Experimental Study of the Polarization and Magnetic Moment of the Antiproton*

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Asymmetries in double scattering of antiprotons of 1.6 BeV/c momentum in the 72-in. hydrogen bubble chamber have been investigated. Analysis of 200 events in which both scatterings occur in the angular region $6.3 \leq \theta_{\text{lab}} \leq 23.6°$ yielded a polarization $P = (50_{-13}^{+8})\%$, at an average angle $\theta_{\text{lab}} = 10°$. The precession of the spin polarization vector in the magnetic field of the bubble chamber between two scatterings decreases the up-down asymmetries by an amount determined by the magnetic moment of the particle. A method for determination of the magnetic moment of the antiproton, using a three-dimensional likelihood function, is described as applied to the above sample of events. The value of the antiproton magnetic moment was determined to be $\mu_{\bar{p}} = -1.8 \pm 1.2$ nuclear magnetons.

INTRODUCTION

THERE have been no direct observations of the polarization and the magnetic moment of antinucleons. An attempt to see if antiprotons are polarized upon production within the Bevatron has given an inconclusive result.[1] Further, prior to this experiment no theory had been developed to predict antiproton polarization.[2] The magnetic moment of the antiproton was expected to be the negative of the proton magnetic moment, in accordance with the *CPT* theorem.

This report treats a measurement of average asymmetry at angles 6° to 25° in the double scattering of 960-MeV (1.61-BeV/c) antiprotons in the 72-in. hydrogen bubble chamber.

EXPERIMENTAL METHOD: I. ASYMMETRY OF DOUBLE SCATTERING WITHIN THE BUBBLE CHAMBER

The expression for intensity after double scattering of spin-1/2 particles is well known to be

$$I(\theta_2,\phi) = I_0(\theta_2)(1 + e\cos\phi), \tag{1}$$

where e is the asymmetry, dependent on θ_1 and θ_2 scattering angles, and $I_0(\theta)$ is the cross section for scattering of an unpolarized beam at the second target. The polarizations characterizing the first and second scatterings, P_1 and P_2, are related to the asymmetry by the expression

$$e = P_1 P_2;$$

the quantities P_1 and P_2 refer to the polarization that that would result from the scattering of an unpolarized beam at angle θ_1 and energy E_1 or at angle θ_2 and energy E_2. The energy difference between first and second scatterings is assumed small enough that $P_1 = P_2$ for the same angles of scattering; since an average is made over the same interval of scattering angle for both targets, the first and second polarizations can be considered equal and the asymmetry e becomes

$$e = P^2. \tag{2}$$

The energy loss due to ionization between scatterings is only 20 MeV on the average, but at the maximum angle accepted in the first scattering, the outgoing antinucleon suffers a scattering loss of 170 MeV. Polarization for nucleon-nucleon scattering can be considered constant over such an energy interval at these high energies, and it was assumed that this situation would also obtain for antinucleon-nucleon scattering.

As can readily be seen from the distribution function, the evaluation of asymmetry e can be made for the cases in which both the first and second scatterings lie in (approximately) the same plane by taking

$$e = \frac{(RR+LL)-(RL+LR)}{(RR+LL)+(RL+LR)}, \tag{3a}$$

where RR, LL, RL, and LR denote right-right, left-left, right-left, and left-right double scattering, respectively. However, for experiments in the bubble chamber, it is desirable for good statistics not to restrict the sample of double-scattering events to those occurring in the same plane; Eq. (3a), then, has to be modified to take into account for each event the azimuthal angle ϕ between the scattering planes. It is evident from the distribution function that an evaluation similar to the above can be made for events with scattering planes which are not parallel, but that the quantity obtained is $e\cos\phi$ rather than e. Thus,

$$e\cos\phi = \frac{I_0(1+e\cos\phi)-I_0(1-e\cos\phi)}{I_0(1+e\cos\phi)+I_0(1-e\cos\phi)}. \tag{3b}$$

A more convenient method for evaluating the asymmetry (for an average $\theta_1 \approx \theta_2 \approx \theta$) is to find the average value of $\cos\phi$ for all events. That this is equal to $e/2$ can be seen by weighting the distribution function by

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† Now at CERN, Geneva.
[1] L. Agnew, thesis, University of California Radiation Laboratory Report UCRL-8785, 1959 (unpublished).
[2] P. K. Srivastava, Nuovo cimento **20**, 172 (1961). This calculation was done after our data were obtained and reported in a preliminary form.

$\cos\phi$ and integrating over all ϕ:

$$e(\theta)=P^2(\theta)=\frac{2\int I(\theta,\phi)\cos\phi d\phi}{\int I(\theta,\phi)d\phi}\approx\frac{2}{N}\sum_{i=1}^{N}\cos\phi_i, \quad (4)$$

where N is the total number of events. It is assumed here that spin precession effects are small.

The fractional error in e as determined by this method is

$$\delta e/e=(1/e)[(2-e^2)/N]^{1/2}. \quad (5)$$

The errors in e and P are of course related by the equation

$$\delta e/e=2(\delta P/P). \quad (6)$$

An alternative method may be used to evaluate the average asymmetry e or P^2. It is possible to write a likelihood function

$$L(P^2)=\prod_{k}^{N}(1+P^2\cos\phi_k), \quad (7)$$

and to find the value of P^2 which gives this function its maximum. This method may be used to find simultaneously the best value of the magnetic moment, also, and is discussed more completely below.

