^\infty\frac{e^{-\sigma(x)}\sin\phi(x)}{x-n^2+i\epsilon}P_s\frac{\tau(E_{dA},(\mathbf{q}-\mathbf{n}_x)^2)}{E_d+E_A-E_{d_x}-E_{A_x}}dx\right]$$
$$-\Gamma_0 T\left[\frac{1}{n^2+\epsilon_B}+\frac{e^{\sigma(n^2)-i\phi(n^2)}}{\pi}\int_0^\infty\frac{e^{-\sigma(x)}\sin\phi(x)dx}{(x-n^2+i\epsilon)(x+\epsilon_B)}\right], \quad (4.19)$$

where we have called the momentum transfer in the stripping reaction $\mathbf{q}=\mathbf{d}-\mathbf{p}$. E_{dx}, n_x, etc., refer to values of these quantities corresponding to a neutron wave number whose square is x.

We must now do the sum in (2.21), which we write as

$$T=\sum_{\mathbf{n}}\frac{\Gamma(n^2)\tilde{F}(n^2)}{n^2+\epsilon_B}. \quad (4.20)$$

We get

$$T=\frac{\gamma_0\Gamma_0}{Z(1-C\Gamma_0^2)}\left[\frac{G(q^2)}{q^2+\epsilon_B}+\frac{e^{-\sigma(q^2)}\sin\phi(q^2)}{\pi q}\int_0^\infty\frac{G(n^2)e^{\sigma(n^2)-i\phi(n^2)}ndn^2}{(q^2-n^2+i\epsilon)(n^2+\epsilon_B)}\right.$$
$$+\frac{1}{2\pi^2}\int_0^\infty\frac{G(n^2)ndn^2}{n^2+\epsilon_B}P_S\frac{\tau(E_{dA},(\mathbf{q}-\mathbf{n})^2)}{E_d-E_A-E_{d_1}-E_{A_1}}+\frac{1}{2\pi^3}\int_0^\infty\frac{G(n^2)ndn^2}{n^2+\epsilon_B}\int_0^\infty\frac{dx\,e^{\sigma(n^2)-\sigma(x)-i\phi(n^2)}\sin\phi(x)}{x-n^2+i\epsilon}$$
$$\left.\times P_s\frac{\tau(E_{dA},(\mathbf{q}-\mathbf{n}_x)^2)}{E_d+E_A-E_{d_x}-E_{A_x}}\right], \quad (4.21)$$

where we have written $\Gamma(n^2)=-\Gamma_0 G(n^2)/Z$, and where

$$C=\frac{1}{2\pi^2}\int_0^\infty\frac{G(n^2)ndn^2}{n^2+\epsilon_B}\left(\frac{1}{n^2+\epsilon_B}+\frac{e^{\sigma(n^2)-i\phi(n^2)}}{\pi}\int_0^\infty\frac{e^{-\sigma(x)}\sin\phi(x)dx}{(x-n^2+i\epsilon)(x+\epsilon_B)}\right). \quad (4.22)$$

The first term of (4.21) is easily recognized as containing the lowest order Born approximation, which is

$$\gamma_0\Gamma_0 f(q^2)/q^2+\epsilon_B, \quad (4.23)$$

neglecting the deuteron structure. This term has a pole at $q^2=-\epsilon_B$ with residue $\Gamma_0\gamma_0$, and so must the entire amplitude.[25] In fact (4.21) has such a pole, but the residue is

$$\frac{G(-\epsilon_B)}{(1-C\Gamma_0^2)}\frac{\gamma_0\Gamma_0}{Z}. \quad (4.24)$$

[25] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959); R. D. Amado, Phys. Rev. Letters **2**, 399 (1959).

To make this residue $\Gamma_0\gamma_0$, it must be that

$$Z(1-C\Gamma_0^2)=G(-\epsilon_B). \tag{4.25}$$

This is fortunate since it allows us to do away with the cumbersome factor $1-C\Gamma_0^2$ and with the unknown wave function renormalization Z. We thus may write for T,

$$T=\frac{\gamma_0\Gamma_0}{G(-\epsilon_B)}\Big[\quad\Big], \tag{4.26}$$

where the square brackets represent the quantity in square brackets in (4.21). Having eliminated Z in this way, we may take the bound-state limit of vanishing Z. Inspection of the quantities involved in (4.26) will show that they all converge and hence no damage is done in taking this limit. That (4.26) is meaningful in this limit may be questioned, but we believe that since the limit exists it represents the T matrix of interest and that the construct of taking B to be an elementary particle may now be dropped.

