Peripheral Collisions at Accelerator Energies and the π-π Interaction

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Recent measurements of inelasticity, angular distributions in the c.m. system, and momentum distributions in the c.m. system at 9 and 24 GeV are interpreted in terms of peripheral collisions involving exchange of an "almost real" pion. In this energy interval the "rest masses" of the fireballs appear to be constant and surprisingly close to the π-π resonances observed at lower energies.

I. INTRODUCTION

In a recent paper from our laboratory,[1] we reported comparative measurements of proton interactions at 9 and 24 GeV. The results showed a significant drop of the inelasticity with primary energy, correlated with a similarly significant increase of the anisotropy in the c.m. system. These features, as well as the low absolute values of the inelasticity coefficients (0.2–0.3), are suggestive of peripheral collisions, involving mainly the pion clouds of the collision partners.

This leads one to suspect that the pion resonances detected at lower energies should also show up, e.g., in some sort of "quantization" of the field energy stripped away from the initial nucleons.[2]

In the present paper, the data of reference 1 as well as later results from CERN[3] and Dubna[4] are analyzed from the viewpoint of the "cloud-collision" two-center model.[5] As will be shown below, different experimental data, obtained and processed in independent ways, fit remarkably well a simple kinematical model in which each nucleon core is assumed to interact independently with a single, "almost real" pion in the other collision partner's meson cloud.

In spite of the relatively large increase of the total energy available in the c.m. system, the total energy contained in the two fireballs appears to remain constant in the investigated energy range; its numerical value is very close to the well-known $\pi-\pi$ resonance detected at lower energies.[6–9]

Units and Notations

Throughout this paper we shall use units such that $c=M=1$, where M is the nucleon rest mass; μ is the pion rest mass. Lorentz factors γ are related to velocities β by $\gamma\equiv(1-\beta^2)^{-\frac{1}{2}}$. For a particle or system of particles emitted at an angle θ to the primary direction in a given system, we define

$$\Delta\equiv\gamma-(\gamma^2-1)^{\frac{1}{2}}\cos\theta; \tag{1}$$

for the special case $\theta=0$ we shall use the notation

$$\Delta(\theta-0)\equiv\delta. \tag{2}$$

Primed quantities refer to the c.m. system, unprimed ones to the lab system; quantities measured in the fireball rest system are denoted by an asterisk.

II. VELOCITY OF THE EMITTING CENTERS

In keeping with the results at very high (cosmic-ray-jet) energies,[10,11] we shall assume that after the collision of a nucleon of velocity β_0 with a nucleon at rest in the laboratory system, two "hot spots" (fireballs) of equal "masses" M^* are formed, trailing behind the nucleons with equal and opposite velocities $\bar{\beta}'$ in the c.m. system of the nucleon-nucleon collision, along the line of flight of the incoming nucleons. Meson emission is assumed to take place independently and isotropically from both "hot spots." Most of the results to be discussed below remain unchanged if only one fireball is created in some interactions, provided forward and backward fireballs occur equally often.

In view of both the relatively low multiplicity and the low $\bar{\gamma}'$ values accessible in accelerator experiments, one should not expect the typical two-cone structure of the cosmic-ray jets ($\gtrsim$1 TeV) to become apparent here. Nevertheless $\bar{\gamma}'$ can be estimated in one of the following ways:

(a) As in the case of cosmic-ray jets, the lab-system angular distribution is analyzed in terms of coordinates

$$x\equiv\log_{10}\cot\theta. \tag{3}$$

The probability density of x is then a superposition of two Gaussians of variance $\sigma_0=0.394$ with a relative shift $2a$; the resulting dispersion of x is then[10]

$$\sigma^2=\sigma_0^2+a^2. \tag{4}$$

[1] E. M. Friedländer, M. Marcu, and M. Spîrchez, Phys. Rev. Letters, 7, 25 (1961).

