Anomalous Magnetic Moments of the Baryons in Global Symmetry Model

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By using the properties of the global symmetry group $SU(2)\times SU(2)\times SU(2)$, the magnetic moments of the baryons have been rigorously derived. The results of earlier authors are shown to follow easily by the present method.

THERE exists in literature a number of calculations[1–3] on the anomalous magnetic moments of the baryons. Various authors have made use of field-theoretic perturbation methods to obtain the magnetic moments of the nucleons and hyperons. However, some predictions on the possible values of and various relationships among the magnetic moments of the nucleons, hyperons, and cascade particles could be made purely from some symmetry considerations. For instance, Marshak, Okubo, and Sudarshan[4] and also Katsumori[2] deduced, purely from charge independence, that $\mu_{\Sigma^+}+\mu_{\Sigma^-}=2\mu_{\Sigma^0}$, where μ stands for the magnetic moment of the particle denoted in its suffix. Feinberg and Behrends,[5] without recourse to any kind of global or cosmic symmetry of interaction other than that allowed by charge independence, deduced some interesting relationships among the magnetic moments of the baryons. Their results were based only on the fact that the interaction should remain invariant under certain kinds of discrete operations. Results on the magnetic moments of the baryons in the global symmetry scheme are usually inferred from the similar isotopic doublet structure of the baryons. The purpose of this note is to provide a simple and rigorous method of obtaining the various above-mentioned relations in the global symmetry scheme of pion-baryon interactions, using the properties of the global symmetry group $SU(2)\times SU(2)\times SU(2)$, as recently used and discussed by Lee and Yang.[6]

To determine rigorously the electromagnetic properties of the baryons in this scheme, it is necessary to know the quantum numbers of the various particle states, with respect to the operations in the global symmetry group. It has already been pointed out and discussed at length by Lee and Yang[6] (see also Pais[7]) that the simplest group G_0 satisfying the requirements of global symmetry is the group $SU(2)\times SU(2)\times SU(2)$, which also contains the discrete element R which mixes all the baryons. One thus introduces three sets of operators L_1, L_2, L_3; M_1, M_2, M_3; and N_1, N_2, N_3, each set satisfies the commutation relations of the angular momentum operators. Further, L_i, M_j, and N_k all commute for any i, j, and k. The explicit matrix representations of these operators are given by Lee and Yang.[6] The usual isotopic spin, hypercharge, and the electric charge are given, respectively, by

$$T_3=L_3+M_3,\quad \tfrac{1}{2}U=N_3,\quad U=S+N,$$

and

$$Q/e=L_3+M_3+N_3, \tag{1}$$

where S is strangeness and N is baryon number. Global symmetry of the strong pion-baryon coupling is then obtained by requiring the conservation of $\sum\mathbf{L}$, $\sum\mathbf{M}$, and $\sum\mathbf{N}$ separately. The various quantum numbers L, L_3; M, M_3; N, N_3 for all the baryon states are given in Table I. The discrete element R is such that

$$R|MM_3, NN_3\rangle=|NN_3MM_3\rangle. \tag{2}$$

From the above table we have, under this operation,

$$p\rightleftarrows\Sigma^+,\quad n\rightleftarrows Y^0,\quad \Xi^0\rightleftarrows Z^0,\quad \Xi^-\rightleftarrows\Sigma^-.$$

In the following derivation, we assume that the photon field A_μ is even under R. Similar assumptions in Gell-Mann's unitary symmetry group have been made by Coleman and Glashow[8] in connection with determining the magnetic moments of the baryons in the unitary symmetry scheme. Noting that the charge $Q=e(L_3+M_3+N_3)$, we can easily write the static magnetic moment operator $\tilde{M}_3$ (we consider here the third component of the magnetic moment vector), as

$$\tilde{M}_3=\frac{ie}{2}\int dV\{\bar{\psi}(L_3+M_3+N_3)(\mathbf{x}\times\boldsymbol{\gamma})_3\psi\}, \tag{3}$$

TABLE I. Quantum numbers for the baryon states.

		L	L_3	M	M_3	N	N_3
p	n	$\frac{1}{2}$	$\pm\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$
Ξ^0	Ξ^-	$\frac{1}{2}$	$\pm\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Σ^+	Y^0	$\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
Z^0	Σ^-	$\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0

