(this property is proved by applying two times the uniqueness of the usual single dispersion relation whose two cuts are separated).

Since single dispersion terms can be represented as (6.20), f_1 minus these terms must be an entire function. Therefore, Theorem VI shows that $\rho_{23}(\alpha,z)$ vanishes except for $z=0$ and $z=1$. The same is true for the other two weight functions. Q.E.D.

In the above theorem the condition (6.23) is very important. Without it even the Mandelstam representation loses its uniqueness property. Our previous results[2] show that this condition is satisfied in almost all practical cases (e.g., equal-mass, nucleon-nucleon, pion-nucleon, and kaon-nucleon scatterings) in every order of perturbation theory.

ACKNOWLEDGMENTS

The author would like to express his sincere thanks to Professor J. Robert Oppenheimer for the hospitality extended to him at the Institute for Advanced Study, and to the National Science Foundation for financial support. He is also much indebted to Dr. Mikio Sato for the mathematical problem on the decomposition (6.17).

PHYSICAL REVIEW VOLUME 127, NUMBER 4 AUGUST 15, 1962

Factorization of the Residues of Regge Poles*

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(Received April 6, 1962)

A proof of the factorizability of the residues of Regge poles, valid for a many-channel potential scattering problem, is given. Unitarity and certain other plausible assumptions about the S matrix allow the proof to be extended to the relativistic S-matrix theory.

RECENTLY, Gell-Mann (private communication) has postulated that the residues of Regge poles of the S matrix for a many-channel problem are factorizable, viz., for

$$\beta_{ij}(E)=\lim_{J\to\alpha(E)}\,[J-\alpha(E)]S_{ij}(E), \tag{1}$$

where i, j label the channels and $\alpha(E)$ is the position of a pole, then

$$\beta_{ij}(E)=\gamma_i(E)\gamma_j(E). \tag{2}$$

Gell-Mann has given a proof of this equation based on the nonrelativistic Schrödinger equation.[1]

In the course of a general study of analyticity in J for the nonrelativistic potential scattering problem we had also obtained a simple proof of this result, which is worth reporting, since the method, being based directly on the S matrix, can immediately be generalized to enable us to say something about the relativistic problem.[2]

For the potential case (and for a wide class of potentials), it can be shown that the S matrix can be written in the form

$$S(J,E)=F_1(J,E)[F_2(J,E)]^{-1}, \tag{3}$$

where F_1 and F_2 are n-by-n matrices,[3] n being the number of channels, with F_1 and F_2 analytic functions of J. Thus, poles of $S(J,E)$ occur for

$$\det[F_2(J,E)]=0. \tag{4}$$

Except for accidental degeneracies, the zeros in

$$\det[F_2(J,E)]$$

are simple zeros. Since the elements of F_2 are analytic in J, it follows that the rank of F_2 is $n-1$.

If we write

$$[F_2]^{-1}=G/\det F_2, \tag{5}$$

where G is the matrix of cofactors, then Sylvester's law of nullity tells us that

$$r(G)+r(F_2)-n\leqslant 0, \tag{6}$$

where $r(A)$ means the rank of the matrix A. Thus

$$r(G)\leqslant 1, \tag{7}$$

i.e., all 2-by-2 cofactors of G are zero. Simple calculation then shows that the residues satisfy

$$\beta_{ii}\beta_{jj}=\beta_{ij}\beta_{ji}. \tag{8}$$

Apart from an irrelevant sign, Eq. (2) follows from Eq. (8) and the fact that S is symmetrical.

If, as seems plausible, the relativistic S matrix can also be written in the form of Eq. (3), with F_1 and F_2

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] Murray Gell-Mann, Phys. Rev. Letters 8, 263 (1962).

[2] Since writing this paper, we have seen an article by V. N. Gribov and I. Ya. Pomeranchuk [Phys. Rev. Letters 8, 343 (1962)] in which the result is proven for a two-channel problem, with the same assumptions that we have made.

