analytic functions of J, then the generalization of the above to the relativistic case is immediate. In fact, however, we may not even need this, since we can proceed directly from unitarity. This tells us[4] that for fixed real E, with $E_N < E < E_{N+1}$,

$$S_N^\dagger(J^*, E+i\epsilon)S_N(J, E+i\epsilon)=1, \tag{9}$$

where S_N is the submatrix containing only those channels which are open in this region of E between the consecutive thresholds E_N and E_{N+1}. Thus,

$$S_N(J, E+i\epsilon)=[S_N^\dagger(J^*, E+i\epsilon)]^{-1}. \tag{10}$$

We now assume that the S-matrix elements are meromorphic functions of J.[5] Then Eq. (10) is of the form of Eq. (3) and we can prove the factorizability relation, provided that at a pole $J=\alpha(E)$ of $S_N(J,E+i\epsilon)$ none of the matrix elements of $S_N(J^*,E+i\epsilon)$ has a pole. The latter condition ensures that poles of $S_N(J,E+i\epsilon)$ correspond to zeros of $\det[S_N(J^*, E+i\epsilon)]$. Note that a further consequence is that all elements of S_N have, in general, the same poles.

[4] That the appropriate continuation of $S(J,E)$ satisfies unitarity for nonphysical J was shown by M. Froissart (unpublished); see also E. J. Squires, Nuovo cimento (to be published).

[5] So far this has been proven only for one-channel problems and in a restricted region of J.

There are two ways in which the above condition might be violated; namely, the function $\alpha(E)$ being real, or there being a pole at $\alpha^*(E)$ in addition to the one at $\alpha(E)$. To rule out the first, consider the matrix of residues, β, defined by

$$S(J,E)=\beta(E)/[J-\alpha(E)]+\text{regular part}, \tag{11}$$

where α is now real. Then Eq. (9) certainly requires

$$\beta^\dagger\beta=0, \tag{12}$$

which is clearly impossible unless $\beta\equiv 0$. We cannot at present rule out the other possibility, but since there is no reason for any symmetry between $S(J,E)$ and $S(J^*,E)$, we believe that it can happen only accidentally, and for isolated values of E.

We have proven the factorizability relation, Eq. (2), for $E_N < E < E_{N+1}$, and for the corresponding submatrix S_N. The relation can obviously be continued to other regions of E, provided we do not cross any cuts. Further, by taking N sufficiently large we can include channels with arbitrarily high thresholds.

It is a pleasure to acknowledge very many useful discussions on the subject of this note with the members of the S-matrix theory group at this laboratory, and at the University of California, Berkeley, Physics Department.

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Possibility of a $[\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)]\cdot\mathbf{L}$ Term in Hyperon-Nucleon Interactions*

B. W. Downs and R. Schrils
University of Colorado, Boulder, Colorado

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The possibility of a term proportional to $[\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)]\cdot\mathbf{L}$ in hyperon-nucleon interactions is suggested, and an experiment is considered in which the presence of such a term might be detected.

The form of the (strong) interaction potential between two spin 1/2 particles, which follows from generally accepted invariance requirements, was first set forth by Eisenbud and Wigner.[1] For the sake of simplicity, these authors limited consideration to potentials which contain the relative momentum of the two particles in powers no higher than the first. The considerations of Eisenbud and Wigner have subsequently been extended to include higher powers of the relative momentum.[2,3] Those terms in the potential which contain the scalar product of the spin operators $\boldsymbol{\sigma}(i)$ of the interacting Fermions and the relative oribtal angular momentum $\mathbf{L}$ are

$$V_1\boldsymbol{\sigma}(1)\cdot\mathbf{L}+V_2\boldsymbol{\sigma}(2)\cdot\mathbf{L}=\tfrac{1}{2}(V_1+V_2)[\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)]\cdot\mathbf{L} +\tfrac{1}{2}(V_1-V_2)[\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)]\cdot\mathbf{L}, \tag{1}$$

where the potential coefficients V_1 and V_2 are functions of the magnitudes of the dynamical variables $(\mathbf{r},\mathbf{p},L)$. The first term on the right-hand side of (1) is the familiar spin-orbit potential. The second term, which is antisymmetric in the coordinates of the two particles, cannot appear in the interaction between two identical particles (two protons, etc.), which must be symmetric in the particle coordinates; nor can it appear in the interaction between a neutron and a proton if the nucleon-nucleon interaction is charge-symmetric, as it almost certainly is. In the charge-symmetric nucleon-

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[1] L. Eisenbud and E. P. Wigner, Proc. Natl. Acad. Sci. U. S. **27**, 281 (1941). They required that the interaction be invariant under (1) translations, (2) spatial rotations, (3) spatial reflections, (4) time reversal, and (5) Galilean transformations.

