Nonradiative $E2$ Transitions in μ Mesonic Atoms*

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An investigation is made of the possibility that a μ^- meson, bound in a $3D$ orbit about a very heavy element, could make a nonradiative transition to the $1S$ state accompanied by electric excitation of the nucleus. A rough calculation, based on recent data on the photofission of U^{238}, indicates that this process may compete favorably with $3D \rightarrow 2P$ x-ray emission.

It has been found[1] that the yields per stopped muon of $2P \rightarrow 1S$ x rays from Th^{232}, U^{235}, and U^{238} are, respectively, 15, 29, and 23% lower than that from Pb. Also, there is evidence[2] that between 5 and 10% of the fissions caused by a μ^- meson stopping in these elements are not due to excitation of the nucleus by the recoiling neutron from the reaction $\mu^- + p \rightarrow n + \nu$.

The possibility that a muon in a $2P$ state could make a transition to the $1S$ state by electromagnetic excitation of the nucleus has been discussed by Zaretskii,[3] who pointed out that the transition energy of $\sim$6.3 Mev in Th, U, and Pu would be slightly above the threshold for fission. From data on photofission of U^{238} at 6 Mev,[4,5] Zaretskii estimated that fission induced by this nonradiative process might occur often enough to be detected. An extensive theoretical investigation of this type of $E1$ transition has been carried out by Zaretskii and Novikov.[6–10]

The purpose of this note is to suggest that nonradiative $E2$ transitions might play an important role in the de-excitation of the muon. For such heavy elements, the nuclear radius is approximately equal to the radius of the $1S$ orbit and is only $\sim$4 times smaller than that of the $3D$ orbit. Consequently, the quadrupole interaction may be important. Recent measurements[11] of the angular distribution of fragments from photofission of U^{238} near threshold show that quadrupole transitions are appreciable and account for approximately 20% of the total cross section. A crude calculation based on these data indicates that the $3D \rightarrow 1S$ transition probability may be large enough to cause a significant decrease in the yield of $3D \rightarrow 2P$ and $2P \rightarrow 1S$ x rays.

Nonradiative dipole and quadrupole transition probabilities will be estimated by the same method which was employed by Zaretskii.[3] The wave function of the nuclear ground state, which for simplicity will be assumed to have spin zero, will be denoted by $\Psi_{0,0}$. The wave function of the initial mesonic state, which will be taken to be a circular orbit with orbital angular momentum l and z component m, will be denoted by $\Phi_{l,m}$. Fine structure will be ignored. A transition of the muon to its ground state $\Phi_{0,0}$, accompanied by an excitation of the nucleus to a state $\Psi_{l,m}$, can be induced by the electric multipole interaction of order l, which is given by

$$H_{\mathrm{E}l} = -e^2[4\pi/(2l+1)] \sum_{p=1}^{Z} \sum_{m'} Y_{l,m'}(\hat{r}_p) Y_{l,m'}{}^*(\hat{r}_\mu) \times \begin{cases} r_p{}^l / r_\mu{}^{l+1}, & r_p < r_\mu, \\ r_\mu{}^l / r_p{}^{l+1}, & r_p > r_\mu, \end{cases} \quad (1)$$

where $\mathbf{r}_\mu$ is the position of the muon and $\mathbf{r}_p$ is that of a nuclear proton. The probability per unit time for this process is

$$P_{nr} = (2\pi/\hbar)(2l+1)^{-2} \times \sum_m \{ |\langle \Psi_{l,m}\Phi_{0,0} | H_{\mathrm{E}l} | \Psi_{0,0}\Phi_{l,m} \rangle|^2 \}_{\mathrm{av}} \rho_l. \quad (2)$$

In Eq. (2), the square of the matrix element is averaged over all excited nuclear levels which can be reached by the transition, and ρ_l is the density of those levels.

If the widths of the excited levels overlap, the inverse of this process is unlikely to occur,[3] and the subsequent de-excitation of the nucleus proceeds by γ-ray emission, neutron emission, or fission. The probability for fission is then given by

$$P_f = [\Gamma_f(l)/\Gamma(l)] P_{nr}, \quad (3)$$

where $\Gamma_f(l)$ and $\Gamma(l)$ are, respectively, the average fission width and the average total width of the levels with angular momentum l.

