

## Optical Model Analysis of 260-MeV $K^+$ -Meson Elastic Scattering\*

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(Received February 15, 1962)

The reaction and differential elastic-scattering cross sections have been measured for  $K^+$  mesons with a mean energy of 260 MeV in nuclear emulsion. Exact diffuse-surface optical model calculations and  $\chi^2$  comparisons with the experimental results have been made to determine the real and imaginary nuclear potential volume integrals per nucleon,  $I_v + iI_w$  for the Saxon well shape with a number of choices for the shape parameters  $a$  and  $R_0$ . Good fits have been obtained for a range of the shape parameters. For example, with  $a=0.57$  F and  $R_0=1.07$  F,  $I_v=145\pm 45$  MeV-F<sup>3</sup> and  $I_w=116\pm 5$  MeV-F<sup>3</sup>. Better fits are obtained for smaller values of 'a,' and equally good fits are obtained with  $R_0=1.20$  F and the smaller 'a' values.  $I_v$  and  $I_w$  are sensitive to the shape parameters  $a$  and  $R_0$ , the relationships:  $I_v a^3 R_0 = \text{const}$  and  $I_w a^3 R_0 = \text{const}$ , holding to within a few percent over a reasonable range of values.

### INTRODUCTION

THE analysis of  $K^+$ -meson elastic scattering from emulsion nuclei has been carried out in several studies at energies below 200 MeV. The earlier calculations used WKB and Born approximations for a square-well optical model<sup>1-3</sup> and indicated a repulsive  $K^+$ -meson nuclear potential.

More recent work with the diffuse-surface optical model<sup>4,5</sup> has confirmed the repulsive character of the potential and has indicated the need for a careful accounting of the spread of  $K^+$ -meson energy and of the effect of the different constituents of nuclear emulsion in order to obtain accurate values for the potential. These diffuse-surface optical model analyses have been carried out over an energy region of from 50 to 150 MeV. The results show that the real potential is not very energy sensitive but that the imaginary potential is gradually rising with energy. Melkanoff *et al.*<sup>5</sup> have also shown that the fitting of the data is insensitive to the choice of surface thickness parameter, that the potentials are radius dependent and that best fits are obtained in the region of  $R_0=1.07-1.20$  F.

In this article we present an extension of the previous studies to the energy region 200-300 MeV. New experimental results are reported here and a diffuse-surface optical model analysis is carried out. In this analysis we have profited by the experience of Melkanoff *et al.* and use one representative energy of 261 MeV, and two nuclear species representative of the nuclei found in nuclear emulsion:  $Z=41$ ,  $A=94$  and  $Z=7$ ,  $A=14$  with weights of 0.426 and 0.574, respectively.

### EXPERIMENTAL DETAILS

A stack of 280 Ilford G5 600- $\mu$  nuclear-emulsion pellicles, 3.25 by 14 in., was exposed to a 620 MeV/ $c$  momentum-selected and degrader-separated  $K^+$ -meson beam at the Berkeley Bevatron. A sketch of the beam layout is shown in Fig. 1. The ratio of  $K^+$  mesons to tracks of minimum ionization was about 1 to 5 and about 3000  $K^+$  mesons entered the stack during the exposure. In scanning, the  $K^+$ -meson tracks were picked up 15 mm from the leading edge of the stack and were followed for 10 cm unless an interaction, scatter, or decay in flight were found.

All inelastic events were noted and all apparently elastic events were recorded for which the projected angle of scattering in the emulsion plane was  $\geq 2^\circ$ . Projected and dip angles of scattering were measured by three different observers in the majority of the cases and by two in the remainder of the sample. An error analysis of these measurements has indicated that the effect of "spill-over" of events from smaller to larger angles, caused by a rapidly decreasing cross section and the uncertainty of angular measurement, is small for the angular intervals used and no correction has been made for this effect.

A count of 400 blobs made both at the pickup point and after following a track for 10 cm eliminated all but

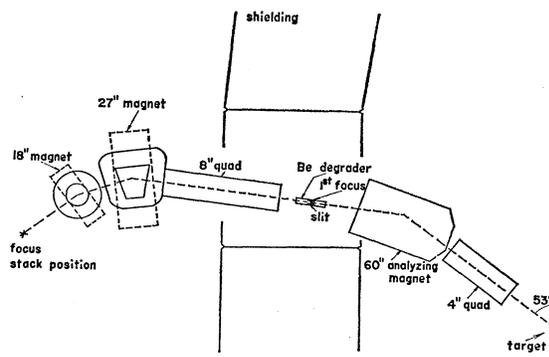


Fig. 1. Sketch of the beam layout at the Berkeley Bevatron. The stack was positioned at the final focus as shown.

\* Supported in part by programs of the U. S. Atomic Energy Commission, the Office of Naval Research, and the National Science Foundation.

<sup>1</sup> C. Marchi, G. Quarenì, A. Vignudelli, G. Dascola, and S. Mora, *Nuovo cimento* **5**, 1790 (1957).