The T matrix for deuteron stripping has now been expressed as an integral over known quantities; the strength of deuteron disassociation, γ_0, the dependence of neutron capture into B on neutron momentum, $f(n^2)$, the d-A and n-A elastic scattering amplitudes on the energy shell, and the one parameter Γ_0. There is no need to solve for scattering wave functions as in the distorted-wave theory,[3] or to assume an optical potential for d-A scattering. The first approximation involved in getting this result occurs at (2.21), where from all the possible states with the quantum numbers of a neutron and A we assume the capture into B to proceed from the n-A state itself. This is the direct-interaction approximation. It leads us to use an optical model for the n-A state and thus reduces this part of the problem to a potential problem. From this point, the approximations involved are the neglect of n-p rescattering on the deuteron break-up, and the neglect of an explicit p-A interaction on the break-up, but not on d-A scattering, since this is taken directly from experiment. This later approximation is explored further in the Appendix. We have also made some plausible, but by no means established conjectures about analytic properties of the amplitudes involved. In doing this, we have been guided in part by relativistic and nonrelativistic[6] theory but largely by the Lee model.[10] This has led to a hybrid formulation in which some amplitudes occur on the energy shell in dispersion relations, while in (2.21) we deal with amplitudes off the energy shell. This technique allows us to introduce wave functions in a simple way—thus, avoiding a discussion of them in terms of dispersion relations, complex intermediate states, and anomalous thresholds.[9] For this we pay the price of an asymmetric formulation and of having to assign simple analytic properties to some very complex objects. It is hoped that in doing so we have adequately described the most important part of the physics. Finally, we have made a number of easily relaxed simplifications, such as dropping spin and treating the deuteron as all asymptotic. In the next section we shall simplify (4.26) further in order to extract some of its meaning. But let us keep in mind that these simplifications are by no means necessary to the use of (4.26). Given the input information it could be used directly to compute T.

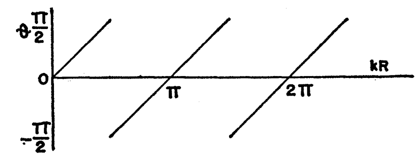

FIG. 2. The form of hard-sphere phase shift chosen in (5.4).

V. DISCUSSION OF RESULTS

In the last section we obtained a form for the stripping T matrix valid for the direct interaction part of the amplitude, that is, when proton-A and neutron-proton interactions are neglected during the stripping process itself. To use this T matrix we need know only neutron-A scattering, deuteron-A scattering, and the neutron bound-state wave function in B. Because the form given in (4.26) is quite complex, it might be well to simplify it still further in order to extract some of its essential features.

First let us investigate the effects of distortion on the vertex as given in Sec. III in a simple model. Consider a particle bound in the lowest state in a very deep, that is, infinite, potential. The bound-state wave function is

$$\begin{matrix} \dfrac{1}{(2\pi R)^{\frac{1}{2}}}\dfrac{1}{\Gamma}\sin(\pi r/R), & \Gamma<R \\ 0, & \Gamma>R \end{matrix} \tag{5.1}$$

where R is the radius of the potential. The corresponding Fourier transform defined as in (3.7) is, up to a multiplicative constant,

$$(\sin kR)/kR. \tag{5.2}$$

We now wish to consider capture into this state without fretting too much about just what it means to have capture into an infinite square well from a scattering state. This model is only meant to represent an exaggerated picture in which to examine the effects of distortion. The s-wave phase shift for wave number k on an infinite square well attractive or repulsive, real or complex, is given by

$$\tan\phi=\tan kR. \tag{5.3}$$

This equation has many solutions, and which one we take depends on the condition of the problem. We wish to consider an optical model potential with some absorption. It is clear that in that case the denominator of (3.15) cannot vanish and hence $-\pi/2 \leq \phi \leq \pi/2$. This restriction will give

$$\begin{aligned} \phi &= kR, & -\pi/2 < kR < \pi/2, \\ &= kR-\pi, & \pi/2 < kR < 3\pi/2, \text{ etc.,} \end{aligned} \tag{5.4}$$

as shown in Fig. (2). We now use this form of ϕ to compute the vertex function of (3.19). Since ϕ as defined above is periodic, it is convenient to expand it in a Fourier series. We obtain

$$\phi(kR) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin 2nkR}{n}. \tag{5.5}$$