EXPERIMENTAL METHOD: II. THE MAGNETIC MOMENT OF THE ANTIPROTON

The formulas (3) and (4) for polarization in double scattering have been derived on the assumption that there is negligible spin precession between the first and the second scattering. However, the 72-in. bubble chamber is in a magnetic field of 17.9 kG, whose direction is perpendicular to that of the incident antiproton beam. After first scattering, the spin-polarization vector $\mathbf{P}$ or magnetic moment $\boldsymbol{\mu}$ $(=g\mathbf{S}=\hat{n}_1 g/2)$ is parallel to $\hat{n}_1$, the unit vector normal to the first scattering plane [see Fig. 1(a)]. Hence, unless the first scattering is horizontal or the path length between scatterings is small, the $\boldsymbol{\mu}$ of the antiproton is subjected to a precession between two scatterings [Fig. 1(b)]. This is equivalent to a depolarization. The Larmor precession is proportional to the *sine* of the angle between $\boldsymbol{\mu}$ and $\mathbf{H}$. If one selects the events in which the first scattering is in the vertical plane (plane parallel to $\mathbf{H}$), the $\boldsymbol{\mu}$ is subjected to a maximum rotation about $\mathbf{H}$, since $\boldsymbol{\mu}$ is perpendicular to $\mathbf{H}$. In contrast, for the events in which the first scattering is in the horizontal plane, the precession does not change the direction of $\boldsymbol{\mu}$, since $\boldsymbol{\mu}$ is parallel to $\mathbf{H}$, and there is no depolarization. The up-down asymmetry e_{UD} is smaller than the right-left asymmetry e_{RL}, given by Eq. (3). It can be shown that $e_{UD}/e_{RL}\approx\cos\delta'$, where δ' is the average angle between the $\boldsymbol{\mu}$ and the scattering normal $\hat{n}_2$ immediately before the second scattering.

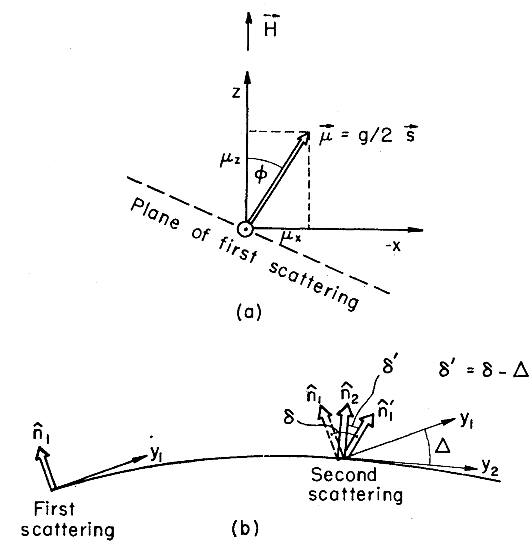

FIG. 1. Effect of a vertical magnetic field on direction of polarization. (a) Components of expected spin or magnetic-moment orientation after first scattering. The vector $\hat{n}_1$ is the unit vector normal to the plane of scattering and defines the direction of spin polarization. The incident particle is in the y direction. (b) Precession of spin direction between first and second scatterings. The vector $\hat{n}_2$ is normal to the plane of second scattering. The angle δ is the Larmor and Thomas spin precession angle. (The diagram represents projection in the horizontal plane, i.e., the precession of μ_x.)

The precession angle δ, which is the rotation of spin due to Larmor precession and to Thomas precession, is given by[3]

$$\delta=\gamma[\tfrac{1}{2}g-1+1/\gamma]\Delta. \quad (8)$$

The magnetic moment μ has the same sign as the charge and its magnitude is given by $\frac{1}{2}g$. The Δ is the deflection between the two scatterings, $\Delta=-(eH/2mc)t/\gamma$. We have not included another relativistic correction in Eq. (8), which comes from a precession about the direction of motion. We have tested the effect of the inclusion of this term and proven it to be undetectable ($<4\%$) within our statistics.

The average deflection of the antiproton momentum vector between two scatterings in our case was $\Delta=7.5$ deg. With $\gamma=2$ and $\mu=-2.79$, the average $\delta'=27$ deg. Thus, the e_{UD} asymmetry observed in vertical-vertical scatters was cos 27° or about 0.9 times e_{RL} of horizontal-horizontal scatters; and the approximation of Eq. (4) seems to have been justified for the evaluation of e.