<sup>2</sup> G. Costa and G. Paternagi, *Nuovo cimento*, **5**, 448 (1957).

<sup>3</sup> G. G. Lim and S. J. Bosgra, *Nuovo cimento* **8**, 340 (1957).

<sup>4</sup> G. Igo, D. G. Ravenhall, J. J. Tiemann, W. W. Chupp, G. Goldhaber, S. Goldhaber, J. Lannutti, and R. M. Thaler, *Phys. Rev.* **109**, 2133 (1958); see also B. Sechi-Zorn and G. T. Zorn, *ibid.* **120**, 1898 (1960).

<sup>5</sup> M. A. Melkanoff, O. R. Price, D. H. Stork, and H. K. Ticho, *Phys. Rev.* **113**, 1303 (1958).

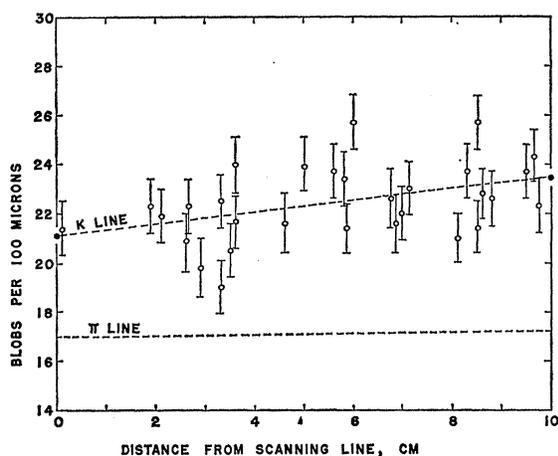


FIG. 2. A typical sample of blob-count measurements made at various points of interaction. The mean values of the blob counts at the pickup point and after a distance of 10 cm are shown as black circles.

a negligible number of noninteracting background tracks. The  $K$ -meson character of interactions and scatters was determined by the following procedure. If a  $K$ -meson secondary emerged, it was identified by observing the decay at the end of its range or by means of a blob count or a blob and hole count as a function of range. A typical sample of blob-count measurements at the points of interaction is shown in Fig. 2. The mean values of the blob counts at the pick-up point and after a distance of 10 cm are shown as black circles. These values are well known as all noninteracting tracks were counted at these two points. If no  $K$  mesons were found emerging from the interaction, the primary track was identified by ionization-multiple-scattering techniques; and if the primary track was indeed a  $K$  meson, such an event was classified as a charge exchange. Only two background events were found whose character had escaped detection in the blob count on the primary track at the pickup point.

#### ELASTIC CRITERIA

An event was provisionally called elastic if but one secondary emerged with scattering angle less than  $30^\circ$  and if there were no apparent changes in ionization. A large sample of such secondaries was followed to rest and their ranges measured. The resulting range distribution is compared to that for noninteracting  $K$  mesons in Fig. 3. The mean range of the  $K$  mesons entering the stack is  $29.35 \pm 0.23$  cm corresponding to a mean energy of  $303 \pm 2$  MeV.<sup>6</sup> The mean range of the group of scattered  $K$  mesons is  $30.05 \pm 0.33$  cm corresponding to an energy of  $308 \pm 3$  MeV. The uncertainty is largely caused by range straggling and the distributions have an average standard deviation of 1.8 cm. The elastic criterion

was determined by a cutoff of 2 standard deviations below the mean range; this cutoff corresponds to an energy loss of 25 MeV or 10% of the mean energy at the point of scattering (261 MeV).

About 40% of the secondaries from elastic scattering events with angles less than  $30^\circ$  interacted a second time, decayed in flight, or left the stack before stopping. The residual range was determined in these cases by ionization measurements. For each event the elastic criterion was determined by a cutoff of 2 standard deviations below the mean range of noninteracting  $K$  mesons. This typically corresponded in energy to 20% of the mean energy at the point of scattering. It was estimated that by this procedure only some 2% of events which were truly elastic were missed from this group of elastically scattered  $K$  mesons and no correction was made.

It is clear from Fig. 3 that there is no gross inelasticity of 10 MeV or more. However, a small number of inelastic events of small energy loss may be included in the resulting elastic distribution. One possible origin of such events is scattering by direct excitation of low lying nuclear levels. Little data is available concerning this effect except in the case of electrons,<sup>7</sup> protons,<sup>8</sup> and  $\pi^+$  mesons<sup>9</sup> inelastically scattered from  $C^{12}$ , where the excitation of the 4.4 MeV level has been directly observed. The ratio of the cross section for excitation of the 4.4 MeV level to the elastic scattering cross section is shown in Fig. 4 as a function of momentum transfer.

