However, because $J(x)$ experimentally has a sharper dependence on T than predicted, we must expect H_c to decrease more rapidly with T than predicted by Eq. (6.5). In the residual resistance range, however, H_c should become temperature independent. To date, the sign reversal at liquid helium temperatures has not been found. In our sample, the expected H_c is much smaller than 0.44 kOe, the smallest field available to us. At liquid nitrogen temperatures, H_c should be about 17 kOe, also inaccessible to us. Some tendency toward a reversal is descernible in the data of Connel and Marcus.[1]

Electrical Properties of n-Type Gallium Arsenide

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As part of a general investigation into the properties of gallium arsenide, the Hall coefficient and resistivity of three pure samples have been measured from 300 to 2°K. The highest mobility measured at 300°K was 7200 cm²/V-sec increasing to 22 000 cm²/V-sec at 72°K, the mobility temperature curve following the theory of Ehrenreich very closely. At helium temperatures impurity conduction was observed with the resistivity $\rho \propto \exp(-\epsilon_1/kT)$ and ϵ_1 varying from 9.8×10^{-5} eV to 5.8×10^{-4} eV. A discussion of these results is given in terms of the theory of Miller and Abrahams and Twose.

In one sample, below about 30°K the current was not a linear function of the voltage. This arises from a dependence of mobility on applied electric field and because carriers can be excited from impurity states to the conduction band. An analysis of the energy loss processes for electrons suggest that piezoelectric scattering is the most important mechanism below about 30°K but that above this temperature optical scattering is dominant. Impact ionization occurs between 4 and 12°K.

A. INTRODUCTION

MEASUREMENTS of the electrical properties of a semiconductor provide much basic information on their properties and particularly on the mechanisms of charge transport and the nature of the scattering processes. This paper describes some measurements of Hall coefficient and conductivity, on samples of low resistivity n-type gallium arsenide over the temperature range 300 to 2°K. Previous investigations have been adequately described in the book by Hilsum and Rose-Innes,[1] but the results described here have been obtained on purer material than was previously available and are thus of special interest.

The discussion of the experimental results falls naturally into three sections. The material available was pure enough for the mobility above about 60°K to be determined largely by lattice scattering. Ehrenreich[2] has recently calculated the theoretical temperature dependence of electron mobility in gallium arsenide on the assumption that scattering by the polar optical modes is the dominant lattice process and our results therefore provide a convenient check on this theory, the agreement being very satisfactory.

At low temperatures the measurements show evidence of impurity conduction in a more pronounced way than has previously been possible in n-type gallium arsenide. The results illustrate the difference between the two possible types of impurity conduction, the "hopping" process which occurs for low impurity concentrations and the metallic type of conduction which takes place in less pure samples.

In one sample at low temperatures, departures from Ohm's law were observed, and to explain these results a discussion is given of the various mechanisms by which electrons lose energy and also of the possibility of electrons being transferred from impurity states to the conduction band under the action of an electric field.

B. EXPERIMENTAL TECHNIQUE

The specimens used were rectangular bars (typically $0.6 \times 0.2 \times 0.1$-cm), cut from single crystal material. After grinding, etching ($1H_2O \ 3H_2SO_4/1H_2O_2$), and washing in de-ionized water, six contacts were formed on the specimen by alloying to it small pellets of indium. Reliable low-resistance Ohmic contacts could readily be made in this way.

A large number of specimens have been examined from room temperature to liquid nitrogen temperature using the simple apparatus described by Ure.[3]

[1] C. Hilsum and A. C. Rose-Innes, *Semiconducting III-V Compounds* (Pergamon Press, New York, 1961).
[2] H. Ehrenreich, Phys. Rev. **120**, 1951 (1960).
[3] R. W. Ure, Rev. Sci. Instr., **28**, 836 (1957).

Measurements on a smaller number of samples have been made in a conventional helium cryostat in which temperatures down to about 1.5°K could be obtained. A carbon resistance thermometer, calibrated against a helium gas thermometer, was used in measuring the temperature of the specimen. The electrical measurements were made by standard potentiometric techniques and current-voltage characteristics were displayed on the X-Y recorder.

C. EXPERIMENTAL RESULTS

The detailed discussion is confined to the results on three specimens whose important properties are given in Table I. The observed variation of Hall coefficient and resistivity is depicted in Figs. 1 and 2.

As the temperature is lowered, the Hall constant in all cases rises to a maximum and then falls again, eventually becoming constant at the lowest temperatures. This behavior is similar to that found in a number of other semiconductors (e.g., Si)[4] and is typical of the situation in which the donors are gradually de-ionized but in which electron transport is possible between the impurity states. The temperature variation of resistivity supports this hypothesis. The resistivity-temperature curve exhibits a minimum just below room temperature but on further cooling it rises first sharply and then less rapidly, the change in slope occurring at about the same temperature as the maximum observed in the Hall constant.

