absolute reduced widths, and from the masking of $l=3$ transitions by dominant $l=1$ groups. The first difficulty could be overcome either by experimental study of the essential empirical constants or, perhaps better, by extracting reduced widths with the help of distorted-wave calculations. The detection of higher l values could be improved by using higher energy resolution or, perhaps, by performing stripping and pickup reactions involving α particles. One problem of interest for future study concerns the shape of energy spectra in pickup reactions up to high excitations (at least 15 MeV) in the residual nucleus.

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Kinetic Energy Distributions of Fragments from the Fission of Au, Tl, Pb, and Bi*†

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Single fragment and total kinetic energy distributions have been measured for fission induced by helium ions and deuterons in Au, Tl, Pb, and Bi targets. The average total kinetic energy release has the same proportionality with $Z^2/A^{\frac{1}{3}}$ found for fission of heavier elements. The widths for the total kinetic energy distributions are comparable to those observed for fission of heavy elements. A preliminary mass distribution curve has been obtained by measuring the energies of coincident fragments.

I. INTRODUCTION

A FEW years ago Fairhall[1] showed that fission of bismuth induced by 20-MeV deuterons results in a mass distribution which is symmetric and rather narrow. The width of this mass distribution was comparable to the widths of the individual light and heavy mass groups resulting from low-energy fission of heavier elements. This predominance of symmetric fission has been found to be characteristic of fission induced in low atomic number targets by protons, deuterons or helium ions.[2,3] Later investigations[4–6] of the excitation functions and angular distributions have provided information about fission thresholds and saddle-point shapes for these systems. The present work is concerned with the kinetic energy distributions, which yield information about the nucleus at scission.

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† Preliminary results were reported by R. Vandenbosch and J. R. Huizenga, Bull. Am. Phys. Soc. **6**, 308 (1961).

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II. SINGLE FRAGMENT KINETIC ENERGY MEASUREMENTS

The energy distributions of the fragments were observed using surface barrier solid-state detectors having a sensitive area of about 0.3 cm². Two detectors were used simultaneously, allowing two independent determinations at the same time. Unfortunately one of the two detectors proved to be unstable in many of the runs, so that final results are often reported for only one of the two detectors. The detectors were located approximately 7 cm from the target and at $\pm150°$ with respect to the beam direction. The choice of a backward angle of observation was made for purposes of reducing pileup pulses from projectile particles scattered by the target into the detectors. The targets were prepared by vacuum volatilization of 300–600 $\mu g/cm^2$ of the metal (natural isotopic composition) onto 0.001-cm Al backing foils. The energy calibration of the detectors was performed with a Cf^{252} spontaneous fission source. As there was an appreciable noise level from the scattered projectile particles even in the backward direction, the calibration was performed with a "beam on" technique. The Cf^{252} source was located on the back of the target holder in such a way that it faced in the opposite direction to that of the target material and was not in the path of the beam. By rotating the target 180° from the normal position, the Cf^{252} source was in a position facing the detectors. The cyclotron beam could still pass through the target to reproduce the noise level of the actual runs, as the target backing, although sufficient to stop the fission fragments, was not thick enough to appreciably change the energies of the scattered particles. The zero point was established using a pulse generator capacitively coupled to the input of the pre-amplifier. The energy calibration was based on the time-of-flight measurements of Milton and Fraser[7] who report most probable kinetic energies of the light and heavy fragments of 104.7 and 79.8 MeV, respectively. There was a tendency for the light fragment to produce a slightly larger pulse height per MeV than the heavy fragment, as has been noted in previous more precise studies. Although this effect, sometimes referred to as the "pulse height defect," can be important if energy calibrations are performed using alpha particles, it does not affect our results to a first approximation as long as the fragment masses and energies being measured are not too different from the fragments used for the calibration.