[2] L. Puzikov, R. Ryndin, and J. Smorodinsky, Nuclear Phys. **3**, 436 (1957).

[3] S. Okubo and R. E. Marshak, Ann. Phys. **4**, 166 (1958).

nucleon interaction, therefore, $V_1=V_2$, and the potential term

$$\tfrac{1}{2}(V_1-V_2)[\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)]\cdot\mathbf{L} \qquad (2)$$

is absent.

It is the purpose of this paper to suggest that a potential of the form (2) may appear in hyperon-nucleon interactions because of the nonidentity of the particles. This possibility has not been considered in the construction of potentials which have been used to predict the results of hyperon-nucleon scattering experiments.[4–8] For this purpose, the Λ-nucleon and Σ-nucleon potentials have been taken to be linear combinations of nucleon-nucleon potentials, in which terms of the form (2) do not appear. In the case of the Λ-nucleon interaction, the justification for the use of such a linear-combination potential is the reasonably good agreement between this and the phenomenological Λ-nucleon interaction deduced from Λ-hypernuclear binding energy data[4,9,10]; in this comparison, a potential of the form (2) would not play a role. The origin of the association of hyperon-nucleon potentials with nucleon-nucleon potentials is the conjecture that there may exist a universal pion-baryon interaction and that the kaon-exchange contributions to the hyperon-nucleon interactions may be negligible compared to the pion-exchange contributions.[4,11–14] Even if these conjectures are correct, differences between the hyperon-nucleon potentials and the relevant nucleon-nucleon potentials could be expected to the extent that the masses of the hyperons differ from that of the nucleons. If these conjectures are not correct, then there is no obvious connection between the hyperon-nucleon interactions and the nucleon-nucleon interaction.[12–14] It would therefore seem that there is no compelling reason for rejecting *a priori* the possibility of a term of the form (2) in the hyperon-nucleon interactions.

It is possible that the presence of an interaction of the form (2) can be detected by measuring the final-state polarizations of the two particles in a hyperon-nucleon scattering experiment. The description of such an experiment has been discussed in detail by Schumacher and Bethe for the case of two identical particles.[15] We reproduce those of their results, modified to include the nonidentity of the particles, which might be most useful for detecting the existence of a term (2) in the hyperon-nucleon interactions.

For the interaction of two spin $\frac{1}{2}$ particles, the most general form of the scattering matrix, which is invariant under space rotations, space reflections, and time reversal, is[15–17]

$$M=A+B\sigma_n(1)\sigma_n(2)+C[\sigma_n(1)+\sigma_n(2)]+D[\sigma_n(1)-\sigma_n(2)]+E\sigma_q(1)\sigma_q(2)+F\sigma_p(1)\sigma_p(2), \qquad (3)$$

where $\sigma_n(i)$ is the component of $\boldsymbol{\sigma}(i)$ in the $\mathbf{n}$ direction, etc.; and $\mathbf{n}=\mathbf{k}_i\times\mathbf{k}_f$ (normal to the scattering plane), $\mathbf{p}=\mathbf{k}_i+\mathbf{k}_f$, and $\mathbf{q}=\mathbf{k}_f-\mathbf{k}_i$ are three mutually perpendicular vectors, $\mathbf{k}_i$ and $\mathbf{k}_f$ being the initial and final relative momenta in the zero-momentum frame. The coefficients $A\cdots F$ are complex functions of the energy and scattering angle. The presence (absence) of the term (2) in the interaction implies the presence (absence) of the term $D[\sigma_n(1)-\sigma_n(2)]$ in the scattering matrix.

In order to investigate the role of the antisymmetric term $D[\sigma_n(1)-\sigma_n(2)]$ in determining the final-state polarizations, we first consider the scattering of unpolarized hyperons by unpolarized nucleons.[18] The polarization vectors of the scattered hyperon and recoil nucleon lie in the $\mathbf{n}$ direction; the magnitudes $P(i)$ of the polarizations are

$$I_0P(1)=\tfrac{1}{4}\,\mathrm{Tr}[MM^\dagger\sigma_n(1)]=2\,\mathrm{Re}C^*(A+B)+2\,\mathrm{Re}D^*(A-B), \qquad (4a)$$

$$I_0P(2)=\tfrac{1}{4}\,\mathrm{Tr}[MM^\dagger\sigma_n(2)]=2\,\mathrm{Re}C^*(A+B)-2\,\mathrm{Re}D^*(A-B), \qquad (4b)$$

where

$$I_0=\tfrac{1}{4}\,\mathrm{Tr}[MM^\dagger]=|A|^2+|B|^2+2|C|^2+2|D|^2+|E|^2+|F|^2 \qquad (5)$$

is the differential cross section. If $D=0$, Eqs. (4) give the well-known result that the polarizations of the scattered and recoil particles are the same. The importance of Eqs. (4) for our purpose is that the difference of the polarizations

$$P(1)-P(2)=4\,\mathrm{Re}D^*(A-B)/I_0 \qquad (6)$$

is proportional to the coefficient D of the antisymmetric term in the scattering matrix. The use of the relation