A. Likelihood Function

A likelihood function can be utilized for the simultaneous determination of polarization and magnetic moment. Without the effect of spin precession, the likeli-

[3] V. Bargmann, L. Michel, and V. L. Telegdi, Phys. Rev. Letters **2**, 435 (1959); D. Nelson, A. Schupp, R. W. Pidd, and H. Crane, *ibid.* **2**, 492 (1959).

hood function for an average P^2 is [as in Eq. (7)]

$$L(P^2)=\prod_{k}^{N}(1+P^2\hat{n}_{1k}\cdot\hat{n}_{2k}), \tag{9}$$

where k is the index number of the event. The precession of $\boldsymbol{\mu}$ caused by the bubble chamber magnetic field changes the spin orientation resulting from first-scattering $\hat{n}_1$ to some new direction $\hat{n}_1'$. The vectors $\hat{n}_1$ and $\hat{n}_1'$ may be described in terms of their components as follows:

With $\hat{k}$ the unit vector along the vertical coordinate and $\hat{\jmath}$ the unit vector in the incident beam direction, the normal to the first scattering plane may be written (with $\bar{K}_{\text{inc}}$ and $\bar{K}_{\text{final}}$ the incident and final momenta)

$$\hat{n}_1=\frac{\bar{K}_{\text{inc}}\times\bar{K}_{\text{final}}}{|\bar{K}_{\text{inc}}\times\bar{K}_{\text{final}}|}=\frac{1}{\sin\theta_1}(\alpha\hat{\imath}+\beta\hat{\jmath}+\gamma\hat{k}). \tag{10}$$

After spin precession through an angle δ, as projected on the horizontal plane, the direction of polarization just before second scattering has been changed from $\hat{n}_1$ to

$$\hat{n}_1'=(1/\sin\theta_1)[(\alpha\cos\delta+\beta\sin\delta)\hat{\imath} +(-\alpha\sin\delta+\beta\cos\delta)\hat{\jmath}+\gamma\hat{k}]. \tag{11}$$

Now, with exact treatment of precession effects, Eq. (9) becomes

$$\begin{aligned}Z_{ijk}(P,\mu)&=1+P^2\hat{n}_{1k}'\cdot\hat{n}_{2k}\\&=1+(P_i^2/\sin\theta_{1k}\sin\theta_{2k})\\&\qquad\times[G_1\cos\delta_j+G_2\sin\delta_j+G_3]_k,\end{aligned} \tag{12}$$

where θ_1 and θ_2 are angles of the first and second scatterings, respectively, and G_1, G_2, and G_3 are geometrical parameters of the events, given in Appendix II. The quantity Z_{ijk} is formed for each event k from nine input quantities: eight geometrical parameters and one momentum (giving the average γ between the two scatterings). Then, one value of P_i is taken, say $P_i=0.50$; with P_i constant, 17 values of μ_j in the range -6 to $+6$ nuclear magnetons are used and 17 Z's are evaluated. In the next step another value of P_i is taken; and again, 17 values of Z evaluated for different assumptions on μ_j. In this fashion, 34 values of P_i^2 in the range -1 to $+1$ are considered, each with 17 assumptions on μ_j for each event; thus, $34\times17=578$ values of Z_{ij} are found for each event k. In practice, we reduced the number of Z_{ij} to 181 for each event, since we narrowed the range of P_i by an iteration procedure. For the next event, say event No. 2, another 181 Z_j are computed. Then each of these 181 Z_{ij}'s from event No. 1 is multiplied by the corresponding Z_{ij} of event 2, etc. The likelihood function so obtained,

$$L(P_i,\mu_j)=\prod_{k=1}^{N}Z_{ijk}, \tag{13}$$

represents a surface in three dimensions. We seek those values for which the surface has a maximum, and so determine the polarization and the magnetic moment of the antiproton simultaneously. The method could be described as follows: we seek the value for μ which corrects the observed vertical asymmetry e_{UD} so as to bring it to a maximum, which cannot be larger than the horizontal asymmetry e_{RL}.

A program called PAP was written to handle this analysis on the IBM 704 computer.

B. Magnetic-Moment Determination from Vertical-Vertical Scatterings Only

By weighting and summing events properly, the value of the magnetic moment can in principle be obtained in a manner similar to the summing of $\cos\Phi_i$ to find asymmetry. The magnetic moment μ of the antiproton appears explicitly in the rotated vector $\hat{n}_1'$ used in the general distribution and likelihood expressions, since the precession angle $\delta=\gamma[\frac{1}{2}g-1+1/\gamma]\Delta=\Gamma\Lambda$. Greatest sensitivity to the value of magnetic moment is obtained by treating only vertical-vertical scatterings, for which (incident track assumed nearly parallel to $\hat{\jmath}$ axis),

$$\begin{aligned}\hat{n}_1&\approx\alpha_1\hat{\imath},\\ \hat{n}_1'&\approx(\alpha_1\cos\delta)\hat{\imath}+(-\alpha_1\sin\delta)\hat{\jmath},\\ \hat{n}_1'\cdot\hat{n}_2&\approx\alpha_1\alpha_2\cos P-\alpha_1\beta_2\sin\delta.\end{aligned} \tag{14}$$

The distribution function for second scattering then becomes

$$\begin{aligned}I(\theta,\mu)&=I_0(\theta)[1+P^2(\alpha_1\alpha_2\cos\delta-\alpha_1\beta_2\sin\delta)/\sin\theta_1\sin\theta_2]\\&=I_0(\theta)[1\pm P^2\sin^2\phi_i(\cos\Delta\cos\delta+\sin\Delta\sin\delta)].\end{aligned} \tag{15}$$