[2] After completion of the first version of this work, similar ideas have been advanced by D. H. Perkins, [Proceedings of the Conference on Theoretical Aspects of Very High Energy Phenomena, CERN 61–22, Geneva, 1961 (unpublished), p. 99.

[3] D. R. O. Morrison, reference 2, p. 153.

[4] T. Visky, I. M. Gramenitzky, Z. Korbel, A. A. Nomofilov, M. J. Podgoretzky, L. Rob, V. N. Streltsov, D. Tuvdendorj, and M. S. Hvastunov, JINR internat. report P-745, 1961 (to be published).

[5] E. M. Friedländer, Phys. Rev. Letters 5, 212 (1960).

[6] J. Derado, Nuovo cimento 15, 853 (1960).

[7] F. Cerulus, Nuovo cimento 14, 827 (1959).

[8] A. Erwin, R. March, W. Walker, and E. West, Phys. Rev. Letters 7, 39 (1961).

[9] E. Pickup, D. Robinson, and E. Salant, Phys. Rev. Letters 7, 192 (1961).

[10] P. Ciok, J. Coghen, J. Gierula, R. Holynski, A. Jurak, M. Miesowicz, J. Saniewska, and J. Pernegr, Nuovo cimento 10, 741 (1958).

[11] G. Cocconi, Phys. Rev. 111, 1699 (1958).

TABLE I. Experimental and expected values for the Lorentz factor of the fireballs with respect to the c.m. system.

Primary energy		9 GeV		24 GeV	
Data from:		Bucharest[a]	Dubna[b]	Bucharest[a]	CERN[c]
Method	a	1.23±0.06		1.44±0.15	
	b		1.35±0.05		1.40±0.12
	c		1.32±0.06		1.37±0.16
Weighted average		1.30±0.03		1.40±0.08	
Computed from the two-center model[d]		1.33		1.41	

[a] See reference 1.
[b] See reference 4.
[c] See reference 3.
[d] See reference 5.

At the relatively low energies considered here the approximation $a \approx \ln(2\bar{\gamma}')$ used in reference 10 is no longer valid and a is given instead by

$$\tanh a = \bar{\beta}'\beta_c, \tag{5}$$

where β_c is the relative velocity of the c.m. system and the lab system.

Thus measurement of σ yields[12] an estimate for $\bar{\gamma}'$ even if no clear-cut two-cone structure can be distinguished.

The $\bar{\gamma}'$ values deduced in this way from the data of reference 1 are shown in the first row of Table I.

(b) The c.m. system angle θ' of a pion emitted with velocity β^* at an angle θ^* in the fireball rest system is given by

$$\bar{\gamma}' \tan\theta' = \sin\theta^* [(\bar{\beta}'/\beta^*) + \cos\theta^*]^{-1}. \tag{6}$$

In view of isotropy in the fireball rest system, we will hence expect that in each of the two c.m. system cones, one-half of the secondaries will be contained in a cone of opening $\theta_{\frac{1}{2}}'$ given by

$$\cos\theta_{\frac{1}{2}}' = (\bar{\gamma}'^2 - 1)^{\frac{1}{2}} (\bar{\gamma}'^2 - \gamma^{*-2})^{-\frac{1}{2}} \approx \bar{\beta}', \tag{7}$$

for $(\bar{\gamma}'\gamma^*)^2 \gg 1$ (which is true in all cases of practical interest).

Row 2 of Table I shows the values of $\bar{\gamma}'$ obtained by this method from p-p collisions at 9 GeV in reference 4 (emulsions) and at 24 GeV in reference 3 (hydrogen bubble chamber).