The temperature variation of Hall mobility $\mu_H = R/\rho$ is shown in Fig. 3. The relative values of the mobilities of specimens Nos. 1 and 2 at 292°K suggest that specimen No. 1 is somewhat purer, and this is in accord with
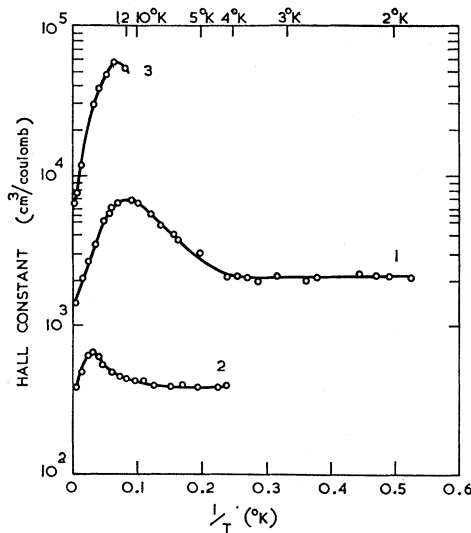
TABLE I. The basic parameters of the three samples discussed.

Specimen number	1	2	3
$R_H(292°K)$, cm³ C⁻¹	1420	417	6650
$\rho(292°K)$, Ω cm	0.2	0.063	2.34
$\mu = R_H/\rho(292°K)$, cm²/V sec	7200	6600	2840
μ_{max}, cm²/V sec	22 000	11 500	7000
T for μ_{max}, °K	72	85	95

the fact that the mobility of this sample at the peak of the μ_H-T curve is larger than for specimen No. 2 and occurs at a slightly lower temperature. At very low temperatures the mobility is determined by impurity conduction, so that the purer specimen has the lower mobility. Specimen No. 3 behaves anomalously because, although the room temperature mobility of 2840 cm²/(V sec⁻¹) suggests an impurity concentration of 10^{18} cm⁻³, the results at low temperatures are inconsistent with such a high value. It is possible that an additional scattering mechanism of unknown origin such as is described by Weisberg and Blanc[5] is operating in this specimen.

For specimens Nos. 1 and 2 the current-voltage curves were linear up to the currents employed (~ 10 mA) but below about 30°K pronounced departures from Ohm's law were observed in specimen No. 3 for electric fields greater than about 4 V/cm.

The interpretation of the results falls conveniently into three sections, the first dealing with temperatures above about 60°K where the interest centers on the temperature variation of mobility, the second with the mechanisms of impurity conduction and the final section dealing with the non-Ohmic effects observed in one sample.

1. Lattice Scattering Range

Ehrenreich[2] has discussed the scattering of electrons in gallium arsenide and concluded that the dominant mechanism for lattice scattering is the polar interaction. In these circumstances, the mobility is given by the expression

$$\mu(\text{cm}^2/\text{V-sec}) = 0.176(T/300)^{1/2}(e/e^*)^2$$
$$\times (m/m^*)^{3/2}(10^{22}M)(10^{23}V_a)$$
$$\times (10^{-13}\omega_l)(e^z-1)[G_1(z,\eta)/(F_{1/2}(\eta))], \quad (1.1)$$

where M is the reduced mass of the two ions in the unit cell, V_a is the volume of the cell, ω_l is the longitudinal optical frequency, $z = \hbar\omega_l/kT$, e^* is the effective ionic charge defined by Callen, and $F_{1/2}$ is the usual Fermi-Dirac integral. The function G_1 depends on the combined effects of the relevant scattering processes and has been calculated by Ehrenreich for a sample of GaAs with an impurity concentration of 2.2×10^{16} cm⁻³. The resulting theoretical curve for the variation of mobility with

FIG. 1. The Hall coefficient for three samples of gallium arsenide as a function of temperature.

[4] H. Fritzsche and K. Lark-Horovitz, Phys. Rev. 113, 999 (1959).

[5] L. R. Weisberg and J. Blanc, Bull. Am. Phys. Soc. 5, 62 (1960).

temperature is plotted in Fig. 3 together with our experimental results. A rough estimate of the total ionized impurity concentration in these samples can be obtained by comparing the room temperature mobilities with the theoretical curve given by Ehrenreich[2] for the variation of mobility with impurity concentration, and this leads to values n_I(specimen No. 2)$=2.5\times10^{16}$ cm^{-3} and n_I(specimen No. 1)$=1.25\times10^{16}$ cm^{-3}.

Apart from n_I, there are no disposable parameters in the theory and the good agreement which exists between the calculated and the experimental curves must there-fore be regarded as highly satisfactory.