For (3.18) we have

$$\begin{aligned} \sigma(k^2)+i\phi(kR) &= (k^2+\epsilon_B)\frac{1}{\pi}\int_0^\infty \frac{\phi(yR)dy^2}{(y^2+\epsilon_B)(y^2-k^2-i\epsilon)} \\ &= (k^2+\epsilon_B)\frac{1}{\pi}\int_{-\infty}^\infty \frac{\phi(yR)ydy}{(y^2+\epsilon_B)(y-k-i\epsilon)(y+k+i\epsilon)}, \end{aligned} \tag{5.6}$$

since ϕ is odd in y. Substituting (5.5) into (5.6) and integrating term by term, we find we must evaluate integrals of the form

$$\frac{(k^2+\epsilon_B)}{\pi}\int_{-\infty}^\infty \frac{ydy \sin 2nyR}{(y-k-i\epsilon)(y+k+i\epsilon)(y+i\kappa)(y-i\kappa)}, \tag{5.7}$$

where we have defined $\kappa = \epsilon_B^{\frac{1}{2}}$. Writing the sine in exponential form and taking the obvious contours in the complex y-plane, we find for (5.7)

$$e^{2inkR} - e^{-2n\kappa R}, \tag{5.8}$$

so that

$$\begin{aligned} \sigma(k^2)+i\phi(k)) &= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(e^{2inkR}-e^{-2n\kappa R}) \\ &= \ln(1+e^{2ikR}/1+e^{-2\kappa R}). \end{aligned} \tag{5.9}$$

We then have

$$\begin{aligned} \sigma(k^2) &= \operatorname{Re}\ln(1+e^{2ikR}/1+e^{-2\kappa R}) \\ &= \ln(2|\cos kR|/1+e^{-2\kappa R}). \end{aligned} \tag{5.10}$$

Putting this into (3.19), we must evaluate the integral

$$g = \frac{1}{\pi}\int_0^\infty \frac{\sin\phi(xR)}{|\cos xR|}\frac{\sin xR}{(xR)}\frac{dx^2}{(x^2-k^2-i\epsilon)}. \tag{5.11}$$

We see from the definition of ϕ that $\sin\phi/|\cos xR| = \tan xR$ so that

$$g = \frac{1}{\pi}\int_{-\infty}^\infty \frac{\tan xR \sin xR dx}{(x-k-i\epsilon)(x+k+i\epsilon)}. \tag{5.12}$$

This integral can again be done by writing the sine in exponential form and taking contours in the complex x plane. The contributions from the poles of the tangent cancel so that

$$g = e^{ikR}(\tan kR)/kR \tag{5.13}$$

and thus

$$\begin{aligned} \Gamma &= C(\sin kR/kR + (\tan kR/kR)|\cos kR|e^{i(\phi+kR)}) \\ &= 2C(\sin kR/kR)\cos\phi e^{i\phi}, \end{aligned} \tag{5.14}$$

where C is a constant depending on Z, Γ_0, etc. In fact, the reduced width is zero for an infinite potential, but as was explained above we ignore these difficulties in this simple model. The absolute square of (5.14), which is what would enter a cross section, is then given by

$$|\Gamma|^2 = 4C^2\frac{\sin^2 kR\cos^2\phi}{(kR)^2} = C^2\left(\frac{\sin 2kR}{kR}\right)^2, \tag{5.15}$$

since $\cos^2\phi = \cos^2 kR$. This is just the form of the undistorted vertex (5.2) except that $R \to 2R$. Such a replacement is familiar in ordinary cutoff stripping theory where the capture probability is written down in undistorted form but the incoming neutron plane wave is cutoff at some radius.[2] This cutoff is supposed to represent the fact that inside it the neutron is so badly scattered about that it cannot be captured. The cutoff radius needed to fit experiment is found to be considerably larger than the nuclear radius. This connection between the cutoff Born approximation for the capture and our formalism is certainly not complete, but that distortion of the incoming neutron is associated with larger effective values of the radius (in our case twice the nuclear radius) seems suggestive.

