Spin-Orbit Splittings in Nuclear Shell Model*

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All available data (mostly from "stripping reactions") on locations of nuclear shell-model states are analyzed to determine energy splittings of spin-orbit doublets, and the systematics is studied. The expected $(2l+1)$ dependence of the splitting is roughly corroborated, but the splitting is considerably larger than expected when the two levels are in different major shells. The mass dependence of the splitting is somewhat stronger than the expected proportionality to $A^{-\frac{1}{3}}$. Applications to "reduced mass" questions, and to the location of unknown levels are discussed.

INTRODUCTION

THE energy splitting of spin-orbit doublets—the energy difference between two shell-model states with the same principal and orbital angular momentum quantum numbers but with total angular momentum $j=l+1/2$ and $l-1/2$, respectively—is of central importance in nuclear shell model. The magnitudes of this splitting, in a few cases, have been known for many years, and several attempts have been made to explain them theoretically from the nucleon-nucleon force.[1]

In recent months, a considerable amount of additional experimental data on spin-orbit splittings have been obtained.[2–7] It is the purpose of this paper to summarize the experimental determinations and to consider what information may be derived from the empirical systematics.

METHOD AND RESULTS

The low-lying energy levels of nuclei with closed shell plus (or minus) one particle may be straightforwardly interpreted as the shell-model levels. Thus, if both members of the spin-orbit doublet are in the same major shell, the splitting may be determined directly from the level spectra of these nuclei. The results for cases of this type are listed in Table I; they may be distinguished as entries in that table for which there is no correction in column (3). (This method is used for the $f_{7/2}$-$f_{5/2}$ splitting in Ca^{41}, even though these levels are in different major shells, as both are completely empty.)

TABLE I. Spin-orbit splitting in various nuclei. All energies are in MeV.

(1) Mass	(2) l	(3) Pairing correction	(4) ΔE	(5) $\frac{\Delta E}{2l+1}$	(6) $\frac{\Delta E}{2l+1}\cdot A^{1/3}$	(7) Reference
207	1	···	0.90	0.30	1.77	8
	2	···	0.96	0.192	1.14	2
	3	···	1.78	0.255	1.51	8
	4	···	2.47	0.275	1.62	2
	6	0.4	5.7	0.44	2.6	2
137, 141	5	0.6	~5.0 (?)	~0.46	~2.4 (?)	3
		0.8	~5.0 (?)	~0.46	~2.4 (?)	3
	3	···	1.88	0.27	1.4	3
	1	···	1.42	0.47	2.4	3
91	2	···	2.9	0.58	2.6	4
	4	0.9	6.8	0.76	3.4	4, 9
59		···	1.7	0.57	2.2	5
49	1	···	2.03	0.68	2.5	6
41			2.1	0.70	2.4	8
41	3	···	6.5	0.93	3.2	7
17	2	···	5.08	1.02	2.60	8

For cases where the two substates of the spin-orbit pair are in different major shells, the problem is more complex. The cases in point are shown in Fig. 1. For Pb^{207}, the $i_{13/2}$ hole state is well known.[8] The position of the $i_{11/2}$ particle state is known in Pb^{209}, and by

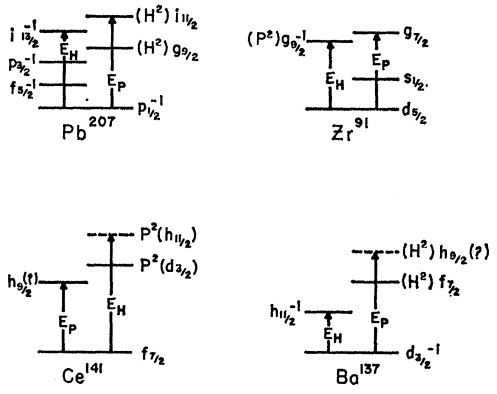

FIG. 1. Level structure in nuclei where member of spin-orbit doublet are in different major shells. See discussion in text.