2. Impurity Conduction

The formal theory of impurity conduction is well established. If conduction is possible by transfer of charge between impurity centers as well as in the ordi-nary conduction band, the conductivity is the sum of two contributions

$$\sigma=e(n_1\mu_1+n_2\mu_2),\qquad(2.1)$$

where n_1,μ_1 refer to the conduction band and n_2,μ_2 refer to the impurity conduction. The Hall constant is

$$R=(1/e)[r_1n_1\mu_1^2+(r_2n_2\mu_2^2)(n_1\mu_1+n_2\mu_2)^2]\qquad(2.2)$$

in which r_1,r_2 express the ratio of the Hall and drift mobilities. Varying the temperature varies n_1 and n_2 subject to the constraint $n_1+n_2=N$ which is constant.
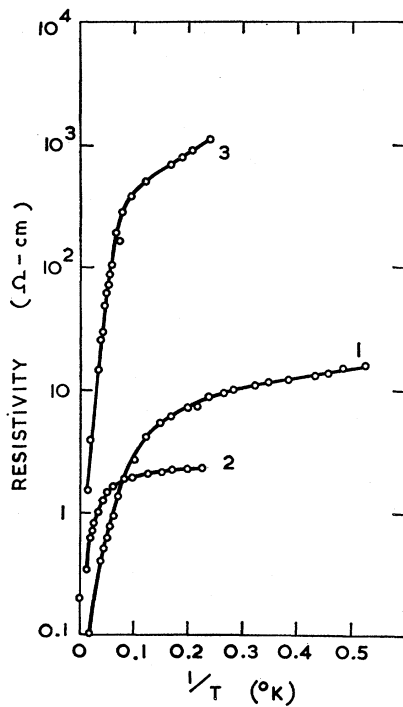

FIG. 2. The resistivity for three samples of gallium arsenide as a function of temperature. The numbers in Figs. 1, 2, and 3 correspond. Values for specimen No. 2 have been multiplied by 10.
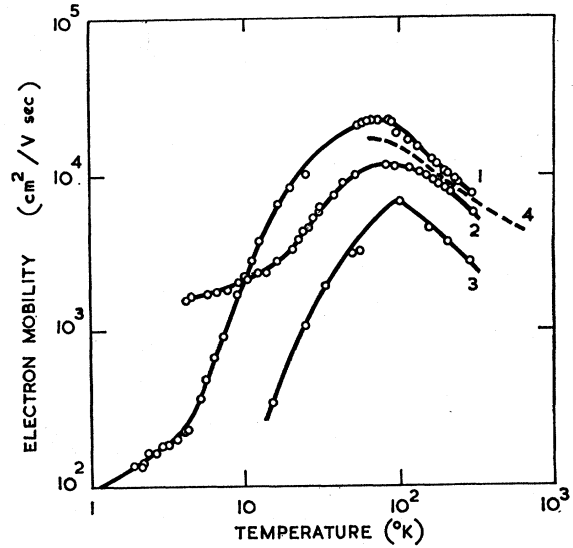

FIG. 3. The Hall mobilities for electrons measured in three samples of gallium arsenide are given in curves 1, 2, and 3. Curve 4 has been derived theoretically by Ehrenreich for an ionized impurity concentration of 2.2×10^{16} cm^{-3}.

The Hall coefficient exhibits a maximum when $n_1\mu_1=n_2\mu_2$ given by $R_{max}=r_1b/4Ne$, where $\mu_1/\mu_2=b\gg1$. In the limits $n_{1(2)}\to N$, $R\to r_{1(2)}/Ne$.

There are two different mechanisms of impurity conduction depending on the degree of overlap between wave functions corresponding to neighboring donor sites (for a general review see Mott and Twose[6]). At high impurity concentrations the overlap is sufficient to give rise to a metallic form of conductivity, independent of temperature. At low concentrations, conduction takes place by a phonon-assisted "hopping" process and is thus temperature dependent. The transition from one mechanism to the other depends on the ratio of the mean distance between the donors r_D to the radius of the modified Bohr orbit for a donor electron a_H^*, where $r_D=(3/4\pi N_D)^{1/3}$ and $a_H^*=\kappa a_H(m/m^*)$ is the effective mass, κ is the dielectric constant, and a_H is the radius of the hydrogen atom. The transition occurs for $r_D/a_H^*\sim3$.

The relevant results for the three specimens, are given in Table II. For specimens Nos. 1 and 2 the values of (N_D+N_A) have been deduced by using the measured room temperature mobility in conjunction with the theoretical relationship between mobility and impurity concentration as explained earlier. If this procedure is carried out for specimen No. 3, a value for the total impurity concentration is obtained which is much greater than for specimens Nos. 1 and 2 and which is inconsistent with the measured values of R_H(max)/R_H(292) and with the activation energy for the resis-tivity in the impurity conduction region. Consequently,

[6] N. F. Mott and W. D. Twose, Advances in Physics, edited by N. F. Mott (Taylor and Frances, Ltd., London, 1961), Vol. 10, p. 107.

TABLE II. Summary of the parameters relevant to the discussion of impurity conduction in gallium arsenide.[a]

Specimen number	1	2	3
R_H(max), cm³/C	7×10^3	6.6×10^2	5.8×10^4
R_H(max)/R_H(292)	4.9	1.6	8.7
T for R_{max}, °K	12	33	14.7
ϵ_1 (eV)	1.6×10^{-4}	9.8×10^{-5}	5.8×10^{-4}
N_D+N_A, cm⁻³	1.25×10^{16}	2.5×10^{16}	2.8×10^{15}
$K=N_A/N_D$	0.39	0.143	0.5
r_D (a.u.)	288	239	500
$\lambda = r_D/a_H^*$	3.79	3.15	6.6
μ (cm²/V sec) at 4°K	220	1700	?

