## Specific Heats of Rubidium and Cesium between 1.3 and 12'K

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The specific heats of rubidium and cesium were measured between  $1.3^{\circ}$  and  $12^{\circ}$ K, and Debye  $\Theta$  values for the two metals were calculated from the data. Each metal exhibited a maximum and a minimum  $\Theta$  value in this temperature range. In the case of rubidium the minimum  $\Theta$  was 50 deg at 3.3°K and the maximum  $\Theta$ was 56 deg at 9°K. For cesium the minimum  $\Theta$  was 36 deg at 2.0°K and the maximum  $\Theta$  was 45 deg at 9°K.

## INTRODUCTION

'HE contribution of lattice vibrations to the energy or specific heat of a solid can be calculated, if the number of normal modes per unit frequency interval is known. Various approximate calculations of such lattice vibrational spectra have been done.<sup>1</sup> It is interesting to compare experimental specific-heat data with the specific heats derived from theory, even though the specific heat is not very sensitive to the finer details of lattice vibrational spectrum.

### APPARATUS AND PROCEDURE

Specimens used in these experiments were obtained from A. D. Mackay, Inc. The rubidium sample weighed 10.2 g, the cesium sample 8.18 g. Both were prepared in the same manner. The metal was melted in a vacuum, poured into a thin-walled Pyrex tube, and the tube was sealed off. A 30-mil-diam tungsten wire was sealed through one end of the Pyrex tube so that the wire extended into the middle of the metal inside. Outside, the tungsten was attached to a small piece of copper that the jaws of a mechanical thermal switch clamped onto, when closed. A carbon resistance thermometer was attached to this copper piece with Wood's metal. Heat was supplied to the specimen from approximately  $1000 \Omega$  of 1.5-mil-diam manganin wire wound around the Pyrex tube. Thermal contact between heater and alkali metal was considered satisfactory as the thermometer response was almost immediate when the heater power was turned on.

Data in the temperature range below  $4^\circ K$  were obtained in an apparatus in which a mechanical thermal switch connected the specimen to the helium bath during cooling and thermometer calibration. Above 4'K a different apparatus was used, in which a mechanical thermal switch connected the specimen to the copper bulb of a gas thermometer of the type suggested by Woodcock.<sup>2</sup> The gas thermometer, which featured a large dead space in an ice bath, was calibrated at the boiling points of nitrogen and helium and then used to

calibrate the carbon-resistance thermometer with which the specific heat measurements were made. The same equipment and techniques yielded results in good agreement with values taken from the literature when used to measure the specific heat of graphite between 1.5 and 10'K.

In both cases, the alkali metal accounted for  $95\%$  of the total heat capacity of the specimen. The contributions of the Pyrex and other materials present were calculated from specific heat values in the literature. It was assumed that the specific heat of the alkali metal can be regarded as the sum of an electronic term and a lattice term. An accurate determination of the electronic term could not be made at the lowest temperatures reached in the present experiments, corresponding to approximately  $\Theta/35$  for rubidium and  $\Theta/25$  for cesium. Recent experiments<sup>3</sup> below  $1^{\circ}$ K yield 2.45 T mJ per mole deg as the electronic term for rubidium, and 3.47 T mJ per mole deg as the electronic term for cesium. These values were used to compute electronic contributions which were subtracted from the specific heats of the alkali metals before calculation of the Debye  $\Theta$ values.

#### RESULTS AND DISCUSSION

Figure 1 shows a plot of Debye  $\Theta$  values vs temperature for the two metals. For cesium,  $\Theta$  has a minimum



FIG. 1. Debye  $\Theta$  vs T. Triangles represent Phillips' data from experiments below  $1^{\circ}K$ .<sup>3</sup>

<sup>3</sup> N. E. Phillips (private communication).