(c) Finally an estimate for $\bar{\gamma}'$ can be obtained in an independent way by comparing the second moments of the distributions of transverse ($p_\perp$) and longitudinal ($p_{\shortparallel}'$) momentum components of the secondary pions. Indeed, irrespective of the shape of the energy spectrum in the fireball's rest system, we have:

$$\langle p_\perp^2 \rangle = \tfrac{2}{3}\mu^2 (\langle \gamma^{*2} \rangle - 1), \tag{8}$$

and

$$\langle p_{\shortparallel}'^2 \rangle = \mu^2 [\langle \gamma^{*2} \rangle (2\bar{\gamma}'^2 - 1) - \bar{\gamma}'^2 (1 + \langle p_\perp^2 \rangle)], \tag{9}$$

[12] The measured value of σ has to be corrected for the nonmonoergic spectrum of the secondaries. [H. H. Aly and C. M. Fisher, Nuovo cimento 17, 98 (1960).] This correction has been taken into account in the computation of the corresponding γ' values of Table I.

where $\langle \gamma^{*2} \rangle$ is the second moment of the energy spectrum in the fireball rest system. Eliminating this unknown quantity from Eqs. (8) and (9), we obtain

$$\bar{\gamma}' = \left[\frac{\langle p_{\shortparallel}'^2 \rangle + \frac{3}{2}\langle p_\perp^2 \rangle + \mu^2}{2\langle p_\perp^2 \rangle + \mu^2} \right]^{\frac{1}{2}}. \tag{10}$$

The $\bar{\gamma}'$ values computed by means of Eq. (10) from the $p_{\shortparallel}'$ and $p_\perp$ values for pions given in references 3 and 4 are shown in row 3 of Table I. It is evident from Table I that: (a) there is a systematic increase of $\bar{\gamma}'$ with primary energy, and (b) the results obtained by all three methods (a)–(c) are in good mutual agreement. The probability that the observed differences arise as statistical fluctuations about a common mean is ~30% at 9 GeV and ~90% at 24 GeV (χ^2 tests). The weighted averages of $\bar{\gamma}'$ at each primary energy are given in row 4 of Table I.

It must be stressed that the good agreement between the $\bar{\gamma}'$ values obtained by methods (a) and (b) (angular distributions) and that obtained by method (c) (momentum measurements) is a strong argument for the validity of the two-center picture.

III. REST MASS OF THE EMITTING CENTERS

The next step in this analysis is to gain information regarding the total energy M^* contained in each fireball (the fireball's "rest mass") by means of the inelasticity measurements. In the lab system, the inelasticity coefficient K is defined as

$$K \equiv (\mu/\gamma_0) \sum \gamma_i, \tag{11}$$

where summation is extended over all *created* particles. In the c.m. system, the analogous definition,

$$K' \equiv (\mu/2\gamma_c) \sum \gamma_i', \tag{12}$$

leads, using energy-momentum conservation, to

$$K' = M^* \bar{\gamma}' \gamma_c^{-1}. \tag{13}$$

Using the standard Lorentz transform, we obtain

$$K\gamma_0 = 2M^* \bar{\gamma}' \gamma_c, \tag{14}$$

whence

$$K = K'(1 + \gamma_0^{-1}) \approx K' \tag{15}$$

for large enough γ_0. At 24 GeV the neglected term amounts to ~4%.

Applying Eqs. (13) and (15) at two primary energies[13] labeled 1 and 2, respectively, we obtain

$$\frac{M_1^*}{M_2^*} = \frac{\gamma_{c1}}{\gamma_{c2}} \frac{\bar{\gamma}_2'}{\bar{\gamma}_1'} \frac{K_1'}{K_2'} \approx \frac{\gamma_{c1}}{\gamma_{c2}} \frac{\bar{\gamma}_2'}{\bar{\gamma}_1'} \frac{K_1}{K_2}. \tag{16}$$

Using the (K_1/K_2) value of reference 1 and the $\bar{\gamma}'$ values of Table I, we obtain finally

$$M_1^*/M_2^* = 1.0 \pm 0.1. \tag{17}$$

[13] In the present case, 24 GeV and 9 GeV, respectively.