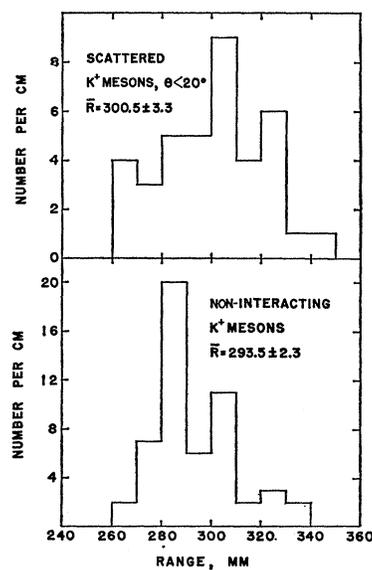


FIG. 3. The range distribution of scattered  $K^+$  mesons compared to the distribution for nonscattered  $K^+$  mesons.

<sup>7</sup> J. H. Fregeau, Phys. Rev. **104**, 225 (1956).

<sup>8</sup> K. Strauch and W. F. Titus, Phys. Rev. **103**, 200 (1956); R. Alphonse, A. Johansson, and G. Tibell, Nuclear Phys. **3**, 185 (1957).

<sup>9</sup> W. F. Baker, J. Rainwater, and R. E. Williams, Phys. Rev. **112**, 1763 (1958).

<sup>6</sup> W. H. Barkas, P. H. Barrett, P. Cuer, H. Heckman, F. M. Smith, and H. K. Ticho, Nuovo cimento **8**, 185 (1958).

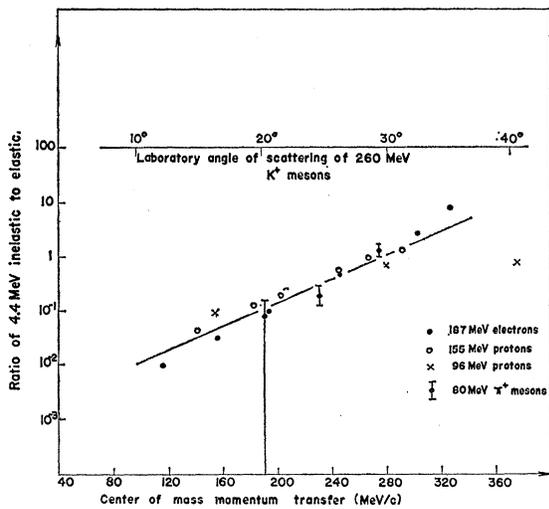


FIG. 4. The ratio of the cross section for inelastic scattering to the 4.4-MeV level of  $C^{12}$  to the elastic cross section plotted against momentum transfer for various particles. The angles of scattering for 200-MeV  $K^+$  mesons which correspond to the lower scale of momentum transfer are shown at the top of the diagram.

It would appear that this ratio depends only weakly upon the nature of the interacting particle. If these results are applied to 261-MeV  $K$  mesons, the ratio is  $\sim 10\%$  for a scattering angle of  $20^\circ$ , where the momentum transfer is about 180 MeV/c, and decreases rapidly for smaller angles. Excitation of higher energy levels in  $C^{12}$  is less probable by a factor of about five.<sup>10</sup> No information is available on the other light elements in emulsion:  $N^{14}$  and  $O^{16}$  with first excited states of 2 and 6 MeV, respectively. The major fraction of the  $K$ -meson scatters occur with Ag and Br for which a large number of low lying levels are expected. In a number of cases, however, the excitation of low lying levels in other heavy nuclei is found to be small<sup>11</sup>; and it is unlikely that the excitation effect would be as large in Ag and Br as that found in  $C^{12}$ . A pessimistic summary of these considerations would be that less than 1% at  $10^\circ$  and less than 10% at  $20^\circ$  of the events classified as elastic are in fact inelastic excitations of low-lying nuclear levels. Because of the tentative nature of this analysis, however, no attempt has been made to correct for this effect.

Inelastic events of small energy loss in which one or more neutral particles escape the nucleus could also go undetected. However, the angular region considered here is such that these events become increasingly unlikely as the energy loss decreases, because of the operation of the exclusion principle as well as the kinematical effects of binding energy. The distribution of residual range of clearly elastic events in Fig. 3 does not suggest that a significant number of such events is included in

<sup>10</sup> See references 7 and 9 and also A. E. Yavin and G. W. Farwell, *Nuclear Phys.* **12**, 1 (1959), for  $\alpha$ -particle scattering.

<sup>11</sup> I. J. van Heerden and D. J. Prowse, *Nuclear Phys.* **19**, 589 (1960).

the elastic distribution as there is no excessive tail on the low-energy side.

