

**(*d,t*) Reaction on the Titanium Isotopes\***

J. L. YNTEMA

*Argonne National Laboratory, Argonne, Illinois*

(Received April 30, 1962)

The (*d,t*) reaction on the five stable isotopes of titanium has been studied at a deuteron energy of 21.4 MeV. The three even isotopes show two strong groups separated by about 2 MeV, both groups corresponding to pickup of a  $1f_{7/2}$  neutron. This indicates that there is considerable seniority mixing in the titanium isotopes. Groups corresponding to the pickup of  $2p$  neutrons were observed in both  $Ti^{50}$  and  $Ti^{48}$ . In the reaction on  $Ti^{48}$ , an  $l=2$  transition to a state near 1.8 MeV in  $Ti^{47}$  was found. The odd-even isotopes show a number of strong groups. The ground-state transitions in  $Ti^{48}(d,t)Ti^{47}$  and  $Ti^{47}(d,t)Ti^{46}$  have not been observed.

**I. INTRODUCTION**

ANGULAR distributions and absolute cross sections have been measured for the triton groups observed in the (*d,t*) reaction on enriched metallic targets of  $Ti^{46}$ ,  $Ti^{47}$ ,  $Ti^{48}$ ,  $Ti^{49}$ , and  $Ti^{50}$  at a deuteron energy of 21.4 MeV. The results are compared with pickup reactions on other targets in this region of the periodic table and with the results from (*d,p*) and (*d,d*) reactions on the titanium isotopes.

**II. EXPERIMENTAL PROCEDURE**

The experiment was performed with the 21.4-MeV deuteron beam of the 60-in. cyclotron bombarding targets in the Argonne 60-in. scattering chamber.<sup>1</sup> The experimental techniques have been described elsewhere. In the present experiment the detector system was different from the one used previously. The NaI crystal which was used as the  $dE/dx$  detector was placed at a 45° angle in front of the photomultiplier which views it through an air light pipe. The improved light collection permitted a reduction in the thickness of the crystal without loss of resolution in the particle-identification system. The new arrangement permits the use of either a NaI crystal or a silicon detector as the *E* detector and extends the energy range of the triton spectra to lower energies.

The target material was obtained as enriched  $TiO_2$  from Oak Ridge National Laboratory. It was reduced by means of the iodide process<sup>2</sup> to titanium metal and was subsequently rolled to a thickness of approximately 1 mg/cm<sup>2</sup>.

The isotopic composition of the targets, determined by mass spectroscopic methods, was as given in Table I.

Since a possibility of calcium contamination was introduced in the reduction process, the spectrum of the  $Ca^{40}(d,t)Ca^{39}$  reaction as well as its angular distribution was obtained. Furthermore, it is possible that some Ta may have diffused from the Ta core wire on which the Ti metal was deposited. In the present experiment the effects of Ta impurities are completely negligible, if present at all.

The energy calibration was obtained from the range-energy relation as well as from the *Q* values of known reactions. In general, the calibration is accurate to about 100 keV over the entire range. The relative cross sections are reliable within the statistical errors except for those peaks where an appreciable contribution from other isotopes of Ti had to be subtracted. The absolute cross sections of the main peaks are reliable within less than 10%.

Energy spectra were obtained in three-degree intervals from 12° to 36°. This angular range is sufficient to identify  $l=1$ ,  $l=2$ , and  $l=3$  transitions.

**III. EXPERIMENTAL RESULTS**

The results are tabulated in Table II. The table gives the energies of the groups with either  $l=3$  or  $l=1$  transitions, together with their absolute differential cross sections measured at 21° for  $l=3$  transitions and at 27° (the approximate location of the second maximum) for  $l=1$  transitions. The last column lists the number of  $f_{7/2}$  neutrons involved in the transitions. This number was obtained by assuming that the combined cross sections for the two  $l=3$  transitions observed in  $Ti^{50}$  correspond to 7.2  $f_{7/2}$  neutrons. This value was chosen as a reasonable estimate of the

TABLE I. Composition of titanium isotopes (in mole percent).