From 9 to 24 GeV, the energy available for meson production in the c.m. system increases by ~94%. Nevertheless the amount of energy stripped away from the nucleons appears to be practically constant, i.e., the fireball behaves like an (unstable) "particle" with well-defined rest-mass M^*. An estimate for M^* can be obtained from the upper limit of K estimated at 24 GeV in reference 1, viz. 0.23±0.06, by means of the $\bar{\gamma}'$ value of Table I:

$$M^* \lesssim (4.2 \pm 1.1)\mu. \tag{18}$$

A $\pi-\pi$ resonance in this energy range (ρ meson) has been predicted theoretically for some time,[14] and has been observed in low-energy pion production experiments,[6,15,16] and invoked in order to explain the features of $p\bar{p}$ annihilation.[7] The M^* value of Eq. (18) suggests that at the higher energies investigated here the process of pion production is determined even to a greater extent by the $\pi-\pi$ interaction. The same interaction seems to be reasonable also for the strong anisotropies of very high energy jets.[17]

In view of the well-known constancy of the average c.m. pion momenta around 0.3–0.4 GeV, one would expect at 24 GeV a multiplicity of charged prongs in p-p collisions of about four. This compares favorably with the result of the Berne-CERN groups,[18] viz., 4.1±0.6.

IV. INELASTICITY

Until now we have made no assumptions whatever as to the mechanism leading to the formation of the two fireballs. Actually, a simple model of nucleon structure in which the nucleon is conceived—at least at the instant of the collision—as predissociated into a core and a loosely bound pion, provides a satisfactory explanation for all the observed facts. If the kinetic energy of these "target pions" in the nucleon rest system is low as compared to the pion rest mass, then, as has been shown before,[5]

$$\bar{\gamma}' = (\mu+1)\gamma_c/(1+\mu^2+2\mu\gamma_0)^{\frac{1}{2}}. \tag{19}$$

The $\bar{\gamma}'$ values computed by means of Eq. (19) for γ_0 values corresponding to primary energies of 9 and 24 GeV, respectively, are given in row 5 of Table I; as can be seen, they are in very good agreement with the observational data ($\chi^2 \approx 1$, with one degree of freedom).

Further, from (16), (17), and (19) we obtain

$$K_1/K_2 = [(1+\mu^2+2\mu\gamma_{01})/(1+\mu^2+2\mu\gamma_{01})]^{\frac{1}{2}}. \tag{20}$$

If in each collision only one fireball is excited, the numerical value of K will be just half as great as in the symmetrical case, but Eq. (20) will still hold. The same goes for Eq. (7) and for its consequences.

For the experiment under consideration,[1] Eq. (20) predicts a ratio

$$K_1/K_2 \approx 0.70, \tag{21}$$

which compares favorably with the experimental result (0.68±0.06).

Another independent check of the model can be made by applying a Birger-Smorodin analysis[19] to the $p_{\shortparallel}'$ and $p_\perp$ data for the recoil protons. According to reference 19, at high enough γ_0, the effective target mass m_τ is given by

$$m_\tau \approx 1-\Delta_r, \tag{22}$$

where Δ_r is the Δ value of the recoil proton in the lab system. Now,[20] Δ_r transforms from the c.m. system to the lab system according to

$$\Delta_r = \delta_c \Delta_r', \tag{23}$$

whence

$$m_\tau \approx 1-\delta_c \Delta_r'. \tag{24}$$

Equation (24) has applied to all the recoil protons in Fig. 4 of reference 3. The m_τ values showed a rather narrow distribution of mean $(1.2\pm0.2)\mu$. Similar data from preliminary measurements[21] carried out in our laboratory on carefully selected p-nucleon collisions of 9-GeV protons in emulsions yielded an average m_τ of $(1.3\pm0.2)\mu$.[22]

It should be stressed however, that the "structural" implications of m_τ are less straightforward then those of the $\bar{\gamma}'$ or K_1/K_2 values discussed above.[23]

Indeed, let β_x be the velocity of the center of momentum for all the *created* particles. The sum of the lab system Δ values for all *these* particles yields

$$\mu \sum \Delta_i = \mu \sum \gamma_i(1-\beta_x). \tag{25}$$

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[14] W. Frazer and J. Fulco, Phys. Rev. **117**, 1609 (1960).