Here, Δ refers to the angle by which the antiproton is turned (cyclotron deflection) in going from first to second scattering and ϕ_i is the azimuthal angle of the particle incident at the first scattering; the plus sign before the P^2 term is appropriate for up-up and down-down scatterings, while the minus sign is appropriate for up-down and down-up scatterings. Evidently, the weighting of events with $\cos\Delta$ and summing should permit an evaluation of the quantity μ. Thus,

$$\sum_i\cos_+\Delta_i-\sum_i\cos_-\Delta_i=\frac{N_+\int(1+P^2\cdots)\cos\Delta d\Delta}{\int(1+P^2\cdots)d\Delta}-\frac{N_-\int(1-P^2\cdots)\cos\Delta d\Delta}{\int(1-P^2\cdots)d\Delta}, \tag{16}$$

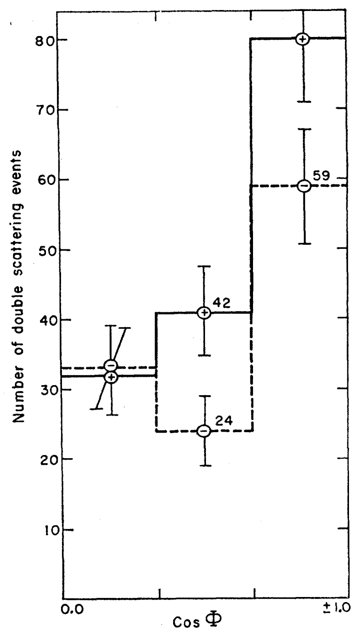

FIG. 2. Number of double-scattering events vs cosϕ. In this diagram, + and − refer to the sign of cosϕ.

where N_+ is the total of up-up and down-down events and N_- is the total of up-down and down-up events. The left side of Eq. (16) represents the sum of experimentally determined deflection-angle cosines; the right side contains expressions for average cosines based on the approximate theoretical distribution (see Appendix III) and weighted by the observed numbers of events.

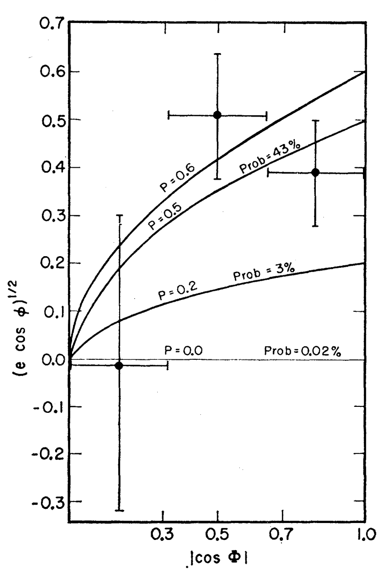

FIG. 3. The quantity $(e\cos\phi)^{1/2}$ vs $|\cos\phi|$ as determined from experimental data with Eq. (3b). Values of polarization P used to calculate the expected $(e\cos\phi)^{1/2}$ or $(P^2\cos\phi)^{1/2}$ dependence on $|\cos\phi|$ are indicated. Evidently $P=0.5$ gives a good fit to the data.

TABLE I. Results on asymmetry and polarization at large angles (from 6.3 to 23.6°).

	Likelihood function	Summation formula 1960	Summation formula 1961
Number of events	197	155	125
$\theta_{av}^{c.m.}$ (deg)	25	25	22
e	0.26±0.10	0.234±0.11	0.189±0.22
P	0.51±0.10	0.485±0.11	0.435±0.25
Momentum selection criterion (BeV/c)	>1.57[a]	>1.60	>1.57
Fitting method	Stereo-plotting	Stereo-plotting	KICK

[a] Including 40 2.0-BeV/c events.

EXPERIMENTAL PROCEDURE

The separated (1.61±0.20)−BeV/c (960-MeV) antiproton beam has been described elsewhere.[4] An integrated flux of 4.6×10^4 antiprotons entered the 72-in. bubble chamber during the exposure. The statistics of the sample of double-scattering events related to this measurement are listed below.

(1)	Number of (2-prong) → (2-prong) events observed at scanning	900
(2)	Rejected on scanning tables as inelastic, unmeasurable, outside the useful volume of the chamber, or due to π meson (δ rays>1.7 cm)	276
(3)	Recoil-proton rescatterings	159
(4)	Number of $2p\rightarrow 2p$ events measured: (1)-(2)-(3)	465
(5)	Rejected as noncoplanar, KICK rejects, incident momentum below 1.6 BeV/c (probable pion), and angles >24 deg (ΔE too large)	172
(6)	Total identified as p-$\bar{p}$, elastic, double-scattering events above 1.60 BeV/c and in the angular region 3 to 24 deg	293

The upper limit of 24 deg lab to the scattering angle corresponds to 54 deg c.m., and to a momentum loss larger than 200 MeV/c (energy loss ≈170 MeV).

We have not made a thorough check on whether the initial antiproton beam was polarized. However, a

TABLE II. Results on asymmetry and polarization at angles from 4.0 to 23.6°, with one or both scatterings at small angles, between 4 and 6.3°.