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[8] *Nuclear Data Sheets*, National Academy of Sciences (National Research Council, Washington, D. C., 1960).

comparing the spectra from $Pb^{206}(d,p)$ and $Pb^{208}(d,p)$, the analog levels in Pb^{207} are clearly indicated.[2] These levels have the configuration $(H^2)\ i_{11/2}$, where H^2 is the configuration from a pair of holes, essentially the ground state configuration in Pb^{206}. The $i_{13/2}$-$i_{11/2}$ splitting is then

$$\Delta E = E_H + E_P + \epsilon(H^2), \tag{1}$$

where E_H and E_P are the energies of the hole and particle states as shown in Fig. 1, and $\epsilon(H^2)$ is the pairing energy of the configuration H^2. This pairing energy has been calculated by True and Ford,[9] and is listed in the third column of Table I.

The mirror situation occurs in Zr^{91}. The location of the $g_{7/2}$ particle state in Z^{91} has been estimated from measurements of the $Zr^{90}(d,p)$ reaction[4]; it is at an excitation energy of about 2.8 MeV. The location of the $(P^2)\ g_{9/2}^{-1}$ level in Zr^{91} (where P^2 is the configuration from a pair of particles, essentially the ground-state configuration of Zr^{92}) may be determined from the $Zr^{92}(p,d)$ reaction[10]; it is the analog of the ground state transition in the $Zr^{90}(p,d)$ reaction. The $g_{9/2}$-$g_{7/2}$ splitting is then

$$\Delta E = E_H + E_P + \epsilon(P^2). \tag{2}$$

The pairing energy $\epsilon(P^2)$ may be crudely estimated[11] as the negative of the lowest eigenvalue of the matrix

$$\begin{pmatrix} 2\epsilon_1 - G(j_1+1/2) & G[(j_1+1/2)(j_2+1/2)]^{1/2} & G[(j_1+1/2)(j_3+1/2)]^{1/2} & \cdots \\ G[(j_2+1/2)(j_1+1/2)]^{1/2} & 2\epsilon_2 - G(j_2+1/2) & G[(j_2+1/2)(j_3+1/2)]^{1/2} & \cdots \\ G[(j_3+1/2)(j_1+1/2)]^{1/2} & G[(j_3+1/2)(j_2+1/2)]^{1/2} & 2\epsilon_3 - G(j_3+1/2) & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix}, \tag{3}$$

where ϵ_1, ϵ_2, ϵ_3, . . . are the energies of the single-particle states available to the particle pair (ϵ_1 is taken as the zero of energy), j_1, j_2, j_3, . . . are the total angular momenta of these states, and G is the strength of the pairing interaction which was taken from Kisslinger and Sorenson.[12] For the particular case considered here, the ground-state configuration of Zr^{92} is known to be almost pure $(d_{5/2})$,[2] so that mixing with the other levels may be ignored and $\epsilon(P^2) = 3G$. Essentially the same result is obtained by the lowest eigenvalue of (3).

The situation in Ba^{137} and Ce^{141} is not as clear since the $h_{9/2}$ state is not definitely known. The center of gravity of this state has been tentatively estimated to be at about[3] 1.9-MeV excitation in Ce^{141}. The $(P^2)\ d_{3/2}^{-1}$ level in Ce^{141} was identified from the $Ce^{142}(d,t)$ reaction[3]; it is the analog of the ground state transition in $Ce^{140}(d,t)$. The $d_{3/2}$-$h_{11/2}$ energy difference in this mass region has been widely studied,[8] so that it can be accurately estimated for Ce^{141} from the systematics. The minimum spin-orbit splitting can thus be determined from Eq. (2) with $\epsilon(P^2)$ calculated from the lowest eigenvalue of (3).

The minimum spin-orbit splitting in this case can also be calculated from the Ba^{137} spectrum if we assume that the $f_{7/2}$-$h_{9/2}$ energy difference is the same as in Ce^{141}. Here, the energies of $h_{11/2}^{-1}$ and $(H^2)\ f_{7/2}$ are known—the latter is found[3] from $Ba^{136}(d,p)$. The pairing correction is obtained from Eq. (1) by finding the lowest eigenvalue of (3). The results indicate complete agreement between these two methods of determining the $h_{11/2}$-$h_{9/2}$ splitting.

All results are shown in column (4) of Table I. The pairing corrections, listed in column (3), are small enough so that their uncertainties are not important sources of error.