From considerations of the phase shifts for  $K^+$ -nucleon scattering processes which were determined independently of direct measurement of the differential nucleon cross-sections, Melkanoff, Prowse, Stork, and Ticho<sup>12</sup> have deduced that the sum of the reactions:  $K^+ + p \rightarrow K^+ + p$  and  $K^+ + n \rightarrow K^+ + n$  for free nucleons in the ratio found in heavy nuclei has an angular distribution which is backward peaked at this energy. In some 60% of our events we can detect an energy loss of 25 MeV, in the remaining 40% the figure is about 50 MeV. The important question is how many inelastic events are expected within these limits which have no visible baryon prongs? Collisions with free nucleons result in energy losses of less than 15% for angles of scatter up to  $30^\circ$  in the laboratory system. We have observed 4 events with prongs in which the energy loss was less than 50 MeV and in which the angle of scatter was less than  $30^\circ$ . Using the phase shifts computed by Melkanoff *et al.* we have calculated, taking the Pauli principle into due account, that in our sample of some 200 inelastic events about 8 events would be expected in this region. Four events would thus seem to have escaped detection, of which two are expected between  $20^\circ$  and  $30^\circ$ . The statistical weight of the results on elastic scattering in the angular intervals  $20^\circ$ - $25^\circ$  and  $25^\circ$ - $30^\circ$  is very poor (7 events) and very little is gained in statistical accuracy by inclusion of these intervals. We therefore choose to limit our investigation to angles less than  $20^\circ$ , in which region we should only have  $\sim 2$  inelastic events as a contaminant. As we have lost some of the elastic events by operation of the 2-standard-deviation cutoff, these effects roughly cancel out.

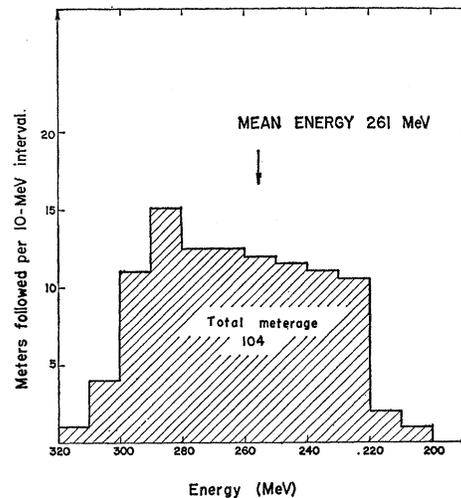


FIG. 5. The  $K$ -meson energy distribution of the track length examined for Elastic scattering and inelastic events.

<sup>12</sup> M. A. Melkanoff, D. J. Prowse, D. H. Stork, and H. K. Ticho, *Phys. Rev. Letters* **5**, 108 (1960); see also E. Helmy, D. J. Prowse, and D. H. Stork, *Nuovo cimento* **19**, 179 (1961).

A further possible contamination are decays in flight of  $K^+$  mesons in which the decay particle is thrown forward and the grain density is within 2 standard deviations of that of the parent  $K^+$  meson. In the track length scanned we observed 22 decays in flight only one of which fell within the "elastic criteria." From the known value of the lifetime<sup>13</sup> about 5 events would be expected within a  $20^\circ$  forward cone, we therefore conclude that our elastic events are contaminated by about 4 decays in flight. As this number is small compared to the statistical errors in every angular interval we have chosen to ignore them.

Events with  $K$ -meson scattering angles of greater than  $20^\circ$  have been classified as inelastic unless they are within two standard deviations of elasticity (there were just 7 events in this latter category). The justification for this procedure is found in the low predicted angular distribution for elastic scattering  $>20^\circ$  in the optical model calculations.

From all the above considerations it appears that this is no serious misclassification of events. The reaction cross section obtained from the number of inelastic events is virtually free from errors other than those of a statistical nature. The differential elastic cross section is likely to be high by an undetermined amount if the direct excitation of low lying nuclear levels is higher than estimated by the elementary means outlined above.

### EXPERIMENTAL RESULTS

A total of 104 m of  $K^+$ -meson track length was followed. The energy distribution of the tracks followed is shown in Fig. 5. We identified 66 charge-exchange events, 126 inelastic non-charge-exchange events and 3 which could not be classified. The mean free path is  $54.1 \pm 3.9$  cm which leads to a reaction cross section of  $396 \pm 28$  mb per emulsion nucleus (excluding hydrogen).

157 elastic scattering events were found with projected angle greater than  $2^\circ$ . These events were individually weighted to correct for the geometrical effect of the  $2^\circ$  projected angle cutoff. Because of the large value of these corrections and their sensitivity to the exact value of the cutoff angle in the interval  $2^\circ$ - $3^\circ$  we have not included this interval in our analysis. The

TABLE I. Differential cross sections for 260-MeV  $K^+$ -meson elastic scattering from emulsion nuclei.

| Angular interval in degrees | Cross section/nucleus in mb |
|-----------------------------|-----------------------------|
| 3-4                         | $16100 \pm 3300$            |
| 4-6                         | $7163 \pm 1075$             |
| 6-8                         | $3171 \pm 580$              |
| 8-10                        | $1678 \pm 380$              |
| 10-15                       | $577 \pm 116$               |
| 15-20                       | $103 \pm 39$                |

<sup>13</sup>L. W. Alvarez, *Ninth Annual International Conference on High-Energy Physics, Kiev, 1959* (Academy of Sciences, Moscow 1960), Alvarez Sec., p. 7.