	Likelihood function	Summation formula 1960	Summation formula 1961
Number of events	69	91	31
$\theta_{av}^{c.m.}$ (deg)	17.5	17.5	13.2
e	0.11±0.8	0.018±0.15	0.199±0.25
P	0.35±0.13	0.133±0.57	0.445±0.28

[4] J. Button, P. Eberhard, G. R. Kalbfleisch, J. Lannutti, G. R. Lynch, B. C. Maglić, M. L. Stevenson, and N. Xuong, Phys. Rev. **121**, 1788 (1961).

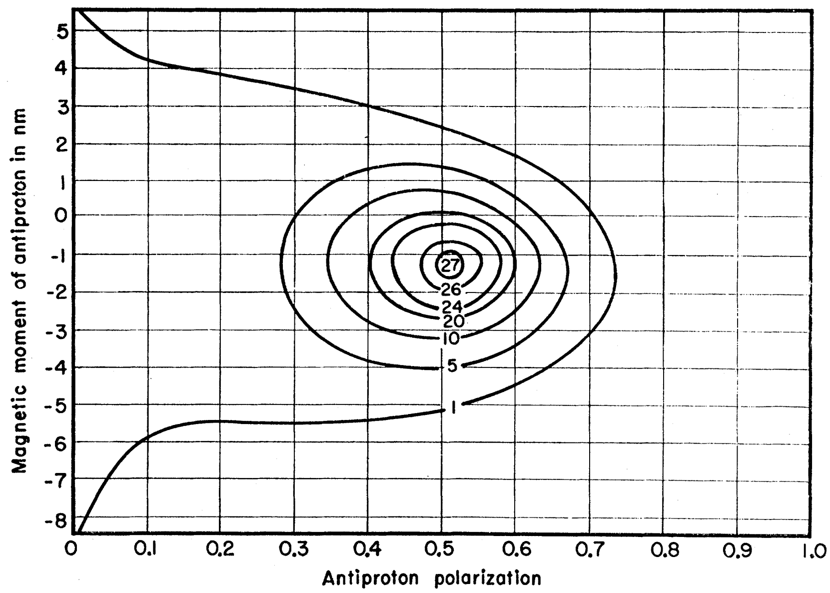

FIG. 4. Dependence of the likelihood function on polarization and magnetic moment of the antiproton.

simple test we made has shown that, if there is polarization, it is not large. This was done by measuring single-scattering events and forming a likelihood function of the type $1+e\cos\phi$, where ϕ is the angle between the normal to the scattering plane and the vertical axis of the chamber. (Any polarization resulting from the production process must necessarily be along the vertical axis.) A sample of 131 single scatterings through an angle $\theta > 6°$ has yielded $e = (6\pm 14)\%$.

We assumed throughout our work that the initial beam was not polarized. Our results will have to be slightly modified if this assumption proves to be unjustified by some later experiment.

RESULTS: I. POLARIZATION

To obtain the polarization, the angle ϕ between the two scattering planes was computed for each event by using fitted values for azimuthal and dip angles. The distribution of $\cos\phi$ is shown in Fig. 2. The values of $e\cos\phi$ and hence P obtained from these events are shown in Fig. 3.

We have divided our sample into two groups, according to the scattering angle θ. The "small-angle" results are probably subject to some bias because of inefficiency in scanning. Results on asymmetry and polarization are given in Tables I and II.

TABLE III. Summation-formula evaluation of magnetic moment.

$(1/N_-)(\Sigma\cos_+\Delta - \Sigma\cos_-\Delta)$	$(1/N_-)(N_+\langle\cos_+\Delta\rangle - N_-\langle\cos_-\Delta\rangle)$			
	$\Gamma = -10.0$	-4.0	0	$+10.0$
	$g/2 = +4.5$	$+1.5$	-0.5	-5.5
0.75 ± 0.30	0.388	0.387	0.385	0.388

RESULTS: II. MAGNETIC MOMENT

An attempt was made to use Eq. (16) to obtain μ from a sample of vertical-vertical scatterings, all of which had been selected not only to satisfy criteria on incident momentum, angles, and kinematic fitting, but also selected to give scattering-plane normals within $\pm 45°$ of the horizontal. The total number of such events was 55, with $N_+ = 32$ and $N_- = 23$. The values of Δ ranged from 0° to 17°; the $\cos\Delta$ sums were weighted to correct for the decrease in the number of events with increasing Δ. The method did not yield a conclusive result even on the sign of μ because of the small number of events, the loose restriction on the normals, and the small range of Δ. The experimental $\cos\Delta$ sums are compared with the theoretical values for different μ or Γ assumptions in Table III.

RESULTS: III. SIMULTANEOUS EVALUATION OF POLARIZATION AND MAGNETIC MOMENT BY THE LIKELIHOOD METHOD

Figure 4 shows the contours of the likelihood function $L(P,\mu)$. Best values for P and μ are given in Tables I, II, and IV.

TABLE IV. Results on magnetic moment.