In order to determine the effect of the finite thickness of the target material on the fragment kinetic energies two runs were always made—one with the target perpendicular to the detector and the other run with the detector at 60° with respect to a line perpendicular to the target plane. This changes the average target thickness by a factor of two, and from the observed change in pulse height a linear extrapolation to zero target thickness was made. This correction varied from 2–10% for different targets employed. The laboratory kinetic energy distributions were converted to center-of-mass coordinates and are illustrated in Fig. 1. The contribution of the target thickness to the widths of the distributions has not been unfolded. The ordinate is proportional to the cross section to show the rapid decrease in cross section as the atomic number is decreased. The total fission cross section[5] integrated over all angles for 43-MeV helium-ion-induced fission of Bi^{209} is approximately 7 mb. The dotted curves for deuteron-induced fission of lead and gold are included, only to show the relative cross sections as it was impossible to obtain a high enough counting rate to define the kinetic energy distribution with any degree of accuracy. The kinetic-energy distributions appear symmetric and when integrated yield an average kinetic energy which is equal to the most probable kinetic energy to within ±2 MeV. The average total kinetic energy releases are tabulated in Table I. Values obtained from independent determinations have been entered. The errors indicated include uncertainties in the energy calibration and finite target thickness corrections. If the number of neutrons emitted per fragment is approximately the same for these systems as for Cf^{252} spontaneous fission, these values represent the total kinetic energy release before neutron emission.

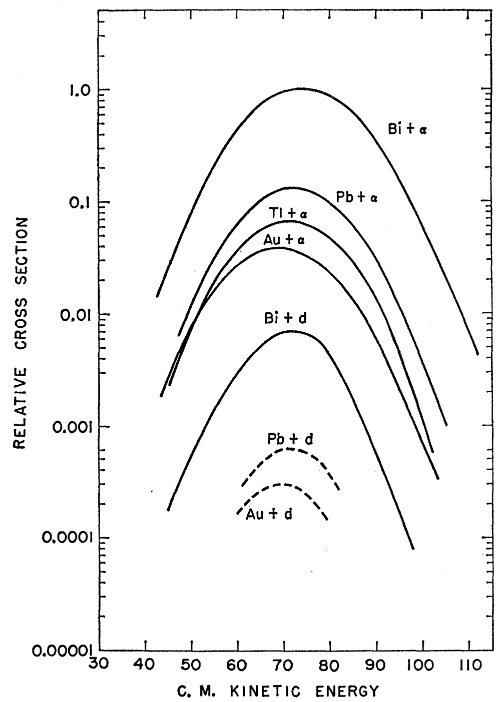

FIG. 1. Single fragment kinetic energy distributions for 43-MeV helium-ion-induced fission and 21.5-MeV deuteron-induced fission. The distributions are placed so that their relative fission cross sections are reproduced.

Table I. Experimentally determined average total kinetic energy release (in the center-of-mass system), in Mev, for 43-MeV helium-ion-induced fission and 21.5-MeV deuteron-induced fission. Independent determinations are given in several cases. The indicated errors include uncertainties arising from calibration and target thickness corrections. The lead and thallium targets were of natural isotopic abundance, and the mass numbers in parentheses indicate the approximate mass number of the fissioning nuclide.

$Bi^{209}+\alpha \rightarrow At^{213}$	149.4 147.5	148±4
$Pb+\alpha \rightarrow Po^{(211)}$	146±5	
$Tl+\alpha \rightarrow Bi^{(209)}$	143±5	
$Au^{197}+\alpha \rightarrow Tl^{201}$	139.3 137.4	138±4
$Bi^{209}+d \rightarrow Po^{211}$	143.0 145.3 139.5	143±5

III. TOTAL KINETIC ENERGY DISPERSION MEASUREMENTS

Conservation of momentum requires that the two fission fragments are emitted at 180° with respect to each other in the center-of-mass system. In the case of particle-induced fission the projectile brings in linear momentum and the fragments are forward folded approximately 8° in the laboratory system. For determining the dispersion in the total kinetic energy, detectors were placed at 112° and 300° with respect to the beam direction. The targets consisted of 100–300 μg/cm² of target material which had been evaporated onto ~175 μg/cm² aluminum backing foils. The target faced

[7] J. C. D. Milton and J. S. Fraser, Phys. Rev. 111, 877 (1958).