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<sup>1</sup> M. Blackman, *Handbuch der Physik*, edited by S. Flügge<br>
(Springer-Verlag, Berlin, 1955), Vol. VII.<br>
<sup>2</sup> A. H. Woodcock, Can. J. Research **A16,** 133 (1938).

value of 36 deg at 2.0'K and <sup>a</sup> maximum value of <sup>45</sup> deg at about  $9^{\circ}K$ . For rubidium,  $\theta$  has aminimum value of 50 deg at 3.3'K and a maximum value of 56 deg at about  $9^{\circ}K$ . Manchester<sup>4</sup> has reported  $\Theta$  values for rubidium between  $1.2^{\circ}$  and  $4.2^{\circ}$ K. He finds an approximately constant  $\Theta$  below 1.6°K and a minimum value of 51.<sup>2</sup> deg at 2.8'K. His results were not corrected for the electronic contribution. The subtraction of an electronic term from the specific heat would make  $\Theta$  rise at lower temperatures and might shift the location of the minimum to a slightly higher temperature, but it could not account for the difference in absolute values of  $\Theta$  minimum found in these experiments.

 $\sigma$  minimum round  $\mu$  and  $\sigma$  values of 61 deg for rubidium at  $25\textdegree K$  and 44 deg for cesium at  $20\textdegree K$ . These results suggest the existence of at least one more minimum in

5T. M. Dauphinee, Douglas L. Martin, and H. Preston-Thomas, Proc. Roy. Soc. (London) A233, 214 (1955).

the  $\Theta$  vs temperature curve for each metal between 12 $\degree$ and  $20^{\circ}$  or  $25^{\circ}$ K.

Bauer<sup>6</sup> and Bhatia<sup>7</sup> have calculated the vibrational spectra of body-centered cubic lattices, using the elastic constants of sodium. Their results are expressed in terms of  $\Theta$  vs temperature curves which typically show a high  $\theta$  at  $0^{\circ}$ K with a minimum at low temperatures followed by an approximately constant  $\theta$  at higher temperatures. These curves are similar to, but less complicated than, those from the experiments on rubidium and cesium.

#### ACKNOWLEDGMENT

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<sup>6</sup> E. Bauer, Phys. Rev. 92, 58 (1953).<br>7 A. B. Bhatia, Phys. Rev. 97, 363 (1955).

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# Effect of Quadrupolar Polarizability upon Field Gradient in Tetragonal Lattices\*

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In the course of investigating sources of electric 6eld gradient in the body-centered tetragonal structure of indium, a study of the effect of quadrupole moments induced in the ions themselves has been made. An eight-figure table covering a wide range of  $c/a$  values and giving the axial field gradient per unit axial quadrupole moment in a tetragonal lattice of quadrupolarly polarized ions has been constructed. The effects of the ions at  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$  and those at  $(0,0,0)$  have been listed separately in the interest of versatility. Using a rough estimate of the quadrupolar polarizability of the indium ion, the net effect of the induced moments in this material is shown to be small. Larger effects could be expected in other substances.

## I. INTRODUCTION

N a recent article,<sup>1</sup> R. R. Hewitt and the author investigated nuclear quadrupole resonance and possible sources of the electric field gradient (EFG) in metallic indium. One possible source, however, was made the subject of a separate investigation and is reported in the present article. That source is the effect of the quadrupole moments induced in the ions themselves.

The principal result of the present work is a detailed table (Table I) of the axial EFG per unit axial quadrupole moment generated in a tetragonal lattice of quadrupolarly polarized ions. This table, which separates the effects of the ions at  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$  from those at  $(0,0,0)$  and which covers the range  $0.50 \le c/a \le 3.20$ , is intended to apply not only to indium but to a variety of tetragonal structures including compounds as well as elements. The extensive numerical work involved in the construction of this table and in other calculations discussed in this article was performed on an IBM 1620 automatic digital computer maintained by the Biometrical Laboratory of the Citrus Research Center and Agricultural Experiment Station located at this campus of the University of California.<sup>2</sup>

Since the quadrupole moment induced in the ions depends upon the total EFG present and at the same time helps to determine this quantity, the total EFG must be found by solving a simple algebraic equation containing the quantity  $F_Q$  given in Table I, the quadrupolar polarizability, and the EFG due to primary sources, e.g., the monopole moments of the ions and the electron density in the interionic space. The only information presently available on the quadrupolar polarizability of the  $In<sup>3+</sup>$  ion is, unfortunately, in the form of a rough estimate. On the basis of this estimate, it is found that the effect of the induced quadrupole

<sup>4</sup> F. D. Manchester, Can. J. Phys. 37, <sup>525</sup> (1959).

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