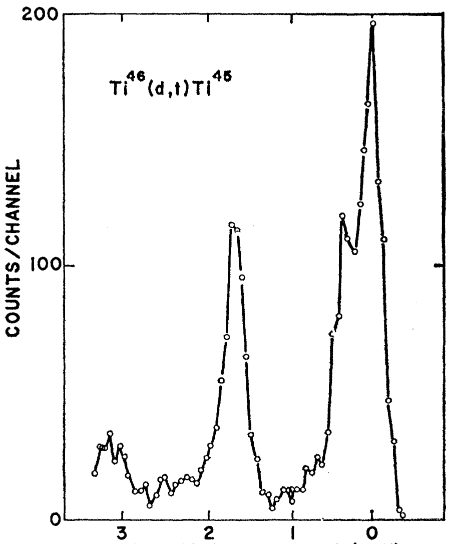

FIG. 6. The spectrum of the $Ti^{46}(d,t)Ti^{45}$ reaction at 21° lab. The spectrum has been corrected for the contributions from the Ti^{47} and Ti^{48} contained in the Ti^{46} target.

Ti^{47}. It appears more likely that the strong peak in Ti^{48} is due to more than one state, the 6^+ state making only a relatively small contribution. In both spectra the strongest group has a Q value which is approximately the same as the Q value for the ground-state (d,t) reaction on the final nucleus. Similar strong groups have been observed in other odd-neutron targets in this region of the periodic table.

Each of the three even-even targets showed two strong $l=3$ transitions separated by about 2 MeV. In the case of Ti^{46} and Ti^{50} it had been anticipated that nearly the entire strength of the $f_{7/2}$ pickup would be found in the ground-state transitions. While core excitation had been observed earlier in nuclei with 28 and 30 protons, it had not been observed in the case of 26 particles. Since an $l=2$ transition is observed in the $Ti^{48}(d,t)Ti^{47}$ reaction to a fairly low-lying state and since there is a strong suspicion for the presence of an $l=2$ transition close to the ground-state transition in the $Ti^{46}(d,t)Ti^{45}$ reaction, it is plausible that calculations assuming an inert core of Ca^{40} and an active $(1f_{7/2})^n$ configuration will yield only approximate agreement with the experimental results.

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Effect of Hard Core on the Photodisintegration Cross Sections of H^3

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The electric-dipole bremsstrahlung weighted cross section (σ_b) and the integrated cross section (σ_{int}) for the photodisintegration of H^3 are calculated using the hard-core wave functions of Kikuta, Morita, and Yamada. A comparison with already published calculations indicates that the introduction of the hard core increases both σ_b and σ_{int} for the H^3 nucleus by about 100 and 8%, respectively.

IN their classic paper Levinger and Bethe[1] derived expressions for the electric-dipole bremsstrahlung weighted cross section $[\sigma_b=\int_0^\infty(\sigma/W)dW]$ and the integrated cross section $[\sigma_{\text{int}}=\int_0^\infty \sigma dW]$ for the nuclear photoeffect on the basis of the generalized Thomas-Reiche-Kuhn sum-rule, using a partially attractive exchange potential of Majorana type. Rustgi and Levinger[2] extended the sum-rule calculations of Levinger and Bethe[1] to include the two-body Heisenberg forces as well. The original expression for the bremsstrahlung weighted cross section for the electric-dipole absorption as obtained by Levinger and Bethe[1] was put in a slightly modified form by Foldy[3] to read as

$$\sigma_b=(4\pi^2/3)(e^2/\hbar c)[NZ/(A-1)]R_c^2, \tag{1}$$

where R_c is the charge root-mean-square radius of the nucleus involved and is given by

$$R_c^2=(1/Z)[\{\textstyle\sum_i(\mathbf{r}_i-\mathbf{R})^2\}]_{00}, \tag{2}$$

where i stands for the proton and $\mathbf{R}$ is the coordinate of the center of mass of the nucleus. Rustgi[4] used the work of Levinger and Bethe[1] to calculate σ_b and σ_{int} for H^3 and He^3 nuclei using a two-body spin-dependent Yukawa potential and the Irving[5] wave function.