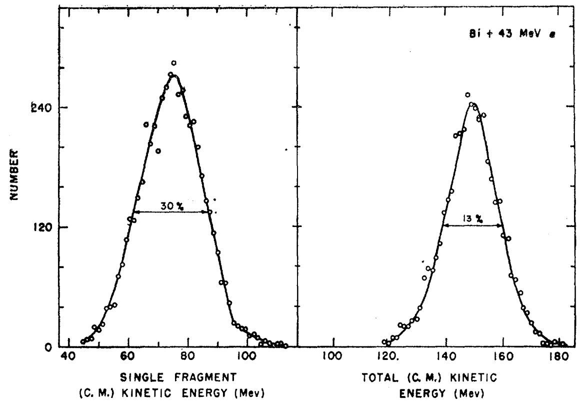

FIG. 2. Single fragment and total kinetic energy distributions for 43-MeV helium-ion-induced fission of bismuth.

the detector at 112° so that the fragments going in the somewhat forward direction had to pass through the target backing. With this arrangement the energy loss of the fragments in the target backing is compensated for to first order by the fact, that in the laboratory system the fragments appearing in the forward direction have higher kinetic energies than do the fragments appearing in the backward direction. The pulses from the two detectors were amplified and presented to the multichannel analyzer. Appropriate routing and coincidence pulses enabled us to display the single fragment kinetic energy distributions from one detector in the first 128 channels and the sum of the two fragment pulses for coincident events in the second half of the analyzer. The results of this type of measurement are illustrated in Fig. 2.

The energy scale was obtained by assuming a most probable total kinetic energy equal to twice the most probable single fragment energy in the center-of-mass system. The measured dispersion is somewhat larger than the true dispersion because the high counting rate of scattered alpha particles broadened the response of the electronics. Tests with a pulse generator, while the cyclotron beam was passing through the target, indicated an instrumental line width of about 3%. This problem was most severe in the counter placed in the forward hemisphere. The finite target thickness also makes a small contribution to the observed width.

IV. PRELIMINARY MASS DISTRIBUTION MEASUREMENT

If one neglects neutron emission, it follows from conservation of momentum that $E_1/(E_1+E_2)=M_2/(M_1+M_2)$, where E_1 is the kinetic energy of fragment with mass M_1, and E_2 is the kinetic energy of fragment with mass M_2. Thus it is possible to determine a mass yield curve by dividing the pulse height from the first detector by the sum of the pulse heights from the first and the second detectors. This technique was developed by Roeland, Bollinger, and Thomas[8] to measure mass distributions for fission following capture of resonant energy neutrons.

A preliminary determination of a mass distribution was obtained using a target and detector arrangement similar to that described in the previous section. The pulses E_1 and (E_1+E_2) were fed into a pulse ratio to time converter circuit. The "start" and "stop" pulses from the time converter were then fed into a multichannel analyzer, bypassing the pulse height to time converter circuit of the analyzer. The results of such a measurement for 42-MeV helium-ion-induced fission of bismuth are shown in Fig. 3. Unfortunately no radiochemically determined mass distributions have been reported for this reaction, but the distribution shown in Fig. 3 has a full width at half-maximum heigh of about 24%, which agrees well with the width of the mass distribution for 43-MeV helium-ion-induced fission of Pb^{206} as measured radiochemically by Neuzil and Fairhall.[2] Further measurements of mass distributions by this technique have been postponed until a multidimensional analyzer becomes available, since with a multidimensional analyzer one can determine simultaneously the mass distributions for different total kinetic energies, etc.

V. DISCUSSION OF RESULTS

The kinetic energy of the fission fragments arises from the coulomb repulsion of the two charged frag-

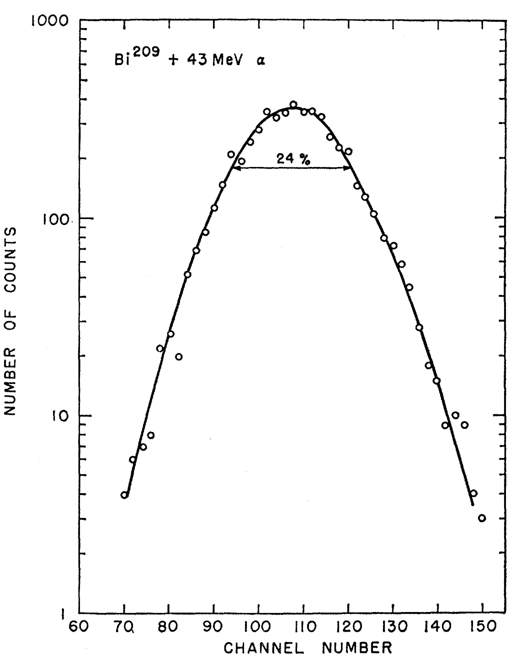

FIG. 3. Mass distribution curve for 43-MeV helium-ion-induced fission of bismuth. The abscissa (in arbitrary units) is proportional to $E_1/(E_1+E_2)\simeq M_2/(M_1+M_2)$.