	Likelihood function 1960	Likelihood function 1961	Summation formula
Momentum-selection criterion (BeV/c)	>1.60	>1.57	>1.57
Fitting method	Stereo-plotting	KICK	KICK
Number of double scattering event	197	125	55
μ (nm)	-1.4 ± 1.4	-3.25 ± 2.75	Inconclusive
Average μ (nm)	-1.8 ± 1.2		

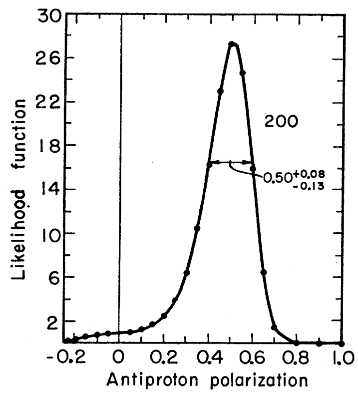

FIG. 5. Likelihood function vs antiproton polarization.

Figure 5 shows the likelihood function vs polarization for the experimental best value $\mu=-1.4$ nm. In Fig. 6(a) is shown the cut through the three-dimensional likelihood surface for the best value of polarization $P=0.5$. This yields $\mu=-1.4\pm1.4$ nm. In Fig. 6(b), a smaller but more carefully selected sample of events has been used to obtain a similar plot yielding the value $\mu=-3.25\pm2.75$ nm. Statistics are evidently inadequate to give a well-defined magnetic moment; however, it seems plausible that the true value lies somewhere between -1.4 and -3.25, since the former answer was obtained with some admixture of (unpolarized) background. An average of the two answers gives a value of -1.8 ± 1.2 nm.

SYSTEMATIC ERRORS

In our first analysis (1960), all events were stereo plotted. Those events which were coplanar were accepted if both $\bar{p}$ and p scattering angles and momenta satisfied kinematics. This was done by using a set of graphs. The minimum momentum cutoff used was 1.60 BeV/c.

In our second analysis (1961), all events were fitted by using KICK, and slightly lower momenta (≥ 1.57 BeV/c) were accepted. Two hypotheses were tested, $\bar{p}+p \rightarrow \bar{p}+p$ and $\pi^-+p \rightarrow \pi^-+p$, for each scattering. The χ^2 distribution for the first hypothesis was characterized by a peak between $\chi^2=1$ and 5 and a long tail extending to $\chi^2\approx50$ with an average $\chi^2\approx6$. We used these criteria: (a) fit must be better in both scatterings for the $\bar{p}$-p hypothesis than for π-p, and (b) $(\chi^2)_{\max}=40$. This and also the angular and momenta criteria were built into the program, so that the events were selected automatically, thus avoiding biases.

We have made checks on possible biases, which would come from the preference of scanners for one side over another. This is particularly plausible in view of the fact that tracks bend in one direction (right) and therefore small-angle scatterings to the right would be less easily seen. It was seen that one can easily eliminate this bias by asking the scanners to note all scatterings above a small angle, about 1°, and in the analysis use two to three times as large an angle as the lower limit, as we have done. Further, we have plotted angular distributions both in projected angle (seen by the scanner) and space angles for left and right, and convinced ourselves that the shapes are equal. Similarly, distribution in azimuthal angle, dip angle, and horizontal angle were studied for left and right separately, to prove that there are no left-right biases.

These exists a "built-in" geometrical bias: The magnetic field deflects particles to the right and hence, those antiprotons scattered the first time to the right (left) after the entrance to the bubble chamber. will have shorter paths (longer) after the first scattering. Thus, the probability is that the second scattering that occurs will be lower (higher) for the events that scatter the first time to the right (left). Since we select only double-scattering events, it will result in a larger number of events that scatter the first time to the left. But, this left-right asymmetry in the first scattering should be equal to the left-right asymmetry in the path length between the two scatterings, which is of the order of 1.15. We have shown that this is indeed so. Because of this bias, it was not possible to determine the polarization of the incoming beam by looking at the double-scattering sample.

TEST OF THE LIKELIHOOD FUNCTION

To test our likelihood-function method, we have done the following fake "experiments":

(a) Instead of the nine parameters from the measured double-scattering events, we have fed random numbers

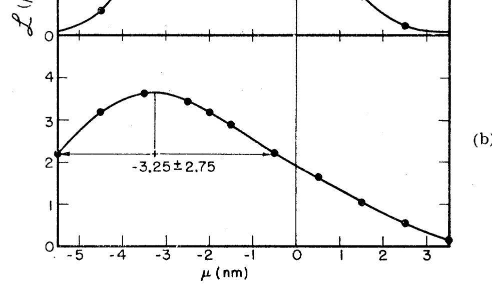

FIG. 6. The likelihood function vs magnetic moment for fixed polarization $P=0.50$. Results shown in 6(a) come from the hand-plotted sample of 197 events analyzed in 1960. Results of 6(b) come from the more carefully selected sample of 125 events analyzed in 1961.