The purpose of the present note is to calculate σ_b and σ_{int} for H^3 using the two-body spin-dependent forces of exponential type with hard core. The effect of the hard core on the binding energy of He^3 and H^3

[1] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[2] M. L. Rustgi and J. S. Levinger, Phys. Rev. **106**, 530 (1957).
[3] L. L. Foldy, Phys. Rev. **107**, 1303 (1957).

has earlier been studied by Kikuta *et al.*[6] and they have found that the hard core gives rise to reasonable values for the binding energy of H^3 and the Coulomb energy of He^3. We find that the hard core increases σ_b by a factor of 2 and σ_{int} by 8% over the corresponding values obtained by using central potentials without hard core.[4] The results obtained in this paper should be applicable to He^3, if one assumes charge symmetry and ignores Coulomb repulsion between the protons.

Kikuta *et al.*[6] assumed a two-body central force expressed by the potential

$$V(r_{ij})=\tfrac{1}{4}(3+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)V_t(r_{ij})+\tfrac{1}{4}(1-\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)V_s(r_{ij}), \quad (3)$$

which is charge independent, and where $V_s(r_{ij})$ and $V_t(r_{ij})$ are potentials for spin-singlet and spin-triplet states, respectively. For even states, V_t and V_s are given by

$$\begin{aligned} V_t^{\text{even}}(r_{ij}) &= -A_t\exp[-\alpha_t(r_{ij}-D)] && \text{for } r_{ij}\geqslant D \\ &=\infty && \text{for } r_{ij}\leqslant D, \end{aligned} \quad (4)$$

$$\begin{aligned} V_s^{\text{even}}(r_{ij}) &= -A_s\exp[-\alpha_s(r_{ij}-D)] && \text{for } r_{ij}\geqslant D \\ &=\infty && \text{for } r_{ij}\leqslant D. \end{aligned} \quad (5)$$

The ground-state wavefunction must be antisymmetric between two like nucleons and can be written as

$$\psi=\chi_a\psi_s+\chi_s\psi_a, \quad (6)$$

where

$$\chi_a=\frac{1}{\sqrt{2}}[\alpha(1)\beta(2)-\alpha(2)\beta(1)]\alpha(3), \quad (7)$$

and

$$\chi_s=[\alpha(1)\beta(2)+\alpha(2)\beta(1)]\frac{\alpha(3)}{\sqrt{6}}-\frac{2}{\sqrt{3}}\alpha(1)\alpha(2)\beta(3), \quad (8)$$

χ_a and χ_s being the antisymmetric and symmetric spin wavefunctions, respectively, and ψ_s and ψ_a are the space wavefunctions which are symmetric and antisymmetric, respectively, with regard to two like nucleons. Using according to Kikuta *et al.*[6] the approximation $\psi_a=0$, the wavefunction ψ becomes

$$\psi=\chi_a\psi_s, \quad (9)$$

where ψ_s, the space part of the wavefunction according to Kikuta *et al.*,[6] is of the exponential type:

$$\begin{aligned} \psi_s &= \prod_{ij}\{\exp[-\mu(r_{ij}-D)]-\exp[-\nu(r_{ij}-D)]\} && \text{for } r_{ij}\geqslant D, \\ &=0 \quad \text{for } r_{ij}\leqslant D. \end{aligned} \quad (10)$$

Using Eq. (2) we get, for the $E1$ bremsstrahlung weighted cross section for H^3,

$$\sigma_b=\frac{4\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\int\psi^*\{(1/9)[2(r_{23}^2+r_{13}^2)-r_{12}^2]\}\psi d\tau, \quad (11)$$

where 1 and 2 denote the neutrons and 3 denotes the proton in the H^3 nucleus. Using the wave function (10) and carrying out the integration following Kikuta *et al.*,[6] one gets

$$\sigma_b=\frac{4\pi^2}{3}\frac{e^2}{\hbar c}\left\{\frac{1}{9N}[3F_3(0)F_1^2(0)-P(2\mu)+2P(\mu+\nu)-P(2\nu)]\right\}, \quad (12)$$