The effect of neutron-A scattering is not taken into account only in the capture vertex, but also in the breakup amplitude, F. The combined effects, independently of deuteron-A scattering are dealt with in the first and second terms of the square bracket in (4.21), that is, in

$$B(q^2) = \frac{G(q^2)}{q^2+\epsilon_B} + \frac{e^{-\sigma(q^2)}\sin\phi(q^2)}{q\pi} \times \int_0^\infty \frac{G(n^2)e^{\sigma(n^2)-i\phi(n^2)}ndn^2}{(q^2-n^2+i\epsilon)(n^2+\epsilon_B)}. \tag{5.16}$$

We can calculate $B(q^2)$ in the infinite well model using the forms of σ, and ϕ found above and putting $G = \Gamma/C$ from (5.14). We must evaluate the integral

$$\frac{1}{\pi R}\int_0^\infty \frac{dn^2 \sin nR\cos^2 nR}{(n^2+\epsilon_B)(n^2-q^2-i\epsilon)}, \tag{5.17}$$

where we have used the fact that $\cos\phi|\cos nR| = \cos^2 nR$. Writing the sine and cosine in exponential form and using the evenness of the integrand in n, we can again evaluate the integrals by choosing contours in the upper or lower half n planes, and we get for (5.17)

$$(e^{2iqR}\cos qR - e^{-2\kappa R}\cosh\kappa R)/2R(q^2+\epsilon_B). \tag{5.18}$$

Substituting this into (5.16) along with the forms of σ and G, we obtain after using some trigonometric identities

$$B(q^2)=(1/q^2+\epsilon_B)(\sin qR/qR)\times(1+e^{-2\kappa R}(\cosh\kappa R/\cos qR)]. \quad (5.19)$$

The first term is just the undistorted vertex. Thus, the combined effects of distortion on the capture and on the deuteron disassociation have canceled. This is disconcerting, but as we shall see not fatal. The second term is even more troublesome since it blows up at $\cos qR=0$, but it clearly has its origins in the singular nature of the potential. It is probably best to assume q sufficiently small and ϵ_B sufficiently large, so that we can neglect it or at worse ignore it. Thus, in the absence of deuteron-A rescattering corrections, we have recovered the Born approximation in the hard-sphere theory. But we need not neglect deuteron-A scattering, and it is just the fact that our formalism is particularly suited to including it which makes it interesting even if the neutron-A rescattering corrections cancel in this model.

Before putting in the deuteron-A scattering, however, we must investigate whether this cancellation occurs in general. To do this, we first obtain a simpler form for $G(n^2)$. We assume that for well-behaved potentials, the Fourier transform of the bound state $f(n^2)$, will be real for positive n^2 and analytic in the entire n^2 plane except for a cut from $-\lambda$ to $-\infty$, where λ is a positive parameter related to the potential parameters, but $\lambda>\epsilon_B$.[9] Thus, we may write

$$f(n^2)=\frac{1}{\pi}\int_{-\lambda}^{-\infty}\frac{D(y)}{y-n^2}dy, \quad (5.20)$$

where $D(y)$ is the discontinuity in f across the cut, and where we have assumed f goes to zero for large n^2. $D(y)$ is real. Using the analytic properties of $\exp(\sigma+i\phi)$ and f, the integral in the expression (3.19) for the vertex may be expressed first as an integral around the cut from 0 to ∞, due to $\exp(\sigma+i\phi)$, and then deformed around to the cut, due to f, so that finally we obtain for the vertex

$$G(n^2)=\frac{e^{\sigma(n^2)+i\phi(n^2)}}{\pi}\int_{-\lambda}^{-\infty}\frac{e^{-\sigma(x)}D(x)dx}{x-n^2}. \quad (5.21)$$

This is clearly a solution of the mapping problem defined by (3.14), since here G has the phase ϕ along the positive real axis and the discontinuity D across the negative cut. This simpler form of G is not appropriate to the hard-sphere case, since the Fourier transform cannot be written as in (5.20), nor is it appropriate to actual nuclear physics calculations, since usually some approximation is used to obtain the wave function which is a good approximation for positive n^2 but may very well not have the correct analytic properties. If reasonable approximations to bound states could be found in terms of their left-hand discontinuities as for example replacing the cut by a pole as is done in the Hulthén wave function for the deuteron,[26] then (5.20) could be used to put the vertex more simply.

