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Kinetic Equation for Plasma

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The collision integral for the kinetic equation describing a qunatum mechanical plasma is set up. It is shown that the classical collision integral derived by Rostoker, Rosenbluth, Balescu, and Lenard is easily obtained from the more perspicuous quantum mechanical result by letting $\hbar \to 0$. The kinetic equations of Pines and Schrieffer, which describe the interactions of the electrons with plasma oscillations, are obtained by isolating the contributions from the plasma oscillations.

1. INTRODUCTION

A NUMBER of recent investigations[1–7] have been devoted to the problem of establishing a kinetic equation (Boltzmann equation) suitable for the description of nonequilibrium phenomena in a plasma. In the case of the classical plasma, several authors,[2–4] proceeding by different methods, have succeeded in deriving a kinetic equation valid in the limit that the number of particles in the Debye sphere is large,

$$n\lambda_D{}^3 \gg 1, \quad \lambda_D = (kT/4\pi ne^2)^{1/2}. \tag{1}$$

For an electron plasma in a uniform positive charge background, one finds the coupled equations

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - \frac{e}{m}\mathbf{E}(\mathbf{x},t)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f(\mathbf{x},\mathbf{v},t) = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}, \tag{2}$$

$$\frac{\partial}{\partial \mathbf{x}}\cdot \mathbf{E}(\mathbf{x},t) = -4\pi en\left[\int d^3v\, f(\mathbf{x},\mathbf{v},t) - 1\right], \tag{3}$$

$$\left(\frac{\delta f(\mathbf{v})}{\delta t}\right)_{\text{coll}} = \frac{2ne^4}{m^2}\frac{\partial}{\partial \mathbf{v}}\cdot\int d^3v' \left\{\int d^3q\, \frac{\mathbf{q}\mathbf{q}}{q^4}\frac{\delta(\mathbf{q}\cdot(\mathbf{v}-\mathbf{v}'))}{|\mathcal{K}(\mathbf{q},\mathbf{q}\cdot\mathbf{v})|^2}\right\}\cdot\left\{\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}} f(\mathbf{v}') - f(\mathbf{v})\frac{\partial f(\mathbf{v}')}{\partial \mathbf{v}'}\right\}. \tag{4}$$

The single-particle distribution function $f(\mathbf{x},\mathbf{v},t)$ is normalized so that

$$\int d^3x \int d^3v\, f(\mathbf{x},\mathbf{v},t) = V, \tag{5}$$

where V is the volume of the system. Equation (2) is the kinetic equation. Equation (3) is Poisson's equation, which determines the self-consistent electric field $\mathbf{E}(\mathbf{x},t)$, and Eq. (4) gives the form of the collision integral appearing in the kinetic equation. The collision integral depends on the dynamic dielectric constant of

[1] N. Bogoliubov, *Problems of Dynamic Theory in Statistical Physics*, AEC-tr-3852 (Federal Publishing House for Technical-Theoret. Lt., Moscow-Leningrad, 1946).
[2] N. Rostoker and M. N. Rosenbluth, Phys. Fluids **3**, 1 (1960).
[3] R. Balescu, Phys. Fluids **3**, 52 (1960).
[4] A. Lenard, Ann. Phys. (New York) **10**, 390 (1960).
[5] O. V. Konstantinov and V. I. Perel', Soviet Phys.—JETP **12**, 597 (1961).
[6] N. Rostoker, Nuclear Fusion **1**, 101 (1960).
[7] D. Pines and J. R. Schrieffer, Phys. Rev. **125**, 804 (1962).

the plasma, which is given by

$$\mathcal{K}(\mathbf{q},\omega)=1+\frac{\omega_p^2}{q^2}\int d^3v\,\frac{\mathbf{q}\cdot[\partial f(\mathbf{v})/\partial\mathbf{v}]}{\omega+i\eta-\mathbf{q}\cdot\mathbf{v}}, \tag{6}$$

where $\omega_p^2=4\pi ne^2/m$ is the plasma frequency and η is a positive infinitesimal.