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[4] D. B. Lichtenberg and M. H. Ross, Phys. Rev. **107**, 1714 (1957).
[5] J. S. Kovacs and D. B. Lichtenberg, Nuovo cimento **13**, 371 (1959).
[6] R. A. Bryan, J. J. de Swart, R. E. Marshak, and P. S. Signell, Phys. Rev. Letters **1**, 70 (1958).
[7] F. Ferrari and L. Fonda, Phys. Rev. **114**, 874 (1959).
[8] J. J. de Swart and C. Dullemond, Ann. Phys. **16**, 263 (1961).
[9] B. W. Downs and R. H. Dalitz, Phys. Rev. **114**, 593 (1959) and other references cited there. The potentials used in references 5–8 contain spin-orbit terms; the analyses of Λ-hypernuclear binding energy data shed no light on the magnitude of such terms.
[10] Some justification has also been provided by G. Alexander, J. A. Anderson, F. S. Crawford, W. Laskar, and L. J. Lloyd, Phys. Rev. Letters **7**, 348 (1961), who report measured Λ-nucleon cross sections which agree with the predictions of Kovacs and Lichtenberg (reference 5) and de Swart and Dullemond (reference 8) within the rather large experimental uncertainties. Their cross-section data do not enable them to detect the presence of the spin-orbit potential used by Kovacs and Lichtenberg.
[11] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).
[12] D. B. Lichtenberg and M. H. Ross, Phys. Rev. **109**, 2163 (1958).
[13] N. Dallaporta and F. Ferrari, Nuovo cimento **5**, 111 (1957).
[14] F. Ferrari and L. Fonda, Nuovo cimento **6**, 1027 (1957); **9**, 842 (1958).

[15] C. R. Schumacher and H. A. Bethe, Phys. Rev. **121**, 1534 (1961). See also reference 2.
[16] R. H. Dalitz, Proc. Phys. Soc. (London) **A65**, 175 (1952).
[17] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).
[18] Unpolarized hyperons are produced, for example, in the capture at rest of K^- mesons by He^4.

TABLE I. Nonzero components of the depolarization and polarization-transfer tensors.

$I_0\mathfrak{D}_{nn}=\lvert A\rvert^2+\lvert B\rvert^2+2\lvert C\rvert^2+2\lvert D\rvert^2-\lvert E\rvert^2-\lvert F\rvert^2$
$I_0\mathfrak{D}_{pp}=\lvert A\rvert^2-\lvert B\rvert^2-\lvert E\rvert^2+\lvert F\rvert^2-4\,\mathrm{Re}CD^*$
$I_0\mathfrak{D}_{qq}=\lvert A\rvert^2-\lvert B\rvert^2+\lvert E\rvert^2-\lvert F\rvert^2-4\,\mathrm{Re}CD^*$
$I_0\mathfrak{D}_{pq}=2\,\mathrm{Im}C^*(A-B)+2\,\mathrm{Im}D^*(A+B)$
$I_0\mathfrak{D}_{qp}=-I_0\mathfrak{D}_{pq}$
$I_0\mathfrak{K}_{nn}=2\,\mathrm{Re}AB^*+2\lvert C\rvert^2-2\lvert D\rvert^2+2\,\mathrm{Re}EF^*$
$I_0\mathfrak{K}_{pp}=2\,\mathrm{Re}AF^*+2\,\mathrm{Re}BE^*$
$I_0\mathfrak{K}_{qq}=2\,\mathrm{Re}AE^*+2\,\mathrm{Re}BF^*$
$I_0\mathfrak{K}_{pq}=2\,\mathrm{Im}C^*(E+F)+2\,\mathrm{Im}D^*(E-F)$
$I_0\mathfrak{K}_{qp}=-2\,\mathrm{Im}C^*(E+F)+2\,\mathrm{Im}D^*(E-F)$

(6) in neutron-proton scattering was originally (1952) suggested by Dalitz[16] to test the correctness of the proposal of the charge symmetry of the nucleon-nucleon interactions. In the experiment being considered here, the polarizations of the recoil nucleon and scattered hyperon can, in principle at least, be measured; that of the former, by an analyzing scatter of the beam of recoiling nucleons[19] and that of the latter, by the asymmetry of the angular distribution of the decay pions.[20–22]