The general effects of distortion are easily formulated in terms of (5.21). We obtain for the general form of $B(q^2)$

$$B(q^2)=\frac{e^{\sigma(q^2)+i\phi(q^2)}}{q^2+\epsilon_B}\times\frac{1}{\pi}\int_{-\lambda}^{-\infty}\frac{e^{-\sigma(x)}D(x)dx}{x-q^2}-\frac{e^{-\sigma(q^2)}\sin\phi(q^2)}{q}\times\frac{1}{\pi}\int_0^{\infty}\frac{ndn^2\,G(n^2)e^{\sigma(n^2)-i\phi(n^2)}}{(n^2+\epsilon_B)(n^2-q^2-i\epsilon)}. \quad (5.22)$$

This defines a function analytic in the cut q^2-plane with a pole at $q^2=-\epsilon_B$ with residue $G(-\epsilon_B)$, which is real. There is a cut from 0 to ∞ due to $\exp(\sigma+i\phi)$ in the first term and the integral in the second. There is also a cut from $-\lambda$ to $-\infty$ from the integral in the first term. The factor $\sin\phi\exp(-\sigma)=-\mathrm{Im}\exp[-(\sigma+i\phi)]$ is real for q^2 real and positive. We shall make the usual assumption about the analyticity of partial wave amplitudes[6] and say that this factor has a cut from $-\mu$ to $-\infty$, where μ is some positive constant generally close to λ and certainly greater than ϵ_B. Thus, we may write for the general form of B

$$B(q^2)=\frac{G(-\epsilon_B)}{q^2+\epsilon_B}+\frac{1}{\pi}\int_0^{\infty}\frac{\mathrm{Im}B(x)}{x-q^2}dx-\frac{1}{\pi}\int_{-\alpha}^{-\infty}\frac{\mathrm{Im}B(x)}{x-q^2}dx, \quad (5.23)$$

where α is the smaller of λ and μ. The imaginary part of the second term of (5.22) for real positive q^2 comes entirely from the small detour introduced by the $i\epsilon$ in the denominator, since all the other factors are real. It is easily seen that this just cancels the imaginary part of the first term so that in fact $\mathrm{Im}B(x)=0$, $x>0$. Thus, only the pole and the left-hand cut survive. This cancellation must occur in order that the stripping amplitude not have a cut in momentum transfer for physical values of the momentum transfer.[27]

[26] L. Hulthén and M. Sugawara, *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 34.

[27] S. Mandelstam, Phys. Rev. **112**, 1344 (1958); R. Blankenbecler, M. L. Goldberger, N. N. Khuri, and S. B. Treiman, Ann. Phys. (New York) **10**, 62 (1960); A. Klein, J. Math. Phys. **1**, 41 (1960).

From the first integral in (5.22), we get that $\mathrm{Im}B(x) = -D(x)$, $-\lambda \leq x < -\infty$, which substituted in (5.23) gives the wave function. In the hard-sphere case the singularities from $\sin\phi \exp -\sigma$ do not add anything significant. In general, there is no reason to believe this will be the case. Only in a simple model where the scattering all proceeds via the intermediate bound state, as in N-θ scattering in the Lee model,[28] will the bound-state and scattering left-hand cuts be trivially related. In general, the cuts will be different, particularly in an optical model where the potentials for the bound state and for the scattering will usually be different. To summarize, it is expected that, in general, $B(q^2)$ will be real for real positive q^2, but it is not expected that the effect of n-A rescattering will disappear completely, and, in fact, we would expect to see the apparent increase in nuclear radius so characteristic of direct interactions emerge from this distortion.

We now turn to the problem of including the deuteron-A interaction. From (4.21), we see that we must compute integrals involving

$$P_s\tau(E_{dA},(\mathbf{q}-\mathbf{n})^2)/[E_d+E_A-E_{d_1}-E_{A_1}+i\epsilon] \tag{5.24}$$

where we have written in explicitly the imaginary part of the denominator coming from (4.5). At moderate energies deuteron-A scattering will be largely diffractive, that is $\tau(E_{dA},(\mathbf{q}-\mathbf{n})^2)$ will be large only for $(\mathbf{q}-\mathbf{n})^2$ small and will oscillate as $(\mathbf{q}-\mathbf{n})^2$ increases. Furthermore, $\mathbf{q}-\mathbf{n}=0$ is the place the denominator vanishes. This can be seen by writing in the momenta

