Note on a Nucleon-Nucleon Potential*

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A nucleon-nucleon potential was used in an approximate representation of the YLAM and YLAN3M phenomenological phase parameter fits to *p-p* and *n-p* scattering data. The potential employs a hard core and is different for singlet-even, singlet-odd, triplet-even, and triplet-odd states. It consists of central, tensor, spin orbit, and quadratic spin orbit parts. The latter are especially indicated by the singlet-even and also by the triplet-even interactions and are used only for even states. Coefficients in formulas for the potential are tabulated and the quality of agreement with experiment is discussed.

Several nucleon-nucleon potentials obtained by fitting scattering data at energies below the meson production threshold may be found in the literature.[1] The phase-parameter fitting of data[2–4] gives a better representation of the experimental material than the potentials and an attempt was made, therefore, to approximate by means of a potential the values of phase parameters obtained in the two Yale searches which appear as the best ones from a statistical point of view.

Several requests for the potential thus obtained have been received privately, and it appears desirable to make it available to a wider group of workers. On account of possible applications to nuclear structure theory, the introduction of velocity dependence of the potential was avoided whenever possible. Since the phase-parameter searches use experimental data explicitly only from 9.69 to 345 Mev for *p-p* scattering and from 13.7 to 350 Mev for *n-p* and since at the lower end of these energy ranges the smoothness of joining to the *s*-wave phase shifts was taken care of only approximately in the phase-parameter work, it appears probable that, in the vicinity of 10 Mev, the potential may be preferable for *s* waves to the fits YLAM and YLAN3M which it is intended to approximate.

The potential used is similar in mathematical form to that used by Bryan.[1] The coefficients of the successive terms, and the hard core radii were varied so as to reproduce YLAM and YLAN3M. Attempts to do so by a gradient search in the space of the coefficients did not prove successful because of the similarity of energy dependence of sensitivity of phase parameters to the coefficients. A modified gradient search was used therefore in which usually one coefficient or the hard core radius was varied at a time and combined changes of several parameters were tried with relative proportions determined on the basis of the one-parameter variation trials.

For the singlet-even potential, values of K_0, the $^1S_0$ phase shift, were used at 2.2, 2.425, 3.3, 4.203, and 5 Mev in addition to those in reference 2. At 2.2, 3.3, and 5 Mev an average of values obtained from the *f*-function analysis[5] was used. At 2.425 the value[6] 48.273° corresponding to the *S*, *P*, *V* type fit[6] was employed. At 4.203 Mev the value 53.912° obtained by pure *S*-wave analysis[7] was made use of. These choices are admittedly arbitrary but it is believed that within the limitations of accuracy of the present work the precise choice is of minor importance. In computations of the weighted mean square deviation *D*, the data were used with a weight equal to that of points taken from the YLAM fit in the energy range 9.69–39.4 Mev. A typical distribution of energies at which comparisons were made with YLAM is as follows: 2.2, 2.425, 4.203, 5.00, 9.69, 19.8, 39.4, 68.3, 147, 210, 312 Mev. The density of points at high energies was purposely taken smaller than at low energies because the sensitivity of experimentally available quantities to K_0 decreases as the energy increases. The weights of the K_0 values from 9.69 Mev on to higher *E* were those corresponding to standard errors of the "parallel shift" procedure.[2]

With potentials of the type tried, it has not proved possible to reproduce K_0 and K_2 by means of the same static potential, the value of K_2 becoming too large above 100 Mev, the disagreement becoming 40% or more at 300 Mev. For this reason a "velocity dependence" was used in the form of the term containing the operator[8,9] Q_{12} in the combination $Q_{12}-(\mathbf{L}\cdot\mathbf{S})^2=(\mathbf{L}\cdot\mathbf{S})^2+(\mathbf{L}\cdot\mathbf{S})-\mathbf{L}^2$. The latter has the value $-L(L+1)$ for the uncoupled states $J=L$ and is zero in all other cases. Terms of this type were used also for the triplet-even interaction but not for odd parity states. Since[9] the energy region below 10 Mev was not considered, it is not

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TABLE I. Values of coefficients a_n in Eq. (2). Triplet and singlet states are designated by 3 and 1, respectively, and parity is distinguished by + and −.