[a] $r_D = (3/4\pi N_D)^{1/3}$ and $a_H^* = \kappa a_H(m/m^*) = 76$ a.u. in GaAs. ϵ_1 is obtained from the relation, valid at low temperatures: $\rho \alpha \exp(\epsilon_1/kT)$.

in the discussion which follows, the value of K ($=N_A/N_D$) has been taken rather arbitrarily as 0.5 leading to a value for $N_A+N_D=2.8 \times 18^{15}$ cm⁻³. From the calculated values of r_D/a_H^* for the three specimens it can be seen that only in specimen 3 is the mean separation between donors sufficiently large for the impurity conduction to be unequivocally ascribed to a phonon assisted hopping process. It is interesting to note, however that a change of about 20% in the value of r_D/a_H^* for $\lambda=3.5$ gives a change of a factor of ~ 8 in the mobility, suggesting that the transition between the two types of conduction occurs quite sharply and supporting the view expressed by Mott.

The recent theory of Miller and Abrahams[7] has been applied to the results on specimen No. 3. The activation energy for impurity conduction is given by

$$\epsilon_1 = f(K)e^2/\kappa r_D,$$

where κ is the dielectric constant, r_D is the mean separation of the donors, and $f(K)$ is tabulated in reference 7. Using $K=0.5$ gives ϵ_1(theor)$=6.35 \times 10^{-4}$ eV which is to be compared with the experimental value of 5.8×10^{-4} eV. The expression for the resistivity applicable to gallium arsenide is (Miller, private communication)

$$\rho = C(T)(r_D/a)^{1/4}[1+18.2(a/r_D)^{3/2}]$$
$$\times \exp[1.09(r_D/a)^{3/2}+\beta\epsilon_1],$$

with

$$C(T) = l_e(T)(8^{-1/2})3^{1/4}\pi^{1/2}2^{-5/4}(0.091)(5 \times 10^3)$$
$$\times \kappa_0^2 \rho s^5 \hbar^4 a^3/e^6 E_1^2,$$

and

$$a = \hbar(2m^* E_D)^{-1/2},$$

where E_D is the observed donor ionization energy, and E_1 is the deformation potential constant. Substituting in the expression for $C(T)$, $E_1=-7.0$ eV, $s=5 \times 10^5$ cm sec⁻¹, $l=1$, and $a=143$A gives $C(T)=1.15 \times 10^{-2}\Omega$ cm. At 4°K this leads to a value for $\rho=1.15 \times 10^3$ Ω cm to be compared with the experimental value of 1.09×10^3 Ω cm. Such close agreement is undoubtedly fortuitous but it does demonstrate that a consistent explanation

of the results of specimen No. 3 can be given with the assumed value of K.

A similar theory due to Twose[8] gives for the root mean square energy difference between any two donor sites

$$E_{rms} = (2.28\pi K)^{1/2}N_D e^2/\kappa,$$

which for the parameters of specimen No. 3 gives $E_{rms}=2.6 \times 10^{-3}$ eV. The dependence of ϵ_1 on E_{rms} has not been calculated but to obtain agreement with experiment it is necessary to assume that $E_{rms}/\epsilon_1=4.5$ which is close to the value ($=4$) assumed by Fritzsche and Cuevas[9] in their work on p-type germanium. The theory does not lead to an expression for resistivity which can easily be compared with experiment.

3. Deviations from Ohm's Law

Specimen No. 3 showed significant deviations from Ohm's law below about 34°K, the conductivity increasing with the applied electric field. For this specimen (Fig. 2), the resistivity at helium temperatures is much greater than for the specimens previously discussed although the general shape of the resistivity temperature curve is the same. The Hall constant rises to a maximum of 5.9×10^4 cm³ C⁻¹ at 14.7°K and below this temperature depends on the current.

A detailed examination of the dependence of current on voltage over the temperature range 34 to 4.2°K (Fig. 4) shows the following features: At the higher temperatures the conductivity increases smoothly from the smallest applied fields. At temperatures close to 4°K the conductivity is constant for small fields, above which it rises, at first gradually and then, at a particular field which increases as the temperature is reduced, very rapidly. In some cases the I-V curve exhibits a negative differential resistance. Between these temperatures the
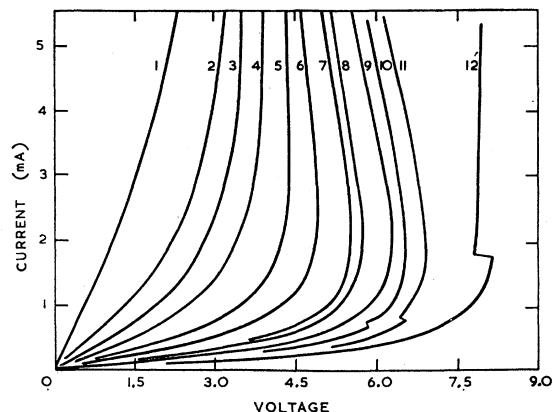

FIG. 4. The dependence of current on applied voltage for specimen No. 3 at different temperatures: (1) 34°K; (2) 23.8°K; (3) 19.3°K; (4) 16.6°K; (5) 11.3°K; (6) 8.1°K; (7) 7.4°K; (8) 6.9°K; (9) 5.9°K; (10) 5.3°K; (11) 4.9°K; (12) 4.2°K.