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where γ is the Dirac matrix and ψ is the baryon field

$$\psi=\begin{bmatrix} p \\ n \\ \Xi^0 \\ \Xi^- \\ \Sigma^+ \\ Y^0 \\ Z^0 \\ \Sigma^- \end{bmatrix},$$

where $Y^0=(\Sigma^0+\Lambda)/\sqrt{2}$; $Z^0=(\Lambda-\Sigma^0)/\sqrt{2}$. It is now easily seen from (3) that the magnetic moment operator $\tilde{M}_3$ can be split into three components, each of which is the third component of the vectors in the L space, M space, and N space, respectively. This is just like its decomposition into an isoscalar and the third component of a isovector part in the usual isotopic-spin space. Thus we have

$$\tilde{M}_3=V_3{}^L+V_3{}^M+V_3{}^N. \tag{4}$$

Now, to determine the magnetic moments of the baryons, we need to take the expectation value of the operator $\tilde{M}_3$ between states characterized by $L\,L_3$, $M\,M_3$, and $N\,N_3$. We thus have

$$\langle\tilde{M}_3\rangle=\langle V_3{}^L\rangle+\langle V_3{}^M\rangle+\langle V_3{}^N\rangle, \tag{5}$$

where

$$\langle\tilde{M}_3\rangle=\langle LL_3,MM_3,NN_3|\tilde{M}_3|LL_3,MM_3,NN_3\rangle,$$
$$\langle V_3{}^L\rangle=\langle LL_3|V_3{}^L|LL_3\rangle,$$
$$\langle V_3{}^M\rangle=\langle MM_3|V_3{}^M|MM_3\rangle,$$
$$\langle V_3{}^N\rangle=\langle NN_3|V_3{}^N|NN_3\rangle.$$

Now, making use of the Wigner-Eckart theorem of tensor algebra, we get

$$\langle LL_3|V_3{}^L|LL_3\rangle=L_3\mu_1,$$

where μ_1 contains the reduced matrix element $\langle L\|V_3{}^L\|L\rangle$ with another factor (independent of L_3) coming from the Clebsch-Gordan coefficients.

Similarly, we write

$$\langle MM_3|V_3{}^M|MM_3\rangle=M_3\mu_2$$

and

$$\langle NN_3|V_3{}^N|NN_3\rangle=N_3\mu_3,$$

where μ_2 and μ_3 contain the corresponding reduced matrix elements with associated factors.

Now taking the expectation values of (5) and making use of Table I we deduce for the magnetic moments μ_p, μ_n, etc., for p and n, etc., the following:

$$\begin{aligned} \mu_p&=\tfrac{1}{2}(\mu_1+\mu_3), & \mu_{\Sigma^+}&=\tfrac{1}{2}(\mu_1+\mu_2),\\ \mu_n&=\tfrac{1}{2}(-\mu_1+\mu_3), & \mu_{Y^0}&=\tfrac{1}{2}(-\mu_1+\mu_2),\\ \mu_{\Xi^0}&=\tfrac{1}{2}(\mu_1-\mu_3), & \mu_{Z^0}&=\tfrac{1}{2}(\mu_1-\mu_2),\\ \mu_{\Xi^-}&=\tfrac{1}{2}(-\mu_1-\mu_3), & \mu_{\Sigma^-}&=\tfrac{1}{2}(-\mu_1-\mu_2). \end{aligned} \tag{6}$$

From (6), and noting that the (Y^0-Z^0) transition magnetic moment, denoted here as $\mu_{Y^0-Z^0}$, must vanish (as T_3 is violated), we at once derive the following relations:

$$\mu_p+\mu_{\Xi^-}=0,\quad \mu_{\Sigma^+}+\mu_{\Sigma^-}=0,$$
$$\mu_n+\mu_{\Xi^0}=0,\quad \mu_{\Sigma^0}=\mu_{\Lambda^0}=0,$$

and

$$\mu_{\Sigma^0-\Lambda^0}=\mu_p-\mu_n-\mu_{\Sigma^+}. \tag{7}$$

The first four results are the same as deduced by Feinberg and Behrends[5] and also by Sakurai[9] using a discrete operation called hypercharge reflection invariance of the interaction. If now we consider the R operation, then we immediately get the following final results:

$$\mu_p=\mu_{\Sigma^+}=-\mu_{\Xi^-}=-\mu_{\Sigma^-};$$
$$\mu_n=-\mu_{\Xi^0};\quad \mu_{\Sigma^0}=\mu_{\Lambda^0}=0;$$

and

$$\mu_{\Sigma^0-\Lambda^0}=-\mu_n. \tag{8}$$

This last result has been used by Chiba *et al.*[10] in connection with $\Sigma\rightarrow\Lambda+\gamma$ decay. It may be noticed that $\mu_{\Sigma^0-\Lambda^0}$ turns out to be $-\frac{1}{2}\mu_n$ in the unitary symmetry[8] model, and it vanishes in Sakurai's[9] case which he derives by hypercharge reflection invariance.

Thus, we have established the well-known results on the magnetic moments of the baryons derived by many authors using different methods. It may be remarked that our present method of calculation is a direct generalization of the method used by Marshak, Okubo, and Sudarshan,[4] who only derived the magnetic moment relations for the Σ triplet. Their relation is satisfied in our model also. Lastly Tanaka,[3] following Feinberg and Behrends,[5] deduced that $\mu_{\Sigma^+}+\mu_{\Sigma^-}=2\mu_{\Sigma^0}=2\mu_{\Lambda^0}$, which is also satisfied in our scheme.

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