The derivations[2–4] leading to the formula (4) for the collision integral, while having the advantage of being precise or rigorous in the limit (1) have at the same time the disadvantage of being quite complicated mathematically. Moreover, the result (4) itself is of a rather complex form and the various factors in it do not at first sight seem to have a simple physical interpretation. One could not easily guess the form (4) for the collision integral.

We would like to point out in Sec. 2 of this paper that if one approaches the problem quantum mechanically, rather, than classically, the whole situation seems much simpler. Thus, the quantum mechanical analog of (4) has a simple form which can be easily guessed. And the classical result can be obtained from the quantum mechanical one by a straightforward expansion in powers of $\hbar$.

Schrieffer and one of the authors of the present paper have recently developed[7] an alternative version of kinetic theory for a plasma which emphasizes the role played by the collective mode or plasma oscillations. These authors start with a quantum mechanical plasma and later reduce their results to the classical limit by letting $\hbar\rightarrow 0$. For the quantum mechanical case one introduces the number of plasmons $N(\mathbf{q})$ with wave number $\mathbf{q}$ or momentum $\hbar\mathbf{q}$ and the number of electrons $F(\mathbf{p})$ with wave number $\mathbf{p}$ or momentum $\hbar\mathbf{p}$. There are then two coupled equations describing the time rate of change of these quantities due to emission and absorption of plasmons by the electrons:

$$\frac{\partial N(\mathbf{q})}{\partial t}=-2\gamma_{\mathbf{q}}N(\mathbf{q})+\frac{4\pi^2e^2\omega_{\mathbf{q}}}{q^2}\frac{1}{V}\sum_{\mathbf{p}}F(\mathbf{p})[1-F(\mathbf{p}-\mathbf{q})]\delta(\hbar\omega_{\mathbf{q}}-\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{p}-\mathbf{q}}), \tag{7}$$

$$\frac{\partial F(\mathbf{p})}{\partial t}=\frac{1}{V}\sum_{|\mathbf{q}|<q_c}\frac{4\pi^2e^2\omega_{\mathbf{q}}}{q^2}[\delta(\hbar\omega_{\mathbf{q}}-\epsilon_{\mathbf{p}+\mathbf{q}}+\epsilon_{\mathbf{p}})\{-F(\mathbf{p})[1-F(\mathbf{p}+\mathbf{q})]N(\mathbf{q})+F(\mathbf{p}+\mathbf{q})[1-F(\mathbf{p})][N(\mathbf{q})+1]\}$$
$$+\delta(\hbar\omega_{\mathbf{q}}-\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{p}-\mathbf{q}})\{-F(\mathbf{p})[1-F(\mathbf{p}-\mathbf{q})][N(\mathbf{q})+1]+F(\mathbf{p}-\mathbf{q})[1-F(\mathbf{p})]N(\mathbf{q})\}]. \tag{8}$$

In these equations $\epsilon_p=\hbar^2p^2/2m$ is the kinetic energy of an electron and $\omega_{\mathbf{q}}$ and $\gamma_{\mathbf{q}}$ are the frequency and decay rate of the plasma oscillations so that $\omega=\omega_{\mathbf{q}}-i\gamma_{\mathbf{q}}$ is the complex frequency of the plasma oscillation. The decay rate is given by the expression

$$\frac{\gamma_{\mathbf{q}}}{\omega_{\mathbf{q}}}=\frac{2\pi^2e^2}{q^2}\frac{1}{V}\sum_{\mathbf{p}}[F(\mathbf{p}-\mathbf{q})-F(\mathbf{p})]\delta[\hbar\omega_{\mathbf{q}}-\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{p}-\mathbf{q}}], \tag{9}$$

which we shall find useful below.