where

$$F_n(u)=D^nA_1(u)+nD^{n-1}A_2(u)+\cdots[n(n-1)\cdots\times2\times1]A_{n+1}(u), \quad (13)$$

$$A_n(u)=\frac{1}{(u+2\mu)^n}-\frac{2}{(u+\mu+\nu)^n}+\frac{1}{(u+2\nu)^n}, \quad (14)$$

and

$$\begin{aligned} P(u)=\exp(-uD)[&(18/u^4)F_1^2(u)+(36/u^3)F_2(u)F_1(u)\\ &+(18/u^2)F_2^2(u)+(28/u^2)F_3(u)F_1(u)\\ &+(24/u)F_2(u)F_3(u)+(12/u)F_1(u)F_4(u)]. \end{aligned} \quad (15)$$

Here N is the normalization constant,[7] given by

$$N=F_1^3(0)-N'(2\mu)+2N'(\mu+\nu)-N'(2\nu), \quad (16)$$

where

$$N'(u)=\frac{3\exp(-uD)}{u}[(1/u)F_1^2(u)+2F_2(u)F_1(u)]. \quad (17)$$

The constants used for the numerical estimate of σ_b are $\mu=0.4\times10^{13}$ cm^{-1}, $\nu=4.5\times10^{13}$ cm^{-1}, and the hard-core radius $D=0.4\times10^{-13}$ cm, which fit the binding energy[6] of H^3 and He^3. One gets, for H^3,

$$\sigma_b=2.72 \text{ mb}. \quad (18)$$

This value of σ_b for H^3 is large compared to Rustgi's[4] value of 1.32 mb because of his small value for the rms radius of H^3 (1.17 F) compared to the large size of H^3 (1.59 F) given by the hard-core wavefunction on assuming point neutron and protons. It is satisfactory to note that the rms size of H^3, obtained from the hard-core wavefunction, lies, unlike the value[4] given by Irving's[5] wavefunction, in between the rms sizes of D^2 (1.96 F) and He^4 (1.40 F) as obtained from electron scattering[8] and photodisintegration experiments.[2,9] Following Rustgi and Levinger,[2] one has for σ_{int} for H^3

$$\sigma_{\text{int}}=\frac{4\pi^2e^2\hbar}{3Mc}\left[1-\frac{M(x+y/2)}{2\hbar^2}\times\int\psi^*\sum_i\sum_j V(r_{ij})r_{ij}^2P_{ij}^M\psi d\tau\right], \quad (19)$$

where i denotes proton and j neutron, the double summation is taken over all pairs of protons and neutrons, x and y are the fractions of Majorana and Heisenberg type of exchange forces and $V(r_{ij})$ is the neutron-proton potential as given by Eqs. (4) and (5).

[4] M. L. Rustgi, Phys. Rev. **106**, 1256 (1957).

[5] J. Irving, Phil. Mag. **42**, 338 (1951).

[6] T. Kikuta, M. Morita, and M. Yamada, Progr. Theoret. Phys. (Kyoto) **15**, 222 (1956).

[7] While the analytical results are the same, it has been noted that there is a factor of 1.55×10^{76} by which the N used in this work differs from that of Kikuta *et al.* (reference 6).

[8] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

[9] J. S. Levinger and M. L. Rustgi, Phys. Rev. **106**, 607 (1957).

The integral on the right-hand side is equal to

$$\int \psi^*[V(r_{23})r_{23}^2+V(r_{13})r_{13}^2]d\tau$$

$$=\frac{1}{N}\{3A_t[K_t(2\mu)+K_t(2\nu)-2K_t(\mu+\nu)$$
$$-\tfrac{1}{2}F_1^2(0)F_3(\alpha_t)]+A_s[K_s(2\mu)+K_s(2\nu)$$
$$-2K_s(\mu+\nu)-\tfrac{1}{2}F_1^2(0)F_3(\alpha_s)]\}, \quad (20)$$