[8] L. W. Roeland, L. M. Bollinger and G. E. Thomas, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), Vol. 15, paper P/551, p. 440.

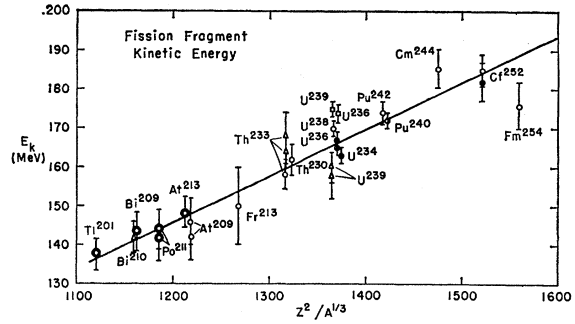

FIG. 4. Average total kinetic energy release as a function of $Z^2/A^{\frac{1}{3}}$. The points indicated by concentric circles are from the present study, and the remaining data are taken from compilations in recent reviews {I. Halpern, Ann. Rev. Nuclear Sci. **9**, 245 (1959) and J. R. Huizenga and R. Vandenbosch, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur [North-Holland Publishing Company, Amsterdam (to be published)], Vol. 2.}. The straight line is taken from Terrell,[9] and represents a least-squares fit to the kinetic energies for fission of elements with atomic number greater than or equal to that of thorium.

ments. Therefore, one expects the kinetic energy to be proportional to a coulomb energy $Z_1Z_2e^2/r$, where r may be thought of as the separation distance of the centers of charges of the two fragments. Letting $Z_1=Z_2=Z/2$ and using the proportionality $r\propto A^{\frac{1}{3}}$ one expects a correlation of the kinetic energy release with $Z^2/A^{\frac{1}{3}}$. Such a correlation has been noted by Terrell,[9] and in Fig. 4, we show a plot of the average total kinetic energy as a function of $Z^2/A^{\frac{1}{3}}$. The points indicated by concentric circles are from the present study. The straight line is taken from Terrell,[9] and represents a least-squares fit to the kinetic energies for fission of elements with atomic number greater than or equal to that of thorium. Present experimental evidence[10,11] suggests that the kinetic energy release is fairly independent of excitation energy. The fact that the kinetic energies for the less fissionable elements correlate so well with $Z^2/A^{\frac{1}{3}}$ is not necessarily to be expected. It has been clearly demonstrated[12–14] that the total kinetic energy release for certain heavy elements, especially for thermal neutron fission of U^{235}, is significantly less for symmetric mass divisions than for the more probable asymmetric mass divisions. In the case of thermal neutron fission of U^{235} the kinetic energy release for symmetric fission is at least 35 MeV less than that for asymmetric fission.[13] Since the kinetic energy releases for the predominantly symmetric fission of the less fissionable elements correlate well with that for the predominantly asymmetric fission of the heavier elements, it would appear that the abnormally low kinetic energies for symmetric fission observed for some of the heavier elements is not a general property of symmetric fission.

It is interesting to attempt to deduce possible scission configurations from the total kinetic energy releases. In this simplified treatment it will be assumed that all of the kinetic energy arises from the coulomb interaction energy of the two fragments at the "scission configuration." The scission configuration will be approximated by two equal tangent prolate spheroids described by major and minor semiaxes C and A. The interaction energy as a function of C/A has been worked out for this and other more general cases by Cohen and Swiatecki.[15] The results of such a calculation, taking the nuclear radius parameter $r_0=1.216\times10^{-13}$ cm, are listed in Table II. A heavy element has been included for comparison. It is seen that the scission stretchings are all very similar, which is to be expected from the correlation shown in Fig. 4. It is also to be noted that the scission stretchings are larger than the saddle-point stretchings as estimated by the equilibrium shape of tangent spheroids in the charged liquid drop model calculations of Cohen and Swiatectki[15] ($C/A\sim1.8$ for Tl^{201}) or as deduced from angular distribution experiments.[6]

Relatively little is known about the dispersion in the total kinetic energy. A summary of the known informa-

TABLE II. Deformation of the fissioning nucleus at the scission point derived from the experimental kinetic energies for the assumption that the scission point can be represented by two equal tangent prolate spheroids. C/A is the ratio of the major to minor semiaxes of the spheroids.