DISCUSSION

It is well known[13] that an $\mathbf{l}\cdot\mathbf{s}$ force gives a spin-orbit splitting proportional to $(2l+1)$; to test this, the empirical splittings divided by $(2l+1)$ are shown in column (5) of Table I. The clearest discrepancy from the $(2l+1)$ dependence is that the splittings are much larger than expected in cases where the two substates are in different major shells. This pattern is consistent for the $i_{13/2}$-$i_{11/2}$ in Pb, the $g_{9/2}$-$g_{7/2}$ in Zr, and the $f_{7/2}$-$f_{5/2}$ in Ca; the $h_{11/2}$-$h_{9/2}$ result in Ba and Ce is not inconsistent. A simple interpretation of this observation is that there is an interaction among the levels of a single major shell which pulls them closer together, and further from levels in other shells. This interpretation would also help to explain the rather small extent of the two major shells near Pb^{208} relative to the energy gap between them.[2] The extents of the 83–126 and 127–184 shell are 3.45 and 2.52 MeV, respectively, as compared to a 3.6-MeV gap between them.[14]

In simple calculations, the spin-orbit splitting is inversely proportional to the radius of the shell model potential,[15] so that one expects an inverse proportionality to $A^{1/3}$. To check this, column (6) of Table I shows the values of column (5) multiplied by $A^{1/3}$. One

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[11] This method was suggested to the authors by R. A. Sorenson.

[12] L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **32**, No. 9 (1960).

[13] *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 969.

[14] This includes an 0.4-MeV correction for pairing energy as discussed in connection with Eq. (1).

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sees that the results decrease monotonically with increasing A, so that if one assumes an A^{-n} dependence, n must be greater than 1/3; the data of Table I indicate that $n=0.53\pm0.09$. It is interesting to note here that Nilsson's calculations assume that the numbers in column (6) of Table I should all be 2.05; this is a reasonable average, but it is certainly a gross oversimplification of the actual situation.

There has been much interest lately in the variation of "effective mass," m^*, of nucleons with binding energy.[16] The variation of spin-orbit splittings (after correction for l-dependence) may perhaps be considered as such an effect. The fact that the splitting of the d states is somewhat smaller than that of the p and f states in Pb^{207} would then indicate that m^* for a neutron is somewhat larger at 1.7-MeV binding energy (d-states) than at 7.0-MeV (p-states) or 5.8-MeV (f-states) binding energy in Pb.[17]

An interesting application of Table I is to use the systematics implied therein to predict locations of unknown or doubtful levels. For example, in Ni^{59} one expects the $d_{5/2}$-$d_{3/2}$ splitting to be about 3.0 MeV. This is somewhat larger than assumed in reference 17, but somewhat smaller than assumed in reference 5.

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Resonance Capture γ Rays from Platinum*

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The high-energy gamma rays following neutron capture in a platinum target have been studied at the Brookhaven-AECL fast chopper facility at Chalk River. The transitions to the 0^+ ground state and the first two excited 2^+ states in Pt^{196} have been measured for 15 resonances from 11.9 to 296 eV. Relative transition probabilities for these cases have been determined and are compared to the chi-squared class of probability distribution functions.

INTRODUCTION

FOR the past several years considerable interest has been growing in the measurement of γ rays accompanying the de-excitation of the states formed by the capture of slow and intermediate energy neutrons.[1] The utilization of pulsed sources and time-of-flight techniques permits a study of the radiation from individual resonances and represents an advance over the thermal capture γ-ray work, where a mixture of capturing states is usually involved. The resonance measurements have been directed toward the determination of quantum numbers of the capturing state[2-5] (principally the angular momentum J), the isotopic identification of resonances,[2,4,6,7] and the study of the size and distribution in size of the widths for de-excitation to the various final states in the product nucleus.[3,4,6,8-13] The present paper is concerned with the latter topic.

Early work in this field showed that capture γ-ray spectra from the various resonances of the same spin state display marked differences.[9] However, initial experiments on the partial radiation widths to states near the ground state indicated little variation from resonance to resonance.[8] In this early work it was not always possible to separate adjacent γ-ray lines. Theoretical considerations indicate that transitions which feed

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