[15] F. Bonsignori and F. Selleri, Nuovo cimento **15**, 465 (1960).

[16] Later experiments (references 8, 9) at $\gtrsim$1-Gev incident pion energy have shown that the ρ-meson mass is ~5.4 μ. This cannot be considered as at variance with the value of Eq. (18), not only because of its relatively large statistical error but also because the M^* values are computed here for an idealized model where each interaction goes via fireball (i.e., practically ρ-meson) production. In fact nonresonant events are known to play a considerable part too; this must necessarily lower the average M^* value for a large number of jets.

[17] E. M. Friedländer, Nuovo cimento **23**, 261 (1962).

[18] G. Cvijanovic, B. Dayton, P. Egli, B. Klaiber, W. Koch, M. Nicolic, R. Schneeberger, and G. Vanderhaeghe, Nuovo cimento **20**, 1012 (1961).

[19] N. Birger, Yu. Smorodin, Soviet Phys.—JETP **37**, 1355 (1959); N. Dobrotin and S. Slavatinsky, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 819.

[20] E. M. Friedländer, Nuovo cimento **19**, 818 (1961).

[21] E. M. Friedländer, M. Marcu, and M. Spîrchez (to be published).

[22] Rigorously speaking, Eq. (22) should be understood as follows: If the nucleon is assumed to be predissociated into a "core" and a loosely bound "target particle" of mass m_τ, the quantity $1-\Delta_r$ will be just equal to the Δ value of this particle, measured in the nucleon's rest system. If $\beta_\tau \approx 0$, we again obtain Eq. (22). Since there is no reason to suspect anisotropies in the nucleon's rest system, the average value of $1-\Delta_r$ will be $\gamma_\tau m_\tau$ [as the $\cos\theta_\tau$ term in Eq. (1) cancels out]. Both experimental results yield $m_\tau > \mu$, although the excess is not significant and sets an upper limit of ~1.5 for γ_τ.

[23] See also Z. Koba and A. Krzywicki, Warsaw, 1961 (to be published). where the limitation of the "target mass" concept is discussed from the field-theoretical point of view.

In view of Eq. (11) we have (at high enough γ_x values)

$$\mu \sum \Delta_i \approx K\gamma_0/2\gamma_x^2. \quad (26)$$

Since[19]

$$\Delta_r + \mu \sum \Delta_i = 1 + (\Delta_0 - \Delta_1), \quad (27)$$

where Δ_0 pertains to γ_0 and Δ_1 to the faster of the outgoing nucleons, we have, for high γ_0 and not too small K, $(\Delta_0 - \Delta_1) \ll 1$, and hence

$$m_r \approx K\gamma_0/2\gamma_x^2. \quad (28)$$

If, furthermore, the secondaries are emitted symmetrically in the c.m. system of the colliding nucleons, $\gamma_x = \gamma_c$ and

$$m_r \approx K\frac{\gamma_0}{\gamma_0+1} \approx K \quad \text{for} \quad \gamma_0 \gg 1. \quad (29)$$

In asymmetrical showers in which only one fireball is created, we have (at high enough γ_0)

$$m_r \approx \mu K, \quad \text{(forward fireball)}$$
$$m_r \approx \mu^{-1} K, \quad \text{(backward fireball)}. \quad (30)$$

Thus, m_r is essentially a measure of the lab system inelasticity,[24] irrespective of its interpretation as effective target mass. Nevertheless, if, at a given primary energy, the spectrum of m_r is peaked, this implies the same for the spectrum of K which is a quantity highly representative of the structural features of the nucleon.

The main weight of the interpretation of the process as interactions with "cloud-pions" is thus borne by the agreement between the independent estimates for m_r and $\bar{\gamma}'$, which has been shown in the preceding sections to be quite satisfactory.