The kinetic equations (7) and (8) describe only the collective effects in the plasma and do not include the effects due to close collisions. Thus, there is a cutoff in Eq. (8), $q<q_c$, such that only plasma oscillations for which the Landau damping (9) is small will be included. In Sec. 3 of this paper we want to show the connection between the quantum mechanical analog of the collision integral (4) and the kinetic equations (7) and (8).

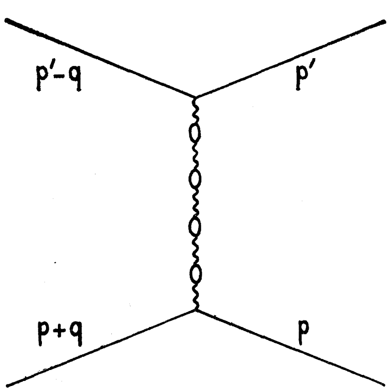

FIG. 1. Feynman diagram for scattering of electrons via a screened Coulomb interaction.

2. QUANTUM MECHANICAL COLLISION INTEGRAL

To describe the quantum plasma, we use single-particle plane wave states

$$\frac{1}{\sqrt{V}}\exp(i\mathbf{p}\cdot\mathbf{x}), \tag{10}$$

where $\mathbf{p}$ is the wave number of the electron so that $\hbar\mathbf{p}$ is its momentum. V is the volume of the system, and the volume in $\mathbf{p}$ space per quantum state is $(2\pi)^3/V$. We let $F(\mathbf{p})$ be the number of electrons with wave number $\mathbf{p}$ in the system. This is related to the distribution function $f(\mathbf{v})$ of Eqs. (2)–(5) by

$$f(\mathbf{v})=(m/2\pi\hbar)^3(1/n)F(\mathbf{p}),\quad \mathbf{v}=\hbar\mathbf{p}/m, \tag{11}$$

where n is the density of particles in the system.

The "golden rule" gives for the transition probability per unit time for scattering of two electrons from states $\mathbf{p}$, $\mathbf{p}'$ to states $\mathbf{p}+\mathbf{q}$, $\mathbf{p}'-\mathbf{q}$ the result

$$(2\pi/\hbar)|M|^2\delta(\epsilon_{p+\mathbf{q}}+\epsilon_{p'-\mathbf{q}}-\epsilon_p-\epsilon_{p'}), \tag{12}$$

where $\epsilon_p=\hbar^2p^2/2m$ is the energy of an electron and M is the matrix element for scattering with momentum transfer $\hbar\mathbf{q}$. For pure Coulomb scattering this matrix element would be

$$M=(1/V)(4\pi e^2/q^2). \tag{13}$$

However, in a plasma, as is well known, it is necessary to take into account the screening effects. This can be done by dividing (13) by the dynamic dielectric constant of the plasma:

$$M=\frac{1}{V}\frac{4\pi e^2}{q^2}\frac{1}{\mathcal{K}(\mathbf{q},\,\hbar^{-1}(\epsilon_{\mathbf{p}+\mathbf{q}}-\epsilon_{\mathbf{p}}))}, \tag{14}$$

where the dielectric constant is given by the quantum analog of (6)

$$\mathcal{K}(\mathbf{q},\omega)=1-\frac{4\pi e^2}{q^2}\frac{1}{V}\sum_{\mathbf{k}}\frac{F(\mathbf{k})-F(\mathbf{k}+\mathbf{q})}{\hbar\omega+i\eta-(\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}})}$$
$$=1-\frac{4\pi e^2}{q^2}\int\frac{d^3k}{(2\pi)^3}\frac{F(\mathbf{k})-F(\mathbf{k}+\mathbf{q})}{\hbar\omega+i\eta-(\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}})}. \tag{15}$$

The approximation (14) is generally called the random phase approximation in the quantum mechanical literature. In field-theory language the matrix element (14) corresponds to the Feynman diagram shown in Fig. 1.