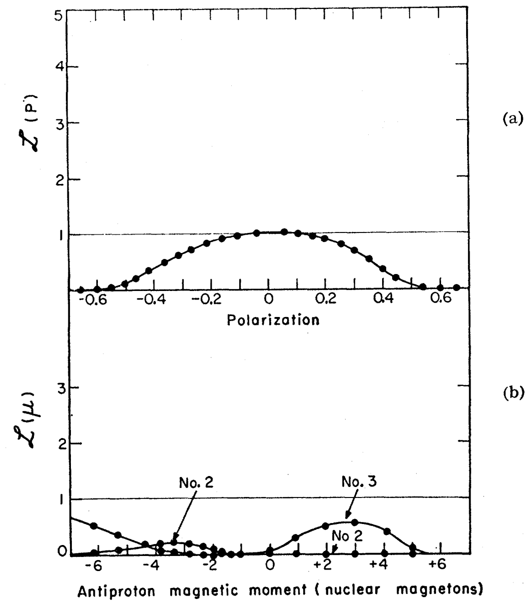

FIG. 7. Random-number experiments. (a) Likelihood function vs polarization. Results using random-number input data from tables and from mixing of experimental measurements were identical. (b) Likelihood function vs magnetic moment. Results labeled No. 2 were obtained with random numbers from tables; results labeled No. 3 were obtained with mixing of experimental data.

into the likelihood function. The numbers were taken out of the Rand Corporation Table of Random Numbers, and their decimal points chosen so as to be within the range of the parameters used in the actual experiment (see Fig. 7, random-number experiment No. 2).

(b) We have "mixed" double-scattering events by using parameters from the second scattering of one event and first scattering of another event, and feeding them into the likelihood function (see Fig. 7, random-number experiment No. 3). Both fake experiments yield zero polarization and no peak in the magnetic moment distribution above $L=1$.

CONCLUSIONS

Our results can be interpreted as evidence for the spin of the anti-proton and for the sign of its magnetic moment.

On the basis of the *CPT* theorem, the magnetic moment of an antiparticle is expected to have a sign opposite to that of the corresponding particle. Our result (Table III) establishes the negative sign of the antiproton magnetic moment.

As for the spin properties of the antiproton, we had hoped to go a little further and draw more specific conclusions than just the statement that the antiproton has a spin (of 1/2). Unfortunately, this experiment has not determined the angular dependence of polarization in the region of Coulomb scattering; hence, no conclusion can be drawn as to the sign of polarization.

APPENDIX I. DERIVATION OF $\delta e/e=(1/e)[(2-e^2)/N]^{\frac{1}{2}}$

This is the fractional error in asymmetry e as determined by the expression

$$e=2\sum_i \cos\phi_i/N,$$

with N the total number of events and ϕ_i the angle between first and second scattering normals for each event. If the number of events for each value of $\cos\phi_i$ is given by n_i, so that the expression is written

$$e=2\sum n_i \cos\phi_i/N,$$

then n_i and N are the statistically distributed (but correlated) quantities in the evaluation of e.

Since

$$(\delta e)^2=\sum_i\left(\frac{\partial e}{\partial x_j}\right)^2(\delta x_j)^2+\sum_{j\neq k}\left(\frac{\partial e}{\partial x_j}\right)\left(\frac{\partial e}{\partial x_k}\right)\delta x_j\delta x_k,$$

with $x_j=n_1, n_2, \cdots, N$, then the average of

$$(\delta e)^2=\sum_i\left(\frac{\partial e}{\partial n_i}\right)^2(\delta n_i)^2+2\sum_i\left(\frac{\partial e}{\partial n_i}\right)\left(\frac{\partial e}{\partial N}\right)\times\delta n_i\delta N+\left(\frac{\partial e}{\partial N}\right)^2(\delta N)^2$$

$$=\left(\frac{2}{N}\right)^2\sum\cos^2\phi_i(\delta n_i)^2+2\left(\frac{2}{N}\right)\sum_i\cos\phi_i\left(-\frac{2}{N^2}\right)\sum_j n_j\cos\phi_j\delta n_i\delta n_i+\left(-\frac{2}{N^2}\right)^2\left(\sum n_i\cos\phi_i\right)^2(\delta N)^2.$$

As (δe^2) should be considered the average of the square of the e error, it is reasonable that the above includes $\delta n_i\delta N$ correlation terms, but no $\delta n_i\delta n_j$ terms. Thus,

$$(\delta e)^2=\left(\frac{2}{N}\right)^2\sum_i(\cos^2\phi_i)n_i-\left(\frac{2}{N}\right)^3\left(\sum_i n_i\cos\phi_i\right)\left(\sum_j n_j\cos\phi_j\right)+\frac{4}{N^4}\left(\sum_i n_i\cos\phi_i\right)^2N$$

$$=\left(\frac{2}{N}\right)^2\frac{N}{2}-\frac{2}{N}e^2+\frac{1}{N}e^2=(2-e^2)/N.$$