It is interesting to remark that, as a consequence of Eq. (28), the "target-mass analysis"[19] is equivalent to the spectrum of Lorentz factors γ_x.[25,26]

Discussion and Conclusions

In trying to summarize the results of the preceding sections, it is important to bear clearly in mind how much of them represents experimental finding and how much is due to model. Indeed, the data show that:

(a) Anisotropy is present and increases with primary energy.

(b) Inelasticity is low and decreases with primary energy.

(c) The lab system four-momentum of the recoil proton is restricted to a narrow range of values such that $\Delta_r \sim (1-\mu)$.

The shape of the anisotropy is such as would be expected from isotropic emission by one or two (slowly) moving centers. Under these assumptions, the agreement with the asymmetry between the longitudinal and transverse momentum components of the created particles is quite satisfactory.

If—as a logical consequence of the above-mentioned agreement—a two-center model is assumed to hold, the total energies of the two fireballs turn out to be constant, i.e., each fireball behaves like a "particle" with well-defined mass.

This phenomenological description can now be related to some model for the structural properties of the nucleon and of its meson cloud. One can account for most of the observed facts by assuming that:

(a) At the instant of collision a loosely bound pion is singled out in the nucleon's meson cloud (clouds) and collides with the bulk of the other nucleon.

(b) In these collisions the main reaction channel is via a ("resonant") $\pi-\pi$ interaction, leading to the formation of metastable states (pion isobars) of about 4–5 pion masses with fast isotropic decay into $\geqslant 2$ pions.

One consequence of such an oversimplified picture is that—if taken literally— it would lead to a constant pion multiplicity, independent of primary energy. This obviously contradicts the experimental findings, viz. an—admittedly very slow—increase of the multiplicity (from 3.25 at 9 GeV to 4.1 at 24 GeV).

The following competing processes could account—individually or collectively—for this discrepancy:

(a) single-pion production, probably mainly via $(\frac{3}{2},\frac{3}{2})$ nucleon isobars;

(b) production of only one fireball in a fraction of the interactions which decreases with increasing energy[27,28];

(c) excitation of higher (than the 5μ) pion resonances as γ_0 increases[29];

(d) central (core-core) interactions with energy-dependent multiplicity and/or cross section.

Process (c) must certainly be invoked if one tries to understand along similar lines the higher multiplicities observed at $\sim$1 TeV or so. It must be stressed that the whole of the available high-energy data is still statistically too poor to distinguish between a continuous increase of multiplicity with energy (as would be expected from the statistical-hydrodynamical theories) and a step-wise set-in of the various resonant channels as soon as their thresholds are reached.

[24] Its significance as a measure of inelasticity in the antilab. system has already been pointed out in reference 19.

[25] E. M. Friedländer, Nuovo cimento 14, 796 (1959).

[26] E. M. Friedländer, M. Marcu, and M. Spîrchez, Nuovo cimento 18, 623 (1960).

[27] F. Salzmann and G. Salzmann, Proceedings of the Conference on Theoretical Aspects of Very High Energy Phenomena, CERN 61–22, Geneva, 1961 (unpublished), p. 283.

[28] In fact the threshold for ρ meson production by a free pion is at $\sim$1 BeV lab system energy. Provided $\gamma_r = 1$, this is just the lab system energy of a virtual pion traveling with the velocity of a 6-BeV proton. Thus, at $\sim$9 BeV the ρ-production process is just above threshold and associated production of ρ-meson pairs is rather improbable. At $\gtrsim$25 BeV one would expect this process to gain importance. It would hence, appear of interest to look for the characteristic angular correlation in stars of higher multiplicities (say $n_s > 4$) at this and higher energies.

[29] F. Selleri, Nuovo cimento 16, 775 (1960).

Another oversimplification of the model discussed here is the neglect of the motion of the "target pion" with respect to its "parent" nucleon. This effect has been shown to be of little importance at accelerator energies, but may gain importance at very high cosmic-ray energies.[17]

Considerably more refined measurements than those available at present are necessary[30] before one can assert knowledge about anything like the cloud's "energy spectrum,"[31] its "temperature," etc.