Using the results (12) and (14), it is simple enough to write down a Boltzmann collision integral for the rate of change of the number of electrons in state $\mathbf{p}$:

$$\left(\frac{\delta F(\mathbf{p})}{\delta t}\right)_{\text{coll}}=-\int\frac{d^3p'}{(2\pi)^3/V}\int\frac{d^3q}{(2\pi)^3/V}\left|\frac{1}{V}\frac{4\pi e^2}{q^2}\frac{1}{\mathcal{K}(\mathbf{q},\,\hbar^{-1}(\epsilon_{\mathbf{p}+\mathbf{q}}-\epsilon_{\mathbf{p}}))}\right|^2\delta(\epsilon_{\mathbf{p}+\mathbf{q}}+\epsilon_{\mathbf{p}'-\mathbf{q}}-\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}'})$$
$$\times\{F(\mathbf{p})F(\mathbf{p}')[1-F(\mathbf{p}+\mathbf{q})][1-F(\mathbf{p}'-\mathbf{q})]-F(\mathbf{p}+\mathbf{q})F(\mathbf{p}'-\mathbf{q})[1-F(\mathbf{p})][1-F(\mathbf{p}')]\}. \tag{16}$$

Here we have included the terms representing the Pauli principle for later use. In the classical limit $F(\mathbf{p})\ll 1$ and we can drop these terms. Changing back to the classical variables, Eq. (11), and dropping the Pauli principle terms, we find in place of (16)

$$\left(\frac{\delta f(\mathbf{v})}{\delta t}\right)_{\text{coll}}=-\frac{4ne^4}{\hbar^2}\int d^3v'\int d^3\mathbf{q}\frac{1}{q^4}\frac{1}{|\mathcal{K}(\mathbf{q},\,\mathbf{q}\cdot\mathbf{v}+\hbar q^2/2m)|^2}$$
$$\times\delta[\mathbf{q}\cdot(\mathbf{v}-\mathbf{v}')+\hbar q^2/m][f(\mathbf{v})f(\mathbf{v}')-f(\mathbf{v}+\hbar\mathbf{q}/m)f(\mathbf{v}'-\hbar\mathbf{q}/m)]. \tag{17}$$

The classical result (4) for the collision integral can now be obtained by expanding (17) in powers of $\hbar$ and keeping only the leading term, which is independent of $\hbar$. Those readers interested in reproducing this expansion should note that it is necessary to keep the first-order correction terms in the expansions of both the delta function and the dielectric constant:

$$\delta[\mathbf{q}\cdot(\mathbf{v}-\mathbf{v}')+\hbar q^2/m]=\delta[\mathbf{q}\cdot(\mathbf{v}-\mathbf{v}')]+(\hbar/m)\mathbf{q}\cdot(\partial/\partial\mathbf{v})\delta[\mathbf{q}\cdot(\mathbf{v}-\mathbf{v}')], \tag{18}$$

$$\frac{1}{|\mathcal{K}(\mathbf{q},\,\mathbf{q}\cdot\mathbf{v}+\hbar q^2/2m)|^2}=\frac{1}{|\mathcal{K}(\mathbf{q},\mathbf{q}\cdot\mathbf{v})|^2}+\frac{\hbar}{2m}\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{v}}\frac{1}{|\mathcal{K}(\mathbf{q},\mathbf{q}\cdot\mathbf{v})|^2}. \tag{19}$$

The term of order $1/\hbar$ vanishes on integrating over $\mathbf{q}$.

This, then, is an easy way to arrive at the classical collision integral (4). One sees how the unfamiliar structure of the classical result—quite different from that of the Boltzmann collision integral—arises just from the Taylor expansion in powers of $\hbar$.