APPENDIX II. LIKELIHOOD FUNCTION

Let $\hat{n}_1$ and $\hat{n}_2$ be unit normals to the first and second scattering planes, respectively, and let $\hat{n}_1'$ define the spin orientation just before the second scattering. An element of the likelihood function for a double scattering is

$$Z=1+P^2\hat{n}_1'\cdot\hat{n}_2. \tag{1}$$

If $\hat{n}_1$ (right after first scattering) is defined with its components along the Cartesian axes of the bubble chamber,

$$\hat{n}_1 = a^{-1}(\alpha_1\hat{\imath}+\beta_1\hat{\jmath}+\gamma_1\hat{k}), \tag{2}$$

the precession of the horizontal spin components through an angle δ [see Eq. (8) in the text] defines the $\hat{n}_1'$:

$$\hat{n}_1' = a^{-1}[(\alpha_1\cos\delta+\beta_1\sin\delta)\hat{\imath} + (\beta_1\cos\delta-\alpha_1\sin\delta)\hat{\jmath}+\gamma_1\hat{k}]. \tag{3}$$

If $\hat{n}_2$ is defined by

$$\hat{n}_2 = b^{-1}(\alpha_2\hat{\imath}+\beta_2\hat{\jmath}+\gamma_2\hat{k}), \tag{4}$$

the likelihood function becomes, from (1),

$$Z_{ij} = 1+(ab)^{-1}P^2[(\alpha_1\alpha_2+\beta_1\beta_2)\cos\delta_j + (\beta_1\alpha_2-\alpha_1\beta_2)\sin\delta_j+\gamma_1\gamma_2]. \tag{5}$$

In these equations, $a=\sin\theta_1$, $b=\sin\theta_2$, where θ_1 and θ_2 are scattering angles of the first and second scattering, respectively; the geometrical parameters α, β, and γ are obtained from the measurements of the dip λ, and the horizontal angle ϕ for the incident (i) and scattered final (f) track at each vertex. The normal $\hat{n}$ may be defined as $(\bar{K}_i\times\bar{K}_f)/|\bar{K}_i\times\bar{K}_f|$, where $\bar{K}_i$ and $\bar{K}_f$ are incident and final momenta. Then the parameter α, or the $\hat{\imath}$ component of $(\sin\theta)\hat{n}=(\bar{K}_i\times\bar{K}_f)/|\bar{K}_i||\bar{K}_f|$, is given by

$$\alpha=\cos\lambda_i\sin\phi_i\sin\lambda_f-\cos\lambda_f\sin\phi_f\sin\lambda_i.$$

We have used the abbreviations

$$\alpha_1\alpha_2+\beta_1\beta_2=G_1,$$
$$\beta_1\alpha_2-\alpha_1\beta_2=G_2,$$
$$\gamma_1\gamma_2=G_3.$$

APPENDIX III. SUMMATION FORMULA FOR MAGNETIC MOMENT

As shown in Appendix II, the expression for double-scattering cross section is

$$I(\theta_1,\theta_2,\delta)=I_0\{1+P^2[(\alpha_1\alpha_2+\beta_1\beta_2)\cos\delta + (\alpha_2\beta_1-\alpha_1\beta_2)\sin\delta+\gamma_1\gamma_2]/\sin\theta_1\sin\theta_2\}.$$

If the two scattering planes are nearly vertical, this reduces to

$$I(\theta_1,\theta_2,\delta)\approx I_0[1+P^2(\alpha_1\alpha_2\cos\delta-\alpha_1\beta_2\sin\delta)/\sin\theta_1\sin\theta_2],$$

where it is assumed that the first normal has only the component α_1. By making the further approximations for initial (i) and final (f) angles

$$\phi_{1f}\sim\phi_{1i},\quad \lambda_{1f}=\theta_1,\quad \lambda_{2i}=\theta_1,$$
$$\phi_{2f}=\phi_{2i}=\phi_{1f}-\Delta,\quad \lambda_{2f}=\theta_1\pm\theta_2,$$

it is possible to reduce this expression to

$$I(\theta_1,\theta_2,\delta)\approx I_0[1\pm P^2\sin^2\phi_i(\cos\Delta\cos\delta+\sin\Delta\sin\delta)]$$

[see Eq. (15)].

Weighting events with $\cos\Delta$ permits the evaluation of the magnetic moment μ. The relationship given in Eq. (16) becomes, with the evaluation of the integrals,

$$\textstyle\sum_i \cos_+\Delta_i-\sum_i\cos_-\Delta_i=N_+A^+/B^+-N_-A^-/B^-,$$

with

$$A^\pm=\sin\Delta_m\pm\langle P^2\rangle_{\text{av}}\sin^2\phi_i[\Gamma/(4-\Gamma^2)] \times\{\sin2\Delta_m\cos\Gamma\Delta_m/\Gamma-\cos^2\Delta_m\sin\Gamma\Delta_m +2\sin\Gamma\Delta_m/\Gamma^2+\sin2\Delta_m\cos\Gamma\Delta_m/2 -\cos2\Delta_m\sin\Gamma\Delta_m/\Gamma\},$$

$$B^\pm=\Delta_m\pm\langle P^2\rangle_{\text{av}}\sin^2\phi_i[\Gamma/(1-\Gamma^2)] \times\{\sin\Delta_m\cos\Gamma\Delta_m/\Gamma-\cos\Delta_m\sin\Gamma\Delta_m +\sin\Delta_m\cos\Gamma\Delta_m-\cos\Delta_m\sin\Gamma\Delta_m/\Gamma\},$$

where Δ_m is the maximum deflection angle.

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