State	V type	$n=0$	1	2	3	4	5	6	7
1+	c	21.925	−19.6303	−194.782	66.4334	−15.2873	−14.5395	1.115	0
	q	0.3333	0.5	0.1	−2.0	−5.1083	−0.2333	0.2	0
3−	c	0	−14.28	18.72	−17.3	0	−5.3	−1.3	0
	T	1.5	50.8984	−83.3812	8.7693	−3.1988	1.6172	0.52	0
[a]	LS	0	0	0	0	−2.9794	−76.4565	43.8285	−7.4186
1−	c	0	−96.0	−71.001	18.0	8.0	125.8	5.01	0
3+	c	0	−47.667	−18.47	−1.00	−3.55	0	0	0
	T	0	17.3933	7.775	13.535	3.0	−1.4971	0	0
	LS	0	0	0	0	14.35	7.4875	0	0
	q	0	0	5.3333	0	−13.5917	−7.4167	−1.6667	0

[a] For $J \geq 3$ no spin-orbit potential was used in the triplet-odd state. Employment of the coefficients listed, for $J \geq 3$ as well, spoils the fits somewhat at high energies.

clear that, in that work, a fit could not have been obtained without the aid of the quadratic spin-orbit operator. The triplet-even system, information regarding which is contained in the n-p fit YLAN3M, does not perhaps definitely require the quadratic $(\mathbf{L}\cdot\mathbf{S})$ dependence. On the other hand, a search for a potential starting from one-pion exchange values and a hard core did not lead to good fits in a natural way. The special difficulty is that too large values of $\delta^{D}{}_2$ and $\delta^{G}{}_4$ are obtained from the potential at high energies. The uncertainties in the determination of $\delta^{G}{}_4$ from fitting data are so large that evidence from this source is only contributory. For $\delta^{D}{}_2$, however, values exceeding those of YLAN3M by 0.2 to 0.5 radians, which remained in the best fits without the quadratic $\mathbf{L}\cdot\mathbf{S}$ terms, appear improbable. There is also a related tendency for $\theta^{D}{}_3$ to come out too large in the calculations from a potential satisfying the requirements of the effective range approximation with Salpeter's values.[10] The quadratic spin-orbit term corrects the deficiencies of the static potential in a marked manner. There may be other possibilities, however. The best potential to be added to the Coulombian has the form

$$V=V^{(2)}+V_c+V_TS_{12}+V_{LS}(\mathbf{L}\cdot\mathbf{S})+V_q[Q_{12}-(\mathbf{L}\cdot\mathbf{S})^2]. \quad (1)$$

Here $V^{(2)}$ is the one-pion exchange potential with $g_0^2/14=0.94$ in singlet-even states and unity otherwise and a pion mass for the neutral pion used for singlet-even and triplet-odd states. For singlet-odd and triplet-even states a weighted mean of charged and neutral pion masses was used in the proportion of 2 to 1, corresponding to Eq. (5.3) of reference 2. The core radius was taken to correspond to $x_c=0.35$; $x=rm_\pi c/\hbar$, $r=$ internucleon distance. The tensor and spin-orbit symbols S_{12} and $\mathbf{L}\cdot\mathbf{S}$ have their standard meaning. Except for $V^{(2)}$ all the V have the form

$$V=\sum_n a_n e^{-2x}/x^n. \quad (2)$$

The values of the a_n in Mev are as in Table I. Plots of phase parameters against energy usually, but not always, fall within the standard deviation error limits of YLAM and YLAN3M. Graphical comparison with experiment is not markedly different regarding quality of agreement from the published results for YLAM and YLAN3M. The weighted mean square deviation, D, used[2,4] in the gradient searches of the data is, however, roughly twice that of the purely phenomenological results. The potential gives an over-all reproduction of scattering data which is appreciably better than that of other potentials in the literature. Attention may be drawn to the difference in sign of the spin-orbit potential for odd and even states. The employment of hard cores and other features of the fitting of the phenomenological searches are arbitrary. No claim of uniqueness for the potential can be made therefore. Furthermore, the phase-parameter fits themselves may require modification as a result of accumulation of additional data.

Well-known arguments make the fundamental significance of a potential used in a nonrelativistic Schrödinger equation rather questionable. It is, nevertheless, believed by many that if a potential accounts satisfactorily for scattering data and has, therefore, a good chance of reproducing the scattering matrix with reasonable accuracy, it also has a good chance of reproducing the properties of nuclear matter and perhaps also of nuclei as scatterers of nucleons. The potential may also be of interest in the theory of hypernuclei and of the hyperon-nucleon interaction. The comparison of K_0 corresponding to the potential with YLAM and YRB1 was shown in a slide at the Rutherford Jubilee International Conference in Manchester and comparisons for other phase parameters as well as with experimental data were shown in unofficial discussions at the same conference.

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