It is clear that only scattering in the nearly forward direction contributes to the classical limit (4), i.e., the momentum transfer $\hbar\mathbf{q}\to 0$. It is interesting that it is possible to obtain another classical limit of (17) in which finite momentum transfers play a role. Thus, if we write the momentum transfer in terms of a velocity variable $\mathbf{u}$,

$$\hbar\mathbf{q}=m\mathbf{u},$$

the collision integral (17) can be rewritten in the form

$$\left(\frac{\delta f(\mathbf{v})}{\delta t}\right)_{\text{coll}}=-\frac{4ne^4}{m}\int d^3v'\int d^3u\frac{1}{u^4}\frac{1}{|\mathcal{K}[\hbar^{-1}m\mathbf{u},\,\hbar^{-1}(\frac{1}{2}m(\mathbf{v}+\mathbf{u})^2-\frac{1}{2}mv^2)]|^2}$$
$$\times\delta[\tfrac{1}{2}m(\mathbf{v}+\mathbf{u})^2+\tfrac{1}{2}m(\mathbf{v}'-\mathbf{u})^2-\tfrac{1}{2}m\mathbf{v}^2-\tfrac{1}{2}m\mathbf{v}'^2][f(\mathbf{v})f(\mathbf{v}')-f(\mathbf{v}+\mathbf{u})f(\mathbf{v}'-\mathbf{u})]. \tag{20}$$

If we let $\hbar\to 0$ in (20), all that happens is that the dielectric constant

$$\mathcal{K}[\hbar^{-1}m\mathbf{u},\hbar^{-1}(\tfrac{1}{2}m(\mathbf{v}+\mathbf{u})^2-\tfrac{1}{2}mv^2)]\to\mathcal{K}(\infty,\infty)=1, \quad (21)$$

approaches its value for infinite arguments, which is unity according to the definition (6). With $\mathcal{K}$ replaced by unity (20) is just the ordinary Boltzmann collision integral with the Coulomb cross section. This apparent ambiguity in the classical limit is neither puzzling nor serious. The classical limit (4) is logarithmically divergent for large q (close collisions) and the classical limit (20) with $\mathcal{K}=1$ is logarithmically divergent for small q (distant collisions). For small q one should use (4) and for large q one should use (20) with $\mathcal{K}=1$. Or one can use (4) with a cutoff at large q which corresponds to taking the maximum scattering angle to be 180°. This latter is the procedure employed in practice.

3. THE CONTRIBUTION FROM THE COLLECTIVE MODES

In this section we want to trace out the connection between the quantum mechanical collision integral as given by Eq. (16) and the version of kinetic theory discussed by Pines and Schrieffer, Eqs. (7), (8), and (9). First we need an expression for the number of plasmons $N(\mathbf{q})$. One way to obtain such an expression is to start from a formula for the density-density autocorrelation function in a plasma with an arbitrary nonequilibrium distribution of electrons $F(\mathbf{p})$. One then approximates this formula, using a method similar to that given below, keeping only the contribution from the poles of $1/\mathcal{K}(\mathbf{q},\omega)$ corresponding to the plasma oscillations. Using the approximate formula for the autocorrelation function, one can calculate the energy in the plasma oscillation fluctuations, which is also given by $N(\mathbf{q})\hbar\omega_{\mathbf{q}}$. We shall not reproduce the details of this calculation, because the result obtained in this way is just what one obtains from Eq. (7) by setting the left-hand side equal to zero, i.e.,

$$N(\mathbf{q})=\frac{1}{\gamma_{\mathbf{q}}}\frac{2\pi^2e^2\omega_{\mathbf{q}}}{q^2}\frac{1}{V}\sum_{\mathbf{p}}F(\mathbf{p})\times[1-F(\mathbf{p}-\mathbf{q})]\delta(\hbar\omega_{\mathbf{q}}-\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{p}-\mathbf{q}}). \quad (22)$$

This gives the distribution of plasmons in an arbitrary nonequilibrium distribution of electrons $F(\mathbf{p})$ *after* sufficient time has elapsed for the plasmons to come to equilibrium with the electrons, but *before* sufficient time has elapsed for the electron distribution function to change appreciably. Of course, the expression (22) makes sense only if the conditions described in the previous sentence are physically realized, and this depends on the problem under consideration. Consider, for example, the problem of calculating the dc electrical conductivity of a plasma. This is a time-independent problem with a nonequilibrium electron distribution, and one would expect that (22) applies.