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If the mesonic wave functions are of the form

$$\Phi_{l,m}=R_{l+1,l}(r_\mu)Y_{l,m}(\hat{r}_\mu),$$

the matrix element in Eq. (2) is

$$-e^2(4\pi)^{\frac{1}{2}}(2l+1)^{-1}\sum_{p=1}^{Z}\int\Psi_{l,m}{}^*\Psi_{0,0}r_p{}^lY_{l,m}(\hat{r}_p)f_l(r_p)d\tau,$$

where

$$f_l(r_p)=r_p{}^{-2l-1}\int_0^{r_p}r_\mu{}^{l+2}R_{l+1,l}(r_\mu)R_{1,0}(r_\mu)dr_\mu+\int_{r_p}^{\infty}r_\mu{}^{-l+1}R_{l+1,l}(r_\mu)R_{1,0}(r_\mu)dr_\mu.$$

Since the region of overlap between the initial and final mesonic states is partially inside the nucleus, the nuclear and mesonic contributions to the matrix element cannot be separated entirely. However, because the quantities $f_l(r_p)$ are slowly varying, monotonically decreasing functions of r_p, it will be assumed that a large error will not be introduced if the matrix element is approximated by

$$-e(4\pi)^{\frac{1}{2}}(2l+1)^{-1}f_l(R)(-1)^mQ_{l,-m},$$

where R is the nuclear radius and

$$Q_{l,-m}=e\sum_{p=1}^{Z}\int\Psi_{l,m}{}^*\Psi_{0,0}r_p{}^lY_{l,-m}{}^*(\hat{r}_p)d\tau.$$

Since the nucleus has zero ground-state spin, $Q_{l,m}$ is independent of m and will be denoted by Q_l. An estimate of $\{|Q_l|^2\}_{\text{av}}\rho_l$ can be obtained from the cross section $\sigma_l(\gamma)$ for excitation of the nucleus by electric multipole radiation of order l. An approximate relationship is given by[12]

$$\sigma_l(\gamma)=\frac{8\pi^3(l+1)}{l[(2l+1)!!]^2}k^{2l-1}\{|Q_l|^2\}_{\text{av}}\rho_l. \tag{4}$$

Numerical calculations were made for U^{238}. In view of the uncertainties regarding the matrix element, it was assumed that it would be sufficient to employ the variational wave functions

$$R_{l+1,l}(r_\mu)\propto(Zr_\mu/a_\mu)^l(1+\delta_lZr_\mu/a_\mu)\times\exp\{-Zr_\mu/[(l+1)a_\mu]\}$$

of Flügge and Zickendraht.[13] The calculation of the parameter δ_l includes relativistic corrections. If the nucleus is assumed to be a uniformly charged sphere of radius $R=7.4$ fermis, the δ_l are 3.7, 0.1, and 0.0 for $l=0$, 1, and 2, respectively. The calculated energies of these states are -11.5, -5.7, and -2.6 Mev. At $r_p=R$, the values of $f_l(r_p)$ are $0.051(Z/a_\mu)^2$ and $0.0038(Z/a_\mu)^3$ for $l=1$ and $l=2$, respectively; at $r_p=0$, they are $0.117(Z/a_\mu)^2$ and $0.0084(Z/a_\mu)^3$.

TABLE I. Transition probabilities for fission and radiation in U^{238}.

Transition	P_f (10^{18} sec^{-1})	P_r (10^{18} sec^{-1})
$3D\rightarrow2P$		1.1
$3D\rightarrow1S$	0.2	
$2P\rightarrow1S$	0.2	1.0

The cross section[3,4] for photofission of U^{238} at 6 Mev is 12 mb. The measurements of Forkman and Johansson[11] at 6.1 Mev indicate that 10 mb is due to $E1$ excitation and 2 mb is due to $E2$ excitation. At higher energies the anisotropy of the angular distribution decreases, presumably because the nucleus is then capable of undergoing fission in a much larger number of channels,[14] and it becomes difficult to separate the dipole and quadrupole contributions.

The data of Forkman and Johansson yield rough values of

$$[\Gamma_f(l)/\Gamma(l)]\{|Q_l|^2\}_{\text{av}}\rho_l$$

at 6.1 Mev. It was assumed that these quantities are approximately constant with increasing energy, and the probabilities for fission were computed. Values of P_f and the radiative transition probability P_r are listed in Table I. The total nonradiative transition probability P_{nr} will, of course, be larger than P_f in both cases due to the possibility of neutron and γ-ray emission.[8]

It has been noted[15] that a μ^- meson in the $1S$ orbit would raise the fission barrier slightly. Calculations made by Zaretskii and Novikov[7,10] indicate that in some cases, U^{238} in particular, the electrostatic coupling between the muon and the deformed, excited nucleus might prevent fission from occurring after a nonradiative $2P\rightarrow1S$ transition. This difficulty does not arise in the $3D\rightarrow1S$ transition, since the excitation is $\sim$9 Mev and is well above the threshold for fission.

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