[7] A. Miller and E. Abrahams, Phys. Rev. 120, 745 (1960).

[8] W. D. Twose, thesis, Cambridge, 1959 (unpublished).
[9] H. Fritzsche and M. Cuevas, Phys. Rev. 119, 1238 (1960).

shape of the characteristic is a varying admixture of the two extremes. It should be emphasized that these results were obtained on a four-terminal specimen and are not due to the properties of the contacts.

Some or all of these effects have been observed in other materials, for example, in germanium by Koenig and Gunther-Mohr,[10] in indium antimonide by Sladek[11] and by Lien Chih-ch'ao and Nasledov,[12] and in silicon by Kaiser and Wheatley.[13] An explanation of these observations can be given in terms of four effects:

(1) The change in the mobility of the conduction electrons which arises from an increase in the mean electron energy produced by the applied field;

(2) The transfer of electrons from impurity states to the conduction band due to a departure of the recombination rate from its thermal equilibrium value as suggested by Koenig and Gunther Mohr[10];

(3) The impact ionization of donors by energetic electrons;

(4) Direct transitions from impurity states to the conduction band produced by the applied field.

At temperatures $\sim 30°$K the conductivity of the conduction band exceeds that of the impurity states. The Hall coefficient is not dependent on the current, within experimental error, so that the initial increase in the conductivity is produced by a change in mobility. On the other hand, at 4°K, practically all the electrons will be in impurity states for vanishingly small electric fields, and the increase in conductivity at this temperature must be due to a transfer of electrons from impurity states to the conduction band. It is not, however, possible to separate these effects completely. Even at 30°K there will be an appreciable number of electrons in impurity states, and the transfer of a small number will produce a large change in conductivity. Thus, in Fig. 5, which shows the dependence of σ on F^2, a linear portion corresponding to a change of mobility with field is observed only for small fields, above which the conductivity rises more rapidly probably due to the electron transfer effect discussed.

The detailed explanation of the results is facilitated, however, by considering the effects separately.

A. Change of Mobility with Field

If the rate at which the electrons gain energy from the applied field exceeds the rate of loss by collisions, the mean electron energy increases and the mobility changes. To first order, the change may be expressed
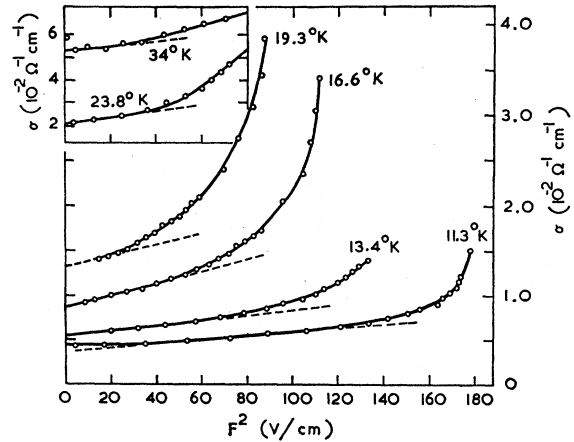
$$\mu = \mu_0(1 + \beta F^2), \qquad (3.1)$$

FIG. 5. The dependence of σ on F^2 for specimen No. 3. The dashed lines indicate the slope of the curve for $F^2 \rightarrow 0$ which has been used in obtaining an experimental value for the increase of electron temperature $(T_e - T)/T = \delta$ with increasing field.

where F is the electric field. The magnitude of β is determined by the mechanism responsible for energy loss. Experimental values for β have been obtained from the results shown in Fig. 5 and compared with the theoretical predictions for various loss mechanisms.

Above a certain critical electron density, interelectronic collisions are sufficiently frequent to enforce a Maxwellian energy distribution characterized by an electron temperature T_e. The critical value depends on the mechanism of energy loss to the lattice and for collisions with acoustic phonons has been calculated by Fröhlich and Paranjape[14] to be

$$n \simeq (1/4\pi)[m^{*\frac{3}{2}}s^2\kappa^2/kT_o e^4 \tau_a(E)] \qquad (3.2)$$

where τ is the collision time between electrons of energy E and acoustic phonons, for a lattice temperature T_0. s is the velocity of sound and κ is the dielectric constant. Putting in the values appropriate to gallium arsenide gives

$$n_A = 3.77 \times 10^{10}(T_0/293)^{1/2}(E/kT_0)^2 \text{ cm}^{-3}. \qquad (3.3)$$