In any event, we shall assume the validity of (22) in what follows. Given this result we can derive Eq. (8) from the quantum mechanical collision integral (16). First, we note that by combining the formula (9) for the decay rate $\gamma_{\mathbf{q}}$ with (22) we obtain another expression for $N(\mathbf{q})$:

$$N(\mathbf{q})+1=\frac{1}{\gamma_{\mathbf{q}}}\frac{2\pi^2e^2\omega_{\mathbf{q}}}{q^2}\frac{1}{V}\sum_{\mathbf{p}}F(\mathbf{p}-\mathbf{q})\times[1-F(\mathbf{p})]\delta(\hbar\omega_{\mathbf{q}}-\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{p}-\mathbf{q}}). \quad (23)$$

Return now to the quantum mechanical collision integral (16). In order to obtain the Pines-Schrieffer Eq. (8) we must pick out the contributions of the plasmons. These come from the poles of $1/\mathcal{K}(\mathbf{q},\omega)$. Now $\mathcal{K}(\mathbf{q},\omega)$ has two zeros:

$$\mathcal{K}(\mathbf{q},\omega)=0$$

at

$$\omega=\omega_{\mathbf{q}}-i\gamma_{\mathbf{q}} \quad\text{and}\quad \omega=-\omega_{-\mathbf{q}}-i\gamma_{-\mathbf{q}}, \quad (24)$$

and near these zeros $\mathcal{K}(\mathbf{q},\omega)$ is given approximately by the first term in its Taylor expansion

$$\begin{aligned}\mathcal{K}(\mathbf{q},\omega)&=\frac{2}{\omega_{\mathbf{q}}}(\omega-\omega_{\mathbf{q}}+i\gamma_{\mathbf{q}}), & \omega&\sim\omega_{\mathbf{q}}\\ &=-\frac{2}{\omega_{-\mathbf{q}}}(\omega+\omega_{-\mathbf{q}}+i\gamma_{-\mathbf{q}}), & \omega&\sim-\omega_{-\mathbf{q}}.\end{aligned} \quad (25)$$

Here, we have used the approximation

$$\left.\frac{d}{d\omega}\mathcal{K}(\mathbf{q},\omega)\right|_{\omega=\omega_{\mathbf{q}}}=\frac{2}{\omega_{\mathbf{q}}}, \quad (26)$$

which is valid as long as the plasma oscillations are weakly damped. The quantity which appears in the quantum mechanical collision integral (16) is

$$\frac{1}{|\mathcal{K}(\mathbf{q},\omega)|^2}=-\left(\mathrm{Im}\frac{1}{\mathcal{K}(\mathbf{q},\omega)}\right)\frac{1}{\mathrm{Im}\mathcal{K}(\mathbf{q},\omega)}, \quad (27)$$

where Im denotes the imaginary part. Using the approximations

$$\begin{aligned}\mathrm{Im}\mathcal{K}(q,\omega)&=\frac{2}{\omega_{\mathbf{q}}}\gamma_{\mathbf{q}}, & \omega&\sim\omega_{\mathbf{q}}\\ &=-\frac{2}{\omega_{-\mathbf{q}}}\gamma_{-\mathbf{q}}, & \omega&\sim-\omega_{-\mathbf{q}}\end{aligned} \quad (28)$$

and

$$\mathrm{Im}\frac{1}{\mathcal{K}(\mathbf{q},\omega)}=-\pi\frac{\omega_{\mathbf{q}}}{2}\delta(\omega-\omega_{\mathbf{q}})+\pi\frac{\omega_{-\mathbf{q}}}{2}\delta(\omega+\omega_{-\mathbf{q}}), \quad (29)$$

obtained from (25) [by letting $\gamma_{\mathbf{q}} \to 0$ in (29)], we find

$$\frac{1}{|\mathcal{K}(\mathbf{q},\omega)|^2} = \pi\frac{\omega_{\mathbf{q}}^2}{4\gamma_{\mathbf{q}}}\delta(\omega-\omega_{\mathbf{q}}) + \pi\frac{\omega_{-\mathbf{q}}^2}{4\gamma_{-\mathbf{q}}}\delta(\omega+\omega_{-\mathbf{q}}). \quad (30)$$

This expression gives approximately the contribution of the plasma oscillations to $1/|\mathcal{K}(\mathbf{q},\omega)|^2$.