Compound nucleus	x[a]	Kinetic energy (MeV)	C/A
At^{213}	0.677	148±4	2.18±0.10
$Po^{(211)}$	0.667	146±5	2.18±0.12
$Bi^{(209)}$	0.657	143±5	2.15±0.12
Tl^{201}	0.651	138±4	2.15±0.10
Pu^{240}	0.734	172±2	2.24±0.05

[a] $x=(Z^2/A)/(Z^2/A)_{\rm crit}$, where $(Z^2/A)_{\rm crit}=50.13$.

TABLE III. Total kinetic energy distributions.

Reaction	Compound nucleus	Full width at half maximum	Reference
Bi^{209} +22-MeV d	Po^{211}	19 MeV, 13%	This work
Bi^{209} +43-MeV α	At^{213}	20 MeV,[a] 13%	This work
Th^{229}+th. n	Th^{230}	16 MeV, 10%	Smith *et al.*[b]
U^{235} +th. n	U^{236}	26 MeV, 15.5%	Safford *et al.*[c]
Cf^{252} S.F.	Cf^{252}	26 MeV, 14%	Milton & Fraser[d]
		32 MeV, 17%	Stein & Whetstone[e]

[a] The value of this quantity obtained from recent measurements [Unik, Bate, and Huizenga, Bull. Am. Phys. Soc. **7** (1962)] with a two-dimensional analyzer is 16±2 MeV and the full width at half-maximum intensity of the mass distribution is 21±2 mass units.
[b] See reference 16.
[c] See reference 17.
[d] See reference 7.
[e] See reference 18.

[9] J. Terrell, Phys. Rev. **113**, 527 (1959).
[10] J. Jungerman and S. C. Wright, Phys. Rev. **76**, 1112 (1949).
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[15] S. Cohen and W. J. Swiatecki, Aarhus University Report, 1961 (unpublished).

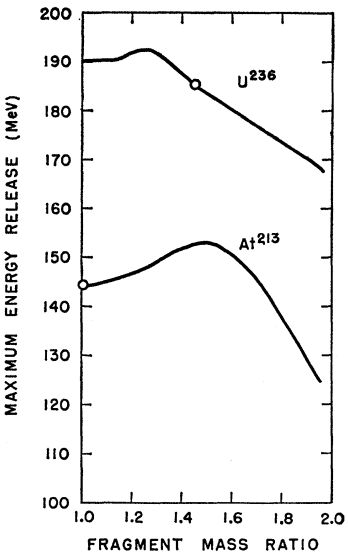

FIG. 5. Calculated maximum energy release as a function of mass ratio for nuclei exhibiting predominantly symmetric or asymmetric fission. The circles indicate the most probable mass divisions observed experimentally.

tion,[7,16–18] including the results of the present study, is given in Table III. The total kinetic energy dispersions reported in the present study are upper limits, as no corrections have been made for instrumental resolution (see earlier section) and the "pulse height defect" of the solid-state detectors. If the "pulse height defect" of each fragment is 10 MeV, for example, the dispersion in the total kinetic energy will decrease by 13%. The dispersions in the total kinetic energy for the fission of the lighter elements appear to be comparable to those reported for the heavier elements.

Concerning the symmetric mass distributions observed for fission of the lighter elements, little can be said. There is at present no theory which adequately explains why heavy element fission is asymmetric or why fission of lighter elements is symmetric. One is often inclined to think of symmetric fission as the normal mode of fission, with asymmetric fisson being anomalous. However from an energy release point of view both situations are somewhat anomalous, as is demonstrated by the energy release calculations illustrated in Fig. 5. The energy releases have been calculated using nuclear masses given by the mass equation of Cameron.[19] The circles indicate the most probable mass divisions observed experimentally. Thus from energy release considerations alone, it is difficult to understand either result.

ACKNOWLEDGMENTS

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