If energy loss to the polar optical modes is important the critical density is[15]

$$n_0 = (eF_0 k\theta/2\pi e^{*4})(T/\theta)^{1/2} \exp(-\theta/T), \qquad (3.4)$$

where $eF_0 = (\kappa_\infty^{-1} - \kappa_0^{-1})m^* e^2 k\theta/\hbar^2$, $k\theta$ is the optical phonon energy, κ_0 and κ_∞ are the static and high-frequency dielectric constants, respectively. This can be rewritten

$$n_0 \simeq 7 \times 10^8 F_0 \theta \kappa^2 (T/\theta)^{1/2} \exp(-\theta/T) \text{ cm}^{-3}.$$

For GaAs, $F_0 = 4.62 \times 10^3$ V cm^{-1}, $\theta = 410°$K, and $\kappa = 13$; so that at 30°K, $n_0 = 4.3 \times 10^{10}$ cm^{-3}. Whatever the mechanism for energy loss, the density of electrons in our samples exceeds the critical value, and it is legiti-

[10] S. H. Koenig and G. R. Gunther Mohr, J. Phys. Chem. Solids 15, 356 (1957).

[11] R. J. Sladek, Phys. Rev. 120, 1589 (1960).

[12] Chih-Ch'ao Lien and D. N. Nasledov, Soviet Phys.-Solid State 2, 729 (1960).

[13] W. Kaiser, and G. H. Wheatley, Phys. Rev. Letters 3, 334 (1959).

[14] H. Fröhlich and B. V. Paranjape, Proc. Phys. Soc. (London) B69, 21 (1956).

[15] R. Stratton, Proc. Roy. Soc. (London) A246, 406 (1958).

mate, therefore, to use the concept of electron temperature to discuss the experimental results.

Assuming that the electrons occupy a standard parabolic energy band and the momentum relaxation time $\tau \propto E^z$, it can be shown (Greene, quoted by Sladek[11]) that the mobility is given by

$$\mu = \mu_0 \left\{ 1 + \delta \left[(z+\tfrac{3}{2}) - 3(z+\tfrac{1}{2}) \frac{F_{\frac{1}{2}}F_{z-1}}{F_{-\frac{1}{2}}F_{z+\frac{1}{2}}} \right] \right\}, \quad (3.5)$$

where $\delta = (T_e - T_0)/T_0$, T_0 being the lattice temperature; F_k are the Fermi-Dirac integrals which are functions of the reduced Fermi energy ζ/kT and $\delta \ll 1$. The errors introduced by the departure of the conduction band in gallium arsenide from standard form are not likely to be serious. In the classical limit, $\zeta/kT \to -\infty$ and $\mu = \mu_0(1+\delta z)$. The value of δ is obtained by equating the average power gain from the applied field to the average energy loss by collisions:

$$e\mu F^2 = \langle dE/dt \rangle_c. \quad (3.6)$$

Three possible mechanisms will be considered.

(i) *Loss of energy to the polar optical modes.* Stratton[15] has obtained the following expression for $\langle dE/dt \rangle_c$:

$$\langle dE/dt \rangle_c = eF_0 N_0 (2k\theta/m^*)^{1/2} [\exp(\gamma_0 - \gamma) - 1] \gamma^{1/2} \\ \times \exp(\gamma/2) K_0(1/2\gamma), \quad (3.7)$$

where, in addition to the symbols previously defined, $N_0 = [\exp(\theta/T_0) - 1]^{-1}$, $\gamma = \theta/T_e$, and $\gamma_0 = \theta/T_0$. Making the approximation $\gamma_0 - \gamma \ll \gamma_0$ and substituting for the lattice mobility due to polar optical scattering in the limit of strong electron-electron scattering

$$\mu_{\rm Op}{}^{e-e} = [3(2\pi)^{1/2}/4](k\theta/m^*)^{1/2}/N_0 F_0 \\ \times 1/\gamma_0^{3/2} \exp(\gamma_0/2) K_1(1/2\gamma_0) \quad (3.8)$$

gives finally

$$\langle dE/dt \rangle_{\rm Op} = (3/2)[(k\theta/m^*)/\mu_{\rm Op}{}^{e-e}] \\ \times \delta [K_0(\gamma_0/2)/K_1(1/2\gamma_0)], \quad (3.9)$$

in which for $\gamma_0 > 5$, $K_1/K_0 \simeq 1$. K_1 and K_0 are modified Bessel functions.

(ii) *Loss of energy by deformation potential scattering.* Using the analysis of Sladek,[11]

$$\langle dE/dt \rangle_{\rm Ac} = (3s^2/\mu_{\rm Ac}{}^{e-e})\delta, \quad (3.10)$$

where

$$\mu_{\rm Ac}{}^{e-e} = (3e/4\sqrt{2})(\rho s^2/E_1{}^2)(\pi\hbar^4/m^{*5/2}) \\ \times [1/(kT_0)^{3/2}]. \quad (3.11)$$

This formula is valid for a classical assembly; E_1 is the deformation potential constant, ρ the density, and s the velocity of sound.