Target	% $Ti^{46}$	% $Ti^{47}$	% $Ti^{48}$	% $Ti^{49}$	% $Ti^{50}$
$Ti^{46}$	83.8 ± 0.1	5.0 ± 0.1	9.76 ± 0.08	0.73 ± 0.03	0.73 ± 0.02
$Ti^{47}$	1.68 ± 0.03	83.05 ± 0.08	13.70 ± 0.07	0.798 ± 0.008	0.76 ± 0.02
$Ti^{48}$	0.161 ± 0.002	0.238 ± 0.004	99.08 ± 0.01	0.227 ± 0.006	0.295 ± 0.003
$Ti^{49}$	1.07 ± 0.02	1.27 ± 0.05	15.66 ± 0.06	77.2 ± 0.2	4.7 ± 0.2
$Ti^{50}$	1.50 ± 0.02	1.32 ± 0.02	12.2 ± 0.1	3.58 ± 0.04	81.5 ± 0.1

\* Work performed under the auspices of the U. S. Atomic Energy Commission.

<sup>1</sup> J. L. Yntema and H. W. Ostrander, Nuclear Instr. and Methods (to be published).<sup>2</sup> This work was done at Battelle Memorial Institute, Columbus, Ohio.

TABLE II. Cross sections for strong groups.

Final nucleus	Exc. energy (MeV)	$d\sigma/d\omega$ at 21° c.m. (mb/sr)	$d\sigma/d\omega$ at 27° c.m. (mb/sr)	Number of particles
Ti <sup>45</sup>	0	2.58		1.83
	0.37			
	1.74			
	3.1			
Ti <sup>46</sup>	0.88	0.95		0.67
	2.07	1.25		0.89
	3.30	0.17		0.12
	3.94	1.94		1.38
	4.40	0.85		0.60
	5.84	0.60		0.43
Ti <sup>47</sup>	0.16	3.22		2.26
	1.6		0.39	
	2.47	2.08		1.48
Ti <sup>48</sup>	0	0.44		0.31
	0.98	1.20		0.85
	2.33	0.85		0.60
	3.30	3.27		2.32
	4.60	1.85		1.31
	4.90			
	5.60	0.72		0.51
	6.20	0.81		0.58
Ti <sup>49</sup>	0	5.91		4.20
	1.45		0.47	
	2.52	4.26		3.02

number of particles when account was taken of the admixture of  $2p$  neutrons and the probability that some of the cross sections would proceed to the higher isobaric-spin states<sup>3</sup> which could not be observed in the present experiment. The normalization process used does not take account of the dependence of reduced width on  $Q$ , nor does it take account of variations of the optical-model potentials between isotopes. Preliminary results on the elastic deuteron scattering indicate that there are differences between the diffraction patterns for different isotopes and that these show a systematic trend with neutron excess and are not explained by a change in the interaction radius. The number of particles given in Table II should, therefore, be treated as a rather tentative quantity.

### A. Ti<sup>50</sup>( $d,t$ )Ti<sup>49</sup> Reaction

The spectrum at 21° lab is shown in Fig. 1. The strong peaks to the ground state and to the states near 2.5 MeV have an  $l=3$  angular distribution, while the group proceeding to states near 1.5 MeV has an  $l=1$  angular distribution. Since Ti<sup>50</sup> has 28 neutrons, the ground-state configuration is expected to be mainly a closed  $f_{7/2}$  neutron shell. From the presence of the  $l=1$  transition to states near 1.5 MeV, it follows that there is some  $2p$  neutron admixture in the ground state. A similar admixture of  $2p$  protons in the ground-state wavefunction has been observed in the Ni<sup>58</sup>( $d,He^3$ )Co<sup>57</sup>

<sup>3</sup> J. B. French and M. H. Macfarlane, Nuclear Phys. **26**, 168 (1961).

reaction<sup>4</sup> which should have a closed  $1f_{7/2}$  proton shell. However, strong  $l=1$  transitions have not been observed in other nuclei with 28 neutrons.<sup>5-8</sup>

The neutron ground-state configuration of Ti<sup>49</sup> is expected to be mainly  $(1f_{7/2})^{-1}$ . Thus, if one assumes that the neutrons and protons do not interact, or alternatively that the neutron-proton eigenfunctions for particles in the  $1f_{7/2}$  shell have well-defined seniority, the Ti<sup>50</sup>( $d,t$ )Ti<sup>49</sup> and Ti<sup>48</sup>( $d,p$ )Ti<sup>49</sup> reaction should proceed only to the ground state of Ti<sup>49</sup>. On the basis of this assumption, Schiffer *et al.*,<sup>9</sup> in their ( $d,p$ ) experiment on natural titanium, identified the  $l=3$  transition observed at 2.5 MeV in the gross-structure spectrum as the  $f_{5/2}$  single-particle state. If this assignment were correct, Ti<sup>50</sup> would contain a very large admixture of  $1f_{5/2}$  neutrons in its ground-state configuration. Such