$$\begin{aligned}E_d+E_A-E_{d_1}-E_{A_1}\\ &=(\mathbf{q}-\mathbf{n})\cdot[(\mathbf{q}+\mathbf{n})/a-(\mathbf{q}-\mathbf{n})/2+d],\end{aligned} \tag{5.25}$$

where a is the atomic number of nucleus A. Thus, we expect the major contribution to the integrals involving (5.24) to come from $\mathbf{q}\approx\mathbf{n}$, particularly for small q^2 where functions like $G(q^2)$ are peaked. Replacing $\mathbf{n}$ or $\mathbf{n}_x$ by $\mathbf{q}$ in the slowly varying parts of (4.26), we get

$$T=\gamma_0\Gamma_0/G(-\epsilon_B)B(q^2)(1+\mathcal{J}), \tag{5.26}$$

where

$$\begin{aligned}\mathcal{J}&=\frac{1}{2\pi^2}\int_0^\infty ndn^2\,P_s\frac{\tau(E_{dA},(\mathbf{q}-\mathbf{n})^2)}{(\mathbf{q}-\mathbf{n})\cdot(\mathbf{d}+2\mathbf{q}/a)+i\epsilon}\\ &=\frac{1}{4\pi^3}\int d^3n\,\frac{\tau(E_{dA},(\mathbf{q}-\mathbf{n})^2)}{(\mathbf{q}-\mathbf{n})\cdot(\mathbf{d}+2\mathbf{q}/a)+i\epsilon}.\end{aligned} \tag{5.27}$$

The angular integration is easily done and we obtain

$$\mathcal{J}=-\frac{i}{2\pi}\int_0^\infty ydy\,\frac{\tau(E_{dA},y^2)}{|\mathbf{d}+2\mathbf{q}/a|}.$$

[28] T. D. Lee, Phys. Rev. 95, 1329 (1954).

To evaluate the remaining integral, let us assume that the diffraction peak of deuteron-A scattering has a narrow width, constant in momentum transfer, so that

$$\begin{aligned}\tau(E,y^2)&=\tau(E,0), & y^2<\alpha^2\\ &=0, & y^2>\alpha^2.\end{aligned} \tag{5.28}$$

We get

$$\mathcal{J}=-(i/4\pi)\tau(E_{dA},0)\alpha^2/|\mathbf{d}+2\mathbf{q}/a|. \tag{5.29}$$

The width α^2 of the diffraction peak should be of the order of the deuteron binding energy.

If deuteron-A scattering is purely diffractive, then $\tau(E,0)$ is pure imaginary and may be related to the total cross-section by the optical theorem. In this case we obtain for the stripping amplitude

$$T=\frac{\gamma_0\Gamma_0}{G(-\epsilon_B)}B(q^2)\left(1-\frac{K_{dA}}{8\pi}\frac{\sigma_T(E_{dA})\alpha^2}{|\mathbf{d}+2\mathbf{q}/a|}\right), \tag{5.30}$$

where K_{dA} is the relative deuteron-A wave member and $\sigma_T(E)$ the total deuteron-A cross section for energy E.

In (5.30) we have succeeded in writing the amplitude for stripping as a function of the momentum transfer $\gamma_0\Gamma_0B(q^2)/G(-\epsilon_B)$ times a function of the energy and momentum transfer. The first factor is just the Born approximation modified by the effects of neutron-A rescattering. It is expected that this term will be very similar to the cutoff Born approximation of Butler[12] and hence will account for the characteristic angular distribution of stripping. The second factor depends only weakly on momentum transfer, but it depends fairly strongly on the energy through the total cross section. The form (5.30) for T then explains, qualitatively at least, how the angular distribution or better distribution in momentum transfer in stripping can be accounted for by a modified Born approximation, while the yield is described by a more violently varying function of energy with variation related to the total deuteron-A cross section.

The question of reduced width Γ_0 obtained from (5.30) is of more doubt. The magnitude of Γ_0 obtained using the correction due to deuteron-A scattering may be greater or less than without, depending on the size of the correction. If $\tau(E,0)$ is not pure imaginary, the situation is more complex. More perpelxing is the fact that the residue will be energy dependent. This is incorrect and arises from the manifestly incorrect analytic structure assigned to τ in getting (5.26) and in (5.30). Nevertheless, it points to the problems which might arise from simple methods for getting the residue which neglect rescattering.[25]

We will not pursue further the properties of (5.30), since it represents only a crude approximation to the form of the full amplitude given in (4.26), but it is clear that many of the qualitative features of (5.30) should persist in a calculation based on the full amplitude.