[3] The matrices F_1 and F_2 are generalizations of Jost functions; see, for example, R. G. Newton, J. Math. Phys. 1, 319 (1960).

analytic functions of J, then the generalization of the above to the relativistic case is immediate. In fact, however, we may not even need this, since we can proceed directly from unitarity. This tells us[4] that for fixed real E, with $E_N < E < E_{N+1}$,

$$S_N^\dagger(J^*, E+i\epsilon) S_N(J, E+i\epsilon) = 1, \tag{9}$$

where S_N is the submatrix containing only those channels which are open in this region of E between the consecutive thresholds E_N and E_{N+1}. Thus,

$$S_N(J, E+i\epsilon) = [S_N^\dagger(J^*, E+i\epsilon)]^{-1}. \tag{10}$$

We now assume that the S-matrix elements are meromorphic functions of J.[5] Then Eq. (10) is of the form of Eq. (3) and we can prove the factorizability relation, provided that at a pole $J=\alpha(E)$ of $S_N(J,E+i\epsilon)$ none of the matrix elements of $S_N(J^*,E+i\epsilon)$ has a pole. The latter condition ensures that poles of $S_N(J,E+i\epsilon)$ correspond to zeros of $\det[S_N(J^*, E+i\epsilon)]$. Note that a further consequence is that all elements of S_N have, in general, the same poles.

[4] That the appropriate continuation of $S(J,E)$ satisfies unitarity for nonphysical J was shown by M. Froissart (unpublished); see also E. J. Squires, Nuovo cimento (to be published).

[5] So far this has been proven only for one-channel problems and in a restricted region of J.

There are two ways in which the above condition might be violated; namely, the function $\alpha(E)$ being real, or there being a pole at $\alpha^*(E)$ in addition to the one at $\alpha(E)$. To rule out the first, consider the matrix of residues, β, defined by

$$S(J,E) = \beta(E)/[J-\alpha(E)] + \text{regular part}, \tag{11}$$

where α is now real. Then Eq. (9) certainly requires

$$\beta^\dagger \beta = 0, \tag{12}$$

which is clearly impossible unless $\beta \equiv 0$. We cannot at present rule out the other possibility, but since there is no reason for any symmetry between $S(J,E)$ and $S(J^*,E)$, we believe that it can happen only accidentally, and for isolated values of E.

We have proven the factorizability relation, Eq. (2), for $E_N < E < E_{N+1}$, and for the corresponding submatrix S_N. The relation can obviously be continued to other regions of E, provided we do not cross any cuts. Further, by taking N sufficiently large we can include channels with arbitrarily high thresholds.

It is a pleasure to acknowledge very many useful discussions on the subject of this note with the members of the S-matrix theory group at this laboratory, and at the University of California, Berkeley, Physics Department.

PHYSICAL REVIEW VOLUME 127, NUMBER 4 AUGUST 15, 1962

Possibility of a $[\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)]\cdot\mathbf{L}$ Term in Hyperon-Nucleon Interactions*

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The possibility of a term proportional to $[\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)]\cdot\mathbf{L}$ in hyperon-nucleon interactions is suggested, and an experiment is considered in which the presence of such a term might be detected.

THE form of the (strong) interaction potential between two spin 1/2 particles, which follows from generally accepted invariance requirements, was first set forth by Eisenbud and Wigner.[1] For the sake of simplicity, these authors limited consideration to potentials which contain the relative momentum of the two particles in powers no higher than the first. The considerations of Eisenbud and Wigner have subsequently been extended to include higher powers of the relative momentum.[2,3] Those terms in the potential which contain the scalar product of the spin operators $\boldsymbol{\sigma}(i)$ of the interacting Fermions and the relative oribtal angular momentum $\mathbf{L}$ are

$$V_1\boldsymbol{\sigma}(1)\cdot\mathbf{L} + V_2\boldsymbol{\sigma}(2)\cdot\mathbf{L} = \tfrac{1}{2}(V_1+V_2)[\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)]\cdot\mathbf{L} + \tfrac{1}{2}(V_1-V_2)[\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)]\cdot\mathbf{L}, \tag{1}$$

where the potential coefficients V_1 and V_2 are functions of the magnitudes of the dynamical variables (r,p,L). The first term on the right-hand side of (1) is the familiar spin-orbit potential. The second term, which is antisymmetric in the coordinates of the two particles, cannot appear in the interaction between two identical particles (two protons, etc.), which must be symmetric in the particle coordinates; nor can it appear in the interaction between a neutron and a proton if the nucleon-nucleon interaction is charge-symmetric, as it almost certainly is. In the charge-symmetric nucleon-

* Supported in part by a grant from the National Science Foundation.

[1] L. Eisenbud and E. P. Wigner, Proc. Natl. Acad. Sci. U. S. **27**, 281 (1941). They required that the interaction be invariant under (1) translations, (2) spatial rotations, (3) spatial reflections, (4) time reversal, and (5) Galilean transformations.

[2] L. Puzikov, R. Ryndin, and J. Smorodinsky, Nuclear Phys. **3**, 436 (1957).

[3] S. Okubo and R. E. Marshak, Ann. Phys. **4**, 166 (1958).