where

$$K_f(u)=\exp(-uD)[(1/u^2)F_3(u+\alpha_f)F_1(u)+(1/u)$$
$$\times F_2(u)F_3(u+\alpha_f)+(1/u)F_1(u)F_4(u+\alpha_f)]$$
$$+\tfrac{1}{2}\exp\{-(u+\alpha_f)D\}\Bigg[\frac{6}{(u+\alpha_f)^4}F_1^2(u+\alpha_f)$$
$$+\frac{12}{(u+\alpha_f)^3}F_2(u+\alpha_f)F_1(u+\alpha_f)+\frac{6}{(u+\alpha_f)^2}$$
$$\times F_3(u+\alpha_f)F_1(u+\alpha_f)+\frac{6}{(u+\alpha_f)^2}F_2^2(u+\alpha_f)$$
$$+\frac{6}{(u+\alpha_f)}F_3(u+\alpha_f)F_2(u+\alpha_f)$$
$$+\frac{2}{(u+\alpha_f)}F_4(u+\alpha_f)F_1(u+\alpha_f)\Bigg], \quad (21)$$

and

$$f=s,t.$$

For the numerical computation, the following parameters were chosen:

$$A_t=475.044 \text{ MeV}, \quad \alpha_t=2.52\times10^{13} \text{ cm}^{-1},$$
$$A_s=235.41 \text{ MeV}, \quad \alpha_s=2.034\times10^{13} \text{ cm}^{-1}, \quad (23)$$

in accordance with Kikuta *et al.*,[6] so as to fit the low-energy data of the two-body system. Using these values, one gets for σ_{int}

$$\sigma_{\text{int}}=\frac{4\pi^2e^2\hbar}{3Mc}[1+0.72(x+\tfrac{1}{2}y)]$$
$$=40[1+0.72(x+\tfrac{1}{2}y)] \text{ MeV mb}. \quad (24)$$

The coefficient of $x+\frac{1}{2}y$ is, thus, 0.72 instead of 0.55 as obtained by Rustgi. It is, thus, observed that the hard core increases the value of σ_{int} by about 8%, which is in agreement with the prediction of Levinger[10] and Okamoto.[11] It is, however, not possible to compare σ_b and σ_{int} with the experiments due to the unavailability of experimental data on the photodisintegration of, or electron scattering from, H^3.

[10] J. S. Levinger, Phys. Rev. **97**, 112 (1955).
[11] K. Okamoto, Phys. Rev. **116**, 428 (1959).

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Study of the $Sm^{149}(n,\alpha)Nd^{146}$ Reaction with Thermal Neutrons*

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The cross section for the $Sm^{149}(n,\alpha)Nd^{146}$ reaction at thermal neutron energies was measured by observing the alpha-particle spectrum of natural Sm and isotopically enriched Sm^{149} targets in the thermal column of a reactor. Alpha-particle groups were observed at 8.72 and 9.12 MeV corresponding to transitions to the first excited state and ground state of Nd^{146}. The 2200-m/sec cross sections (σ_0) for the two groups were calculated to be 121 ± 15 mb and 22 ± 10 mb, respectively, from the experimental results. The 4− resonance at 0.0967 eV accounts for most of the cross section to the first excited state, but cannot contribute to the population of the ground state. It is postulated that the population of the ground state arises from a contribution of a bound 3− state in Sm^{150}. Calculated values for the (n,α) cross section are compared with the experimental results.

I. INTRODUCTION

ALTHOUGH the (n,α) reaction at thermal neutron energies is energetically possible for several nuclides throughout the periodic table, the probability for alpha-particle emission from the capturing state of the compound nucleus is usually very small. Only for those reactions in which the energy available for α-particle emission is comparable with the Coulomb-barrier height can this mode of decay compete favorably with γ-ray emission. The thermal neutron (n,α) cross sections which have been reported are for nuclides with $Z\leqslant30$.[1]

In the region of the rare-earth elements, there are a

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[1] D. J. Hughes and R. B. Schwartz, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (U. S. Government Printing Office, Washington, D. C., 1958), 2nd ed.