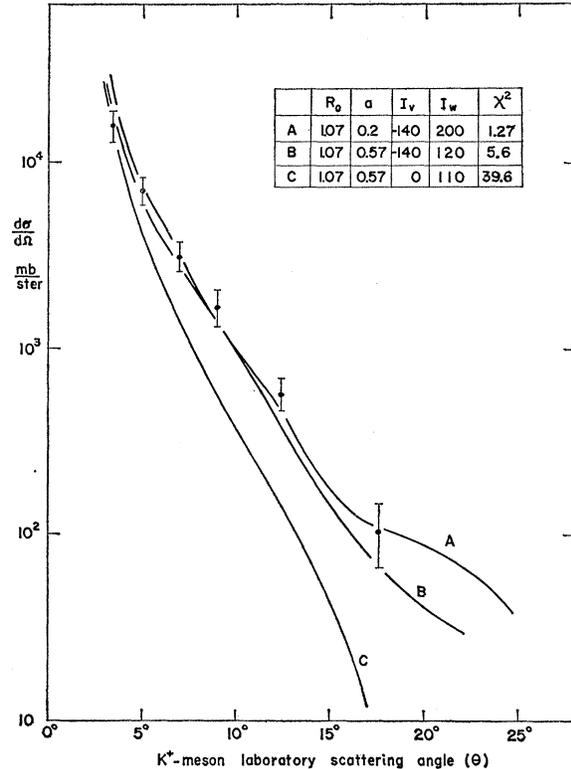


FIG. 6. The experimental differential cross sections. The three curves labeled A, B, C are the theoretical results for the sets of parameters listed in the body of the figure. It is clear that curve C is a very bad fit although it is not easy to choose between A and B.

resulting differential cross sections are listed in Table I and are plotted in Fig. 6.

### OPTICAL MODEL ANALYSIS

Recently Lipperheide and Saxon<sup>14</sup> have shown that the volume integral of the optical potential  $V_{opt}(\mathbf{r})$  is related in first order (neglecting correlation and exchange effects) to the forward scattering amplitude  $f(0)$  of the elementary two-body scattering process:

$$V_{opt}(\mathbf{r})d\mathbf{r} = (-2\pi/m)h^2 A f(0) = (I_v + iI_w)A,$$

where  $A$  is the atomic number of the nucleus and  $m$  is the reduced mass of the two-body system.

For a potential of a square well shape, this implies that

$$(4/3)\pi R^3(V+W) = (2\pi/m)h^2 A f(0),$$

where  $+V$  and  $+W$  are the depths of the real and imaginary parts of an attractive absorbing potential. If  $R = R_0 A^{1/3}$  the atomic number cancels out;  $V$  and  $W$  would, therefore, not be expected to vary with  $A$ . However, if a more realistic Saxon shape is chosen,

$$V_{opt}(\mathbf{r}) = -(V + iW)(1 + e^{+(r-R)/a})^{-1},$$

<sup>14</sup>R. Lipperheide and D. S. Saxon, *Phys. Rev.* **120**, 1458 (1960).

we have the following relationship implied:

$$(4/3)\pi R^3(1+9.88a^2/R^2)(V+iW)=(2\pi/m)h^2A f(0).$$

Here the atomic number does not cancel out and one would expect that  $V$  and  $W$  should therefore vary with  $A$  because the value of  $f(0)$  is the fundamental quantity. The values  $V$  and  $W$  to be used for the two nuclear species in emulsion will thus differ for each value of  $f(0)$  or  $I_v+iI_w$  chosen, these being the quantities that the analysis determines. This point has been mentioned previously in the literature<sup>4</sup> but it has been claimed that since the light nuclei contribute little to the elastic scattering, the effect of using the same value of  $V$  and  $W$  for  ${}^7\text{N}^{14}$  as for  ${}_{41}\text{Nb}^{94}$  is small. This statement is of doubtful validity because at some angles and for some values of  $I_v$ , the contribution to the elastic scattering from  ${}^7\text{N}^{14}$  is not negligible and actually exceeds that from the heavy nuclei. Furthermore, the reaction cross section per average nucleus is affected equally by light and heavy nuclei, and as far as the determination of  $I_w$  is concerned it is incorrect to use the same value of  $W$  for both  ${}^7\text{N}^{14}$  and  ${}_{41}\text{Nb}^{94}$ .

Exact diffuse surface optical model calculations were carried out on the IBM 709 of the Western Data Processing Center, UCLA. Details of the method are given by Melkanoff *et al.* in reference 5. Profiting by the experience of these workers, we have chosen a single energy of 261 MeV (the mean  $K$ -meson energy) and two representative emulsion nuclei: a heavy element with  $Z=41$  and  $A=94$  with 42.6% numerical weight and a light element with  $Z=7$  and  $A=14$ .