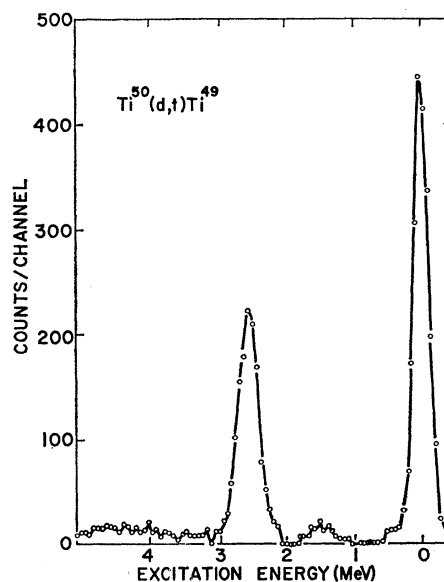


Fig. 1. The spectrum of Ti<sup>50</sup>( $d,t$ )Ti<sup>49</sup> at 21° lab. The spectrum has been corrected for the contributions from the Ti<sup>49</sup> and Ti<sup>48</sup> contained in the Ti<sup>50</sup> target.

an admixture would appear to be unlikely in view of the relatively small admixtures of  $1f_{5/2}$  neutrons in targets with 30 and 32 neutrons. Furthermore, the fact that the ratio of the intensity of the 2.5-MeV transition to that of the ground-state transition are the same in Ti<sup>50</sup>( $d,t$ )Ti<sup>49</sup> and in Ti<sup>48</sup>( $d,p$ )Ti<sup>49</sup> strongly

<sup>4</sup> J. L. Yntema, T. H. Braid, B. Zeidman, and H. W. Broek, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company Ltd., London, 1961), p. 521.

<sup>5</sup> B. Zeidman, J. L. Yntema, and B. J. Raz, Phys. Rev. **120**, 1723 (1960).

<sup>6</sup> M. H. Macfarlane, B. J. Raz, J. L. Yntema, and B. Zeidman, Phys. Rev. **127**, 204 (1962).

<sup>7</sup> C. D. Goodman, J. B. Ball, and C. B. Fulmer, Phys. Rev. **127**, 574 (1962).

<sup>8</sup> A. G. Blair and H. E. Wegner (to be published).

<sup>9</sup> J. P. Schiffer, L. L. Lee, Jr., and B. Zeidman, Phys. Rev. **115**, 427 (1959).

indicates that the 2.5-MeV transition involves the transfer of a  $1f_{7/2}$  neutron. This indicates that only about 58% of the ground-state eigenfunction has seniority 1 and that the remaining 42% is distributed between the  $J=7/2$  states at about 2.5-MeV excitation. States with nondefinite seniority have already been used to calculate some properties of nuclei in the  $1f_{7/2}$  shell.<sup>10</sup> Lawson and Zeidman<sup>11</sup> have found that the eigenfunctions used in reference 10 can also explain the transition strengths obtained in the  $(d,t)$  experiment. Similar effects have been observed in other nuclei with 28 neutrons. In the  $\text{Fe}^{54}(\text{He}^3, \alpha)\text{Fe}^{53}$  reaction,<sup>8</sup> three peaks with  $l=3$  angular distributions were observed. Satchler<sup>12</sup> has pointed out that the relative intensities of the groups in this case can be expected to be different

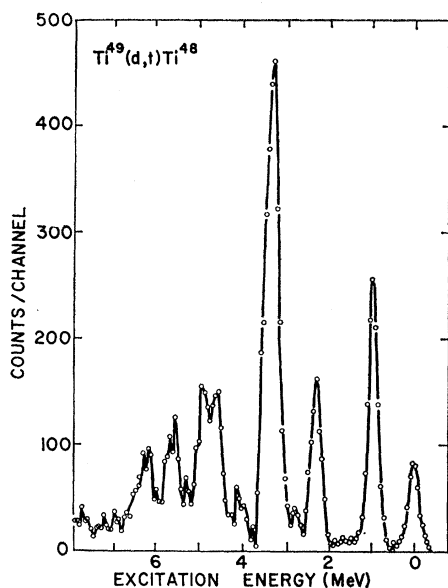


FIG. 2. The spectrum of  $\text{Ti}^{49}(d,t)\text{Ti}^{48}$  at  $21^\circ$  lab. The spectrum has been corrected for the contributions from the  $\text{Ti}^{50}$  and  $\text{Ti}^{48}$  contained in the  $\text{Ti}^{49}$  target.

from those which would be obtained in the  $\text{Fe}^{54}(d,t)\text{Fe}^{53}$  reaction.