Substituting the result (30) in the quantum mechanical collision integral (16) we obtain

$$\left(\frac{\delta F(\mathbf{p})}{\delta t}\right)_{\text{coll.}} = -\frac{1}{V}\sum_{\mathbf{q}}\frac{1}{V}\sum_{\mathbf{p}'}\left(\frac{4\pi e^2}{q^2}\right)^2\left[\pi\frac{\hbar\omega_{\mathbf{q}}^2}{4\gamma_{\mathbf{q}}}\delta(\hbar\omega_{\mathbf{q}}-\epsilon_{\mathbf{p}+\mathbf{q}}+\epsilon_{\mathbf{p}}) + \pi\frac{\omega_{-\mathbf{q}}^2}{4\gamma_{-\mathbf{q}}}\delta(\hbar\omega_{-\mathbf{q}}+\epsilon_{\mathbf{p}+\mathbf{q}}-\epsilon_{\mathbf{p}})\right]\delta(\epsilon_{\mathbf{p}+\mathbf{q}}+\epsilon_{\mathbf{p}'-\mathbf{q}}-\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}'})$$
$$\times\{F(\mathbf{p})F(\mathbf{p}')[1-F(\mathbf{p}+\mathbf{q})][1-F(\mathbf{p}'-\mathbf{q})]-F(\mathbf{p}+\mathbf{q})F(\mathbf{p}'-\mathbf{q})[1-F(\mathbf{p})][1-F(\mathbf{p}')]\}. \quad (31)$$

Using the formulas (22) and (23) for $N(\mathbf{q})$ and $N(\mathbf{q})+1$, it is easy to carry out the sums over $\mathbf{p}'$ in (31) and obtain precisely the result (8).

Thus, we see that the quantum mechanical collision integral (16) and consequently its classical limit (4) contain the collective effects described by the equations of Pines and Schrieffer insofar as the plasmons can be regarded as in equilibrium with the electrons. The collision integrals (16) and (4) contain also, of course, the effects of close collisions.

We remark that a certain amount of care must be exercised when considering the plasma wave contributions which are implicitly described in Eq. (2). As we have seen, the quantum version of (4) contains plasma wave contributions, which are, in fact, just the right-hand side of Eq. (8). On the other hand, as Drummond and one of the authors have shown,[8] one obtains precisely the induced emission and absorption part of these plasma wave contributions by means of a perturbation-theoretic treatment of the higher order contributions to the nonlinear term on the left-hand side of (2), i.e., those beyond the usual linearized version. Thus, it might appear that the same class of terms are included twice in (2): once in the nonlinear terms associated with the left-hand side; once in the plasma wave contribution to the right-hand side.

The resolution of this paradox is comparatively simple. In both cases one is dealing with fluctuations in the electric field associated with the plasma waves. One must be careful not to count these fluctuations twice. One can either describe these effects as fluctuations in the electric field which appears on the left side of Eq. (2), in which case one should eliminate them from the right-hand side and keep only the single-particle effects or close collisions in the collision integral (4). Or one can leave the plasma wave fluctuations in the collision integral as in the treatment above, in which case $\mathbf{E}(\mathbf{x},t)$ is to be regarded as an averaged self-consistent field without fluctuations.

[8] W. E. Drummond and D. Pines, Proceedings of the Salzburg Conference on Plasma Physics, Nuclear Fusion (to be published).