There are still a large number of questions. To what extent will (4.26) account for the experimental data? What is its relation to distorted-wave theory?[3] How can it be improved? What modifications are needed for other direct reactions? Can one give a better justification of the analytic properties assumed in Sec. IV? What is the relation of the simplified forms discussed in this section to the semiclassical ray-tracing theories?[29] It is hoped they will soon receive answers.

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APPENDIX. PROTON-A INTERACTION

In a potential formalism the only interactions essential to deuteron stripping are the neutron-proton interaction (to bind the deuteron) and the neutron-A interaction (to bind B). Of course, in nature there is also a proton-A interaction. In the formalism of this paper we have allowed implicitly for the contribution of this interaction to deuteron-A scattering by using the full experimental amplitude for this process, but have neglected with the equal time commutator of (4.3) its contribution to the stripping process itself.

In our formulation of stripping, the deuteron and A come together and interact in all possible ways until finally the proton goes off freely leaving behind an interacting neutron-A state which coalesces into B. To this point the analysis is exact, whether or not there is a proton-A interaction. We have claimed that the signature of a direct interaction is determined by what happens just previous to this disassociation into free proton and neutron-A state. We are not claiming that all sorts of things do not happen, but rather that if they do a direct reaction will not occur. In order for the neutron and A to form B they must interact. Thus, if the deuteron disassociation is brought about by the proton interacting with A, it is necessary for the proton to escape and for the neutron to "find" A. This seems an unlikely way for the direct reaction to proceed, but this is just the process described by the equal-time commutator. To see this let us suppose that the proton-A interaction is described by the potential U. The equal time commutator E of (4.3) can be then written,

$$E=\int d^3x\, e^{-i\mathbf{A}_1\cdot\mathbf{x}}\langle N|[\psi_A(\mathbf{x}),J_p(0)]|dA^{(+)}\rangle$$

$$=\int d^3x d^3y\, e^{-i\mathbf{A}_1\cdot\mathbf{x}-i\mathbf{p}\cdot\mathbf{y}}U(|\mathbf{x}-\mathbf{y}|)\times\langle N|\psi_A(\mathbf{x})\psi_p(\mathbf{y})|dA^{(+)}\rangle. \quad \text{(A.1)}$$

This describes the process in which an interacting deuteron and A turn into a free neutron, free proton, and free A. The last process before this being an interaction of the proton and A via U. If we analyze this further by contracting the A from the right and inserting states, we obtain,

$$E=\int d^3x d^3y\, \langle N|\psi_p(\mathbf{x})|d\rangle U(|\mathbf{x}-\mathbf{y}|)e^{i(\mathbf{A}-\mathbf{A}_1)\cdot\mathbf{x}-i\mathbf{p}\cdot\mathbf{y}}$$

$$+\int d^3x d^3y\, e^{-i\mathbf{A}_1\cdot\mathbf{x}-i\mathbf{p}\cdot\mathbf{y}}U(|\mathbf{x}-\mathbf{y}|)$$

$$\times\sum_S \frac{\langle N|\psi_A(\mathbf{x})\psi_p(\mathbf{y})|S\rangle\langle S|J_A^\dagger(0)|d\rangle}{E_S-E_A-E_d-i\epsilon}. \quad \text{(A.2)}$$

The first term comes from an equal-time commutator and represents the lowest order contribution to the process we have described, that is, break-up of the free deuteron through an interaction of the proton with A. It is easily evaluated in terms of the Fourier transform of U and of the deuteron wave function and may be added to the inhomogeneous term in (4.16) if one wishes. The other term may be described in terms of the states S, the most obvious being dA, pB, and npA. All of these represent complicated contributions to the basic deuteron break-up via the proton-A interaction and can probably be evaluated in some simple cases, for example, if the deuteron-A interaction occurs strongly in some partial wave, but not in general. We again insist that neglect of the p-B state in this place is not neglect of final state rescattering but rather to say that the chain of processes $d+A \to p+B \to [n+p+A] \to p+B$, where in the step $[n+p+A]$ the neutron is free and the proton interacts with A, is unlikely.

[29] See S. T. Butler, N. Austern, and C. Pearson, Phys. Rev. **112**, 1227 (1958).