A number of states in  $\text{Ti}^{49}$  have been observed in the  $\text{Ti}^{48}(d,p)\text{Ti}^{49}$  reaction in the region in which the  $l=1$  transition is observed in the  $\text{Ti}^{50}(d,t)\text{Ti}^{49}$  reaction. In their  $\text{Ti}^{48}(d,p)\text{Ti}^{49}$  experiment, Rietjens *et al.*<sup>13</sup> observed strong  $l=1$  transitions to states at 1.38 and 1.72 MeV. The spins of these states have been shown to be  $3/2^-$  and  $1/2^-$ , respectively.<sup>14</sup> The resolution in the present experiment is not good enough to separate these two states. However, the intensity of excitation of the 1.72-

MeV state is estimated to be less than 30% of the intensity of the transition to the  $3/2^-$  state.

It is possible to obtain an estimate of the admixture of  $p$ -shell particles if one uses the approach of Macfarlane *et al.*<sup>6</sup> This indicates that the admixture of  $2p$  neutrons is approximately  $0.4 \pm 0.2$  particles.

### B. $\text{Ti}^{49}(d,t)\text{Ti}^{48}$ Reaction

The spectrum from the  $\text{Ti}^{49}(d,t)\text{Ti}^{48}$  reaction at  $21^\circ$  lab is shown in Fig. 2. The angular distributions of all groups show the typical behavior of  $l=3$  transitions. Transitions to the ground state and to the  $2^+$  and  $4^+$  states are observed. The transition to the group near 3.3-MeV excitation occurs at a  $Q$  value of  $-5.3$  MeV while the ground-state  $Q$  value for the  $\text{Ti}^{48}(d,t)\text{Ti}^{47}$  reaction is  $-5.36$  MeV. While the  $6^+$  state in  $\text{Ti}^{48}$  has been found at 3.34 MeV<sup>15</sup> it is not at all clear that the 3.3-MeV group involves transitions to this state only.

### C. $\text{Ti}^{48}(d,t)\text{Ti}^{47}$

The spectrum obtained at  $21^\circ$  lab, is shown in Fig. 3. The transition to the ground state of  $\text{Ti}^{47}$  is considerably weaker than the transition to the 160-keV level if it is present at all. The ground-state spin of  $\text{Ti}^{47}$  is  $5/2^-$  and if the neutron configuration of this state is  $(f_{7/2})^{-3}$  one would not expect to observe the ground-state transition. The group leading to states near 2.4 MeV has an  $l=3$  angular distribution. Transitions to this state were not observed by Rietjens *et al.* in their  $\text{Ti}^{46}(d,p)\text{Ti}^{47}$  experiment. However, an  $l=3$  transition in this energy region would have been completely masked by contributions from the  $\text{Ti}^{48}(d,p)\text{Ti}^{49}$  reaction. The peak near 1.8 MeV shows a shoulder near 1.6 MeV. In the  $\text{Ti}^{46}(d,p)\text{Ti}^{47}$  reaction, strong  $l=1$  groups were observed

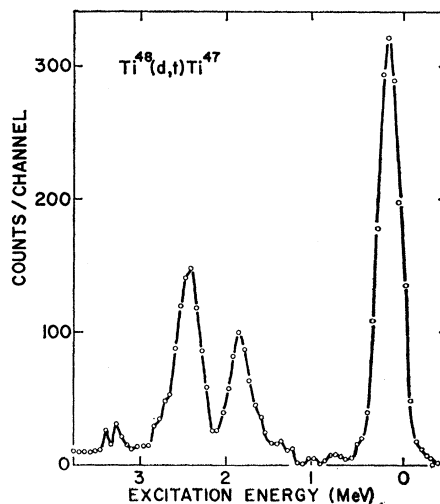


FIG. 3. The spectrum of  $\text{Ti}^{48}(d,t)\text{Ti}^{47}$  at  $21^\circ$  lab.