[30] Accurate measurements of $\bar{\gamma}'$ by the $p_{\parallel}'$ vs $p_{\perp}$ method seem the most promising, especially if coupled with measurements of Δ_r on the same event.

[31] T. Yajima, S. Takagi, and G. Kobayakawa, Progr. Theoret. Phys. (Kyoto) 24, 59 (1960).

Finally, it is important to stress that all the classical notions and the picture developed here need not, and must not, be taken too literally. Probably some new physical concepts will have to be developed before deeper insight becomes possible.

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Extrapolation of Proton-Proton Scattering Data to the One-Pion Exchange Pole*

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The dependence of various *p-p* scattering parameters on the one-pion exchange contribution and the Coulomb scattering amplitude is investigated. Two parameters, $I_0(1-D)$ and I_0Y, have second-order one-pion poles and no Coulomb singularity in the forward direction. The extrapolation procedure frequently applied to *n-p* cross sections to determine the pion-nucleon coupling constant can be applied to these parameters. This is done with measurements of $I_0(1-D)$ at 142 MeV, giving a value of $8.8_{-2.6}^{+2.0}$ for g^2, compared with the currently accepted value of 14. The relative sensitivity of the various *p-p* parameters to the one-pion exchange contribution is discussed.

INTRODUCTION

In the past, proton-proton scattering data have been used to determine the pion-nucleon coupling constant by means of the "modified phase shift analysis" of Moravcsik *et al.*[1] In a phase-shift search, the higher angular momentum phase shifts are given by the one-pion exchange contribution, and the coupling constant is varied to obtain the best fit to the data.

Neutron-proton data have been used in a much more direct procedure.[2] The cross section is expected to have a second-order pole, due to the one-pion exchange contribution. The coefficient of the second-order term at this pole, simply related to the pion-nucleon coupling constant, is obtained by an extrapolation in $\cos\theta$, where θ is the center-of-mass scattering angle.

This simple extrapolation procedure cannot be used for the proton-proton cross section, because the Coulomb amplitude completely distorts the cross section at small angles. In this note, the possibility of extrapolating some parameter other than the cross section is investigated. Two suitable parameters are found: $I_0(1-D)$ and I_0Y. The extrapolation procedure is applied to the parameter $I_0(1-D)$, giving a reasonable value for the coupling constant.

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[1] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **116**, 1248 (1959).

[2] P. Cziffra and M. J. Moravcsik, Phys. Rev. **116**, 226 (1959).

SINGULARITIES OF *p-p* PARAMETERS

Following Wolfenstein,[3] we write the *p-p* scattering matrix as

$$M=B(\theta)S+C(\theta)[\sigma_{1n}+\sigma_{2n}]+\tfrac{1}{2}G(\theta)[\sigma_{1k}\sigma_{2k}+\sigma_{1p}\sigma_{2p}]T +\tfrac{1}{2}H(\theta)[\sigma_{1k}\sigma_{2k}-\sigma_{1p}\sigma_{2p}]T+N(\theta)[\sigma_{1n}\sigma_{2n}]T, \quad (1)$$

where

$$S=\tfrac{1}{4}(1-\sigma_1\cdot\sigma_2),\quad T=\tfrac{1}{4}(3+\sigma_1\cdot\sigma_2),$$

are the singlet and triplet projection operators. The amplitudes B, C, G, H, and N, considered as functions of $x=\cos\theta$, have singularities in the complex x plane located as follows: simple poles at $x=\pm(1+\mu^2/MT)$, from one-pion exchange, the singularities we wish to make use of; singularities at $x=\pm1$, from the Coulomb amplitude $f_c(\pm x)$, the singularities we wish to avoid; and branch cuts for $x>1+4\mu^2/MT$ and $x<-(1+4\mu^2/MT)$, from exchange of two or more pions, the singularities we wish to ignore. (T is the kinetic energy

[3] L. Wolfenstein, Ann. Rev. Nuclear Sci. **6**, 43 (1956).