(iii) *Loss of energy by piezoelectric scattering.* A similar expression for $\langle dE/dt \rangle$ has also been derived by Sladek[11]:

$$\langle dE/dt \rangle_{\rm PE} = (3s^2/\mu_{\rm PE}{}^{e-e})\delta, \quad (3.12)$$

where

$$\mu_{\rm PE}{}^{e-e} = \frac{3\sqrt{2}}{16\pi} \frac{\hbar^2 \kappa^2}{m^{*3/2} e\, e_{14}{}^2} \frac{1}{(kT_0)^{1/2}} \\ \times \left[\frac{16}{13(c_{11}+c_{12}+4c_{44}+16\pi e_{14}{}^2/\kappa)} \\ + \frac{6}{13(c_{44}+4\pi e_{14}{}^2/\kappa)} \right]^{-1}; \quad (3.13)$$

e_{14} is the piezoelectric constant obtained by calculation from the relation

$$e_{14}{}^2 = (\kappa_0 - \kappa_\infty/4\pi)(c_{12}-c_{44})(c_{44}/c_{12}), \quad (3.14)$$

and the c_{ij}'s are elastic constants in the standard notation.

The electron temperature can now be calculated theoretically by combining the rates of energy loss for the three mechanisms discussed and using equation (3.6).

Since the separate rates have a different energy dependence for each of the three processes, the combination should be carried out before averaging over the electron distribution. However an approximate result can be obtained by adding the already averaged expressions for $\langle dE/dt \rangle$ given previously, We, thus, obtain

$$\delta/F^2 = \mu_I / \sum_i A_i V_i{}^2/\mu_i, \quad (3.15)$$

where the A_i's are numerical constants and the V_i's have the dimensions of a velocity.

In the impurity scattering range $\tau \propto E^{3/2}$ so that, using Eq. (3.5) in the classical limit and Eq. (3.1), we have $\delta/F^2 = 2(\sigma - \sigma_0)/3\sigma_0 F^2$. The experimental and theo-
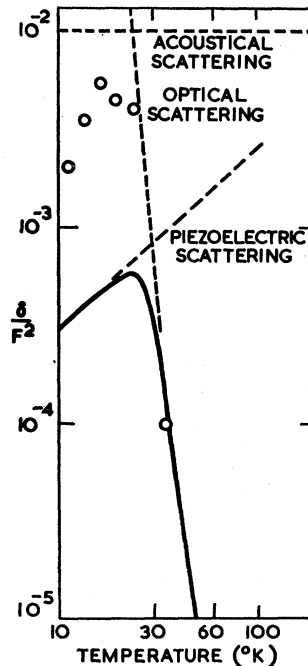

FIG. 6. The dependence of $(T_e - T)/TF^2$ on lattice temperature. The dashed lines show the theoretical values of this parameter assuming various energy loss mechanisms operate singly. The full line is obtained by combining the contributions of the three mechanisms. The circles are experimental values for specimen No. 3 derived from the slopes of the σ vs F^2 curves.

retical values are plotted together in Fig. 6. The only disposable factor in the theoretical expression for δ/F^2 is μ_I which is the mobility in the limit $F \to 0$. The experimental values for this quantity at the lower temperatures are determined in some measure by the mobility of electrons in impurity states. Consequently, values of μ_I have been obtained by fitting a relation $\mu_I = BT^{3/2}$ at the peak of the experimental mobility-temperature curve.

It will be seen that agreement between theory and experiment is not quantitatively very good but, in view of the approximations made, it is doubtful whether better agreement could be expected.

The main features of the results are, however, represented by the theory which suggests that, above about 30°K, energy loss will be dominated by optical scattering which is so efficient that an immeasurably small temperature rise will take place. Below this temperature piezoelectric scattering is most important but deformation potential scattering does make a small contribution to the energy loss.

B. Changes of Carrier Concentration with Field

The differential equation governing the concentration of carriers in the conduction band can be written, following Koenig,[16]

$$A_T(T)(N_D - N_A - n)$$
$$+ n\{A_I(T_e)(N_D - N_A - n) - B_T(T_e,T)(N_A + n)\}$$
$$- n^2 B_I(T_e,T)(N_A + n) = dn/dt, \quad (3.16)$$

where A_T, A_I represent rates of carrier production by thermal or impact ionization and B_T, B_I are the rates for the corresponding inverse processes. The discussion is facilitated by referring to an electron temperature T_e, although it may not in all cases be strictly legitimate to define one. Assuming that $n \ll N_A$, $N_D - N_A$, and neglecting the term in n^2 the conductivity can be written as the sum of contributions due to the impurity states and the conduction band:

$$\sigma = \sigma_I + \sigma_C, \quad (3.17)$$

where

$$\sigma_C = ne\mu = \frac{e\mu A_T(N_D - N_A)}{B_T(T_e,T)N_A[1 - A_I(T_e)(N_D - N_A)/B_T(T_e,T)N_A]}. \quad (3.18)$$

Three regions of the current-voltage characteristic can now be distinguished.