<sup>10</sup> R. D. Lawson, Phys. Rev. **124**, 1500 (1961).

<sup>11</sup> R. D. Lawson and B. Zeidman (to be published).

<sup>12</sup> R. G. Satchler (private communication).

<sup>13</sup> L. H. Rietjens, O. M. Bilaniuk, and M. H. Macfarlane, Phys. Rev. **120**, 527 (1960).

<sup>14</sup> J. F. Vervier, Nuclear Phys. **26**, 10 (1961); O. Hansen, *ibid.* **251**, 140 (1961).

<sup>15</sup> Nuclear Data Sheets, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C., 1959).

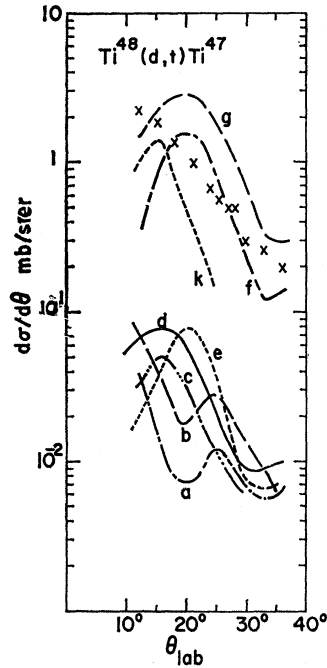


FIG. 4. Angular distributions for the  $Ti^{48}(d,t)Ti^{47}$  reaction. Curves *f* and *g* are the angular distributions (in the laboratory system) of the transitions to the 0.16- and 2.4-MeV states of  $Ti^{47}$ . The crosses represent the angular distribution of the 1.8-MeV group. Curve *a* is the angular distribution from the portion from 1.5 to 1.6 MeV in the 1.8-MeV group, curve *b* is the portion from 1.6 to 1.7 MeV, curve *c* from 1.7 to 1.8 MeV, and curve *d* from 1.8 to about 1.9 MeV. Curve *k* is the angular distribution of the  $Ca^0(d,t)Ca^{39}$  ground-state transition. Curve *e* is a section of the 2.4-MeV group.

to states at 1.56 and 1.80 MeV. In the present experiment the peak shape of the 1.8-MeV group changes with angle. Therefore, various portions of the peaks were analyzed separately. The resulting angular distributions are shown in Fig. 4. Curves *a* and *b* involve the low-energy end of the 1.8-MeV group and show the typical  $l=1$  behavior. Curves *c* and *d* are the curves derived from the high-energy end of the spectrum, while curve *e* is part of the 2.4-MeV group. Curves *f* and *g* are the angular distributions from the transitions to the 0.16- and 2.4-MeV groups, respectively. It is seen that these latter curves have similar angular distributions. This distribution is characteristic of  $l=3$  transitions in this region. Curves *c* and *d* are definitely different. In order to establish the angular distribution for  $l=2$ , the  $Ca^{40}(d,t)Ca^{39}$  ground-state reaction was used. It is shown in curve *k*. From this, it follows that the transition to about 1.6 MeV is  $l=1$  and the one to the 1.8-MeV state has an  $l=2$  angular distribution. It is clear that a weak admixture of  $l=1$  in the 1.8-MeV state cannot be excluded on the basis of the present data. From the experimental data the admixture of  $2p$  neutrons in the ground-state configuration of  $Ti^{48}$  is estimated to be approximately half of the  $2p$  neutron admixture in  $Ti^{50}$ .

#### D. $Ti^{47}(d,t)Ti^{46}$

The spectrum is shown in Fig. 5. As is to be expected from the spins of the ground states, there is no evidence for the ground-state transition. The transitions to the  $2^+$  and the  $4^+$  states of  $Ti^{46}$  are quite strong. The transition to the 2.96-MeV state is, however, very weak if present at all. The determination of the cross section of the transition to this state is made difficult

by the  $Ti^{48}(d,t)Ti^{47}$  transition to the 160-keV state. Subtraction of the  $Ti^{48}$  contribution virtually eliminates the 2.96-MeV contribution. A strong group is observed with an excitation energy of about 3.9 MeV. The  $Q$  value for this group is approximately  $-6.5$  MeV, while the  $Q$  value for the ground-state transition of  $Ti^{46}(d,t)Ti^{45}$  is  $-6.9$  MeV. It appears to be a general rule in this region of the periodic table that the  $(d,t)$  group with the largest cross section from targets with odd neutron number has a  $Q$  value a few hundred kilovolts higher than the  $Q$  value for the ground-state transition of the final even-even nucleus.