The imaginary potential integral per nucleon  $I_w$  was adjusted to fit the reaction cross section for chosen

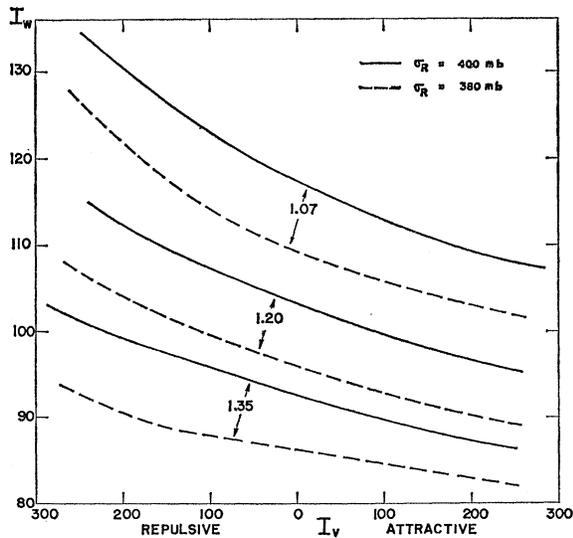


FIG. 7. The variation of  $I_w$  and  $I_v$  shown for three radius parameters (1.07, 1.20, and 1.35 F). The solid curve relates the  $I_w$  and  $I_v$  necessary to maintain a theoretical reaction cross section of 400 mb while the dashed lines refer to a reaction cross section of 380 mb.

values of the radius parameter  $R_0$ , the surface thickness parameter  $a$ , and the real potential integral per nucleon  $I_v$ , for the Saxon form factor given above. The value of the reaction cross section was obtained from a compilation of data from the laboratories of Bristol,<sup>15</sup> Brookhaven,<sup>16</sup> Dublin and Padua,<sup>17</sup> and UCLA, as shown in Table II. The best value of  $\sigma_R$  is  $380\pm 13$  mb from 824 events found in some 460 m of track length. The required value of  $I_w$  is shown in Fig. 7 as a function of other parameters. The Coulomb radius was held constant at  $1.07A^{1/3}$  F, independent of the value of  $R_0$ . The effect of a change in the Coulomb radius parameter from 1.07 to 1.35 F was found however to have an effect of less than 5% at all angles.

A  $\chi^2$  comparison to the differential elastic scattering cross sections for the six angular intervals between  $3^\circ$  and  $20^\circ$  was then carried out. The calculated cross sections were averaged over each angular interval by means of a three-point Simpson's rule. Curves of  $\chi^2$  as a function of  $I_v$  are shown in Fig. 8 for several values of  $R_0$ . Two  $\chi^2$  minima are exhibited, one in the repulsive region and one in the attractive. The attractive fit is poor and combined with a reasonable extrapolation from studies at lower energies,<sup>4,5</sup> we feel it can be rejected for sensible values of the parameters. The smallest radius parameter  $R_0=1.07$  is preferred by the data. Figure 8 is for a rounding parameter  $a$  of 0.57 F. When this is changed the  $\chi^2$  curves change in the following way: lowering the value of  $a$  lowers the minimum  $\chi^2$  in the repulsive region for all radii and makes the minimum very broad; for  $R_0=1.07$  and  $a=0.2$  F the  $\chi^2$  value does not exceed 6.0 (a Pearson probability of 10%) anywhere between  $I_v=240$  and 80 MeV F<sup>3</sup>/nucleon. The minimum is lowered less for the larger radii than for the smaller radii. The  $\chi^2$  values at the minimum being 24 and 18 for  $R_0=1.35$  and  $a=0.57$  and 0.20, respectively, while for  $R_0=1.07$  they are 5.5 and 1.0

TABLE II. Experimental data used for determination of the reaction cross section for 260-MeV  $K^+$  mesons with emulsion nuclei.

| Laboratory              | Energy interval (MeV) | Meters followed | No. of events |
|-------------------------|-----------------------|-----------------|---------------|
| Bristol <sup>a</sup>    | 200-300               | 92              | 152           |
| Brookhaven <sup>b</sup> | 218-295               | 85.7            | 166           |
| Dublin <sup>c</sup>     | 240-300               | 184.5           | 311           |
| UCLA                    | 200-300               | 104             | 195           |
| Total                   | 200-300               | 466.2           | 824           |

Mean free path =  $56.7\pm 2.0$  cm, corresponding to a cross section of  $380\pm 13$  mb

<sup>a</sup> See reference 15.

<sup>b</sup> See reference 16.

<sup>c</sup> See reference 17.

<sup>15</sup> D. Evans, F. Hassan, K. K. Nagpaul, and M. Shafi, *Nuovo cimento* **16**, 476 (1960).

<sup>16</sup> B. Sechi-Zorn and G. T. Zorn, *Phys. Rev.* **120**, 1898 (1960).

<sup>17</sup> D. Keefe, A. Kernan, A. Montwill, M. Grilli, L. Guerriero, and G. A. Salandin, *Nuovo cimento* **12**, 241 (1959).

for  $a=0.57$  and  $0.20$ , respectively. If the value of  $a$  is raised, the values of  $\chi^2$  at the minima get larger and larger. There is no doubt that the experimental data is best fitted with a small radius and/or a small rounding parameter.