(i) For low fields, σ is constant because A_I is negligible and B_T (hence, n) has its thermal equilibrium value.

(ii) For intermediate fields, σ increases slowly because B_T varies inversely with the mean electron energy while A_I is still negligible. On general grounds, it would be reasonable to assume such a variation for B_T, and Lax[17] has discussed a specific mechanism in which $B_T \propto T^{-3.5}$ approximately. Some measurements of B_T for germanium by Koenig[16] seem to fit in with these predictions.

(iii) At a particular field, values of which are plotted in Fig. 7, σ increases rapidly. This occurs when the mean energy of the electrons approaches the ionization energy of the impurities. As A_I increases and $(A_I/B_T)(N_D - N_A)/N_A \to 1$, n rises rapidly. For further increases in n, the term representing Auger recombination has to be included and Eq. (3.18) must be modified. Chuenkov[18] has attempted a precise analysis of this situation by solving the Boltzmann equation including all the effects that have been discussed. It was not possible, however, to derive an expression for the conductivity for the case in which charged impurity scattering is the dominant mechanism.

A plausible calculation of the critical breakdown field can be made by equating the energy gained from the field to the energy lost to the lattice when the mean electron energy equals the ionization energy. This

leads to

$$F_C^2 = (3s^2/\mu_C\mu_L)(E_D/kT). \quad (3.19)$$

With $E_D = 2.7 \times 10^{-3}$ eV and $T = 4$°K, this equation is satisfied if $\mu_C\mu_L = 4.75 \times 10^{10}$ (cm²/V sec)². In this, μ_C is the conduction-band mobility for the "hot" electron distribution and will be determined to a large extent by piezoelectric scattering and scattering by neutral impurities since, before impact ionization sets in, most of the donors will be de-ionized; μ_L is the lattice mobility arising from the mechanism responsible for energy loss, i.e., piezoelectric scattering. Thus $\mu_C\mu_L \simeq C\mu_L^2$, where C
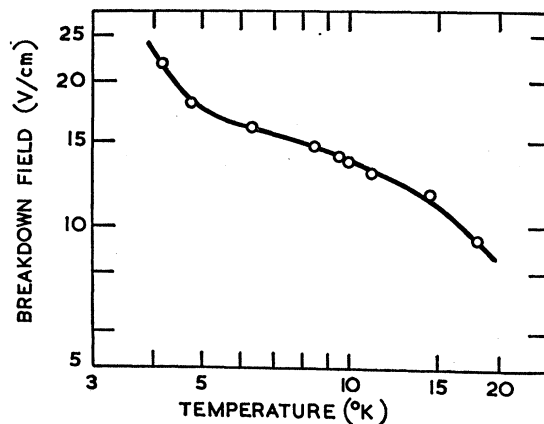

FIG. 7. The dependence of breakdown field on temperature for specimen No. 3. Breakdown field is here defined as the field for which $dI/dV = \infty$.

[17] M. Lax, Phys. Rev. 119, 1502 (1960).
[18] V. A. Chenkov. Fizika Tverdogo Tela U.S.S.R., 2, 799 [translation: Soviet Phys—Solid State 2, 734 (1960)].

[16] S. H. Koenig, Phys. Rev. 110, 986 (1958).

represents the reduction in the piezoelectric mobility μ_L brought about by the contribution from other forms of scattering. The electron temperature corresponding to breakdown $\simeq 32°K$, and at this temperature the calculated value of μ_L is 3.5×10^5 cm^2 (V sec)$^{-1}$ which gives for C a value of 0.39. The argument given is not intended as a precise analysis of the situation which is obviously quite complicated. The analysis suggests that $F_C \propto T^{-1/2}$ which is not exactly true but explains the general trend of the results with temperature.

It is not considered that direct transitions between impurity states and the conduction band under the action of the electric field are important (Zener effect). The transition probability is

$$P \simeq \exp[-\pi m^* a E_D^2 / \hbar^2 eF] = 10^{-63/F},$$

for gallium arsenide, F being in V cm^{-1} so that, at the critical field, the transition probability $P \simeq 10^{-3}$.

D. CONCLUSIONS

The measurements of mobility on two specimens give strong support to the idea that in pure samples of gallium arsenide the polar optical modes exert the dominant scattering effect. There are, however, some unexplained anomalies in the variation of mobility with impurity concentration as found in specimen No. 3 and also in high-resistivity samples, which suggest the need for an improved theory of impurity scattering applicable to semiconductors where the density of free carriers is small.

The low-temperature results illustrate the two forms of impurity conduction. In one sample, where the hopping form of conduction is clearly present, the results agree well with the recent theory of Miller and Abrahams.

A study of the departures from Ohm's law can yield a good deal of extra information on scattering processes. In the case of the work on gallium arsenide described here, there seems to be a number of other effects present as well which complicate the analysis. It is hoped that further experiments at present being carried out will enable the significance of these separate effects to be elucidated.

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