#### E. $Ti^{46}(d,t)Ti^{45}$ Reaction

The spectrum at  $21^\circ$  lab is shown in Fig. 6. Here again there are two strong  $l=3$  transitions separated by about 2 MeV. The group near 3-MeV excitation could be due to Ca contamination. From the angular distribution, however, it appears that this is an  $l=3$  transition and that it should, therefore, be assigned to  $Ti^{46}$ . There is a possibility that an  $l=2$  admixture is present to a state at about 300-keV excitation. No groups with an  $l=1$  angular distribution have been observed.

#### IV. CONCLUSIONS

Previous experiments on targets in the  $1f-2p$  shell have been mainly interpreted through the use of sum rules in order to obtain information on the neutron configuration of the ground states. In the present experiment one may expect that a more detailed interpretation will be feasible. There are some noticeable similarities in the results on the titanium isotopes of odd mass. Both show the expected large number of peaks. On the assumption that the 2.96-MeV state in  $Ti^{46}$  is the  $6^+$  state, it is apparent that the transition probabilities to the  $2^+$ ,  $4^+$ , and  $6^+$  states in  $Ti^{46}$  are very different from those in  $Ti^{48}$ —at least if the strong peak near 3.4 MeV in  $Ti^{48}$  were to be identified with the  $6^+$  state at 3.34 MeV. Some difference in the ratios might be expected because of the ground-state spin of

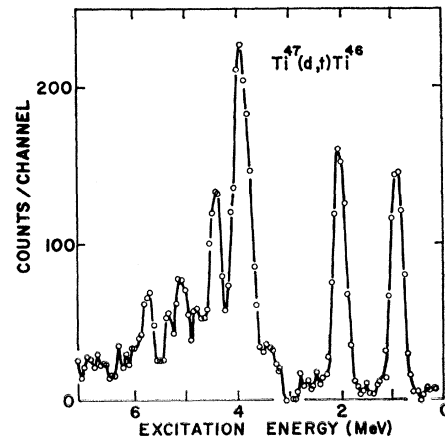


FIG. 5. The spectrum of the  $Ti^{47}(d,t)Ti^{46}$  reaction at  $21^\circ$  lab. The spectrum has been corrected for the contributions from the  $Ti^{46}$  and  $Ti^{48}$  contained in the  $Ti^{47}$  target.

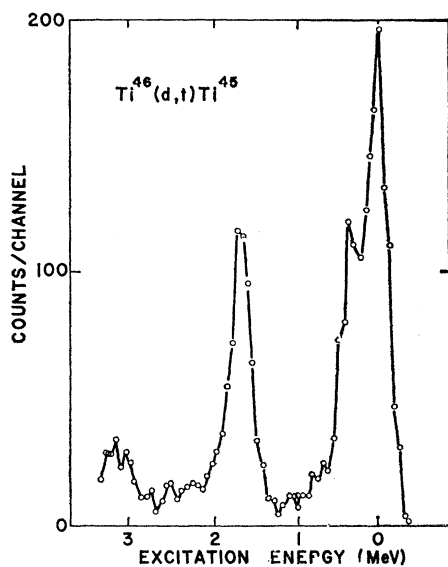


FIG. 6. The spectrum of the  $\text{Ti}^{46}(d,t)\text{Ti}^{45}$  reaction at  $21^\circ$  lab. The spectrum has been corrected for the contributions from the  $\text{Ti}^{47}$  and  $\text{Ti}^{48}$  contained in the  $\text{Ti}^{46}$  target.

$\text{Ti}^{47}$ . It appears more likely that the strong peak in  $\text{Ti}^{48}$  is due to more than one state, the  $6^+$  state making only a relatively small contribution. In both spectra the strongest group has a  $Q$  value which is approximately the same as the  $Q$  value for the ground-state

$(d,t)$  reaction on the final nucleus. Similar strong groups have been observed in other odd-neutron targets in this region of the periodic table.