DISCUSSION OF RESULTS

It is clear that the repulsive potential is a best fit to the data for all reasonable shape parameters. The exact value of the potential in terms of  $I_v$  however is not well defined. There is a tendency for the best value of  $I_v$  (value at the minimum) to decrease with increasing radii and to decrease with increasing value of  $a$ . In fact an empirical relationship appears to hold between  $I_v$ ,  $a$ , and  $R_0$  of the form:  $I_v a^3 R_0 = \text{const}$ . A similar relationship holds for the value of  $I_w$  (required to fit the reaction cross section) over a reasonable range of  $a$  and  $R_0$  values. It is therefore not possible to obtain a value of  $I_v$  or  $I_w$  from the data as long as  $a$  and  $R_0$  remain unknown. The data is best fitted by a small radius ( $\sim 1.07$ ) and by a small value of  $a$  but acceptable fits are obtained for radii up to  $1.35$  provided  $a$  is kept small ( $\sim 0.2$ ) or up to  $a=0.85$  if the radius is kept small ( $\sim 1.07$ ).

The fact that all the optical model curves have to be averaged over the light and heavy nuclei in order to compare with experiment somewhat obscures some of the features of the behavior of  $I_w$  and  $I_v$  for the individual nuclei  ${}^7\text{N}^{14}$  and  ${}_{41}\text{Nb}^{94}$ . In Fig. 9, we show the variation of  $I_w$  on  $I_v$  and on  $R_0$  necessary to maintain arbitrary constant total cross sections of  $700$  mb for the heavy nucleus and  $150$  mb for the light nucleus. There is

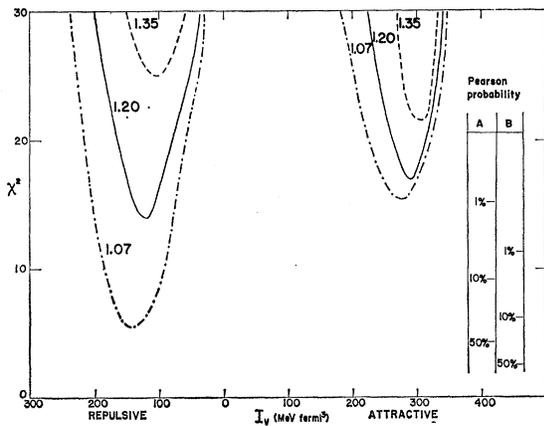


FIG. 8. Curves of  $\chi^2$  vs  $I_v$  for various radial parameters keeping  $a$  fixed at  $0.57$  F. There are two regions of low  $\chi^2$ , one is the repulsive region and one is the attractive region. The repulsive solution is slightly preferred by the data and reasonable extrapolation from lower energies would indicate that this is the correct solution. On the right-hand side of the figure we give the Pearson probabilities associated with the  $\chi^2$  values for (A) 5 degrees of freedom, and (B) 3 degrees of freedom. There are 6 experimental points used in the analysis and a value for the reaction cross section, 2 quantities have to be determined  $I_v$  and  $I_w$  if  $a$  and  $R_0$  are assumed known. This corresponds to 5 degrees of freedom. If however we do not admit a knowledge of  $a$  or  $R_0$  there are 3 degrees of freedom.

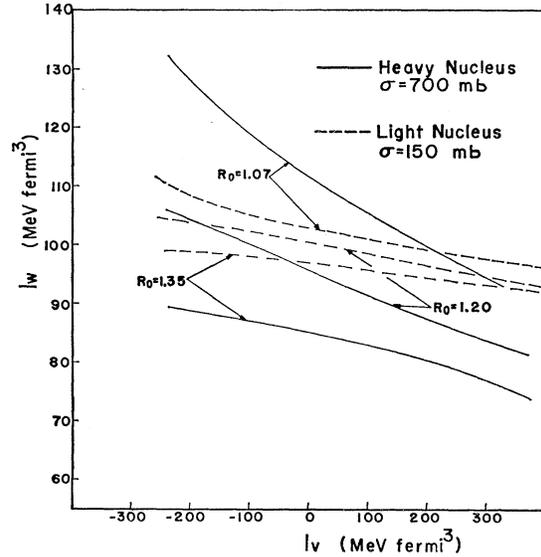


FIG. 9. Values of  $I_w$  required to fit reaction cross sections of  $700$  and  $150$  mb for heavy and light nuclei, respectively, plotted against  $I_v$  for the three radius parameters  $R_0=1.07$ ,  $1.20$ , and  $1.35$  F.

considerable variation of  $I_w$  on both  $I_v$  and  $R_0$  for the heavy nucleus which is not exhibited for the light nucleus. Reversing the logic of the statement this implies that the reaction cross section for the light nucleus well determines  $I_w$ , independent of a knowledge of  $I_v$  or  $R_0$  whereas for a heavy nucleus  $I_w$  is not well determined by the reaction cross section unless the radius is known. This is in the direction which one would expect. For large negative  $I_v$  (corresponding to a repulsive potential) the  $K$  meson cannot penetrate the full density region of the nucleus so easily—this means that the average point of interaction is further out from the