In the scattering of polarized hyperons by unpolarized nucleons,[23] the effect of the term $D[\sigma_n(1)-\sigma_n(2)]$ in determining the final-state polarizations is, to some extent, masked by the presence of other terms in the scattering matrix. By proper choice of the component of the final-state polarizations to be measured it is, however, possible to isolate the coefficient D as it was in Eq. (6). In this case of polarized incident particles, the polarizations $P_k'(i)$ of the scattered hyperon and recoil nucleon in the $\mathbf{k}$ direction are

$$P_k'(1)=[P(1)\delta_{kn}+\textstyle\sum_i \mathfrak{D}_{ik}P_i^0(1)]/[1+\mathbf{P}^0(1)\cdot\mathbf{P}(1)], \quad (7a)$$

$$P_k'(2)=[P(2)\delta_{kn}+\textstyle\sum_i \mathfrak{K}_{ik}P_i^0(1)]/[1+\mathbf{P}^0(1)\cdot\mathbf{P}(1)], \quad (7b)$$

where $\mathbf{P}^0(1)$ is the initial polarization of the incident hyperon (1 = hyperon), and $P(1)$ and $P(2)$ are the polarizations produced when the incident particle is unpolarized [Eqs. (4)]. The depolarization tensor $\mathfrak{D}_{ik}$ and the polarization-transfer tensor $\mathfrak{K}_{ik}$ are defined by

$$I_0\mathfrak{D}_{ik}=\tfrac{1}{4}\,\mathrm{Tr}[M\sigma_i(1)M^\dagger\sigma_k(1)], \quad (8a)$$

$$I_0\mathfrak{K}_{ik}=\tfrac{1}{4}\,\mathrm{Tr}[M\sigma_i(1)M^\dagger\sigma_k(2)]. \quad (8b)$$

The components of these tensors for which $i=n$ or $k=n$ (but not $i=k=n$) are zero; the nonzero components are given in Table I. For an initial polarization $\mathbf{P}^0(1)$ of arbitrary orientation with respect to the scattering plane, the final-state polarizations depend upon all the coefficients in the scattering matrix. If the initial polarization lies in the scattering plane, however, the component of the polarization of the scattered hyperon and that of the recoil nucleon in the $\mathbf{n}$ direction are given by Eqs. (4), and the difference of these polarization components is given by Eq. (6).

It would appear that the use of Eq. (6) in the scattering of either unpolarized or polarized hyperons by unpolarized nucleons offers some promise for investigating the existence of the antisymmetric term (2) in hyperon-nucleon interactions.[24]

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[19] See, for example, L. Rosen, J. E. Brolley, and L. Stewart, Phys. Rev. **121**, 1423 (1961).

[20] See, for example, F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, J. Steinberger, P. Bassi, V. Borelli, G. Puppi, G. Tanaka, P. Woloschek, V. Zoboli, M. Conversi, P. Franzini, I. Mannelli, R. Santangelo, V. Silvestrini, D. A. Glaser, C. Graves, and M. L. Perl, Phys. Rev. **108**, 1353 (1957).

[21] The asymmetry parameters for Λ decay and Σ decay measured by J. Leitner, L. Gray, E. Harth, S. Lichtman, J. Westgard, M. Block, B. Brucker, A. Engler, R. Gessaroli, A. Kovacs, T. Kikuchi, C. Meltzer, H. O. Cohn, W. Bugg, A. Pevsner, P. Schlein, M. Meer, N. T. Grinellini, L. Lendinara, L. Monari, and G. Puppi, Phys. Rev. Letters **7**, 264 (1961), and by E. F. Beall, B. Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters **7**, 285 (1961) might well need considerable refinement before the relation (6) can be used even if the polarization of the recoil nucleon can be measured accurately.

[22] De Swart and Dullemond (reference 8) have calculated the polarizations to be expected in Σ^+-proton scattering at energies of 40, 100, and 140 MeV. For these energies, the expected maximum (as a function of scattering angle) polarizations are less than 1%, 21%, and 38%, respectively. See also reference 6. If the hyperon-nucleon interaction arises predominantly from a universal pion-baryon interaction, the first term in (4a) and (4b) (divided by I_0) could be expected to be of this magnitude.

[23] The hyperons produced in associated production processes are polarized in the direction of the normal to the production plane; see reference 20.

[24] The relation $\mathfrak{K}_{pq}+\mathfrak{K}_{qp}=4\,\mathrm{Im}D^*(E-F)/I_0$ probably cannot be exploited to detect the presence of the antisymmetric term in the scattering matrix.