Each of the three even-even targets showed two strong  $l=3$  transitions separated by about 2 MeV. In the case of  $\text{Ti}^{46}$  and  $\text{Ti}^{50}$  it had been anticipated that nearly the entire strength of the  $f_{7/2}$  pickup would be found in the ground-state transitions. While core excitation had been observed earlier in nuclei with 28 and 30 protons, it had not been observed in the case of 26 particles. Since an  $l=2$  transition is observed in the  $\text{Ti}^{48}(d,t)\text{Ti}^{47}$  reaction to a fairly low-lying state and since there is a strong suspicion for the presence of an  $l=2$  transition close to the ground-state transition in the  $\text{Ti}^{46}(d,t)\text{Ti}^{45}$  reaction, it is plausible that calculations assuming an inert core of  $\text{Ca}^{40}$  and an active  $(1f_{7/2})^n$  configuration will yield only approximate agreement with the experimental results.

#### ACKNOWLEDGMENTS

I am indebted to F. Karasek for the rolling of the target foils, to C. M. Stevens and his group for the mass spectroscopic analysis of the targets, and to W. J. O'Neill for the improvements in the detection system. I wish to thank R. D. Lawson, M. H. Macfarlane, and B. J. Raz for helpful discussions and W. Ramler and the cyclotron group for their cooperation.

### Effect of Hard Core on the Photodisintegration Cross Sections of $\text{H}^3$

V. S. MATHUR, S. N. MUKHERJEE, AND M. L. RUSTGI  
*Physics Department, Banaras Hindu University, Varanasi, India*  
 (Received April 20, 1962)

The electric-dipole bremsstrahlung weighted cross section ( $\sigma_b$ ) and the integrated cross section ( $\sigma_{\text{int}}$ ) for the photodisintegration of  $\text{H}^3$  are calculated using the hard-core wave functions of Kikuta, Morita, and Yamada. A comparison with already published calculations indicates that the introduction of the hard core increases both  $\sigma_b$  and  $\sigma_{\text{int}}$  for the  $\text{H}^3$  nucleus by about 100 and 8%, respectively.

IN their classic paper Levinger and Bethe<sup>1</sup> derived expressions for the electric-dipole bremsstrahlung weighted cross section [ $\sigma_b = \int_0^\infty (\sigma/W) dW$ ] and the integrated cross section [ $\sigma_{\text{int}} = \int_0^\infty \sigma dW$ ] for the nuclear photoeffect on the basis of the generalized Thomas-Reiche-Kuhn sum-rule, using a partially attractive exchange potential of Majorana type. Rustgi and Levinger<sup>2</sup> extended the sum-rule calculations of Levinger and Bethe<sup>1</sup> to include the two-body Heisenberg forces as well. The original expression for the bremsstrahlung weighted cross section for the electric-dipole absorption as obtained by Levinger and Bethe<sup>1</sup> was put in a slightly modified form by Foldy<sup>3</sup>

to read as

$$\sigma_b = (4\pi^2/3)(e^2/\hbar c)[NZ/(A-1)]R_c^2, \quad (1)$$

where  $R_c$  is the charge root-mean-square radius of the nucleus involved and is given by

$$R_c^2 = (1/Z)[\{\sum_i (\mathbf{r}_i - \mathbf{R})^2\}]_{00}, \quad (2)$$

where  $i$  stands for the proton and  $\mathbf{R}$  is the coordinate of the center of mass of the nucleus. Rustgi<sup>4</sup> used the work of Levinger and Bethe<sup>1</sup> to calculate  $\sigma_b$  and  $\sigma_{\text{int}}$  for  $\text{H}^3$  and  $\text{He}^3$  nuclei using a two-body spin-dependent Yukawa potential and the Irving<sup>5</sup> wave function.

The purpose of the present note is to calculate  $\sigma_b$  and  $\sigma_{\text{int}}$  for  $\text{H}^3$  using the two-body spin-dependent forces of exponential type with hard core. The effect of the hard core on the binding energy of  $\text{He}^3$  and  $\text{H}^3$

<sup>1</sup> J. S. Levinger and H. A. Bethe, *Phys. Rev.* **78**, 115 (1950).

<sup>2</sup> M. L. Rustgi and J. S. Levinger, *Phys. Rev.* **106**, 530 (1957).

<sup>3</sup> L. L. Foldy, *Phys. Rev.* **107**, 1303 (1957).