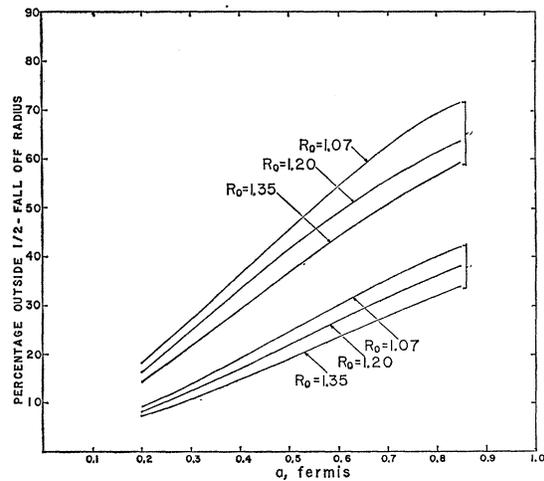


FIG. 10. The percentage of the nuclear volume which is outside the half-falloff radius plotted as a function of  $a$  for the three radius parameters  $R_0=1.07$ ,  $1.20$ , and  $1.35$  F. The top three curves refer to nitrogen and the lower three to the hypothetical nucleus,  $A=94$  and  $Z=41$ .

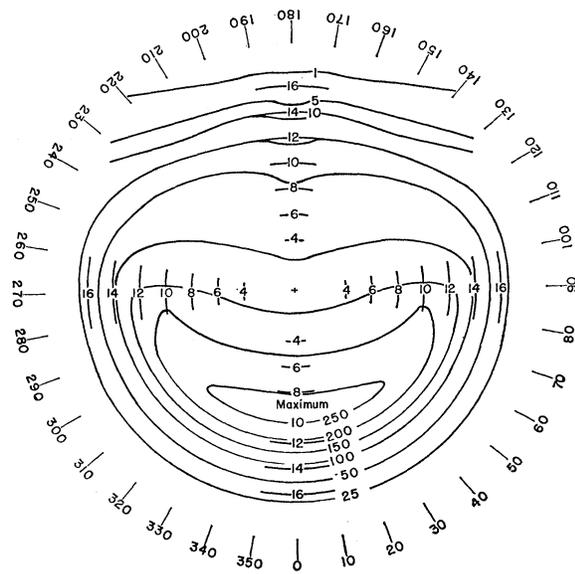


FIG. 11. Contour lines of the quantal flux divergence as a function of azimuthal angle and of distance from the center  $r$  multiplied by the wave number of the  $K$  meson ( $\rho = kr$ ). The divergence is in arbitrary units. The half-falloff radius has a  $\rho$  value of about 14 for the set of parameters used.

nuclear center. This region has a lower nuclear density and so to give the same reaction cross section  $I_w$  has to be raised. This is most marked when the ratio of the rounding parameter to half-falloff radius is large ( $\sim 0.5$ ). To illustrate the different surface characters of nuclei as functions of  $a$  and  $R_0$ , in Fig. 10 we show the fraction of the nucleus which is outside the half-falloff radius of (a) the heavy nucleus and (b) the light nucleus. For  $a = 0.57$ , the light nucleus has between 48 and 50% of its volume outside the half-falloff radius and the heavy nucleus only between 22 and 28% for radius parameters

between 1.35 and 1.07 F. It is clear from these considerations that the front illuminated surface of a heavy nucleus is very important as far as the reaction cross section is concerned. For light nuclei it is not relatively so important because the attenuation of the beam by the time it has reached the region of maximum density is less. To demonstrate the importance of the front surface of the nucleus we have calculated the divergence of the quantal flux as given by the optical model calculation as a function of  $\rho$  ( $=kr$ ) and of azimuthal angle  $\phi$ . Contour lines of this quantity (in arbitrary units) are shown in Fig. 11 for the following set of parameters:  $I_v = 160$ ,  $I_w = 92$ ,  $A = 94$ ,  $Z = 41$ ,  $a = 0.57$  F, and  $R_0 = 1.20$ . It is clear that the region of importance for interaction is a considerable distance from the center. The effect of "refraction" can also be seen in the accentuation of the outer sides of the nucleus at the expense of the center, the particles being refracted away from the center regions.

### CONCLUSIONS

The nuclear potential for  $K^+$  mesons is repulsive, the exact value depending on the nuclear parameters assumed. Best fits are obtained for  $I_v$  values close to 140 MeV-F<sup>3</sup> and for radius parameters which are small ( $\sim 1.07$  F) and for rounding parameters which are small. The importance of the nuclear surface even for the rather weakly interacting  $K^+$  mesons has been demonstrated by calculating the divergence of the quantal flux.

### ACKNOWLEDGMENTS

We would like to thank Miss Janet Schlan, Mrs. Claire Swan, and Miss Joyce Naftulin for much of the microscope work. We are greatly indebted to Dr. E. J. Lofgren for Bevatron exposure facilities. Numerous discussions with Professor H. K. Ticho and Professor D. S. Saxon are gratefully acknowledged.